"But what about the oneths?"

A Year 7 student's misconception about decimal place value

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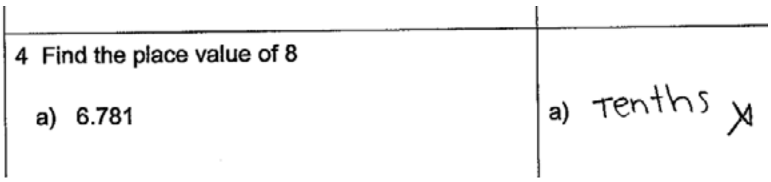

Figure 1. Trissy's response to an examination question.

Amy: Can you explain to me why you thought the place value of the 8 was tenths?

Trissy: Well, because the 7 is in the oneths column, so the 8 must be in the tenths column.

The "oneths" column?

As revealed in the above dialogue, I recently identified a misconception about decimal place value from the examination response of a Year 7 student named Trissy. Trissy and I had been going over a recent decimals examination, when I noticed that Trissy's answers concerning place value were consistently off by one place. After questioning Trissy about the question pictured above, I realised that she had been mistakenly thinking that the first place after the decimal point had the value "oneths". I proceeded to explain to Trissy the structure of decimal notation (Figure 2):

...	Thousands	Hundreds	Tens	Ones	.	Tenths	Hundredths	Thousandths	...

Figure 2. Decimal notation structure.

Still confused, Trissy responded "But what about the oneths?"

I felt it was important to get to the bottom of this misconception, so I asked Trissy to explain to me why the first place after the decimal point was "oneths."

Trissy	When you have a whole number, like 346, the 3 is the hundreds column, the 4 is in the tens column, and the 6 is in the ones column, right? So then when you have a decimal, the decimal point is like the middle, so you have the same columns on the other side but they go the opposite way and they have "-ths" on the end.
Amy:	Do you know what the "-ths" mean?
Trissy:	Yeah, like, if it was 8 hundredths, it means 8 out of 100, and if it was 8 tens, it'd be 8 out of 10, so 8 oneths is 8 out of 1.
Amy:	Trissy, can you write "8 out of 1" as a fraction for me?
Trissy:	Isn't 8 over 1 just 8?
Amy:	So is it a whole number or a part of a number?"
Trissy:	A whole number… Ohhhh, I get it. Oops!

The mirror metaphor

After further investigation, it was interesting to find that other teachers and researchers had also identified this misconception and suggested that it was related to the "symmetry" of the decimal notation (Ball & Bass, 2000; Chinn, 2008; Hiebert, Wearne & Taber, 1991; Ministry of Education, Ontario, 2006; Moskal & Magone, 2000; Resnick, Nesher, Leonard, Magone, Omanson & Peled, 1989). Ball and Bass (2000) suggest that a student's asking, "Where is the oneths place?" emanates from the reasonable expectation that if there is a ones place to the left of the decimal point, and a tens place to the left of that, there should be a symmetry to the right of the decimal point. The decimal point becomes a "mirror," reflecting the place values to the left of the decimal point on to the right side. In their study of students' understandings of decimals, Stacey, Helme and Steinle (2001) describe the confusion between decimals, fractions and negative numbers as a consequence of the so-called "mirror metaphor." However, in their discussion of the applications of this conceptual metaphor, they do not highlight the effect it has on decimal place value. It can be argued that the "oneths" misconception is a prime example of the application of the mirror as a conceptual metaphor. Interestingly, when applied correctly, the mirror metaphor actually explains the structure of decimal place value, however the important distinction to be made is that the symmetry is based around the ones column and not the decimal point.

"Oneths": A rational error

Talia Ben-Zeev (1998) offers an insight into the psychology of the errors students make in mathematics, describing many of these errors as rational: errors which are "logically consistent and rule based rather than being random" (p. 366). She suggests that when faced with an unfamiliar problem, rather than give up, people will construct their own rules or strategies in order to solve it. These strategies make sense to those who created them, however the procedure can lead to erroneous solutions (Ben-Zeev, 1998). The strategies constructed in order to assist in solving an unfamiliar problem are usually based on prior knowledge and experience. But as Resnick and colleagues (1989, p. 6) suggest, "prior knowledge of whole numbers and fractions can both support and interfere with construction of a correct concept of decimals." Trissy's application of her understanding of the place value of whole numbers to be mirrored in the place values of fractions of whole numbers is an example. The fact that decimal place value concepts are embedded in a structure that shares key features of place value with whole numbers could suggest to students that the extension of this system is identical to the existing one and lead them to ignore the differences between the two, and this could even be heightened by teachers' attempts to help students use their prior knowledge of whole numbers to facilitate learning the decimal system (Resnick et al., 1989).

