Building informal inference with TinkerPlots in a measurement context

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Several issues surround the continuing implementation of the Chance and Data component of the mathematics curriculum in Australia. First is its survival. Second is the question of how far toward formal inference should the curriculum take students (assuming it survives). Third is what contexts are amenable for understanding the concepts in the curriculum. Fourth is what tools are available to save time and assist in the learning process. One of the ways of ensuring survival is to convince decision makers that Chance and Data can be taught and learned successfully.

As is probably true of other parts of the mathematics curriculum, there is sometimes a tendency in Chance and Data to focus on small components, without spending time to fit them into the overall picture of handling data to answer questions and draw conclusions. Playing games with dice might be fun but how does the activity lead to answering meaningful questions? Finding the mean of a set of numbers might be good practice in addition and division but what does it convey about the set of numbers and how can it be useful in answering a question? Drawing a graph might create an attractive, colourful picture but what story is told about a data set, its variation and its clusters of values? Statistics is about telling stories and answering questions based on various types of data. For statisticians the questions involve collecting samples from populations and drawing inferences about the latter from the former, usually based on random selection. For school students statistics is likely to be more what Tukey (1977) called exploratory data analysis, perhaps answering questions limited to their own experience on a known population from which a convenience, rather than random, sample is drawn. One of the aims across the middle years of school should be to provide students a pathway for asking questions about populations within which they see themselves as members. This pathway is signposted with the techniques, such as finding middles, drawing representations, and describing variation, which assist in telling stories and answering questions. Associated with these techniques there are now software packages that will ease the computational burden and provide visual representations to make decision making more intuitive than in the past. The packages can change the focus from performing computations to interpreting and explaining. Experiencing this process is part of informal inference, which will lay the foundation for formal inference in later years.

This article uses a familiar setting to explore the issues associated with developing ideas of informal inference and introduces the software package, TinkerPlots (Konold & Miller, 2005), as a tool to facilitate this development. Those wishing to follow up on information about TinkerPlots can download a trial version at www.keypress.com. An evaluation of TinkerPlots as an educational data analysis package is provided by Fitzallen (2007). The value of one representation provided in TinkerPlots, the hat plot, is explored in detail by Watson, Fitzallen, Wilson, and Creed (in press).

The activities suggested in this article are intended for use with middle and secondary students (grades 6 to 10). It is acknowledged, however, that teachers in a school might need to work together to gain an appreciation of the expected development of understanding and plan for the background and level of the students they teach. The data and suggestions presented here have arisen mainly from workshops with inservice middle school teachers and preservice primary teachers, and hence may provide models for similar sessions, as well as for activities in the classroom. Examples of student work from grade 7 are also included.

The context chosen for the investigations is body measurement. Activities based on measuring hand span, foot length, arm span, and height have been described by others (e.g., Clarke, 1996; Lovitt & Clarke, 1992) and the famous drawing by Leonardo da Vinci of the Vitruvian Man is often used as a motivation for asking a question about arm span equalling height. The recent Australian Bureau of Statistics CensusAtSchool survey asked students for measurements of the height of their belly button from the floor, the length of their right foot, and their total height; these measurements hence provide an excellent data base from which random samples can be collected ("2006 CensusAtSchool Questionnaire", 2006).

When planning a unit of work that aims to develop ideas associated with informal inference, the starting point and questions need to be considered carefully. Some mathematics educators, for example, would suggest beginning with the da Vinci drawing and asking a question about the population at large: Do you think it is true for all the people in the world that their arm span lengths are equal to their heights? Discussion would evolve into how this question could be answered, with suggestions about appropriate kinds of data collection. Most students will be interested in checking themselves and collecting data from their classmates. Most high school teachers would assume that collecting these data will lead to the production of a scatterplot with arm span measured on one axis and height on the other. Jumping straight into this type of investigation may be appropriate for students with some previous experience in data handling and graphing (perhaps in grade 9 and above) but for younger students it seems more appropriate to begin with a less complex scenario in terms of the data handling expectations. Thus, even though the question that begins with a population is quite easy to understand, the techniques required to provide an answer may be relatively sophisticated.

