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Factors Affecting the Algebra I Achievement of Academically Talented Learners

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Algebra I is considered a gateway course to higher levels of study in mathematics (U.S. Department of Education, 1997; Walker & Senger, 2007). Successful completion of algebra coursework taken in middle school has been shown to lead to improved performance on mathematics proficiency tests and increased understanding of advanced mathematics, as well as higher rates of enrollment in advanced coursework in high school (Wang & Goldschmidt, 2003) and beyond (Benbow & Arjmand, 1990). Therefore, it is important to understand how different educational and individual variables may be related to algebra achievement, both to guide future study and to consider potential causal relationships that might allow educators to foster these beneficial outcomes.

Although authors often describe mathematics performance as consisting of distinct skills such as problem solving (Gavin et al., 2007), reversing processes (Wagner & Zimmermann, 1986), and adaptive reasoning (Kilpatrick, Swafford, & Findell, 2001), relatively few researchers have considered these skills directly in researching mathematics achievement. More commonly, rather than addressing specific skills in isolation, such research

summary

Understanding student performance in Algebra I is important because this course serves as the gateway to advanced coursework in mathematics and science through the remainder of high school and into postsecondary education. In the current study, we analyzed secondary data to evaluate the relationship between selected indicators of mathematics and the Algebra I performance of academically able and gifted learners who participated in above-level talent search testing. We used structural equation modeling to examine the relationship among selected variables and students' scores on a standardized measure of Algebra I achievement. Variables included prior mathematics ability, parental education level, whether a student was identified as gifted, participation in afterschool activities, the time spent on homework, and the amount of class time spent on discussions and lectures. Results indicate the strongest relationships were between mathematics reasoning and Algebra I achievement. Although gifted status was a strong predictor of mathematics reasoning, it was not strongly related to Algebra achievement, which supports the need for differentiated instruction for gifted learners. The amount of class time spent on discussion had a significant effect on the amount of time spent weekly on Algebra I homework. Rather than reliance on traditional lecture-based instruction, teachers should consider incorporating more classroom discussion on mathematical topics.

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implicitly relies upon a psychometric definition in which results from mathematics tests are equated with mathematics ability or achievement (Mayer, 1995). Our study is consistent with this psychometric perspective in that it operationally defines global scores from two different tests as indicators of mathematics ability and mathematics achievement, respectively.

Mathematics Ability and Mathematics Achievement

As might be expected, prior achievement in mathematics covaries strongly with current performance. Prior achievement likely depends on ability, as well as on the influence of motivation and academic self-concept or self-efficacy on performance (Gavin et al., 2007; Lubinski & Benbow, 2006; Ma, 1999; Siegle & McCoach, 2007). The relative contribution to prior achievement of the educational environment versus individual differences in ability remains unclear, but access to effective educational practices likely plays an important role in student achievement. Siegle and McCoach have recently published some promising work in this area, identifying several teaching practices that can increase student self-efficacy in mathematics; these authors cite a study by Zarch and Kadivar (2006), which found that self-efficacy judgments modify the relationship between mathematics ability and mathematics performance.

The effects of motivational differences on the mathematics performance of high-ability learners also have been investigated using large education data sets. Ma (2002) used the Longitudinal Study of American Youth (LSAY) database to examine the development of self-esteem among gifted, honors, and regular students accelerated in mathematics, and found that the self-esteem of males in the gifted group and minority learners in all three groups benefited from early acceleration. Ma (2003) used the same LSAY database to investigate the role of acceleration in the growth of students' mathematics anxiety, and found that gifted learners' mathematics anxiety did not increase upon placement in accelerated coursework. The mathematics anxiety of honors students grew at a similar rate in both accelerated and nonaccelerated settings, while regular students' mathematics anxiety grew more rapidly in accelerated than in nonaccelerated settings.

Many other researchers have studied educational interventions and outcomes for students with high abilities in mathematics (Gavin et al., 2007; Kilpatrick et al., 2001; Lubinski & Benbow, 2006; Stanley & Benbow, 1982; Subotnik & Steiner, 1994; Wagner & Zimmermann, 1986). This attention probably is due in part to the attention granted to mathematics and science preparation for the purpose of promoting economic competitiveness.

Wagner and Zimmermann (1986) described an enrichment program in mathematics for 12-year-old students in Germany. These authors defined mathematical giftedness as follows:

> Mathematical giftedness is a set of testable abilities of an individual. If he/she scores high in nearly all of these abilities, there is a high probability of successful creative work later on in the mathematical field and related areas. These abilities are defined by the mathematical parts of the SAT. (p. 246)

Wagner and Zimmerman also developed an additional test for mathematical giftedness, which drew upon the six areas of mathematics ability they identified: organizing material; recognizing patterns or rules; changing the problem to a new representation and recognizing the patterns or rules that apply to the new representation; comprehending and working within complex structures; reversing processes; and finding or constructing related problems.

