Transforming Identities:
Understanding Teachers across Professional Development and Classroom Practice

By Dan Battey & Megan L. Franke

Despite the prevalence of professional development in schools and the variability in its implementation, little research has been conducted on how professional development makes its way into the classroom (Wilson & Berne, 1999). Even when teachers participate in high-quality professional development, there remains a large and often undocumented variability in how teachers make use of ideas learned (Enyedy, Goldberg, & Muir, in press; Kazemi, 2004; Franke, Carpenter, Levi & Fennema, 2000). Therefore, educational researchers and professional developers need to better understand the dilemmas and choices teachers face in making use of learned practices.

We have recently turned to identity as a way to help us document, analyze, and understand teacher learning and classroom practice. "We take identity to be a central means by which selves and the sets of actions they organize form and re-form over personal lifetimes and in the histories of social collectives" (Holland, 2001, p. 270). Focusing on identity as a part of learning has enabled us to see teacher learning as both situated in practice and as an integrated, complex system embedded in the structures, histories, and cultures of schools. We use identity to differentiate how teachers participate in and make sense of professional development in practice. The construct of

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identity allows us to begin to understand why professional development can look very different as teachers take new ideas and put them into classroom practice.

Characterizing Identity

Identities are constructed in relation to history, cultural practices and communities, and the broader contexts in which we participate (Wenger, 1998; Holland, 2001). How one thinks of herself is conceived of in relation to a particular context, with a particular history, with others who have ideas about themselves. These histories (and the structures in which they are embedded) contribute to how a teacher comes to make sense of what it means for her or him to be a teacher, what it means to be a “White” or “African-American” teacher, what it means to be a “traditional” or “reform” mathematics teacher, as well as what it means to be a “good” teacher. We do not develop our identities as teachers in isolation. Ever changing histories, cultural and historical events, create and continue to create space for particular identities and shape how teachers navigate their everyday practice (Holland, 2001). In creating this space, they can both open and constrain how identities develop.

Through participation in social practice, identity shapes how one participates and how one participates shapes identity (Lave & Wenger, 1991; Rogoff, 1997, 1994). Lave (1996) states it this way,

> Crafting identities is a social process, and becoming more knowledgeably skilled is an aspect of participation in social practice, who you are becoming shapes crucially and fundamentally what you know. (p. 57)

Identity is shaped by the knowledge and skills we acquire and shapes the knowledge and skills we seek to develop. So identity does not sit separately from knowledge and skills; acquiring new knowledge and skill play a critical role in reshaping identity (Franke & Kazemi, 2001). The relational nature of identity arises, in this sense, as a way of contextualizing knowledge and skill.

Instead of assuming that teaching is the sum of knowledge, beliefs, and skill, we assume that teaching occurs through participation in a community of practice. Therefore, participation constantly leads to the formation of a new identity (Wenger, 1998). Teaching is a process of becoming a member in a defined group of practitioners with specific skills, with the important marker of learning being the adoption of an identity as a full member. For us, professional development is a space for acquiring new knowledge, re-crafting identities, and challenging existing cultural and social practices.

Teaching is a highly contextualized social practice (Goodson, 1991). As teachers tell stories in professional development, they situate themselves and the narratives they use to define themselves. Through these storied identities we can view how the teacher sees herself in relation to teaching, to the content of mathematics, to her students, and to her community. Teachers’ stories and how these stories are practiced exhibit knowledge and skill use. These relations also can be viewed through classrooms as teachers practice their identities, engaging with students and the content. Teachers enact identities as they engage in practice, broadly conceptualized here as including professional development and the classroom.

Teacher Learning within a Community of Practice

Teachers follow a trajectory of learning similar to the apprenticeship model. It is the day-to-day participation over years of engaging in classroom practice (both as a student and a teacher) where teachers develop what it means to teach (Enyedy et al, in press). Through participating in schools and classrooms, apprentice teachers learn how to work with students, talk about students, and how to organize their practice. Although much of this trajectory is not done in-line with the way reformers would have teachers learn their practice, the contexts in which teachers work is a social construction, a form of a community of practice.

Within a community of practice, there are shared ways of talking about students, content, and teaching. New teachers are in the process of appropriating the ways in which experienced teachers talk. Those forms of talk, or discourses, bring certain kinds of knowledge, values, and possible identities with them (Gee, 1996). The process of learning to teach is a social process of identity transformation, mediated by talking to others who collectively define, through language, what it means to teach (Deneroff, 2005). The language practices that teachers use in talking about the profession of teaching both hold the acceptable identities for teachers and carry the important knowledge, skills, practices, and values for teaching.

Identity is in itself a tool that mediates action. From this perspective, teachers use their professional identity to navigate dilemmas, to decide what knowledge to use, and to make sense of other/new contexts (Enyedy et al, in press). In professional development, teachers make choices about how new practices fit into the existing context of their classroom. Their identity mediates what makes its way into the classroom by how consistent or inconsistent the new practices are with how they think about teaching content. A teacher's identity connects the contexts of the classroom and professional development, and helps the teacher decide how much of the newly learned knowledge and skills are appropriate for students.

Similarly, the available professional development and the school community deem certain practices appropriate. So while identity serves as a pivot to navigate dilemmas of practice, local communities limit the variety of practices that teachers have access to. The communities teachers participate in, the histories, structures, as well as colleagues provide contexts for teachers to construct possible identities, and allow new teachers the opportunity to "practice their identity" (see Holland, 2001), for full membership in the community of practice. In this sense, the contexts in which we participate guide us in developing who we are. Contexts can constrain or open up new possibilities as other teachers practice mathematics instruction in particular ways, the curriculum embodies a particular take on what mathematics is, and students bring their own notions of what it means to do mathematics. A school that uses a traditional mathematics curriculum may guide a new teacher to focus on student learning in particular ways. Or, new professional development at a school may reveal new ways of enacting what it means to be a mathematics teacher.

