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The transitory phase to the attainment of self-regulatory skill in mathematical problem solving

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Three phases of development of self-regulatory skill in the domain of mathematical problem solving were designed to examine students' behaviour and the effects on their problem solving ability. Forty-eight Grade 4 students (10 year olds) participated in this pilot study. The students were randomly assigned to one of three groups, each representing a different developmental level of self-regulatory skill: (a) the observation level, (b) the emulation level and (c) the self-control level. The first findings advocate that novices and medium solvers performed better in cooperative than in traditional environments in contrast with experts. Moreover, the first findings advocate that learning environments which provide peer modelling may contribute to the development of self-regulatory skills in medium problem solvers.

Self-regulation, mathematics, problem solving, medium state solvers

INTRODUCTION

A productive and independent learner tends to be one of the first priorities of a contemporary educational system almost all over the world. It is recognised that the capability of self-regulation exists in all human beings and this enables us to survive, evolve and improve ourselves. Researchers have indicated that self-regulation can be taught, but the period of the attainment is often long. However, a start must be made and the earlier the better. While problem solving is placed in the heart of mathematics (NCTM, 2000), it is considered to contribute to the whole thinking processes. Nevertheless, differences in mathematical problem solving ability begin to appear in Grade 1 and increase gradually throughout the formal years of schooling. These differences have already been recorded and are known as ‘novice’ compared to ‘expert’ differences. However, there is a period of unknown duration which raises some questions in which novices are transformed into experts. Students who pass through this period in mathematics are called ‘medium’ solvers by their teacher. The present pilot study examines this period in three different environments in an effort to describe under what conditions the transformation is achieved more effectively. Additionally, the study explores the main characteristics of this category of students.

THEORETICAL BACKGROUND

The main feature of human behavior is that it is goal-oriented thereby eliminating constraints and hindrances that may hinder individuals attaining their goals (Anderson, 1995; Glover et al. 1990). Thus, it can be said that all cognitive activities are problem solving examples by their nature, as they are based on the individual’s attained, codified, well-organised and registered experience (Koliadis, 2002). In order to attain their goals, individuals regulate their behavior and functioning.

Problem solving is characterised as an essential and complex activity in mathematics (Hembree, 1992). It takes time to develop the problem solving ability (Lester, 1994). Becoming an expert in problem solving means that you have been engaged in this for at least 10 years, in order to perform well (Ericsson and Lehmann, 1996).

During our daily contact with students, as teachers, we are faced with three categories of students in mathematics problem solving: (a) the novice solvers, (b) the expert solvers and another group of students that includes the majority of students. This category is called 'medium' solvers as they can be classified neither in the first nor in the second category. Actually, they are in a transitory phase between novices and experts. It seems impossible for such students to be classified in a category. Nevertheless, from time to time they may display either a positive or a negative mathematical performance. But what do we know about them? How can we help them, as teachers, to go beyond this phase?

Expert Compared with Novice Solvers

According to the expert-novice literature there are striking differences between the declarative, the procedural and the conditional aspects of knowledge. Specifically, in the domain of problem solving, the novice solvers have little memory for relevant problem components. They classify the problem types according to their surface structure, display no automation in their procedural function, prefer the means-end-analysis strategy for the manipulation of the problems, need more time than the experts, rarely reach the solution of a problem, face difficulties with the 'innovative' problems, can not drive and monitor the process they follow in order to attain the desired goal (Matlin, 1998; Sternberg, 1999) and show no self-discipline (Ericsson and Lehman, 1996). Apart from the cognitive, there is also the affective aspect. It seems that during the problem solving activity a core of attitudes and emotions is present, which contributes to the successful engagement of problem solving as well. Beliefs, values, emotions and attitudes play an important role in the problem solving activity. For instance, if someone believes that he is not capable of solving a certain type of problem then he is likely not to solve it. Beliefs are important determinants of students' learning, thinking and performance (Boekaerts, 1997; Pintrich, Marx and Boyle, 1993).