A less demanding approach for students may be to start with a measurement activity, asking how accurately the arm span of a particular member of the class (or the teacher) can be measured (Konold & Pollatsek, 2002; Shaughnessy, 2006). In this way, questions of accuracy and variation can be introduced: What does it mean to make an accurate measurement? What variation can we expect in a measurement? Why is accuracy important? These questions can in fact be considered in a narrow classroom context, such as suggested here or expanded to consider wider social or scientific contexts. From this initial investigation, students can be encouraged to think about what a typical arm span measurement for a particular age or grade level might be, before comparing the arm span measurements of two groups, such as boys and girls. Finally, students can investigate the association of two variables, arm span and height, and draw conclusions from this. The questions discussed in the previous paragraph about a larger population can now be explored with more confidence. The following few investigations provide examples of how investigations might proceed.

Investigation 1: Measuring accurately

Four questions similar to the following can be used to begin an investigation of accuracy in measurement.

1. What does it mean to make an accurate measurement?
2. What variation can be expected if a measurement is repeated?
3. Why is accuracy important?
4. How confident can we be that we have the "true" measurement?

Although these questions appear to be about measuring, not statistics, statistics can be used to help answer them. The following steps in the investigation provide starting points for teachers to adapt for their classes.

Setting the question

How accurately can the arm span of a person be measured? What method should be used? What would be a reasonable estimate?

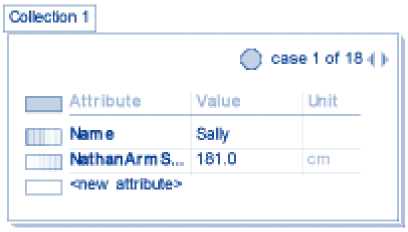
Discussion of various methods of measuring is likely to be a good place to begin to answer the question. Why might more than one measurement be needed? All students in the class can contribute by suggesting how they would make the measurement. Would students expect all measurements to be the same? Issues might include whether a person would stand or lie on the floor, what instruments would be used to make the measurements, and what accuracy of measurement should be recorded.

Data collection (interval data)

Each person measures the arm span of a single selected person (say, with arms spread out, to nearest 0.5 cm). Discussion can focus on how many measurements would be needed for a good estimate of the actual value.

Table 1. Example of how the data can be recorded.

No.	Measurer's Name	Arm Span Length	No.	Measurer's Name	Arm Span Length
1			11		
2			12		
3			13		
4			14		
5			15		
6			16		
7			17		
8			18		
9			19		
10			20		



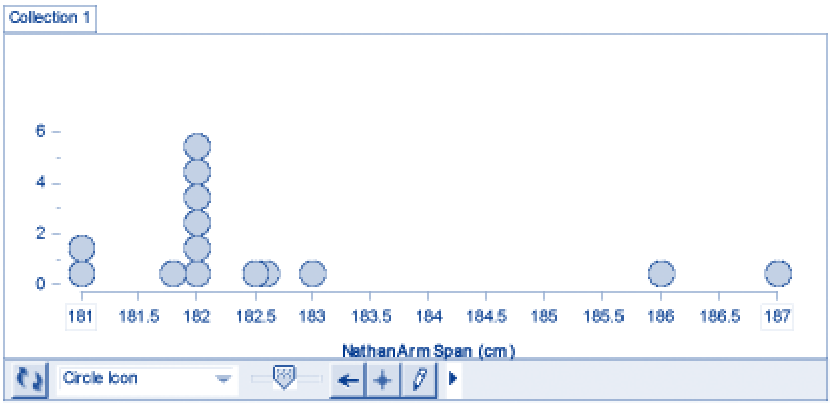


Figure 1. Example of a TinkerPlots data card and a stacked dot plot showing the arm span length of a single person measured by 14 different people.

Representing data

The collected data can be listed and ordered in a table similar to the one in Table 1 and initial discussion based on values observed in the table. What is the largest value measured? What is the smallest value? Are any values repeated? Students can enter the data on TinkerPlots data cards and create a graphical representation for the measurements. Figure 1 displays an example of a stacked dot plot.

Summarising data

Using the tools available in TinkerPlots, students can mark the mean, median, mode, and range on the line plot. Any interesting features can then be discussed. Are any of the averages the same? Are there any outliers? Can they be explained? In Figure 1 for example, the values of 186 cm and 187 cm were measured by one person with a shorter ruler than the other people used and by another person who measured "over Nathan's body" rather than flat on the floor under him.