Apprenticing to a New Community of Practice

Seen this way, professional development is about taking on new ways of talking, relating, and acting in relation to students and the teaching of mathematics. This focuses attention not only on the ways teachers take on new knowledge, but also how they make use of that knowledge as they participate in their teaching practice. As teachers participate they develop new knowledge, skills, and ways of talking, and these facilitate a shift in identity of what it means to be a mathematics teacher.

In our professional development, teachers join a community working to establish a particular set of goals and practices (Fullan, 1991; Lieberman & Miller, 1990; McLaughlin & Talbert, 1993; Secada & Adajian, 1997; Tharp & Gallimore, 1988). In Holland and her colleagues' (2001) study of Alcoholics Anonymous they point out how there is explicit work necessary for people to come to identify as 'alcoholics who do not drink' rather than 'non-alcoholics who drink'. In our professional development, we too are creating a community in which teachers are working toward a particular way of being. We ask teachers to begin to see themselves as learning from their students; as teachers who learn about their students and students' mathematical thinking in ways that change how they participate in interactions with students, how they talk with parents or engage with the mathematics. We ask them to become teachers who challenge their own assumptions about students. Working to become a part of such a community of teachers often sits in contrast to what exists within teachers' schools.

We work in urban schools with the school staff, not just with teachers that seek us out. This is important for a range of reasons, however here it is critical as we discuss the transformation of teacher identities that might begin by looking quite oppositional to the professional development goals. The teachers we work and write about in this paper have not sought out professional development opportunities. In addition to thinking differently about who is participating, we need to consider the cultural practices of the school community. We have to find ways to connect the professional development to the established forms of teaching mathematics in schools. This might mean asking teachers to try something for 5-10 minutes a day or one day a week to begin with, so they can find ways to tweak the well-established cultural practices towards practices that help them see and hear their students' mathematical thinking. We think these issues are important in understanding the discussion that follows and how we think about professional development as opening the possibility of developing new identities.

A Professional Development Community on Algebraic Thinking

The mathematical focus of the professional development for the work reported here was on algebraic thinking (Jacobs, Franke, Carpenter, Levi, & Battey, 2007). We built on the growing body of research on how elementary school students think about algebra and the ideas they find accessible (Bastable & Schifter, 1998; Blanton & Kaput, 2005; Carraher, Schliemann, & Brizuela, 2000; Davis, 1964; Kaput & Blanton, 2000; NCTM, 1997, 1998; National Research Council, 2001; Schifter, 1999). We also identified ideas that would be most useful to teachers during instruction—in particular, ideas that would connect with students' current thinking and have immediate face validity to teachers (for more information, see Carpenter et al., 2003; Franke, Carpenter, & Battey, 2007).

We focus on developing relational thinking through the understanding of the equal sign, the use of number relations to solve problems, conjecture, and justify (Carpenter et al., 2003). This approach views the equal sign as an indicator of a relationship between two expressions. Unfortunately, many students hold an alternative view in which the equal sign is a signal to carry out computation and the number after the equal sign is the answer to that calculation. For example, students without a relational view of the equal sign may solve the equation 57+36=___+34 by putting 93 in the box. Although researchers have documented the prevalence of this problematic view of the equal sign (Behr, Erlwanger, & Nichols, 1980; Erlwanger & Berlanger, 1983; Kieran, 1992), evidence exists that even young children can learn to think of the equal sign as an indicator of a relation (Carpenter & Levi, 2000; Falkner, Levi, & Carpenter, 1999; Kieran, 1981; Saenz-Ludlow & Walgamuth, 1998). Developing a relational view of the equal sign is critical for learning algebra, and a lack of such understanding is a major stumbling block for students when they move from arithmetic to algebra (Kieran, 1981; Matz, 1982). The way we have chosen to focus our algebraic thinking work also has implications for the types of conversations that support the development of student thinking (Franke, Battey, & Carpenter, 2007). Conversations centered on reasoning together around whether a mathematical idea is always true, or under what parameters it is true (Jacobs et al., 2007). These types of conversations require students to put forth arguments and support them, challenge each others arguments, and continue the conversations over time. Not only does this form of practice require knowledge and skills of teachers that run counter to how we usually practice mathematics instruction, it requires a reframing of student learning in mathematics and a new relationship with classroom practice.

In the case of the professional development session discussed in this paper we used the handshake problem below (Kaput & Blanton, 2001). Discussing the same problem together in the workgroup and then having teachers make choices about how to enact it in their classrooms allowed the content to be as similar across classrooms as possible. In this sense, we could focus on how teachers engaged in the same content, the changes they made, and why they made these changes.

> Twenty people are at a party. If each person is to shake everybody else's hand once, how many handshakes will take place at the party? How many handshakes

> will be needed for 21 people? How does the number of handshakes grow every time someone new arrives at the party?

This problem allows multiple strategies to be generated as well as possessing the possibility for generating generalizations. The idea is that the first person at the party shakes 19 people's hands (they are person 20 and can't shake their own hand). The second shakes 18 because they can't shake their own and already shook hands with the first person. The pattern that emerges is that the number of handshakes decreases by one for each subsequent person (19+18+17+16…). In thinking about 21, 22, or 100 people at the party, we can begin to think of generating an equation that we could apply no matter how many people are at the party. In this way we can generate that the total number of handshakes would be the number of the people at the party (n) multiplied by the average number of handshakes (n-1)/2. So no matter how many people are at the party we can figure out the total number of handshakes (h) from the equation: $h=n(n-1)/2$.