Self-Regulation

Another important issue is related to effective self-regulation. Self-regulation lies at the core of successful and lifelong learning. Self-regulated learners tend to be active, reflective and productive in their own thinking and learning (Zimmerman and Kitsantas, 1996). They think critically, use problem solving strategies and memory techniques when appropriate. Self-regulatory processes and accompanying beliefs are said to fall into three cyclical phases: (a) forethought, (b) performance or volitional control and (c) self-reflection processes (Schunk and Zimmerman, 1998). The first phase precedes the effort thath contributes to the attainment of a goal. The second phase involves processes that occur during motor efforts and affects attention and action (Zimmerman, 2000, p.16). Nevertheless, there is a last phase which impacts on a person's response to that experience.

The forethought phase consists of task analysis (goal setting and strategic planning) and self-motivational beliefs (self-efficacy, outcome expectations, intrinsic interest/value and goal-orientation) subclasses. The performance or volitional control phase consists of self-control (self-instruction, imagery, attention focusing and task strategies) and self-observation subclasses. Finally, the self-reflection phase consists of self-judgment (self-evaluation and causal attribution) and self-reaction (self-satisfaction, affect and adaptive or defensive inferences) (Zimmerman, 2000).

Even if the self-regulation components seem to be clarified, it has not been proven yet how the self-regulatory competence is grown deliberately for instance in a scholarly context. From a social cognitive perspective there is an answer, but it is still important to pinpoint the differentiating outcomes among the different levels of the development of self-regulatory skills.

The Developmental Levels of Self-Regulation

In order to incorporate the effective self-regulatory skills into one's behavioural repertoire, a social cognitive perspective implied that this skill passes through four developmental levels (Ericsson and Lehman, 1996; Kitsantas, Zimmerman and Cleary, 1999; Schunk and Zimmerman, 1997): The observational level of the skill refers to the induction of the skill from a proficient model. Further development of the observed skill depends on the perceived similarity to the model and the positive outcomes of the model's use of this skill (Zimmerman and Rosenthal, 1974). The emulation level goes beyond the simple copying of the vicarious learning. After observing the proficient model, a learner generates a similar behavior in a social context. The similarity to the model's behavior is improved as the observer receives feedback, guidance and social reinforcement during practice (Kitsantas, Zimmerman and Cleary, 1999). The first two levels overcome the need for social guidance. Their practice tends to be more self-directed. The third level, self-control, refers to the independent display of the model's skill under structured conditions. During this phase, learning strategies that focus on fundamental processes rather than outcomes are most beneficial in producing mastery (Zimmerman, 2000, p.30).

The last phase refers to the student's systematically adaptive use of the skill across changing personal and environmental conditions in order to attain the goal. During this phase the behavior is outcome-oriented and self-efficacy plays an important role as it defines the sustainability of this level of skill. These levels are introduced for easy and effective learning. However, it has not escaped our observation that there is a degree of resemblance between this theory of development of self-regulatory skill and the cognitive apprenticeship scheme in which learning scaffolds from extensive social guidance to self-relied efforts.

Self-Regulation and Metacognition

The term 'self-regulated' is mostly associated with metacognitive guided learning forms that are at least partly motivated and strategic in nature (Winne, 1995, 1997). Metacognition involves the learners' awareness about their strengths and weaknesses as well as their knowledge about how to regulate engagement in tasks to optimise learning processes and outcomes. Motivation refers to self-regulated learners' beliefs in proximal learning, the efficacy of learning and attributions that link outcomes to factors under their control. The term 'strategic' describes the way in which these learners approach challenging tasks and problems by choosing from a repertoire of tactics those they believe are best suited to the situation and applying them appropriately (Winne and Perry, 2000, p.533).

