PRE-SERVICE GEOMETRY EDUCATION IN SOUTH AFRICA: A TYPICAL CASE?

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Abstract

A two year study investigating the state of pre-service teachers' (PTs') (n=224), teachers' (n=18) and students' knowledge (n=123) of Grade 7 geometry (using the van Hiele theory (1986) and acquisition scales of Gutierrez, Jaime & Fortuny (1991)) is reported. Results indicate that both teachers and PTs fail to reach the expected level of geometric thinking and degree of acquisition. A relationship between student learning and the teachers' pre-service education and years of teaching experience seems to exist. Some pre-service preparation programs had no significant impact by either maintaining or positively impacting on the already attained thought levels. Results may hold implications for pre- and in-service training. Features of an improved program are suggested.

Introduction

Teachers carry the responsibility in partnership with parents to prepare learners to be independent, literate, numerate and multi-skilled leaders and decision-makers of tomorrow by equipping learners with relevant knowledge and allowing them to form suitable attitudes and beliefs. Teachers are therefore seen as mediators of learning, interpreters and designers of learning programs and material (Department of Education, 2002). The responsibility to adequately prepare prospective teachers (PTs) in their behavior as teachers and in teaching lies with tertiary institutions. The level and degree of teachers' and prospective teachers' content knowledge will be examined as proof of the adequacy of pre and in-service teacher education. Consequently, certain conclusions and some recommendations will also be made.

The Van Hiele model of geometric understanding

The Van Hiele theory (Van Hiele, 1999, 1986) proposes levels of geometrical thinking, with the lowest level being the visual level. At this level of thinking figures are judged by appearance alone, and learners at this level may respond by saying: "It is a rectangle because it looks like a door". At the next level, the descriptive level, properties of figures are recognized but properties are not yet logically ordered (Van Hiele, 1999). At the third level, the informal deductive level also called the Abstract / Relational level, properties are logically ordered and deduced from one another. Learners at this level use already known properties to formulate definitions despite not understanding the intrinsic meaning of deduction (the role of axioms, definitions and theorems). At the next level, formal deduction, theorems within an axiomatic system are established. The last level, the Metamathematical level, is characterized by the formal reasoning about mathematical systems by manipulating geometric statements such as axioms, definitions, and theorems.

Various researchers (Pegg & Davey, 1991; Wilson, 1990; Burger & Shaughnessy, 1986) reason that learners could be in transition between levels and Gutiérrez, Jaime and Fortuny (1991) present a method to evaluate learners in transition between levels. Gutiérrez et al. (1991) propose the qualitative utilization of the different ways in which learners' reason for placement within a proposed range of 0 to 100 – thus creating a scale of degrees of acquisition. Within this continuous range five stages or degrees of acquisition (no, low, intermediate, high and complete) are also identified, for example a high degree of acquisition is between 60 and 85.

It is believed that development through these levels is more dependent on instruction (Koehler & Grouws, 1992) than on age or biological maturation (Piaget, 1970) and that types of instructional experiences can foster or impede development (Van Hiele, 1999).

Teacher's Knowledge

A teacher is viewed as someone that possesses (or should possess) specific and adequate content knowledge. Jaworski and Wood (1999) note that in various countries the need to improve the experience of classroom mathematical learning through the development of teachers' knowledge of mathematics and knowledge of pedagogy is still relevant. Farah-Sirkis (1999, p.44) notes that both experienced and novice teachers view subject matter knowledge as a priority for in-service training programs and mentions that 80% of teachers viewed subject matter knowledge as the number one qualification for a good mathematics teacher.

Shulman's work (1986) confirmed the complexity of research into teaching and teacher behavior and focussed on elements of teacher knowledge (content and *pedagogical content knowledge*) – with teacher knowledge being one of the factors that influence teacher behavior. Teacher knowledge and the possible role it plays in the classroom are well documented (for e.g. Ernest, 1989; Koehler & Grouws, 1992; Kong & Kwok, 1999), but what and how much knowledge a teacher needs to be successful remains a question for debate. The solution requires a change in the nature of training rather than more content courses as "course taking is not a proxy for knowledge" (Ball, 1999, p.21).