The handshake problem allows teachers to discuss possible student strategies, the mistakes they might make, as well as the different ways to teach it such as acting it out or starting with smaller numbers. The problem affords the types of discussion central to the community—discussion of student thinking, mathematics, and new forms of instruction.

Investigating Participation across Communities of Practice

The goal of this particular study was to understand the relationships between professional development and classroom practice.[1] We looked at the ways teachers participated in each of these communities and their identities as teachers of mathematics across the communities. Examining teachers in both settings allowed for a comparative analysis of teachers' participation in and identities around their professional development and classroom community. To highlight the ways in which identity help us make sense of the relationship, we focus here on detailing the participation and identities of two of the particular workgroup teachers.[2]

Study Context

School Community

Westland elementary school was designated a low performing school the year before we started the algebraic thinking professional development with teachers. The school was designated an II/USP (Immediate Intervention/Underperforming Schools Program) school because it's API (Academic Performance Index) ranked between the 10th and 20th percentiles of California schools. The student body, roughly 1300 students, was primarily Latino (85% Latino, 15% African American). All instruction was in English only by state law. Few teachers spoke Spanish. Much of the faculty, especially in the upper grades, was new. When we came to the school, the culture of teaching was extremely isolated. Not only did teachers not talk to each other much about teaching, they rarely talked to each other period. Almost every classroom door in the school was locked throughout the day. When the principal made time for grade-level group meetings, the teachers generally thought they would be a waste of time. The teachers reported feeling very stressed. They talked about the pressure of the multiple professional development programs and test scores, and worried about what the new principal was going to ask them to do next.

The Workgroup Community

The professional development plan separated the teachers into three workgroups based on grade levels. The fourth and fifth grade teacher workgroup served as the focus for this study. The fourth and fifth grade teachers were generally inexperienced. Out of the 10 teachers, one of them had a credential at the beginning of the year and two at the end. Eight of the teachers were in their first 4 years of teaching. One of the remaining two teachers taught for over 30 years, including 20 at Westland Elementary School. Not including this teacher, the other nine averaged 2 years of mathematics teaching experience. Another teacher had taught for 20 years, the last three in public school. He'd taught P.E. for 17 and only had 3 years experience teaching mathematics. One teacher was fired towards the end of the year and the teacher who had taught for 30 years chose not to participate for personal reasons as the year ended. Eight teachers participated in the professional development both during the year and in this study. These eight teachers averaged 5.3 years of teaching experience and 1.8 years experience teaching mathematics.

The workgroups provided a setting for engaging teachers in inquiry connected to their daily classroom practice. The workgroups met with the same group of teachers once a month throughout the year. Typically, meetings lasted between an hour and an hour and a half. As professional developers we participated in the workgroup meetings and the school. We led the workgroup meetings, planning the tasks and facilitating the discussion. We also participated in the school by spending one morning each week on-site. This allowed informal time with teachers in the lunch room and hallways as well as time in classrooms to provide support. Spending time in their classrooms gave us the opportunity to learn more about the teachers' practices and their students. School visits provided a chance to check in with teachers, hear their concerns, and share some thoughts that might be helpful. Our goal was to be seen as a part of both the workgroup and school communities and to engage with teachers in both settings.

Data Collection

We used three primary data sources to understand the participating teachers: videotaping professional development, videotaping classrooms, and interviewing teachers.

Professional Development Observation

We examined teacher workgroup participation through a focused examination of one workgroup session. The final workgroup of the year was designed to capture participation in the professional development. During this session we engaged the group in working with a problem related to, but different from, any we had worked with during the year: the handshake problem mentioned earlier. This workgroup lasted about 60 minutes. Teachers solved the problem themselves, talked about how their students might approach the problem, and discussed issues that could arise in teaching the problem to students. The video was set to capture the workgroup professional discourse. The camera lens focused so all teachers were seen. The video started as teachers walked into the room and ended just before we left.

Classroom Observation

We followed the assessment of professional development participation by documenting teachers' classroom practice—all around solving the problem discussed in the last workgroup meeting. We videotaped and took field notes as teachers engaged students with the handshake problem. Video of the classroom gave evidence as to how the teachers engaged with students and the content. As teachers engaged students in the algebra problem, the video focused on the teachers and the discourse of the classroom. We placed the camera in the back corner in order to see all of the students, the board, and the teacher. The camera followed the teachers' interactions during the lessons. If teachers were talking to individual students, the camera focused on this interaction. If the teachers were talking to the whole class, the camera captured as much of the class as possible including the white board. We also took field notes that detailed the work of the students that the video could not capture. The student work was also collected.

Teacher Interview

We interviewed the teachers following the professional development and classroom observation; giving them a chance to share what they learned about their students, the mathematics, and their teaching practice. The interview was semi-structured, allowing space to refer to specific instances that occurred in the classroom. This gave teachers an opportunity to share their standpoint on the practices they engaged in during the observed lesson.

Three main questions focused the teacher interview. First, what do you think the students learned in the handshake lesson? This allowed teachers to talk in detail about their students. Second, what are the mathematical ideas underlying the problem? This provided an opportunity to get at the mathematical knowledge of teachers. It also revealed information on how teachers engaged students and the way they presented the problem. And third, what would you do differently if you taught this again? This gave teachers an opportunity to explain what they learned about their practice of teaching mathematics. These three questions helped situate teachers' participation during the workgroup and classroom. The interview allowed teachers to make explicit their relationship to student thinking, the content, and their instruction.