THE STUDY

The experimental groups were formulated according to the developmental levels of self-regulatory skill as shown in Figure 1. This pilot study does not include any control group due to the fact that we intended to track down the first reactions to the following design of the study. However, a control group was used for the experimental phase in which a larger number of students participated. In the first experimental group of the pilot study the students were taught the self-regulatory skill using only the teacher's model. Then they were asked to repeat what they had observed while solving a mathematical problem individually. In the second experimental group the students, after observing the teacher's model, were asked to cooperate in a four member group in order to solve a mathematical problem emulating the teacher's model enriched by their own tactics. This meant that the students were free to reinforce the model's general pattern with their inspired and positive experience and strategies. In the last experimental group the students received exactly the same treatment as in the second group with the addition of some cognitive and metacognitive tools they were equipped with. These included their own metacognitive cards on which they wrote the main steps of problem solving while observing the teacher's model and their peers' attitudes. In addition, they were equipped with a portfolio in which they recorded their own individual information in the mathematics course such as goals divided into sub-goals, the estimated time they needed, their progress and the difficulties they had faced.

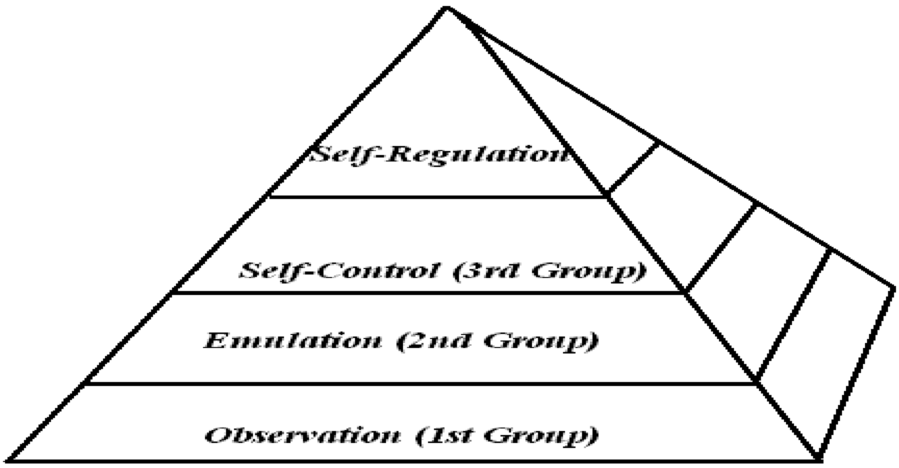

Figure 1. The design of the Pilot Study with reference to the formulation of experimental groups

In the students' case, a set of steps are required to approach the solution of a problem. Thus, a variety of models had been developed in order to support students' efforts when facing a problematic situation. Some of these were based on Polya's model (1957), the GSP model (1972) and Sternberg's model (2003). For example, Polya's model (1957) described the steps to be taken in tackling a problem as follows:

- Understand the problem,
- Devise a plan,
- Carry out the plan, and
- Look back.

The writings of Polya (1988), while not specifically aligned with problem-based learning emphasised the need for metacognitive reflection on learned heuristics as a problem solving tool. GSP is due to a momentary lapse, the right acronym is General Problem Solver (or **GPS**) and it comes from Newell and Simon's (1072) work. It refers to a general problem solver's model. According to this model the problem solving process is defined as an interaction process between the task and the solver who functions as an Information Processing model. GPS model is based on Gestaltists' psychologists emphasising the problem solving space and frame (both represent the possible 'extensions' of a problem) such as:

- The initial situation of a problem,
- The final goal,
- The potential and permissible handlings, and
- The constraints.

Each of these models has its own value depending on its theoretical background. Sternberg's model (1999) is a general model that can be applied in almost all problematic conditions and it is appropriate for well-structured as well as ill-structured problems. It consists of seven steps: (a) problem identification, (b) definition of problem, (c) constructing a strategy of problem solving, (d) organising information about a problem, (e) allocation of resources, (f) monitoring of problem solving, and (g) evaluating of problem solving. In the mathematics domain the first step is not required as it is obvious to the students that they are faced with a problem. This step concerns poorly structured problems, such as the problems we face in daily life or problems in which the issues are nor clearly defined. In mathematics, students generally have to solve well-structured problems. Thus, in our intervention students were taught only Sternberg's last six steps. However, it is questionable if a series of steps make students capable of solving problems. According to some researchers, students are not able to solve problems due to their deficiency as metacognitive thinkers (Artzt and Armour-Thomas, 1992; Carr and Biddlecomb, 1998; Schoenfeld, 1983). Consequently, there was a need to enrich the six steps with some metacognitive questions such as those found in Table 1.