The current state of South African primary school (Gr.1-7) teachers' content knowledge and the impact on classrooms have previously been investigated by South African researchers, including Webb, Bolt, Austin, Cloete, England, Feza, Ilsley, Kurup, Peires and Wessels (1998), in subjects such as science and mathematics. These studies brought the state of South African teachers' knowledge to the fore, but none focussed solely on Geometry – a problematic topic in secondary school (Gr.8-12). These studies also did not account for the influence and adequacy of pre-service training received by teachers.

This article will endeavour to summarize a two year study investigating both the state of pre-service teachers' (PTs'), teachers' and learners' knowledge of grade 7 geometry (using the Van Hiele theory (1986) and the acquisition scales of Gutiérrez, Jaime and Fortuny (1991)). Furthermore, the article will investigate the possible relationship between a teacher's content knowledge and the learners' learning gain. The conclusion will point to the implications of the findings, but will also make some recommendations for both pre and in-service teacher education.

Methodology

Population

This study is a two year study in which Grade 7 mathematics teachers from five towns were invited to participate in the first year of the study. All invited teachers were qualified to teach grade 7 mathematics and schools were deemed privileged (if taking the historical background of South Africa into account, e.g. all invited schools had electricity, water and qualified teachers). These teachers were requested to complete a) a questionnaire investigating their views and beliefs regarding their own knowledge, and the teaching and learning of Geometry; and b) a questionnaire investigating their own geometry subject knowledge. From the eighteen teachers volunteering to participate in the first year of the study, four teachers were selected (two with high and two with less geometry content knowledge) to continue their participation during the second year of the study. Each of these four teachers selected one of their grade 7 classes to be part of the year-long study. A total of 123 learners participated and were tested in the beginning as well as at the end of the second year in order to determine learner learning gain in each of the four classrooms. All learners received instruction and were tested in their mother tongue.

The population of PTs in the first year consisted of all final year education major students (*n=100*) at one of the five participating higher education institutions in the North-West province (South Africa). Seventy-eight PTs received second language instruction and followed a 3-year college curriculum at a college of education. The other PTs (*n=22*) followed a four year curriculum and received instruction in their first language at either a college of education or university. Each of the five tertiary institutions was visited in the final quarter of the first year where PTs completed a questionnaire investigating their geometry content knowledge. In the second year of the study, only one college of education remained in the North-West province due to restructuring of higher education in South Africa. From this college all freshmen (*n=27*) and junior year mathematics major PTs (*n=34*) (math majors only take 3 years of math and spend their fourth year focussing on the education part of their pre-service training) and all final year education major PTs (*n=63*) were included in the study during the second year of the study. The participating freshmen mathematics major PTs were tested during the first quarter while the junior mathematics major PTs and the final education major year PTs were tested during the final quarter of the academic year.

Research Design

An ex post facto research design (Leedy, 1997) was used to determine the state of PTs', teachers' and learners' knowledge of grade 7 geometry (by determining their geometric thought level attainment). Data was gathered directly from individual teachers and PTs by using two questionnaires and from groups of learners, by using a geometry subject knowledge questionnaire, in their natural environments to investigating possible cause-and-effect relationships.

Instruments

Two questionnaires were utilized; the first questionnaire completed by the eighteen teachers consisted of 142 items and investigated educators' views on their own knowledge (content, pedagogical and curriculum), pre-service preparation, beliefs and attitudes regarding geometry as well as the teaching and learning of geometry. Items required a response on a 4 point Likert-type scale (e.g. Disagree strongly; Disagree; Agree; Strongly agree). The following are examples of items included in the first questionnaire: "I was adequately prepared by the college/university with regard to academic knowledge (Geometry knowledge); If learners struggle with Geometry, it is a good idea to give them more opportunity to do self-exercises in class; To do well in Mathematics, it is important for learners to understand how Geometry is applied and used."