Data Analysis

Participation was a focus of the analysis. How did teachers and students engage or not engage in the mathematics? How did they engage the classroom in mathematical discourse? How did teachers structure the mathematical practice of students in the classroom? How did students engage with the established classroom practice? The form of their participation revealed one element of practice.

Content-specific knowledge provided another element of practice. What content did teachers engage students in? Could teachers anticipate how students would solve the problem? What strategies did students use to find a solution? Knowledge and skill intertwine with the forms of participation that teachers and students engaged in. Without mathematical knowledge, the content of the discourse would look much different.

Teachers' relationships with students, mathematics, and colleagues offered a context to place participation and knowledge. Were teachers confident in their mathematics knowledge? How did their math knowledge influence the mathematics students had an opportunity to engage in during class? How did their knowledge push participation towards or away from the center of the mathematical practice? These relationships provided a context to understand the direction of participation and mathematical learning in the classroom.

These three main ideas guided the coding of both the workgroup and classroom data: participation, content-specific engagement, and relationships. These categories are not mutually exclusive, but capture the dynamics of the community from different places of reference. Together they painted a picture of the practice that teachers engaged in.

The Workgroup Session

Typically, it was difficult to get the teachers talking in the workgroup. Ms. Spencer, Mr. Dylan, and Mr. Thompson participated the most regularly unless the veteran 30-year teacher was present. With her there, Ms. Spencer became quieter. Mrs. Brown and Mrs. Taylor would add comments here and there. Mr. Gray and Mr. Jones usually sat completely silent. Mr. Lopez's participation was somewhat erratic, sometimes interacting often, but always very attentive. We use Mrs. Brown and Mr. Gray in this paper as cases for understanding how teacher identity shapes participation in professional development and classroom practice.

We focused on one algebraic reasoning topic in each meeting (e.g., equality, writing true/false number sentences around place value, multiplication, or division). During the workgroup, we would solve the problems in a number of ways trying to get the teachers to come up with strategies other than the standard algorithm. We would talk about what was important for students to understand in terms of the mathematics and discuss different ways that they could develop their own students' understandings. Although this workgroup usually went more smoothly than the other grade-level groups in the school, it was still difficult to get teachers to participate and talk to each other. Some of this was due to a lack of mathematics knowledge for certain teachers and some of it was due to a very different idea of professional development that teachers held. Many teachers wanted us to tell them how to teach and did not see any reason for talking with other teachers; they wanted answers.

The meeting around the handshake problem went fairly typically, except for the fact that the problem was more mathematically challenging for them than usual. The professional development began with us reading the problem and asking teachers to solve the first part of the problem (20 people), before moving on (21 and 22 people). They spent about 10 minutes solving it on their own, checking answers with each other, and trying to generalize a formula. After they solved it, two teachers shared their strategies. The teachers quickly figured out what they would get for 21 and 22 people and we talked about why that was. We went through other numbers of people at the party to figure out a mathematical pattern. The teachers generated a formula for any number of people and tested it with different numbers to assess its accuracy.

The teachers talked about different ways that they could work on this problem with their students including acting it out, drawing pictures, and starting with smaller numbers. This conversation took about ten minutes and the discussion turned to the difficulties students might run into when solving the problem. Some thought students would have problems realizing you have to go back one each time (19+18+17…). Others teachers commented that students wouldn't know to start with 19, and a few said students would multiply first. This took another ten minutes. We then helped them see links between two of the teachers' strategies and the formula that the teachers generated. Since much of their generation process included guessing, we focused on understanding the relationship between their strategies, the formula they generated, and why it worked. We ended by scheduling times to videotape them teaching the same problem to their own students.

Teacher Identity in Practice

We begin by presenting analysis of Mrs. Brown's identity as she tells stories about herself in relation to the teaching and learning of mathematics. We then look more closely at her identity in relations to both the professional development and classroom settings. We follow with the same for Mr. Gray. For the purpose of this paper we have chosen not to keep our commentary until the end; instead we weave the teachers' words, findings from the video analysis, as well as contextual information from field notes to draw a more cohesive story. This allows for a greater understanding of the teachers within the context they're working and allows us to relate their perspectives to broader histories of teaching mathematics.

Mrs. Brown's History with Mathematics

> Math is like a typing class, you have to warm up and do your timing tests. I'm an old typing teacher and you have to drill.

Mrs. Brown taught 4th grade. She had taught for 4 years, two in middle school and two in elementary school, and she only taught math in her elementary school years. Prior to working in K-12 schools, she worked at a community college as a typing teacher. She is an African-American female, in her forties, without a credential. The recurring story in the telling of her identity was the notion of being a typing teacher. Mrs. Brown saw a strong connection between her work as a typing and a mathematics teacher as evidenced in the quote above. The identity of a typing teacher crossed over to her seeing drill, timed tests, repeated practice, speed, and applying formulas as central in her being a mathematics teacher.

Mrs. Brown enjoyed mathematics because, like typing, she saw it as structured and determined, involving sequences of steps to solve the problem. She called herself an "algebra person" because she saw herself as someone who could work with formulas, "Just give me the formula and I can use it, but don't ask me to come up with it." She did not want to generate strategies, formulas, or mathematics, but felt she could apply a strategy or formula. Mrs. Brown was confident in her ability to apply procedures and formulas, and viewed this as the important aspect of mathematics to know.