Constructing a hat plot in TinkerPlots is often helpful in summarising a data set. A default hat plot covers the middle 50% of the data values under its crown and the bottom and top 25% under its brims. Does the hat plot help describe the spread of the data? Figure 2 shows a hat plot for the plot in Figure 1 with the data remaining visible. How does the graph help answer the question about how accurately the arm span can be measured? What is the best estimate of the selected person's arm span length from the data collected? Looking at the crown of the hat should help narrow the value of the estimated arm span without forcing the choice of a single value, such as the mean or median.

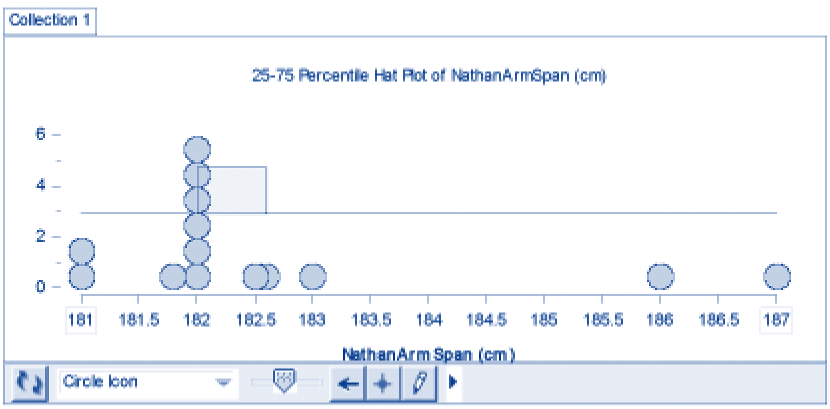


Figure 2. Example of a Hat Plot showing the spread of information when measuring arm span length.

Chance and sampling questions

How could a better estimate of the selected person's arm span be obtained? Discussion could focus for example on selecting another sample of measures, collecting a larger sample of measurements, or using a more consistent measuring instrument or technique.

Ways of randomly choosing a sample of measurements for this problem could be discussed. Would the same data set, mean, median or mode be obtained each time? How chance selection of a sample from a much larger set of measurements might affect the mean, or other values, could be an interesting topic of discussion.

Drawing a conclusion

Students should finally write a summary report, including all of the assumptions made, to explain how accurately the group measured the arm span of a single person and what the best estimate is. Decisions about the potential outliers and their inclusion in or exclusion from the analysis need to be included in the report. Suggestions for further investigation are also valuable to include. This report can be written in a text box in TinkerPlots to include with the plots created or the plots can be copied and pasted into Word documents. An "informal" inference reached should include a "best" estimate for the person's arm span, perhaps expressed as a range and with some statement about the degree of confidence with which the estimate is made.

Investigation 2: Measuring arm spans of a group

A natural progression from Investigation 1 is to consider the typical arm span of a class of students, or of students of a certain age. Students should have a feel for the accuracy of their individual measurements (and may want to take several measurements and average them in some way). The following steps suggest a possible pathway.

Setting the question

What is the typical arm span measurement of grade X students?

Data collection (interval data)

Students should discuss how their particular class can contribute to answering the rather general question. After discussing and deciding on a method of measurement, the next issue is how many measurements would be needed for a good estimate of the typical arm span length for the class. All students then have their arm spans measured (say, with arms spread out, to nearest 0.5 cm).

Representing data

The next step is to create a table similar to Table 1 for students to record their data (or add to the previous data set, perhaps as "My_armspan"). This information can be entered into TinkerPlots by each student and representations created for the class data. Students should be given freedom to create their own preferred graphical form. Again there may be occasion to discuss outliers if there are some very unusual measurements recorded.

Summarising data

Students should be asked to fill in a text box in TinkerPlots summarising what their representation tells them about the typical arm span of their class. This may involve discussion of the mean, median, mode, or range as found on their graphs. They may use the hat plot to discuss the shape of the data and the variation in the data. Figure 3 shows an example. If Investigation 1 has preceded Investigation 2, a discussion point would be the difference in the variation shown in the two stacked dot plots in Figures 2 and 3. Why would the second be expected to show more variation?