Other researchers studying talent development also have focused on understanding achievement in mathematics and science (e.g., Hoffer, 1992; Subotnik & Steiner, 1994). This tradition may have originated in the extensive program of research conducted by the Study of Mathematically Precocious Youth

(SMPY), a longitudinal study founded by Julian Stanley in the early 1970s (Benbow, 1988; Benbow & Arjmand, 1990; Lubinski & Benbow, 2006; Stanley & Benbow, 1982; Stanley, Keating, & Fox, 1974) to study and promote talent development among mathematically talented youth. The SMPY studies collectively support the effectiveness of academic acceleration in mathematics. They also underscore the influence of early mathematics coursework on subsequent achievement in mathematics and science, an effect that continues into students' postsecondary and graduate education.

Extrinsic Sources Influencing Mathematics Achievement

Research suggests that a variety of other factors in addition to prior differences in achievement or ability also may impact academic achievement in mathematics. Parents' educational level is positively correlated with mathematics performance (Grigg, Donahue, & Dion, 2007; Ma, 1999). Parental education may contribute directly to mathematics achievement through parents' ability to help with homework (Cooper, Lindsay, Nye, & Greathouse, 1998) or their familiarity with the organization, norms, and structure of public education (Nokelainen, Tirri, & Meremti-Välimäki, 2004), but it also likely operates indirectly via the strong relationship between parental education level and socioeconomic status (Ma, 1999). It also appears that parental involvement in their children's schoolwork makes children more likely to take challenging mathematics courses such as Algebra I at an early point in their academic career (U.S. Department of Education, 1997).

Research also supports that students engaged in structured extracurricular activities, including nonacademically focused activities such as sports, service clubs, and art activities, show higher achievement in academic coursework (Gerber, 1996). Although the Gerber study did not specifically focus on high-ability learners, a qualitative study by Reis and Diaz (1999) found that high-achieving culturally diverse female students who were academically successful also participated in many afterschool activities.

The nature and quality of classroom instruction received during one's years in school may influence achievement over time, as well as influence one's current performance. Interactive approaches to instruction, such as class discussions, appear to be correlated positively with mathematics achievement, while less interactive approaches, such as lectures, are negatively associated with achievement (House, 2005). Ability grouping in mathematics may be most beneficial for students in high-ability groups, although the magnitude of this effect may be small (Hoffer, 1992).

Outside of class, time spent on homework also is related to performance in academic coursework in general (e.g., Cooper & Valentine, 2001), and the amount of homework completed is positively related to achievement, particularly in the upper grades (Cooper et al., 1998). Therefore, it is reasonable to assume that the more time students spend interacting with class assignments, whether inside or outside of the classroom, the better their achievement will be.

Aims of the Study

In the present study, we included measured and latent variables related to each of these areas identified in the literature as influences on mathematics achievement. We used structural equation modeling (SEM) to examine the variables in this study in order to test an overall model with multiple independent variables, rather than simply looking at individual coefficients as would be done using multiple regression (Garson, 2008; Musil, Jones, & Warner, 1998). Further, SEM allowed us to explore both structural and measurement variables simultaneously (Musil et al., 1998), as well as allowing for multiple meditational paths (Garson, 2008; Raykov & Marcoulides, 2006).

Our questions specifically address a population identified as academically/intellectually able via test scores in the areas of academic achievement or intelligence. Although best practices suggest that students should be identified in the specific area in which services would be provided, this is not done in talent searches because available above-level tests address multiple areas of academic ability (i.e., Verbal and Mathematics, in the case of the SAT; the ACT incorporates additional content areas including science). This broader approach to gifted identification also is the case in practice, as gifted education programs only rarely focus assessment efforts on a single field of ability such as mathematics. Therefore, our study design probably is consistent with how many schools approach gifted service delivery.

Study research questions included the following:

1. What is the effect of mathematical reasoning ability on Algebra I achievement?
2. What are the effects of the amounts of time spent on class lecture versus discussion and time spent on homework on Algebra I achievement?
3. What is the effect of parents' educational level on Algebra I achievement?
4. What is the effect of being identified as gifted on Algebra I achievement?

Methods

Sample and Data Sources

Data were collected from two sources. The initial data source was comprised of all seventh-grade North Carolina students participating in above-level testing through the Duke University Talent Identification Program (TIP; Matthews, 2006; Matthews & McBee, 2007; Talent Identification Program, 2007) in 1996 or 1997. Eligibility requirements for participation in TIP testing at that time required students to document full-scale or subscale scores at or above the 95th percentile on a grade-level achievement or intelligence test. Although these scores represent different constructs, achievement test scores are readily available, whereas intelligence test scores often are not, so Duke TIP and other talent search programs traditionally have accepted both types of scores as evidence of high performance (Lee, Matthews, & Olszewski-Kubilius, 2008; Matthews, 2007). The Duke TIP participants from North Carolina took the SAT (or in approximately 4% of cases, the similar ACT exam) during their seventh-grade school year. Both tests are college entrance examinations normally taken during a student's junior or senior year of high school, and thus are above-level tests when taken in seventh grade (Stanley, 1988).