Implications for practice

When considering student misconceptions, it is important for teachers to consider what can be incorporated into their lessons in order to address, or avoid, these misconceptions. For example, it might be helpful for teachers to make references to the "endless Base 10 chain" (Steinle & Stacey, 1998, p.418; see Figure 3) when explaining the structure of decimal notation to students. This is one of the important properties of the Base 10 numeration system that seems to be overlooked by students with misconceptions (Steinle & Stacey, 1998).

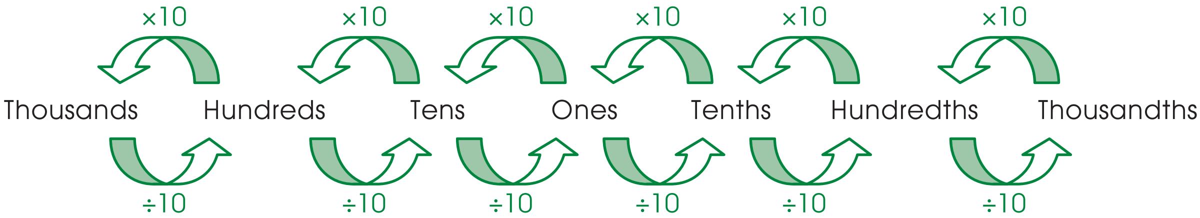

Figure 3. The endless Base 10 chain (adapted from Steinle & Stacey, 1998).

By highlighting this chain, emphasis is taken away from the decimal point and is placed on the role of, and relationships between, the individual digits within the decimal. As Chinn (2008) has explained, students tend to focus on the decimal point partly because that is how they are taught e.g., "line up the decimal points; then you can add" (p.19), and partly because it is different. Placing too much emphasis upon the decimal point could increase the likelihood of misconceptions, such as that held by Trissy, developing. Encouraging students to recognise that the symmetry of the decimal revolves around the "ones", and thus highlighting the fact that role of the decimal point is to identify which digit falls into the "ones" position, may help students overcome the desire to label incorrectly the first position after the decimal point.

The key to understanding the development of student misconceptions is to ask students to explain their thinking. Time constraints of classroom teaching make it difficult to consult with each and every individual student about their thought processes. However, when a particular error keeps surfacing, such as that revealed in Trissy's decimals examination, simply marking the response as incorrect will not assist the student. A two-minute conversation, such as that reported at the beginning of the article, can help a student's mathematical understanding.

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From Helen Prochazka's

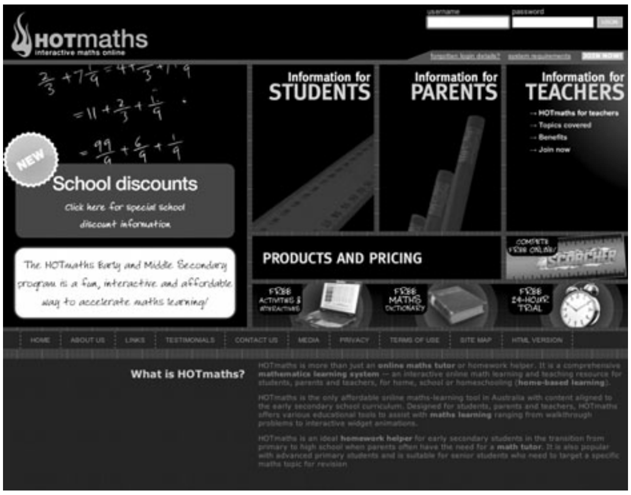
Scrapbook

The structures with which mathematics deals are more like lace, the leaves of trees and the play of the light and shadow on a human face than they are like buildings and machines, the least of their representatives.
American educator and philosopher Scott Buchanan (1895–1968)

An equation for me has no meaning unless it expresses a thought of God's.
Indian mathematician Srinivasa Ramanujan (1887–1920)

Reflections on Resources

with Donna Miller



HOTmaths

Available via the Internet at
http://www.hotmaths.com.au
Pricing varies depending on use

I am naturally suspicious of Web-based mathematics programs, particularly those that purport to be a “comprehensive mathematics learning system,” as is claimed on the Welcome page. “HOTmaths is more than just an online maths tutor or homework helper. It is a comprehensive mathematics learning system — an interactive online math learning and teaching resource for students, parents and teachers, for home, school or homeschooling (home-based learning).” However, when I logged in as a guest teacher, student and parent many of my preconceptions disappeared.

HOTmaths does indeed provide a very comprehensive program that covers the junior secondary curriculum. It is not state-specific, but then junior secondary mathematics content is very similar throughout Australia. A quick glance at the list of topics shows that it covers content in number, space, measurement, algebra, and chance and data, with at least as wide a range as in standard textbooks. Each