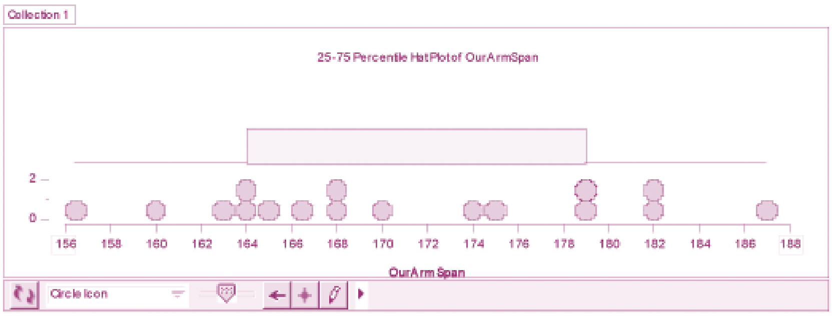


Figure 3. Example of a Hat Plot showing the shape of and variation in the data when measuring the arm span lengths of a group of people.

Chance

Students should discuss ways of randomly choosing a sample for answering this question. Perhaps there are other grade X classes in the school that could be measured. How would chance and a different sample affect the mean, the median, the variation, and the shape of the hat plot?

Drawing a conclusion

Students should then write a report, complete with graphs, including all of the assumptions made, to explain how the class arrived at its estimate of a typical arm span length for grade X students and to indicate its degree of confidence in the estimate.

Investigation 3: Comparing measurements on two groups

A natural extension of Investigation 2 is to ask a question that compares two groups, perhaps boys and girls, or students in different grades. Questions to consider might be: Do boys have greater arm spans than girls? Do students across the middle years have increasing arm spans with higher grades? To make formal inferences about these questions for a state or country would require random samples and advanced techniques but much can be learned about the processes in the informal inference arena.

The data collection and representation tasks would be similar to Investigations 1 and 2. As an example, Figure 4 shows a portion of a TinkerPlots table with data from 58 students in grades 5 to 8 with gender also included. Two interesting comparisons are possible from this data set. Figure 5 shows the stacked dot plots for the boys and girls in the middle years, whereas Figure 6 shows the stacked dot plots for the grades.

Collection 1

	Name	Grade	Armspan	Gender
1	Aden	5	136.0	M
2	Tom	5	151.0	M
3	Emily	6	145.0	F
4	Casie	6	145.5	F
5	Taylor	5	153.5	M
6	Dillon	8	170.5	M
7	Dylan	5	140.0	M
8	Matthew	6	160.0	M
9	Cheyenne	5	151.5	F
10	Sharn	6	152.5	F
11	Tai	8	159.0	M
12	Nick	7	177.0	M

Figure 4. An example of a TinkerPlots table showing data from students in grades 5 to 8.

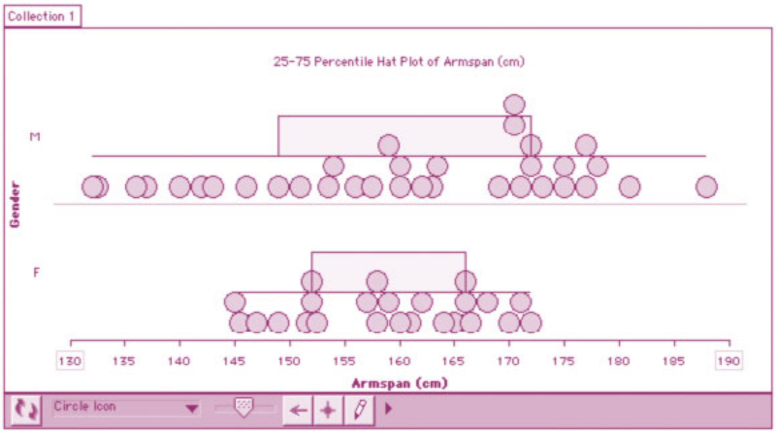


Figure 5. Stacked dot plot of arm span length by gender.