Algebra I scores for students were collected through the North Carolina Education Research Data Center (NCERDC; Center for Child and Family Policy, 2007). The Algebra I exam is one of 10 standardized end-of-course exams administered to students in North Carolina public schools upon completion of core academic subject courses. The NCERDC records also included demographic information and self-reported questionnaire responses about participation in afterschool activities, parents' educational level, whether students were identified as academically/intellectually gifted by their school following eligibility requirements set forth by the State of North Carolina, the amount of class time spent on discussions and on lectures, and the amount of time spent on homework. Questionnaire items were administered at the same time as the Algebra I exam, and course-specific responses therefore refer to the Algebra I course. Each of the 10 available North Carolina end-of-course exams included a similar questionnaire specifically focused on the course in question.

Records matched initially included all students from the 1996 and 1997 TIP talent search who took Algebra I in North Carolina public schools in 1998. These 3,817 students took the North Carolina Algebra I end-of-course exam in the spring of 1998. Because very few students in the North Carolina sample had chosen to take the ACT exam, 92 records having ACT scores (2.4% of those taking the Algebra I test in 1998) were eliminated from the initial sample, leaving 3,725 records from the 1998 end-of-course testing having SAT scores from the talent search records. These students were almost evenly divided between males and females and were predominantly White (93.03%). Other ethnicities included: 3.66% Black, 2.12% Asian, and 0.35% or less of each remaining group (American Indian, Hispanic, multiracial, or other). An additional 37 students were excluded due to being identified as having a learning disability, which was not included as a population of interest in this study, further reducing the sample size to 3,688.

Talent search students tend to come from relatively affluent home backgrounds, which is also true of academically gifted populations in general. Individual-level socioeconomic data were unavailable in the present study. However, Lee et al. (2008) reported that talent search programs awarded need-based financial aid to an average of 7.01% of students participating in talent search testing, with a range of .01% to 16.9% across the six talent search programs surveyed. This tabulation of the amount of aid awarded likely underestimates the proportion of participants who would have been judged eligible, as talent search funds for financial aid typically were exhausted before all eligible individuals were served. In the Duke TIP program, eligibility for such aid was based on students' eligibility for free or reduced-price school lunch services, an indicator also frequently used in educational research as a proxy for socioeconomic status.

Evaluation of Missing Data

Individual records having missing data were compared to records having complete data using chi-square and t tests. Results indicate there were statistically significant differences between the missing and complete data sets in ethnicity (χ^2 [6] = 29.42, p < .0001, Φ = .09), gifted status (χ^2 [1] = 8.34, p = .004, Φ = -.05), and Algebra I score (t [60.2] = 2.56, p = .01, d = .42). Further examination of the frequency counts and percentages indicated that the numbers of students with missing data were lower than expected for each non-White ethnicity and for gifted status, which were statistically significant yet practically trivial differences that we judged would not unduly influence the study results. The data for Algebra I scores indicated the complete data had a higher mean score (69.09) compared to the missing data (66.15); however, as defined by North Carolina, both these scores represented "Superior Performance." To assist in determining how to proceed, the model (see discussion in Models section) was run using the gifted, not identified as gifted, and combined data samples to see if the results would be influenced. After analysis of the results, we determined to delete the missing data using listwise deletion (Allison, 2003; Garson, 2008). The final sample included 3,622 students having records with no missing data. This sample included 2 students in the seventh grade (.05%), 3,266 in eighth grade (88.58%), and 418 in the ninth grade (11.34%), with an overall mean age of 14.3 years (SD = 6 months). Seventy-nine percent of these students (n = 2,915) were identified as academically/intellectually gifted by their school.

Study Variables

Mathematics ability was measured using the mathematics score from the SAT. Benbow (1988) has suggested that when used with seventh graders, achievement tests such as the SAT may be considered to provide a measure of mathematics ability, because they include content that students have not yet been exposed to in school. This definition of mathematics ability is also consistent with the definition of mathematical giftedness expressed by Wagner and Zimmermann (1986), who used the SAT Mathematics as a measure of this construct. Therefore, we refer interchangeably to students' standardized SAT Mathematics scores from the talent search testing as mathematics ability through the remainder of this paper.

The number of self-reported afterschool activities, SAT Mathematics scores, and Algebra I end-of-course scores are continuous variables based on the total number of activities reported or the score obtained through testing. The SAT Mathematics and Algebra I end-of-course numbers are scaled scores, which have a possible range from 200 to 800 on the former and 36 to 87 on the latter measure.

Other study variables were measured at the ordinal level (see Table 1). School-reported records in the NCERDC data identified students' exceptionality status as not receiving additional educational services or as academically/intellectually gifted. Self-reported variables that were taken from the NCERDC data included measures of time spent on homework, participation in afterschool activities, amount of time in class spent on lecturing, amount of time in class spent on discussion, and the parents' educational level. Although self-reported data have been criticized due to the unknown nature of possible response biases, there is some evidence supporting the accuracy of these types of data. Cooper et al. (1998) compared students' self-reported homework assigned and time spent on homework to reports by classroom teachers and parents, respectively, and found statistically nonsignificant differences between the student and adult responses.