The second questionnaire was used for all three population groups and investigated geometry subject knowledge. The version for teachers and PTs consisted of 56 items and the version for learners consisted of 40 items (a subset of the teacher and PTs version). Items were selected from the Mayberry test (Lewin & Pegg version as published by Lawrie, 1998) and a test developed from Research Unit for Mathematics Education of the University of Stellenbosch (RUMEUS) (1984). Test items dealt with various concepts and shapes relevant for grade 7 over the first three Van Hiele levels for the learners' questionnaire and over the first four Van Hiele levels for the teachers and PTs. The following is an example of items that appeared in both the teacher and learners versions: "Does a right-angled triangle always have a longest side? If so, which one? (Make a drawing and indicate on your drawing)." Answers were evaluated using the acquisition scales of Gutiérrez et al. (1991, p.237-239). The reliability (Cronbach Alpha values) was calculated for the PTs' and teachers' version and is 0.77 for level 1, 0.71 for level 2 and 0.68 for level 3 with 0.56 for level 4. The reliability for the learner version on level 1 was 0.60, 0.52 for level 2 and 0.24 for level 3. All the computations for this article were done with SAS® (SAS Institute Inc., 1999). As the populations were not random samples, inferential statistics could not be used and statistical significance (p-values) is not applicable (Cohen, 1988).

Results and Discussion

In analyzing the data from both teachers and PT, it became clear that neither group was capable of achieving a *complete degree of acquisition* (>85%) for any of the Van Hiele levels tested (see Table 1). The 18 grade 7 mathematics teachers only achieved a low degree of acquisition for both Van Hiele level 3 (the relevant level for grade 7 - Department of Education, 2002) and Van Hiele level 4 (level relevant for high school mathematics) (Spear, 1993).

Table 1

Degrees of acquisition of teachers and prospective teachers

Year tested		Participants	N	Van Hiele Level and Degree of Acquisition			
				Level 1	Level 2	Level 3	Level 4
Year 1		Teachers	18	High	Intermediate	Low	Low
Year 1	Teachers	Junior Ed. Major (3 year college)	78	Intermediate	Low	Low	Low
Year 1	Teachers	Senior Ed Majors (4 year college)	22	Intermediate	Low	Low	Low
Year 2	Prospective	Freshmen Math Majors	27	High	Intermediate	Low	Low
Year 2	Prospective	Junior Math Majors	34	Intermediate	Low	Low	Low
Year 2	Prospective	Senior Education Majors	63	Intermediate	Low	Low	Low

Pre-service teachers, irrespective of subject specialization or time spent in pre-service education, could not reach an intermediate degree of acquisition for Van Hiele levels 3 or 4. PTs in their first year of pre-service education exhibited the highest degree of acquisition in nearly all the Van Hiele levels, when compared to any other group of prospective teachers.

In investigating the effect teachers have on their learners' learning gain (as revealed by their degree of acquisition), teachers' influence resulted in (at most) a single degree of acquisition shift (see Table 2).

Table 2

Degrees of acquisition of four classes and their teachers with some teacher biographical information

		Van Hiele Level & Deg of Acquisition				*Education & Training*		*Mathematics teaching experience*
		Level 1	*Level 2*	*Level 3*		*Academic Mathematics*	*Mathematics Methodology*	
Class 1	Learners (n =36)				Teacher Carl	4 years	4 years	9 years
	Beginning of Year 2	Intermediate	Low	Low		*Level 1*	*Level 2*	*Level 3*
	End of Year 2	High	Low	Low		High	High	Intermediate
		Level 1	*Level 2*	*Level 3*		*Acad Math*	*Math meth*	*Exper*
Class 2	Learners (n =29)				Teacher Peter	0 years	4 years	8 years
	Beginning of Year 2	Low	Low	Low		*Level 1*	*Level 2*	*Level 3*
	End of Year 2	Intermediate	Low	Low		High	Intermediate	Low
		Level 1	*Level 2*	*Level 3*		*Acad Math*	*Math meth*	*Exper*
Class 3	Learners (n =29)				Teacher Sarah	0 years	4 years	2 years
	Beginning of Year 2	Intermediate	Low	Low		*Level 1*	*Level 2*	*Level 3*
	End of Year 2	Intermediate	Low	Low		High	Intermediate	Low
		Level 1	*Level 2*	*Level 3*		*Acad Math*	*Math meth*	*Exper*
Class 4	Learners (n =29)				Teacher Paul	0 years	0 years	6 years
	Beginning of Year 2	Low	Low	Low		*Level 1*	*Level 2*	*Level 3*
	End of Year 2	Intermediate	Low	Low		High	Intermediate	Low