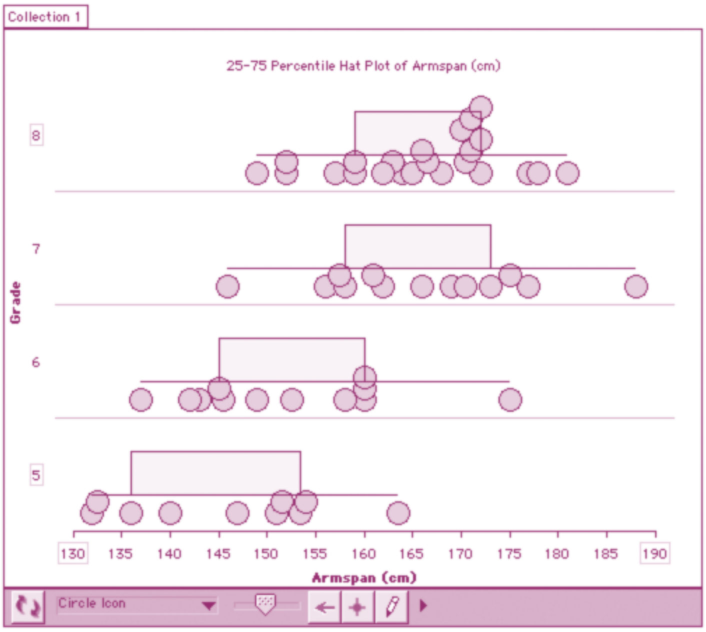


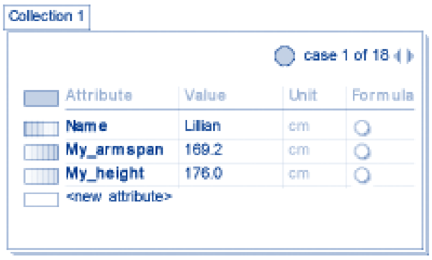
Figure 6. Stacked dot plot of arm span length by grade.

For this data set some very interesting observations about differences in variation as well as typical arm span can be made. The students in this school, for example, concluded that the variation in arm spans of boys in the middle school was greater than the variation in the arm spans of girls in the same grades. They also concluded that arm span increased from grade 5 to grade 6 and from grade 6 to grade 7, but then levelled off, probably related to growth spurts up to grade 7. Including hat plots in the graphical representations further enhances the discussion of "middles" and spread. For this school as the population, the students could make definitive statements about the data sets, to answer the questions and speculate about causes; but for a larger population, they would have to reach informal inferences and acknowledge uncertainty.

Investigation 4: Comparing measurements on two variables

An extension to Investigation 3 is to ask a question about the association between two variables within the same group, namely arm span length and height. In this investigation, students can be introduced (or reintroduced) to da Vinci's *Vitruvian Man* and asked to consider the questions: Is there an association between people's arm spans and their heights? Are they the same or nearly the same? Is there a "cause" of the association?

Data collection can involve students measuring their heights (say, with shoes off, to nearest 0.5 cm) and their arm spans (if not measured before). These data need to be recorded, perhaps on a whiteboard or worksheet, in



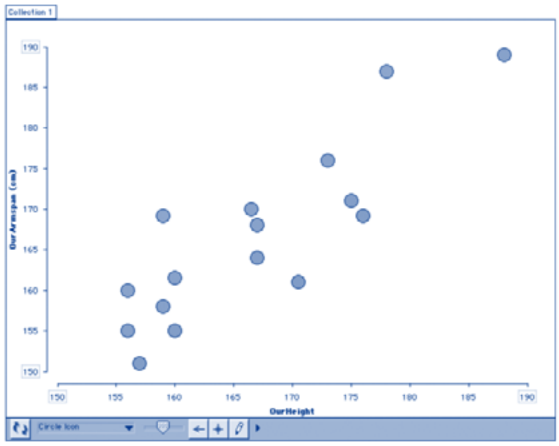


Figure 7. Data card and graph showing the association between arm span and height in a group of preservice teachers.

a manner that makes it easy for students to enter them into new data cards in TinkerPlots (or into the data cards from Investigation 1).

If they have the appropriate background, students can then produce association graphs such as the one represented in Figure 7, with height on one axis and arm span length on the other. Older students can also use a calculator and see if there is a significant correlation between the two attributes.

Summarising the data may involve finding the mean of the data on each axis and discussing any outliers. Using the drawing tool in TinkerPlots, it is possible to draw a “line of best fit” showing the association between height and arm span (an example of this is shown in Figure 8).