All Local Education Agencies (districts) in North Carolina offered gifted education services in at least one school at the time of the study, but gifted programs were not necessarily available in all schools in each district. For study purposes, giftedness was defined operationally as students' gifted status as reported in North Carolina Department of Education records maintained by the NCERDC. Because all students who qualify to participate in talent search testing would likely also meet state criteria for gifted identification, some students in the nongifted group may have attended schools where such services were unavailable. Alternatively, given the unknown magnitude of the correlation between achievement test scores and IQ performance, it is also possible that these students either were not referred, or somehow failed to qualify for academically/intellectually gifted services even when such services were available in their schools.

Descriptive statistics were examined for these variables and are summarized in Table 2. Review of the results indicates the variables have values that are consistent with a normal distribution. The correlations between variables were examined (see Table 3). SAT Mathematics is moderately correlated with Algebra I scores.

Table 1

Categorical Variables and Frequencies for Algebra I Participants

Variable	Category	Frequency	Percent
Afterschool Activities [a]	Does not participate in afterschool activities	289	7.84
	Participates in one type of afterschool activity	1,292	35.04
	Participates in two types of afterschool activities	1,105	29.97
	Participates in three types of afterschool activities	718	19.47
	Participates in four types of afterschool activities	224	6.08
	Participates in five types of afterschool activities	54	1.46
	Participates in six types of afterschool activities	5	0.14
Gifted Status	Did not finish high school	18	0.49
	High school	476	12.91
	Trade or business school	82	2.22
	Community/technical/junior college	516	14.00
	Four-year college	1,444	39.16
	Graduate school degree	1,099	29.81
Exceptionality	None	772	20.94
	Academically or intellectually gifted	2,915	79.06
Algebra I Homework	None	13	0.35
	I have homework but do not do it	42	1.14
	Less than 1 hour per week	549	14.92
	One to 1 ½ hours per week	826	22.45
	One and a half to 3 hours per week	700	19.02
	Three to 5 hours per week	867	23.56
	More than 5 hours per week	683	18.56
Class Discussions	Several times each class period	1,833	49.72
	About once each class period	665	18.04
	Teacher occasionally encourages discussion	723	19.61
	Teacher hardly ever encourages discussion	315	8.54
	Teacher never encourages class discussion	137	3.72
Class Lecture	Teacher never lectures	52	1.41
	Teacher hardly ever lectures	532	14.43
	Teacher lectures less than half of class time	970	26.31
	Teacher lectures about half of class time	997	27.04
	Teacher lectures more than half of class time	676	18.33
	Teacher lectures almost all of class time	436	11.83

[a] **Types of afterschool activities include sports, academic, art, vocational, service, and other.**

Table 2

Descriptive Statistics for Variables

Variable	Mean	*SD*	Skew	Kurtosis	Range
Afterschool Activities	1.86	1.11	0.53	-0.001	0–6
Parent Education	4.70	1.30	-1.05	0.11	1–6
Class Discussions	4.02	1.17	-0.91	-0.24	1–5
Class Lecturing	3.18	1.27	-0.09	-0.80	1–6
SAT Mathematics	450.28	63.48	0.02	0.39	210–690
Time Spent on Homework	5.04	1.39	-0.16	-1.00	1–7
Algebra I Score	69.08	6.89	-0.19	0.10	40–87

Table 1 also lists the frequencies for each response by variable. For afterschool activities, 50% of students participate in one to three afterschool activities. A majority of the students reported that their parents had attended a 4-year university or graduate school. In addition, most students reported engaging in class discussions several times each class period and reported spending less than half the class period listening to lectures. The majority of students reported spending 1 to 3 or more hours on Algebra homework each week. Half of the students in the sample scored between 400 and 490 on the SAT Mathematics subscale, with 25% scoring above 490. Finally, examination of the Algebra I scores indicates 57% of students in the combined sample scored between 64 and 74 on the state exam, which is substantially higher than the statewide mean score of 56.3 obtained on this test. Mean scores above 65 on the Algebra I exam are counted in the highest of the state's four performance levels, labeled "Superior Performance." Scores at this level indicate that students in the talent search sample consistently are achieving mastery of the Algebra I content.

Data Analysis

Assumptions for SEM include a large sample, indicator variables with multivariate normal distribution, valid specification of

Table 3

Correlation Matrix

	Afterschool Activities	Parent Education	Gifted Status	Time Spent on Homework	Class Discussion	Class Lecture	Algebra I Score	SAT Mathematics
Afterschool Activities	1.00							
Parent Education	0.14	1.00						
Gifted Status	0.14	0.13	1.00					
Time Spent on Homework	0.12	0.06	0.11	1.00				
Class Discussion	0.07	-0.05	0.009	0.07	1.00			
Class Lecture	0.03	-0.002	0.05	-0.008	0.05	1.00		
Algebra I Score	0.09	0.12	0.21	0.10	0.03	0.05	1.00	
SAT Mathematics	0.09	0.17	0.30	0.0004	-0.03	0.04	0.52	1.00

the model, and continuous indicator variables (or ordinal variables having at least five categories) without strong skew (Raykov & Marcoulides, 2006). In addition, variables may be dichotomous (i.e., identified gifted, not identified gifted) if they are exogenous. After examining the distribution of each variable and verifying that all assumptions for SEM were met, we conducted SEM using maximum likelihood estimation in SAS 9.1 to examine the effects of these variables on students' Algebra I achievement.