Table 1. The steps of problem – solving cycle according to Sternberg's theory and their corresponding metacognitive questions

Sternberg's Six Steps for Well-Structured Problems	Examples of Metacognitive Questions
1. Definition of Problem	Have I solved a similar problem before? Are there any key points that help me? How is the specific problem connected to what we have been taught?
2. Constructing a Strategy	What would happen if I chose another way of solving the problem? Is there any? How could I use the related theory I've been taught? How would I secure the correctness of my procedure? What would be that thing that would make me choose x way instead of z way?
3. Organising Information	Should I draw the problem or tables in order to avoid any misunderstanding? How would the related formulas be useful in my problem?
4. Allocation of Resources	How would I manage the available time? At which point should I intensify my concentration and my efforts to make up for my weaknesses?
5. Monitoring the Process	To what extent does my followed process differ from my initial planning? How would I improve the allocated solving phases? Why did I make this mistake?
6. Evaluating the Process	Why did I choose x way instead of z way? When would I choose z way? How useful would this problem be in my daily life? In what way could I use this problem in solving other problems?

THE METHOD

Aims of the Study

This study was designed to examine the procedural characteristics of the medium problem solvers in comparison with their classmates who are characterised either as novices or as experts. A two month study was planned to research their capacities in three different and successive levels of self-regulation: (a) observation, (b) emulation and (c) the self-control level in mathematics problem solving.

Student Sample

The sample of the pilot study included 48 students, all of whom were in Grade 4 of primary school. The study encompassed three classrooms of three different schools in the centre of Athens. Each group (called the 'experimental group') represented different levels of self-regulation. The choice of these particular schools was made in the light of the students' earlier familiarisation with co-operative projects. Thus, it was expected that the usual functional problems in initiating cooperation would be avoidable.

Procedure

The treatment lasted for two months. The entire gathering of data took place during one academic year. All the groups were exposed to instruction of metacognitive strategies. The students were unaware of the different treatments and so were their teachers. The study supervisor traced the implementation of the pilot experiment through the observers' daily reports. The observers had been properly trained for over two months before participating in the pilot project.

In order to check the effect of the treatment on problem solving, the students were asked to solve four mathematical problems constructed to have parallel mathematical structure but different problem contexts. After that students completed a questionnaire. This procedure was repeated twice: before and after the pilot study. Yet, students were asked to use thinking aloud protocol during one problem solving activity, and this procedure was also repeated twice. Students were classified into three categories 'novice', 'medium' and 'expert' problem solvers, according to their teacher's judgment. The analysis of thinking aloud protocol was based on Sternberg's six steps of well-structured problem solving described in Table 1.

Measurements

During the pilot study, we used a set of measurements recorded here as the self-regulation components that were complex and multidimensional. First of all, the thinking aloud technique was used in order to decode the thinking process of the solvers, especially of the medium solvers. The note-taking was done by four fourth graders students of the University of Piraeus. They were familiarised with the technique beforehand, and the reason for that choice was made on the grounds of the objectivity they were able to provide.

Secondly, a manual recording of the co-operation in the second and third experimental groups was used, done by the experts of the groups. However, as we did not have unusual data to analyse no further reference to it is intended. The students found it awkward to have to register all the peers' contributions. They avoided it and we had to use a more automatic way of recording all the peer group interactions in order to analyse them in future investigations. Through this analysis, we hoped to derive useful information about the quantitative and, mainly, the qualitative parts of the cooperation. The scaffolding of the cooperation and the outcome of group work could be collected through the decoding of the interrelationships among the participants. Having data about the development of individual methods of problem solving might then be extracted concerning the time or the conditions needed to develop a self-regulatory skill.