Mrs. Brown's view of students and the world outside of school fit with this perspective. She elaborated her stance further in the interview, "They [students] don't care how they get there, they just want an answer. I mean that's how it is in the real world. We don't care how we get an answer, we just want an answer." This comment says more about why she is interested in formulas, speed, and answers. In the real world, she saw people as applying established strategies to get answers.

In addition to being concerned with answers, drill, and speed, she saw steps and procedures as critical mathematics to learn. She further stated her ideas about mathematics and who can do mathematics in the interview, "Some kids have a knack at looking at something and it just comes to them. They're process people or step people naturally and others are not. It just doesn't come naturally to a good majority." There is a blending in this statement of her views of mathematics (steps and procedures) as well as her belief that most students can't do math. Embedded in this statement are common notions that there are two kinds of people: math people and non-math people. Math people think in sequenced steps according to her and non-math people—a good majority of students—had difficulty thinking this way. This take on mathematics ability intertwines with her framing of mathematics and teaching typing.

Within her telling of her identity, we see a connection to her personal history as a typing teacher, a particular relationship to mathematics as organized steps, and framing of ability that allows a belief that many students can't do mathematics. We now turn to understand how Mrs. Brown's storied identity guides her participation in professional development and her classroom.

Mrs. Brown's Identity in Practice

Professional Development

Mrs. Brown turned to look at Mr. Jones' paper, saw that he was multiplying, and turned away. She then turned to Mr. Thompson and looked at his strategy.

Mrs. Brown: And once the hand is shaken you don't go back?

Mr. Thompson: The first person can't shake their own hand. So you have to start with 19, not 20

Mrs. Brown: You have to go back one each time?

Mr. Thompson: Yes.

She solved the problem using the same strategy as Mr. Thompson.

Mrs. Brown: I got it [the answer] first.

Two of Mrs. Brown's colleagues, however, already had the answer.

Mrs. Brown: Um, should we just hold our answers.

Battey: Why don't you discuss your strategy with someone else.

She turned to Ms. Spencer and Mr. Thompson and tells them how she did it, but they'd already solved it in a similar way. Ms. Spencer was trying to generalize to a formula.

Mrs. Brown: Is there a quicker way. I'm an algebra person, but don't ask me, I don't know [how I came up with it].

This excerpt from the professional setting documents Mrs. Brown's participation and highlights her quite consistent participation pattern. Within professional development Mrs. Brown produced procedures and gravitated toward symbolic forms of solving problems. In the midst of professional development challenging her to think about multiple strategies, student thinking, and the deeper mathematics underlying procedures, Mrs. Brown's participation stood in stark contrast. She often exhibited substantial knowledge of content, but wanted fast, symbolic ways to solve problems. This epitomized an algebra person to her. In addition, she did not engage the strategies other teachers generated unless they involved more speed or abstraction. She would only engage with student thinking and other ways of teaching if they could enlighten her about how to progressively step students towards these 'algebraic' strategies.

Mrs. Brown did not generate a strategy to solve the handshake problem, but applied one picked up in interacting with a colleague. After solving the problem Mrs. Brown told the group "I got it [the answer] first," denoting her consistent interest in speed. She asked the professional developer for a faster way to solve the problem three times, further demonstrating her focus on speed and answers. In actuality, two others in the workgroup solved it before her, but were working on generating a second strategy on the problem. Mrs. Brown started to find a formula, but then asked if there was a quicker way as someone else generated the general formula that would work for all numbers. She stated to another teacher as we worked on the formula that, "I'm just an algebra person," meaning she could apply formulas, but not generate them. Similarly to the interview she stated her stance another way, "Just give me the formula and I can use it, but don't ask me to come up with it. Formulas are all kids need and want to do anyway." She reiterated this by saying "teaching the formula is easier."

With respect to discussing student thinking, Mrs. Brown only engaged when it came to considering whether they had a procedure to solve the problem or not. She stated that "kids need a sign to help them know minus one" so she wanted to put a one exponent to remind students that the number of people shaking hands decreases by one each time. This is somewhat reminiscent of carrying a one when regrouping in addition or subtraction, a procedure by which children often misinterpret the meaning. Mrs. Brown also misinterpreted the strategy of "going down by one," which a fellow teacher generated, "as soon as you've shaken a hand, you're out." Mrs. Brown preferred her students to use the formula rather than solve the problem on their own.

After the workgroup generated the formula together, Mrs. Brown said, "So that's the formula? So we can just teach them that? I'll start with that… That's [the formula] so much easier." Her focus was not just on speed in mathematics, but speed in teaching. Again, we see the connection to her history as a typing teacher with time and drill being major concerns.

Classroom

After posing the problem Mrs. Brown called five students to the board.

Mrs. Brown: I want you to shake each other's hands.

The students start shaking each other's hands randomly.

Mrs. Brown: No, line up. You [*pointing to a girl*] shake all of their hands. The girl shakes the hands of the other four students.

Mrs. Brown: Now, how many hands did you shake?

Student 1: 4?

Mrs. Brown: Good. Write that on the board. Now, you (different student) shake the other's hands.

The next boy shakes the other four students' hands.

Mrs. Brown: No, you already shook her hand. Do it again.

The boy shakes three other's hands and skips the last girl.

Mrs. Brown: How many hands did you shake?

Student 2: 4?

Mrs. Brown: Wrong. You shook three. Write a 3 on the board. Next.