Note. Pseudonyms have been used; and teachers (Carl, Peter, Sarah and Paul) was tested in year 1 as part of the teacher population.

All learners exited grade 7 with a low degree of acquisition, with some learners exhibiting no learning gain, on Van Hiele level 3 (level relevant for this grade - Department of Education, 2002) after a year of instruction. Results of the second questionnaire that the four teachers completed illustrate their views and beliefs about the nature of geometry as well as the teaching and learning of geometry (see Table 3).

Table 3
Four teachers' views about Geometry, teaching and learning

	Geometry	Teaching	Learning
Carl (class 1)	Should be understood and used as a practical and structured guide for approaching situations	Uses every day items; Giving problems with guidance.	Solve problems; Give reasons for answers; Require creative thinking.
Peter (Class 2)	Viewed as formal method to present reality, should be learned as set of rules/ algorithms.	Give explanations to struggling learners; Individual or group work; Whole class teaching.	Remember formulas and procedures; Think in sequential manner according to procedures
Sarah (Class 3)	Is viewed as set of rules/ algorithms.	Individual tasks; Whole class teaching; Show learners how to solve problems.	Remember formulas and procedures; Think in sequential manner according to rules learnt. Copy notes from black board.
Paul (Class 4)	Viewed as an abstract subject.	Show learners how to solve problems and give answers to struggling learners; Whole class teaching; Learners work individually.	Remember formulas; Think in sequential manner according to procedures

Reflecting on the results (Table 1 and 2), the current state of the participating teachers' and PTs' content knowledge levels may be an issue requiring further scrutiny. Both teacher and PT populations failed to reach the expected level of competence (Van Hiele level 3) and degree of acquisition (high or complete) – the level expected from grade 7 learners.

Research done on the geometrical thought level acquisition of teachers in 2000 (see Table 1) brought the bleak state of teachers' knowledge to the fore. Teachers' low degree of acquisition on Van Hiele level 3 and 4 was unexpected and emphasizes the ineffectiveness of pre-service training. It seemed reasonable to expect practicing grade 7 teachers would have completely mastered the Van Hiele level 3 (the level relevant for grade 7) and would have at least exhibited some significant degree of acquisition of the work their learners are to learn in the future (Van Hiele level 4).

A person becomes a mathematics teacher only after further education and training, and if the degree of geometry acquisition of teachers is lacking (as was indicated in this study), then a possible root for poor learner achievement would surely lie in pre-service training received by teachers. Investigating the knowledge (geometrical thought) level attainment with which teachers enter the career (i.e. final year PTs – see Table 1) would give an indication of the effectiveness of pre-service training in preparing PTs for a career of mathematics teaching. The results seem to indicate mathematics teacher education, irrespective of years of education (3 years or 4 years) does not adequately prepare PTs. All PTs achieved a low degree of acquisition for both Van Hiele levels 3 and 4 – the same degree of acquisition attained by teachers. PTs following the four year education model slightly outperformed PTs following the 3 year education model, but were not able to achieve a degree difference on any of the Van Hiele levels.