Thirdly, important conclusions were drawn through the qualitative analysis of the students' portfolios of the third experimental group. The conclusions were related to the students' progress as well as to the excising of some parts of the questionnaire they completed such as self-efficacy and goal setting.

Lastly, the questionnaire that measured goal setting and strategic planning, awareness, self-efficacy, self-monitoring and self-reflection was used at the beginning and the end of this study. The questionnaire was developed from O'Neil and Abedi's (1996) study on metacognition, specifically, the items which measured the awareness and the monitoring. The parts which measured the planning and the self-efficacy were abstracted from O'Neil and Schacter's (1997) work on problem solving assessment. The items of self-regulation were formulated according to the social cognitive theory of self-regulation mentioned above. The entire inventory consisted of 30 items with an alpha reliability of 0.91. The reliability coefficients were compared too. The split-half reliability gave us encouraging results (Guttman Split-half=0.92).

Teacher's Role

Three teachers worked with the different groups in the pilot study. These teachers were considered qualified by the researcher as they were very experienced and had participated in other research in the past. The latter was very important because the teacher's style was not included in the study and consequently, there was a great need for formal application of the main principles of the pilot study. A factor that was not measured but it is worth mentioning was that these three teachers showed an increased interest in the development of self-regulatory skills and asked for references to improve their understanding. After the 'intervention' the teachers described the positive impressions their students gained while participating in this program.

RESULTS

The quantitative analysis of the sample showed that half of the participants were called ‘medium’ problem solvers according to their teacher’s judgment. The results showed that 39.6 percent of the participants were classified in the ‘expert’ category, and only 10.4 percent were classified in the ‘novice’ category of problem solver.

Table 2 analyses the mean scores of problem solving ability. A significant increase was observed in the first and second groups and a slight increase in the third group of novice problem solvers after the pilot study. Before the pilot study, it appeared that the students of the third group far exceeded those of the first or second group. After the study, all the students had increased their problem solving ability skills. This was confirmed by a one-way analysis of variance (ANOVA) on the scores of the pre-test and post-test. Before the treatment there were significant differences among the groups ($p=0.019$, $F=4.32$). A Scheffe test showed that the third group had high achievement on problem solving in comparison with the first group before the treatment. After the treatment no significant differences were found among the groups ($p=0.243$, $F=1.46$).

Table 2. Means scores and standard deviations of the three problem solving groups of students by different developmental levels of self-regulation

Groups	Pre	Post
1st	4.94 (3.39)	8.17 (3.07)
2nd	5.94 (3.86)	8.69 (2.89)
3rd	8.50 (3.01)	9.71 (0.83)

Specifically, the medium solvers increased their problem solving ability in all groups with most significant results in the second group. The latter showed that mere cooperation contributed to the increase of encouraging outcomes for the medium solvers. However, when new cognitive tools were introduced, even in cooperative activities, medium solvers had to struggle in order to incorporate them directly into their behavioural repertoire. Nevertheless, Figure 2 showed that when the means of the metacognitive strategies used in three groups were compared, it was found that they increased in the second and third group but decreased in the first group.