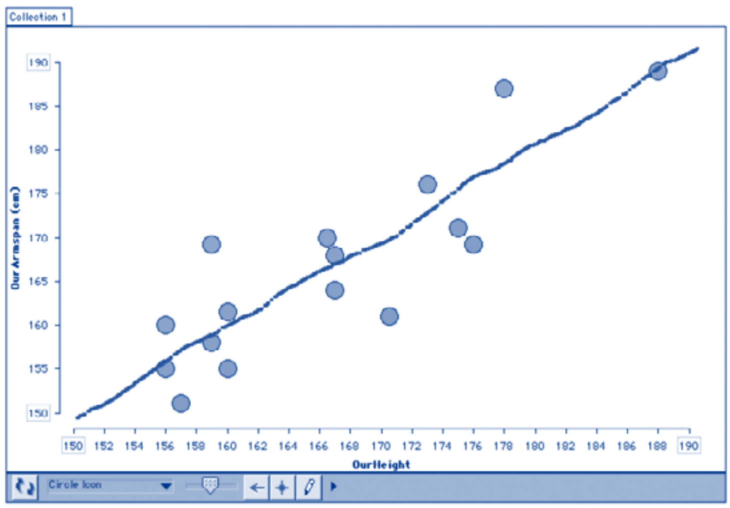


Figure 8. The “line of best fit” for arm span length and height in a group of preservice teachers.

Younger students who may not be familiar with scattergraphs may suggest subtracting arm span length from height to see if the results are zero or close to zero. This can be easily done using a special formula in TinkerPlots that provides the difference between the two attributes. Figure 9 contains the formula box showing how this can be achieved. The difference can then be represented graphically, as in Figure 10. Of interest in Figure 10 is how many differences (My_Height –

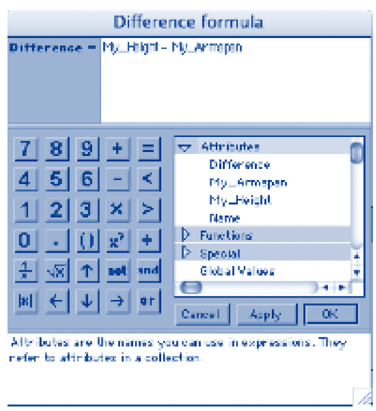


Figure 9. Data card and formula box showing how to find the difference between arm span length and height.

My_Armspan) are equal to zero, positive, or negative. What is a reasonable difference from zero that students could observe and still be able to say that the two measurements are "roughly the same"? The hat plot might be useful here. There is no definitive answer and again it might be necessary to check for (and explain) outliers.

Other students, perhaps at an age between those who would subtract and those who would draw a scattergraph, might suggest dividing one measurement by the other and seeing how close the ratios are to one. This idea is of course related to how close the points on a scattergraph lie to the straight line drawn at 45° from the origin. Figure 11 shows the formula box and what the associated graph would look like for the preservice teachers' data.

Using a TinkerPlots text box, students can write a report, setting the context for the question, answering the question, and explaining how the analysis was carried out for their inferences. They can also speculate on the "cause" of this association, being careful to use probabilistic rather than declarative language. It would be interesting in a class if different groups of students presented these three representations (or others) and their associated arguments to answer the question. Students could discuss which was the most convincing.

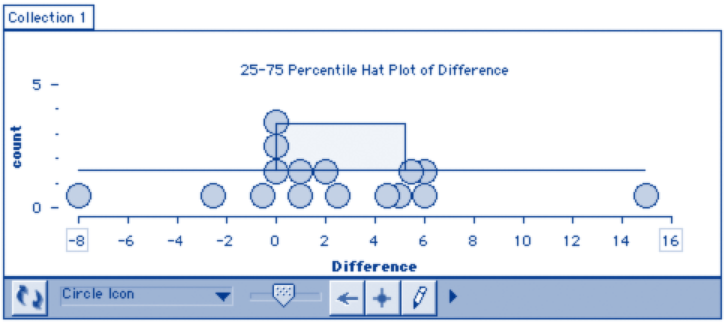


Figure 10. Graph showing the difference between arm span length and height in a group of preservice teachers.