Models. An initial model (see Figure 1) was run using a data set with similar variables from another study (Matthews & Dodge, 2008). This sample included 673 students (38% Black, 25% Asian, 25% White, and 12% Other) with a mean grade of 8 and a mean age of 14 years. The initial model attempted to predict Algebra I achievement through a latent variable of class activities, which consisted of class discussions and lectures and was mediated by the quality time spent on class assignments (indicated by time spent on algebra homework each week). The amount of afterschool activities indicated a latent variable of extracurricular activities, which was modeled as directly affecting Algebra I achievement. In addition, Algebra I achievement also was predicted by mathematics reasoning (indicated by SAT Mathematics scores). Parents' educational levels and whether a student was identified as gifted predicted mathematics reasoning. This initial model could not reach convergence. Several attempts were made with similar models (removing and adding paths using modification procedures available in SAS); however, the result was the same.

We were able to arrive at a second model (see Figure 2) by beginning with Algebra I achievement and adding one predictor at a time. During this process, it was determined that convergence had not been achieved in the initial models due to the latent variables of class activities and extracurricular activities. When these latent variables were removed, leaving indicators of class discussions, class lectures, and afterschool activities, the model reached convergence. Because the theory underlying the original model is still represented by the final model, we determined it would be acceptable to proceed with the analysis. In this model