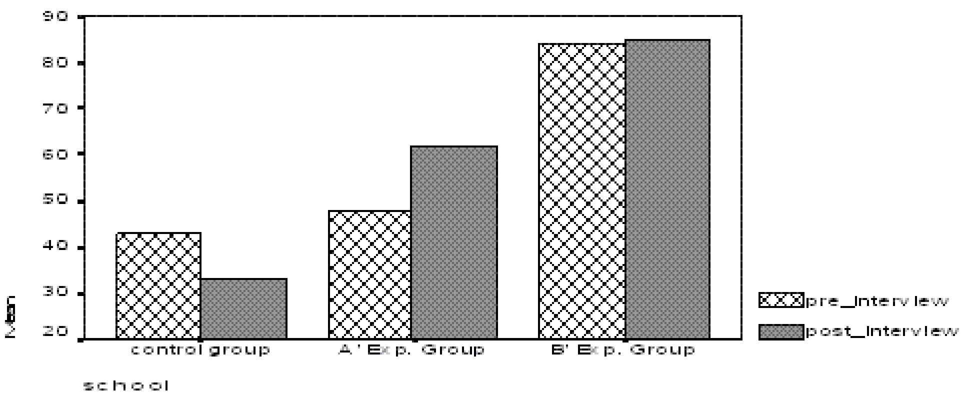

Figure 2. The trends of using metacognitive techniques by “medium” solvers in the different treatment groups

It seemed paradoxical that medium problem solvers decreased in the use of metacognitive strategies but increased their problem solving ability while observing the teacher’s model that is the control group in the Greek version of the study. However, the sample was too small to draw definite conclusions. It was only an intimation which needed more specific data. A very important issue in this point was the contribution of the participants and their interactions in group problem solving and the teacher’s role which was not included in our study. This might have given us an answer to this paradox. Analysing the other participants’ results, it is noticed that novices did not have any benefit from the observation of the model. Instead, they benefited by cooperation and from peer group modelling. Their increased achievement was accompanied by an augmented use of metacognitive strategies. The experts decreased their achievement and the use of metacognitive strategies while cooperating; unlike them, the expert problem solvers noticed a significant increase in their achievement and the metacognitive use of strategies either by observing the teacher's display of the problem solving strategies or developing their level of self-control.

Through the thinking aloud protocol, it was observed that medium solvers slightly decreased the monitoring and strategic planning of the problem solving process and increased metacognitive techniques which referred to problem definition, organisation of the information, resources management and evaluation. The differences are shown in Figure 3.

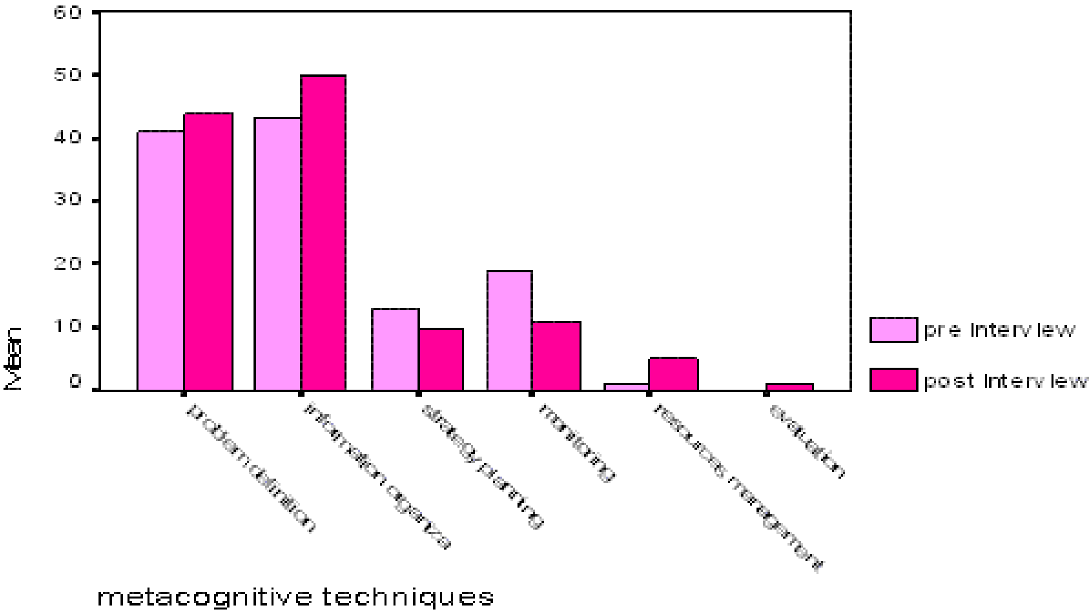

Figure 3. The drawn results through the thinking aloud protocol technique about the meta-cognitive techniques that are used by the students before and after the pilot study