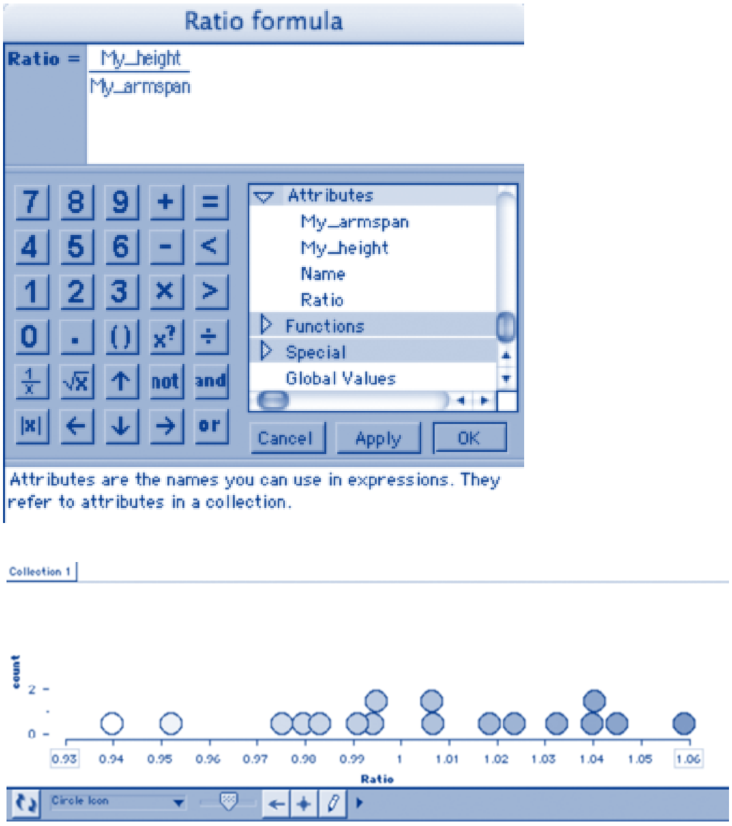


Figure 11. Formula box for the ratio of height to arm span and the associated graph.

Conclusion

The purpose of this article has been to motivate teachers to present their students with meaningful investigations that lead to an appreciation of the types of questions that informal inference can help to answer. The chance and data curriculum is about much more than finding averages and drawing graphs. All three averages found in curriculum documents — mean, median and mode — can be illustrated in these investigations. In Figure 1, for example, the median and the mode are both 182, whereas the mean ranges from 182.6 to 182 depending on whether the two potential outliers are included or not. These observations should not, however, be the only focus of Investigation 1: variation observed, reasons for it, and consequent qualified statements about accuracy are essential to a meaningful report. Although these investigations could be carried out without the use of TinkerPlots, the package can save time and add creativity and student ownership to the production of evidence and the creation of a final report answering the initial questions.

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References

2006 CensusAtSchool Questionnaire. (2006). Canberra, ACT: Australian Bureau of Statistics. Retrieved 9 October 2007, from http://www.abs.gov.au/websitedbs/CaSHome.nsf/Home/Students+Area

Clarke, D. M. (1996). *The case of the mystery bone: A unit of work on measurement for grades 5 to 8.* North Ryde, NSW: The Mathematical Association of New South Wales, Inc.

Fitzallen, N. (2007). Evaluating data analysis software: The case of TinkerPlots. *Australian Primary Mathematics Classroom, 12*(1), 23–28.

Konold, C. & Miller, C. D. (2005). *TinkerPlots: Dynamic data exploration.* Emeryville, CA: Key Curriculum Press.

Konold, C. & Pollatsek, A. (2002). Data analysis as the search for signals in noisy processes. *Journal for Research in Mathematics Education, 33*(4), 259–289.

Lovitt, C. & Clarke, D. (1992). *MCTP Activity bank volume 2.* Carlton, Vic.: Curriculum Corporation.

Shaughnessy, J. M. (2006). Student work and student thinking: An invaluable source for teaching and research. In A. Rossman & B. Chance (Eds), *Proceedings of the Seventh International Conference on Teaching Statistics: Working cooperatively in statistics education* [CD-ROM]. Voorburg, The Netherlands: International Association for Statistical Education and the International Statistical Institute.

Tukey, J. W. (1977). *Exploratory data analysis*. Reading, MA: Addison-Wesley.

Watson, J., Fitzallen, N., Wilson, K. & Creed, J. (in press). The representational value of hats. *Mathematics Teaching in the Middle School.*