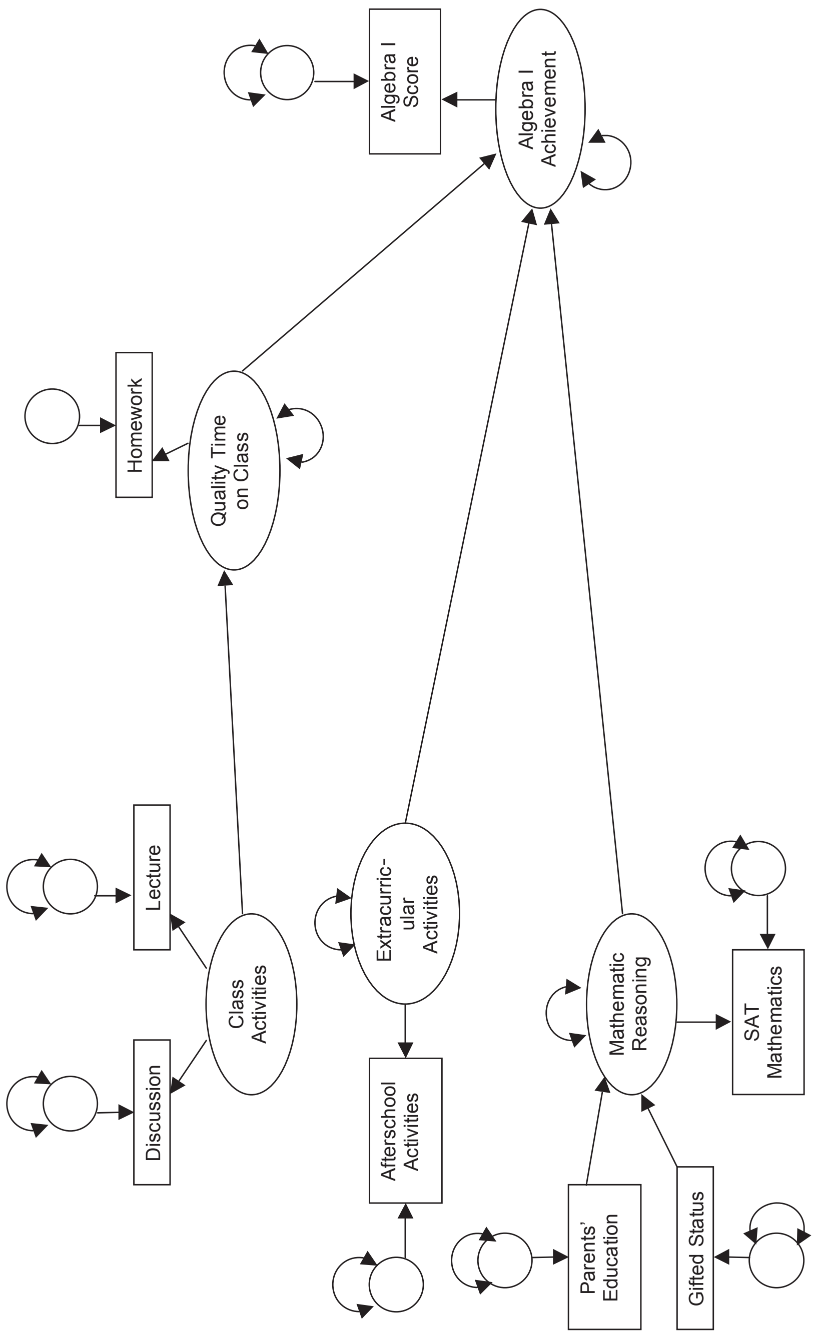

***Figure 1*. Initial model.**

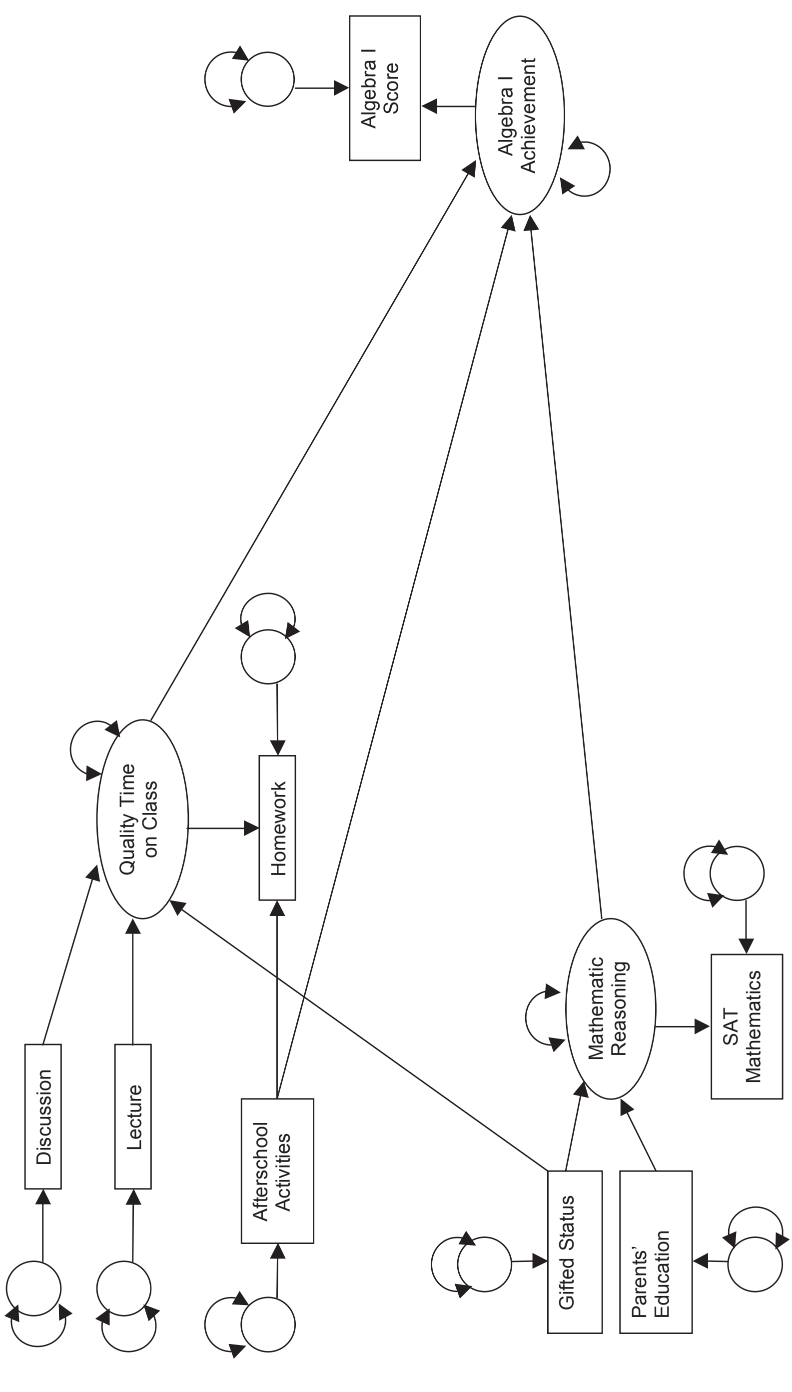

Figure 2. **Model using sample data.**

we fixed the error terms for SAT Mathematics and Algebra I score, because these are single indicators of latent variables. The values of these error terms were determined by subtracting their estimated reliability scores from one, then multiplying by the variance. Because exact reliability could not be determined for these measures within the study sample, reliabilities for both the SAT Mathematics and Algebra I scores were estimated at .85 due to the extensive research and development that goes into these exams (e.g., "Test Characteristics," 2006). The use of these reliability estimates produced estimated respective error values of 8.86 and 792.02 for the Algebra I and SAT Mathematics scores, respectively.

The model was then run using our current sample (N = 3,622). Upon both the advice of reviewers and further theoretical consideration, the latent variable of quality time spent on class was removed, leaving the measured variable of self-reported time spent on Algebra I homework. Specifically, the latent variable of quality time spent on class was suggestive of additional observed variables related to educational quality that were not included in the dataset; therefore, we decided to include in the model only the observed variable of self-reported time spent on Algebra I homework. Previous research indicated a link between being identified as gifted and academic achievement (Davis & Rimm, 2004; Schreiber, 2002), leading us to theorize an additional path between gifted status and Algebra I achievement. Using the same method as outlined above, the error values for scores on Algebra I and SAT Mathematics were fixed at 7.13 and 604.39, respectively.