Examining the students' self-efficacy beliefs, it was found that their self-efficacy increased in all groups, especially in the second group of learners. Overconfidence might have been one reason for the decrease in monitoring and strategic planning noted above. A decrease in strategic planning was recorded in the novice group, while a decrease in monitoring was recorded in the expert group. It seemed that strategic planning remains an important issue for the experts, but the same does not apply for the monitoring phase because as thestudents' self-efficacy is increased, the need for monitoring appeared to decrease.

The analysis of self-efficacy beliefs in the third group, according to the students' portfolio records, indicated that as long as the students monitored their daily achievement, their self-efficacy beliefs were more restrained than those of the students of the first or second groups, in which there was no daily monitoring.

Also, it was found that there was a strong correlation ($\rho=1.00$, $df=2$, $p<0.001$, Spearman) between the mathematics problem solving ability and goal-setting. The scatter plot analysis showed no signs of curvilinear relation between the two variables. This meant that when students searched for the answer to a problem, they solved it correctly. This could have been a factor that was predictive of their success.

CONCLUSIONS

This pilot study is a companion to a further study on the development of students' self-regulatory skills in the domain of mathematics in a computer supported collaborative learning environment.

Specifically, this article presents the initial phase of recording the behaviours in the transitory phase from novice to expert problem solvers while passing through the three developmental levels of self-regulation. In this article we argue that the use of models provided by both the teachers and student peers during mathematics instruction is an important contextual factor that can promote elementary children's mathematical self-regulation, metacognitive ability, self-efficacy and ultimately achievement. It is shown that self-regulation could be taught through modeling for the novices and medium problem solvers and through self-control level for the expert solvers. In addition to cognitive strategies, models may have an impact on students' beliefs so as to help them overcome the emerging difficulties and remain goal-oriented. Of course, this does not imply that we must return to traditional teaching methods; we can enrich them with new cognitive tools promoting their benefits in higher order thinking results. In order to help self-regulatory solvers improve in the domain of mathematics we have to take into account their prior state of knowledge and ability and finally offer them the appropriate instruction. It seems that there is no one formal method which can be applied to all mathematical problem solving categories.

Focusing on the process rather than the outcome may have implications beyond the domain examined in this paper. The attainment of self-regulatory skill requires an altered focus on the process of thinking, which is an integral part of problem solving and can be transferred to other kinds of problems (Kappa, 1999). The instruction of the metacognitive thinking process through the different phases of problem solving (pre-during-post phases) seems to contribute to the emergence and the integration of new techniques in all solvers' behavioral repertoire. Concerning the medium solvers, it seems that as far as they are taught the integration of metacognitive strategies into their repertoire, they advance the points of representation and evaluation by a decline in their planning and monitoring efforts. However, an extension in the size of the sample seems to be necessary in order to draw reliable conclusions and more specialised research on these points is needed in order to find out the reason why it happens. The latter can be the reason for keeping these students in this category and not let them slip into the next category.

The important increase of medium solvers' problem solving ability and use of metacognitive techniques in the second group reveals that this category of solvers benefits most when they cooperate with their classmates. Taking this finding into account, teachers could help students to progress by giving them more opportunities for cooperation and interaction. They seem to enjoy participating in these activities rather than sitting on a chair and observing their teacher. This is confirmed by their teachers' informal statement in a related conference.

A significant finding of this pilot study is related to the effectiveness of the intervention program with regard to novice and medium problem solvers. By the end of the intervention these solvers had improved their ability to solve word problems correctly. Despite the fact that expert solvers only slightly increased their ability, their tendency was not as spectacular as it was expected. This is in accordance with a phenomenon called a 'ceiling effect'. That is the expert solvers are *a priori* at a higher level than other groups and the remaining margins for improvement are more constrained.

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