Teachers, on the other hand, did outperform the PTs on Van Hiele levels 1 and 2, which could lead to a conclusion that teaching experience could be a positive factor in the acquisition of the lower Van Hiele thought levels. A general conclusion, of the research done in 2000, is that teachers and PTs are not adequately in control of the subject matter they have to teach. This study verifies the study of Webb et al. (1998) regarding the state of South African teacher knowledge of the subject matter they are to teach to learners, but also confirms findings by American researchers. For example, Mayberry (1983) found that of the 19 prospective elementary teachers in her study that she tested and interviewed, 13% where at level 0, 20% at level 1, 19% at level 2, 24% at level 3, 25% at level 4 and none at level 5. Mason and Schell (1988) investigated 67 prospective elementary school teachers' geometry knowledge and found that 38% of these prospective teachers were functioning below level 4 with 8% not even attaining the lowest level, recognition (Van Hiele level 1). This correlation could imply that teachers, irrespective of educational history, could currently be teaching without the necessary subject content knowledge and also that PTs enter the teaching career with less than adequate knowledge.

The research done in the second year at the four selected teachers' classes revealed the learning gain learners exhibited during the year (see Table 2). All four classes completed grade 7 while remaining on a low degree of acquisition for both Van Hiele levels 2 and 3. Three of the four classes made a degree improvement on Van Hiele level 1, with only one class achieving a high degree of acquisition on that level. There seems to be a possible relationship between the learning gain made by learners and the teachers' pre-service education and years of teaching experience. The class whose teacher completed a four year pre-service education program with mathematics and mathematics methodology combined with nine years of teaching experience was the only class to achieve significant learning gains on all three of the Van Hiele levels. The class whose teacher completed a four year pre-service education program without any mathematics and mathematics methodology education with only six years of teaching experience, was the only class who showed no learning gains on Van Hiele level 3. Despite the different education received by the four teachers or the years of experience, only one teacher could reach an intermediate degree of acquisition on Van Hiele level 3. The other three teachers themselves (just as their learners at the end of the year) exhibited a low degree of acquisition of Van Hiele level 3. These findings regarding the inadequate state of teachers' knowledge confirms the findings on South African teachers by Taylor and Vinjevold (1999) and is consistent with an international phenomenon illustrated in Ma's (1999) study. These findings of teachers' knowledge and the possible impact on learning, raise the question of the adequacy of pre-service education, but also confirm the need for intensive in-service programs to assist teachers to teach on the relevant and expected level.

Teachers' behavior is not only influenced by their knowledge, but by their views and beliefs. The four teachers in this study revealed their views and beliefs about the nature of geometry, teaching, learning and their own self-efficacy to effectively teach grade 7 geometry. The teacher who received both mathematics and mathematics methodology education also exhibited quite different views about the nature of geometry, teaching and learning from the other teachers who received only mathematics methodology or no methodology or mathematics education (see Table 3). A belief all teachers shared, was that they believed they were successful in teaching, that is, all four teachers exposed a positive self-efficacy. This false sense of self-efficacy could emanate from teachers aiming teaching only on the level they are confident in or by only teaching the lower Van Hiele levels.

In an effort to find the possible cause for teachers' low level of geometrical acquisition, but also a possible starting point to correct the state of affairs, a more in-depth investigation of the impact of pre-service teacher education was done at the only remaining college of education in the North West province. PTs with no training in either mathematics or methodology education outperformed PTs that had completed three years of mathematics and methodology education as well as PTs that had completed 4 years of methodology education (see Table 1). This revelation could imply that pre-service education had no significant impact by either

maintaining the geometrical thought level acquisition attained through schooling or by positively impacting on the already attained thought level.

The inadequate and fragmented nature of PTs' and teachers' knowledge of school mathematics has been noted before by various researchers, for example Cooney (2001), and has in this study been confirmed for the participating teachers and PTs. The study furthermore confirms the findings of Ball (1990) that relying on what PTs learned in the pre-service training mathematics and methodology classes is "unlikely to provide adequate subject matter preparation for teaching mathematics with understanding" (Ball, 1990, p.142). If the teachers' and PTs' knowledge (geometrical though level attainment) is considered, both pre-service and in-service training of mathematics teachers need attention and/or renewal. The current education programs at tertiary institutions seem not to have the desired impact on PTs' level or degree of geometric acquisition expected (and required) to teach effectively – with the subsequent effect reflected in learners' achievement in this study as well as in international surveys (such as the TIMSS). The need for change in teacher pre-service education is clear.