Standardized covariances for the exogenous variables were reviewed. These values ranged from -0.08 (parent education and discussion) to 0.09 (discussion and afterschool activities). Omission of these covariances did not impact the model fit negatively, and consequently, we removed them from the model. Further, we examined modification indices. We determined that further addition or deletion of any of the paths did not have a positive impact on the fit of the model. Specifically, a path from parent education to Algebra I achievement was added but

Table 4

Fit Indices

Chi-square	RMSEA	Bentler's CFI
37.93, $df = 8, p < .0001$	0.03	0.98

a chi-square difference test did not indicate statistically significant results ($\Delta\chi^2 = .20$, $\Delta df = 1$), meaning the two models were approximately equivalent. In such cases, the more parsimonious (less parameterized) model is preferred.

Due to concerns regarding the statistically significant differences between the missing and complete data sets, the model also was run separately for the gifted and not identified as gifted populations to check for differences in the fit and strength of path coefficients. Results indicated that the model demonstrated similar overall fit when the full combined sample was used, versus the two analyses run separately with gifted and nongifted samples. In addition, standardized path coefficients appear to be similar among the samples. Based on these results, we decided to continue utilizing the combined sample in the model.

Fit indices. Fit indices are provided in Table 4. While the chi-square is statistically significant, that is not unusual given the large sample size ($N = 3{,}622$). A statistically significant chi-square does suggest the possibility of improper fit and, therefore, results should be interpreted cautiously. Examining additional output, the root mean square error of approximation (RMSEA) and Bentler's comparative fit index (CFI) are both within acceptable levels (Raykov & Marcoulides, 2006). Therefore, it can be concluded provisionally that these paths do appropriately model the study data.

Results

Results of standardized path coefficients are reported in Figure 3. Several paths were statistically significant (see Table

5). Overall, gifted status had a moderate effect on mathematical reasoning (path coefficient = 0.31), while parent education had a lesser effect (path coefficient = 0.14). The amount of time spent weekly on algebra homework had a small effect on Algebra I achievement, with a path coefficient of 0.09. After controlling for the other variables in the model, mathematical reasoning had a moderate effect on Algebra I achievement (path coefficient = 0.60).

The amount of class time spent on discussion and afterschool activities, as well as whether a student is identified as gifted, had small effects (path coefficients = 0.07, 0.10, and 0.09, respectively) on the amount of time spent on homework.

It is important to note the *r*-squared values for the endogenous variables. Although the measured variables (SAT Mathematics Reasoning and Algebra I score) have acceptable *r*-squared values (0.85 and 0.85, respectively), these values were set during the analysis. The *r*-squared value for mathematics reasoning (0.13) is relatively low; however, algebra achievement had a moderate *r*-squared value (0.39).

Discussion

Although the model presented shows an adequate fit to the study data, the effects of these variables on algebra achievement are relatively weak. The *r*-squared value for mathematics reasoning is relatively low, while the *r*-squared value for Algebra I achievement is moderate, which is not unexpected due to the nature of the model. There may be several possible explanations for the relatively weak magnitude of these relationships in this sample. We used single indicators for both mathematics reasoning and Algebra I achievement, and we relied upon estimated reliabilities for both due to the secondary nature of these data. Additional indicators for each latent variable may produce stronger relationships. The way the data were collected may provide another explanation. The data were collected using self-report measures with varied scales. Using additional data collection