Although Mrs. Brown began the lesson by having student act out the problem, she directed the steps needed to model the handshakes. She continued the lesson by showing a procedure step by step and then had her students replicate those steps. She had her students plug 10 different numbers into the problem for practice. Repeating each of Mrs. Brown's steps was the only acceptable strategy for students. In reflecting on teaching the lesson, Mrs. Brown shared this,

> We did it in three steps. We really didn't just jump over to anything, formula, uh formula-wise. But I think the best thing was to make them understand every step that we did, every step of the way… They [students] don't understand formulas because of the number of operations. They get lost, and I do too. I mean how do you keep from getting lost? They have to know, they do not know how to separate the steps. (Mrs. Brown, teacher interview)

When students used other strategies or skipped a step, Mrs. Brown regularly cut them off with "Wrong," "No," "Incorrect," or "You weren't listening."

After explaining the formula to her students Mrs. Brown had her "best student" show the class her strategy step-by-step, but called out "No William" if he veered from her way. She had the class plug in a number and "do it" on individually while she walked around and told students "wrong" or "right." Most students (we observed) used correct strategies, but she told them they were doing it wrong because it was not the strategy she taught. It was not clear if she understood that students were actually using correct strategies and wanted them to do it her way or if she did not understand that they were appropriate strategies. Mrs. Brown was consistent in her focus on promoting one strategy to solve the problem, getting the steps in the right order, and producing a correct strategy.

She showed a formula for the handshake problem but used an incorrect one. The only variable in the formula was the number of handshakes, not the number of people, so it did not correctly model the problem situation. Mrs. Brown went through the incorrect formula plugging in 20 for the variable. She tried to change the equation, but had the same problem. She wanted students to develop the ability to use the formula, so she again had them plug in different numbers of people, show her strategy, and write the total number of handshakes at each party.

In Mrs. Brown's practice we see her personal history as a typing teacher as well as embedded notions of what mathematics is and the important ways to do mathematics. This was evidenced in her applying procedures to solve problems in the professional development and classroom. She focused students on steps, her steps, as a way to successfully and speedily navigate to correct answers. We now shift direction to Mr. Gray and how he framed his history with mathematics.

Mr. Gray's History with Mathematics

> For me, I learned it [the handshake problem] algebraically so I could teach it to them. I love simple ways to break things down, just in case some [students] don't get it algebraically… I was surprised, I know I shouldn't have been, but when you're not an expert in math you know, I was wondering if it was going to work out the way it was supposed to. But when I did it, it worked out. When it came out, the kids literally clapped! I don't know what the other teachers did, but my class did it 4 different ways. (Mr. Gray, teacher interview)

Mr. Gray taught 5th grade and was a first year teacher without a credential. He is an African-American teacher in his early thirties. As evidenced by the story he told about himself, Mr. Gray stated that he struggled to understand mathematical concepts, but his lack of understanding served as a source of motivation because he wanted to insure his students had more access to mathematics than he did. Mr. Gray had very little confidence in his mathematics knowledge. However, in the interview, he said that he worked at mathematics and students could too, but they had to be willing to work hard; learning math was about hard work and sticking with it.

He also stated that, "For me, I learned it algebraically so I could teach it to them." Algebra was major a gatekeeper for him in pursuing mathematics. He was currently taking an algebra class at a local community college so he could understand and help his students understand the underlying algebraic reasoning within the elementary curriculum. For Mr. Gray, understanding mathematics algebraically meant understanding abstract mathematics. In his own schooling, abstract mathematics held very little meaning and he didn't want his own students to experience this. He felt the more he understood the concepts underlying the mathematics, the more he could get it across to more students and this motivated him to engage in further mathematics learning.

He would develop multiple strategies for any problem that he was going to pose to his students prior to instruction so that he could understand both the mathematics as well as any difficulties that they might encounter. Multiple strategies were a very important part of mathematics and instruction for Mr. Gray, "You should know at least two ways to solve any problem." He also commented students could come up with a lot of different ways to solve problems and that he learned different ways to see the mathematics and solve problems from them. In addition to learning mathematics from his students, getting them to believe that they could do the mathematics and giving them respect were very important to him. He always called his students 'Doctor' as he discussed in the interview because he "Wanted to build his students' confidence in math."

Mr. Gray's identity focused on struggling to understand important mathematics. Although he acknowledged having little confidence in his mathematics background, he used this to further his own mathematics learning. This also shaped his priorities for student learning. He wanted students to believe they could engage with mathematics, to struggle with abstract algebra, and to be able to solve problems multiple ways. Again, the story of lacking confidence but being willing to struggle with learning more mathematics was central in Mr. Gray's professional practice.

Mr. Gray's Identity in Practice

Professional Development

Mr. Gray did not say one word to any of the other teachers during the entire workgroup meeting. He instead chose a seat in the back corner, as far away from his teachers as he could get. His only comment was to the first author.

Mr. Gray: This is way too hard, I can't teach this.

Mr. Gray said virtually nothing in professional development. He kept to himself and solved problems and although he wanted to talk about mathematical concepts and student thinking, he was worried that others would see his lack of mathematics knowledge. However, he would take notes on different strategies and ideas shared by colleagues in the workgroup. As evidenced by the brief excerpt above, his participation in this particular workgroup looked much the same.

Mr. Gray solved the problem, but did not engage the mathematics or his colleagues further during the workgroup. It took him about 25 minutes to solve the handshake problem and by that time the workgroup had moved on. Mr. Gray struggled to understand the handshake problem conceptually. He did not say a word to any of the other teachers the entire time but did ask me for help in solving the problem and said "This is way too hard, I can't teach this." At the end of the professional development, he admitted to me that he did not understand the formula and I told him that we could work on the problem more together. He did not solve the problem a second way, discuss student thinking, or pedagogy in the professional development. Mr. Gray told me after the professional development that he did not say anything in the workgroup because he was worried that the other teachers would know he was weak mathematically. His limited content knowledge and lack of confidence restrained his participation with his colleagues and the mathematics.