Recommendations

Cooney (2001) indicates how crucial pre-service education is by noting that the role of teacher educators is to reveal and make evident the complexity of teaching and to propose alternatives for dealing with that complexity as teachers have neither the luxury, nor the resources, to experiment with or fantasize about a different school environment. With the results seemingly indicating that pre-service training is not adequately preparing PTs and thus not assisting teachers, the following implications and recommendations could be useful for teacher educators and tertiary institutions (in the US and internationally) wanting to make a meaningful contribution to the preparation of mathematics teachers.

A long term teacher education program is suggested that incorporates constructive learning environments that: Firstly, enables PTs to develop knowledge of mathematics that permits the teaching of mathematics from a constructive perspective with courses provoking PTs to confront their possible "naïve notions of teaching mathematics" (Lerman, 2001). Secondly, offer teachers and PTs an opportunity to reflect on their experience as learners of mathematics (Cooney, 2001), but also as teachers/mentors, in an environment where they experience the learning in the same way as they will be expected to work with learners. The learning environment could include relevant technology, such as dynamic software that could assist in creating relevant and powerful learning environments. Thirdly, balancing mathematics content knowledge with pedagogical competency in combination with collaboration and reflection. Fourthly, provide a context in which teachers and PTs develop expertise in identifying and analyzing the constraints they face in teaching and explore strategies to deal with such constraints. Lastly, afford contexts in which PTs and teachers can gain experience in assessing learners' understanding and learning of mathematics. Franke and Kazemi (2001) suggest that PTs are transformed from teachers into learners by listening to learners' mathematical thinking/explanations with the benefit of PTs learning about the teaching and learning of mathematics in the context of their practice.

The advantages of following a program of this nature could include teachers being enabled to become less textbook dependant with less emphasis given to computational tasks and a focus shifting from teaching to learning. By providing opportunity to apply instructional strategies and techniques, teachers acquire knowledge that enables them to reflect on their own learning and knowledge base and so generate the realization for the need to relearn forgotten knowledge and gain new knowledge.

Cooney (2001) theorises that the greatest moral dimension of teacher education is the challenge which enables teachers to see knowledge acquisition as power they can use to enable learners to acquire the same kind of power. If teachers are to teach according to the visions of reform, they must be convinced of the value of reform and have exposure to similar learning environments first-hand as learners. This places great responsibility on the shoulders of tertiary

institutions to reform teacher education by establishing a theory for their practice and by giving attention to content and pedagogy while training teachers to be reflective, problem solving, intelligent professionals.

Concluding Remarks

Educational research and teacher education are means of educational improvement. The results suggest several directions for future research, for example, future research could include comparative and longitudinal studies with bigger study populations (in South Africa, the USA and internationally) to determine the applicability of the results on a wider population and subject area, especially focussing on pre-service training. The correlation between the nature of the pre-service training and ultimate teacher instructional behavior is also a point worth further investigation. Follow PTs through their years of training and into their first few years of teaching as this could produce a clearer picture of the impact of pre-service training on both teaching and learning. Research could thirdly be conducted into the cohesiveness between college instruction and instruction in the school setting. Furthermore, an investigation into the possible hierarchical structure of the elements of teachers' beliefs could contribute to an understanding of teachers' instructional behavior. Finally, the search and study of teachers with adequate knowledge and their subsequent behavior could also provide useful insights into the areas on teacher training that need addressing – with geometry being an area ripe for inquire (as evident in the lack of research into geometry teaching and learning).

In conclusion, it is hoped that the findings and recommendations of this study would assist to convince teacher educators that reform in pre and in-service programs is urgently needed. As educators of teachers, we have a responsibility to our children to assist their teachers to become the best teachers they can be –in order for our children to become the best they can be.

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