Mr. Gray told us in the interview that he was very nervous about teaching the handshake problem. He was scared because it was a complicated problem and was worried about whether the lesson would work out the way he wanted it to. Because of this, he solved the problem multiple ways on his own. Not surprisingly, his classroom centered on sharing different strategies.

Classroom

Mr. Gray: Does anyone have a strategy they would like to share with the class?

A number of students raise their hands and Mr. Gray chose one boy in particular. The boy goes to the board and used the standard algorithm for multiplication.

Student 1: I multiplied 19 times 20 and got 380.

Mr. Gray: Thank you, doctor. How about (Student 2)?

This boy wrote the numbers from 1 to 20 on the board with a line under each one. He then made 19 tally marks in bundles under *1*, 18 under *2*, and so on. The boy added up the tallies by fives and then added the ones. He got 190.

Student 2: 5, 10, 15, 20… 150. 151, 152, 153, 154… 190. I counted all of the fives and then the ones.

Mr. Gray: Does everyone understand his strategy? (*No one raises their hand or says anything*) My class is so quiet today. How about you, doctor.

He calls on another boy who wrote the numbers 19 to 1 vertically on the board. He added up 19 and 18 first, then 17 and 16 and so on.

Student 3: I added up every two numbers, like 19 and 18 and got 190.

Mr. Gray: Does everyone understand the problem now?

One girl raises her hand.

Student 4: I don't.

Mr. Gray: That's okay because math is hard work. You have to stick with it.

In his classroom, students could choose any strategy they wanted. Six different students presented their thinking. All but one student explained their strategy. The student with a procedural solution was also incorrect, but Mr. Gray responded with "Thank you, doctor." Mr. Gray did not ask any questions to develop the boy's strategy nor did he ask him to solve it another way. He did not question any of the students who shared and only posed questions to the class like, "Does everyone understand now?" After one student said no, he modeled it with 20 students in the front of the room and the whole class counted the number of handshakes to 190. Mr. Gray did not ignore the fact that a student did not understand, but instead of questioning the student about the strategy she used or how she understood the problem, he engaged the class in solving it a different way. He wanted his students to know that you could solve math problems in many ways and that any way was as valuable as the next.

Although there was extensive focus on student strategies, Mr. Gray did not guide students to look at similarities or differences, or to uncover the deeper mathematics in the problem. The idea of mathematical sophistication and the important mathematics to learn were not central to his classroom. Also, although he felt it was important to solve problems multiple ways, he did not ask any students to solve it another way. First, Mr. Gray did not extend the lesson towards more conceptual mathematics. He did not develop the mathematics any more deeply than the students did in their explanations. Therefore, the sophistication of mathematical thought went only as far as that which students shared. Second, without questioning students for conceptual explanations, the classroom allowed a procedural explanation to be satisfactory. His classroom was dominated by numerous solution strategies and limited explanations without mathematical connection.

Interestingly, while his lack of understanding limited his engagement in professional development, his stance on struggling with mathematics was a resource to draw on in engaging students in more substantial mathematics than he engaged in during the workgroup.

Summary

Mrs. Brown engaged in her classroom and the professional development in ways that supported the stories she told about herself as a mathematics teacher. She saw mathematics as straightforward, a set of steps to be taught, as in typing. She engaged in mathematics this way and she engaged her students the same way. Supporting her students to learn mathematics meant making sure they knew the steps. Engaging with her colleagues around becoming teachers that listened to their students' algebraic thinking was about getting to the most efficient ways to solve the problems. Given these relations, it is not surprising to see how she takes up the professional development and makes use of the ideas in her classroom. She takes on algebra as a set of steps to teach. However, she does begin to draw on some of the practices of the workgroup, like acting out the problem, though she breaks this instructional practice into a procedure. For us, this marks a shift in her participation and an opening for becoming a different kind of teacher within this community; one who listens to students' algebraic thinking.

Mr. Gray on the other hand, had a difficult history with the content of mathematics and saw himself as lacking the content knowledge he needed. This relation to the mathematics shaped his participation with his colleagues in the workgroup. He listened closely to his colleagues, tried the math without sharing, but said little to nothing to his colleagues. He was a peripheral participant. However, he did see himself as someone who could help his students. Because of his history he saw the value in multiple strategies. He even made sure he could produce them before teaching. He worked on developing respectful relationships with his students and saw them as capable of success with abstract algebra. His limited content knowledge is seen within the classroom, but we also see a different piece of Mr. Gray's history in action—increasing mathematics access and confidence for students.

Discussion

Given this, how might we provide opportunities for Mrs. Brown's and Mr. Gray's change? Rather than conceding that Mrs. Brown has a different stance than the professional development or won't learn, we ask the question: can we design professional development in a way that provides different possible identities for her? In a classroom that looks so different from the kind we were trying to develop, we might not expect that much would change across the first nine months we worked together. At the beginning of the year, Mrs. Brown showed students the strategy for solving each problem and students recorded answers on worksheets. In our final workgroup lesson, she had students record their thinking in a journal and carry out her strategy on the board. While Mrs. Brown's classroom remained focused on teacher directed procedures and steps she took on fundamentally new practices. She had students act out the problem and used the routines of journaling and student sharing to make sure students understood and followed her procedure. However, the introduction of these new routines and artifacts might open up opportunities for further change in Mrs. Brown's teacher identity.

In the case of Mr. Gray, his classroom looked very different at the beginning of the year than it did in the episode shared here. He followed the lesson in the book verbatim, even in moments when it didn't make sense to him. He was more comfortable doing this, trusting the curriculum to provide the content. His lack of confidence and lack of knowledge led him to practice the teaching of mathematics in a fundamentally scripted, teacher-directed way. Now, Mr. Gray's classroom is centered on the practice of student generation and sharing of strategies. He studies the content and solves problems prior to instruction to understand the mathematics and to understand the student thinking that will be involved. In fact, he took the algebra course at the community college because of the professional development. He saw his mathematics understanding as central for his students' access and sought out further learning. Seen this way, his classroom routines and his identity have already substantially changed. In some ways, he has the kind of identity we're looking for in reform mathematics, but needs to further develop the knowledge and skill to support his stance.

We can see these identities in isolation, but they were actually in the same professional development workgroup. If we are creating a community of practice that opens up new identities for these teachers, the importance of drawing on different expertise to further one another's learning is central. The knowledge that Mr. Gray is developing about student thinking might prove valuable for Mrs. Brown's intellectual development. We could use Mrs. Brown's journals in the workgroup in order to explore student thinking and see if students recorded other viable strategies. This could challenge her notions of natural math ability students as "process" people and show that students who aren't tied to steps have legitimate mathematical thinking as well as challenging relations to learning to type. Similarly, although Mrs. Brown is primarily procedurally in mathematics, she has a lot more mathematical knowledge than that. Pushing her to share that knowledge of mathematical ideas would shift her workgroup participation to something critical in the community as well as be vital to Mr. Gray developing a deeper understanding of mathematics. Providing opportunities for both teachers to participate in different ways in the workgroup would open new opportunities for them to become different kinds of teachers.

In this way, both could open up the possibility of being a different kind of mathematics teacher for each other by shifting the norms of participation in the workgroup. It is the differences and diversity of teachers in the workgroup that pushes the possibilities of transformation and the development of common goals allows for a community to emerge. Developing a new identity is not just about gathering new ideas; it is also about developing new frameworks for understanding those ideas and reinterpreting past experiences.

Implications

We present these teachers and unpack their identities not to critique either one of them, but to raise issues with how professional development is taken up by different teachers and what this means for us in designing learning opportunities. Both teachers brought identities as teachers of mathematics to professional development and classroom practice that carried ways of being that did not fit well with the work we were doing together. Teachers' identities carry personal histories, emotion, values, and knowledge and they shape how teachers participate in professional development and their classrooms. While we could see shifts over time in participation, the shifts were small and slow in coming. Understanding these shifts in relation to our work and the teachers' identities has shown in detail why "implementing ideas" from professional development is difficult.

Drawing again on the work of Wenger (1998) can help illuminate teacher learning in terms of what they learn in professional development that supports their classroom work. Wenger's perspective is quite consistent with the findings here. He argues that the norms of the setting, or the rules of participation, guide the parameters of participation and identity development. In the case of this professional development, norms were established in a negotiation between the teachers and the professional developer. The workgroup stressed an inquiry type of participation rather than sharing information and activities. Certain teachers participated along these norms, some remained on the periphery, and some were still learning to engage with these ideas. But, at the beginning of the year, teachers' norms of participation looked quite different because they were accustomed to a different way of participating in professional development.

In the classroom, teachers negotiated norms with their students. Teachers and students brought histories, identities, and past participation in mathematics with them as they negotiated what it meant to do mathematics in the classroom context. Norms are continually negotiated and reestablished as the year continues. It takes a different identity, supported by significant knowledge and skills, to reestablish existing classroom norms to better match what was learned in professional development. It is not simply a matter of taking a problem from the professional development and teaching it to the class as worked on in the professional development-especially if existing classroom norms are quite different from those that build on students' mathematical thinking.

Many professional development efforts that attempt to reform the classroom require students to explain their thinking, generate strategies, and respond to questioning from the teacher. In a classroom where norms include repeating procedures or doing worksheets, the professional development practices probably will not work well at first. Students may not have developed knowledge and skills related to talking mathematically and teachers might not know how to question students or facilitate the sharing of strategies. These classrooms already have ways of thinking, doing, and talking math, supporting the development of certain possible identities that may be out of line or at odds with implementing new practices. When teachers return to their classrooms with new ideas and perspectives on engaging in mathematics, they interact with an established context, containing established norms, co-constructed with a group of students. Renegotiation is challenging work as we see in the practices of the teachers shared here. Teachers are taking practices from one context and dropping them into a different social setting that can have very different meanings about engaging in content.

Reform efforts sometimes take for granted the conditions that need to be in place for the new practices to work. Practices recommended by reforms often require more of a teacher in terms of facilitating classroom discussions and guiding students to learn with understanding. These reforms require teachers to have a different relationship to practice and content or new identities. It should not surprise us then when so many reform efforts lead to superficial changes, co-opting practices to fit preexisting structures, or no change at all. If part of teachers work is reconstructing the participation structure or norms within their classrooms, we need to rethink what we are doing in professional development to help teachers incorporate new practices. Professional developers must start to find ways to help teachers reestablish classroom norms that align with their developing identities on what it means to teach and learn mathematics.

The data presented here and the theory that supports it argues that professional development needs to be reconceived. We must work to provide opportunities for teachers to work together to become particular kinds of mathematics teachers in ways that allow teachers to make sense of their knowledge, skills, and identities in relation to norms in both professional development and classroom practice.

Notes

[1] This study was part of a larger study exploring professional development on algebraic thinking (see Jacobs et al., 2007).

[2] For details on the remaining teachers and the larger study see Battey, D. (2004).

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