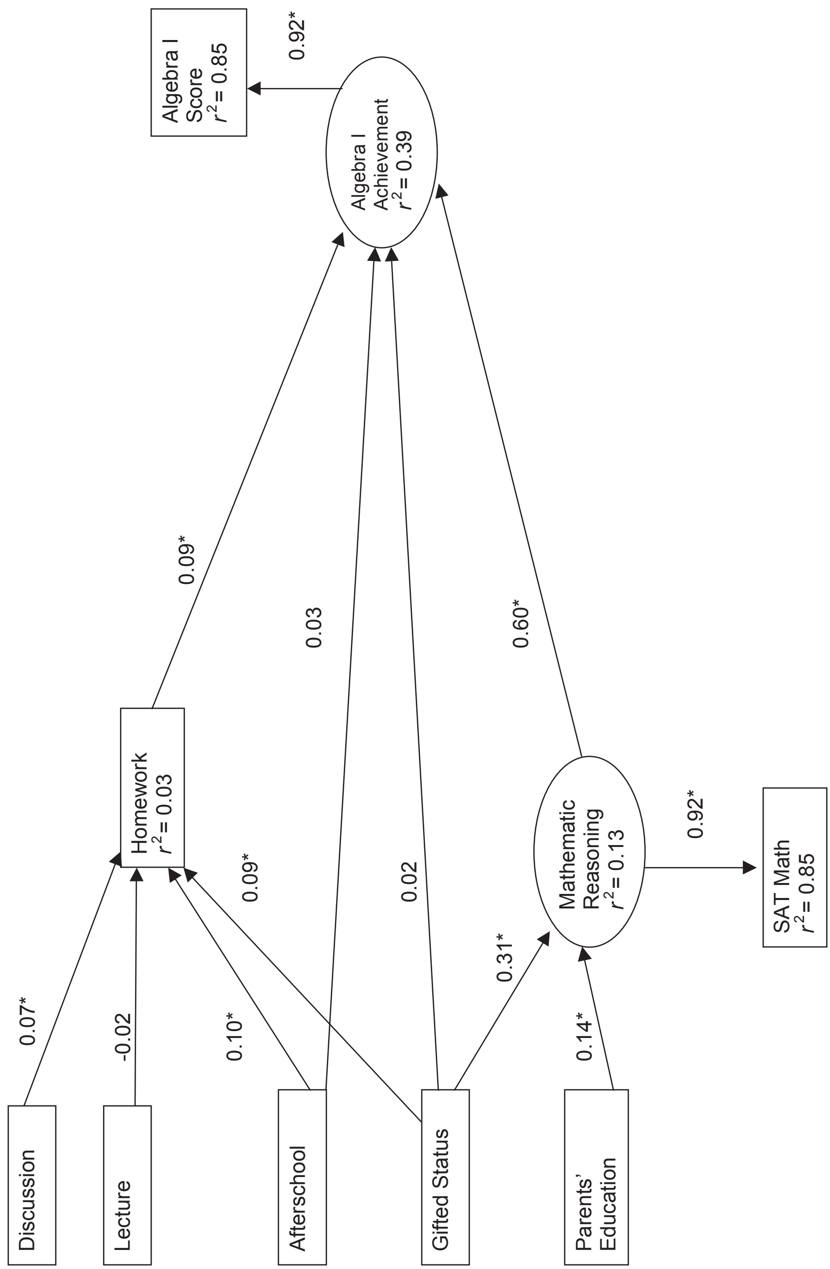

***Figure 3*. Final model with standardized path coefficients.**

* $p < .05$.

Table 5

Path Coefficient Estimates, Standard Error, and *t*-values

Path	Path Coefficient Estimate	Standard Error	*t*
Gifted Status to Time Spent on Homework	0.32	0.06	5.68
Discussion to Time Spent on Homework	0.08	0.02	4.15
Lecture to Time Spent on Homework	-0.02	0.02	-1.12
Afterschool to Time Spent on Homework	0.13	0.02	6.06
Parents' Education to Mathematic Reasoning	6.29	0.77	8.21
Gifted Status to Mathematic Reasoning	44.77	2.48	18.04
Mathematic Reasoning to Algebra I Achievement	0.07	0.002	33.75
Gifted Status to Algebra I Achievement	0.28	0.26	1.09
Time Spent on Homework to Algebra I Achievement	0.43	0.07	6.11
Afterschool to Algebra I Achievement	0.16	0.09	1.78

methods with the self-report measures and providing similar scales for responses may lead to increases in the magnitude of the observed relationships.

Research Question 1: What Is the Effect of Mathematical Reasoning Ability on Algebra I Achievement?

After controlling for the other variables in the model, mathematical reasoning ability had a moderate effect on Algebra I achievement for the students in this sample. The standardized path coefficient indicates that an increase of one standard deviation in mathematical reasoning (i.e., an increase of 63 points on the SAT-Mathematics score) would correspond to an increase of 0.60 standard deviation units in algebra achievement, or an increase of just over four points on the scaled scores reported for the Algebra I end-of-course exam. For students whose performance would be categorized as *Inconsistent Mastery* (scale scores ranging from 45 to 54) or *Consistent Mastery* (scores from 55 to 65) on the state's four-level performance scale, a four-point gain would therefore represent nearly half the span between the bottom of these levels and achieving the next higher performance category.

Research Question 2: What Are the Effects of the Amounts of Time Spent on Class Lecture Versus Discussion and Time Spent on Homework on Algebra I Achievement?

Although the amount of class time spent on discussion had a statistically significant effect on the amount of time spent weekly on Algebra I homework, the amount of time spent lecturing did not. When class discussion and lectures are mediated by the time spent on homework, there is a small statistically significant effect on Algebra I achievement. Further, the correlation matrix indicates these two variables have a low correlation, rather than being inversely correlated as might be expected. This may be due in part to the nature of permissible responses on the self-report questionnaire (i.e., class discussion had five possible answer choices dealing with the teachers' encouragement of class discussion, while class lecture allowed six answer choices dealing with the frequency of lectures). The majority of responses on each question are not inconsistent with each other, suggesting that more than half of the students responding reported spending about half to less than half of class time in lecturing, and an overlapping half of those responding reported engaging in discussion several times each class period. Although the low observed correlation is not ideal, as a secondary data analysis the researchers had no control over the development of these items. Future research should explore the effects of class time spent in lecture versus discussion on Algebra I achievement using other indicator items or even direct observation of classroom instruction.

Research Question 3: What Is the Effect of Parents' Educational Level on Algebra I Achievement?

Parents' educational level had a small effect on mathematical reasoning, which had a moderate effect on Algebra I achievement. This finding is consistent with previous research regarding the impact of parent education on academic achievement (Grigg et al., 2007). It seems likely that the restricted range of parents' educational levels in the talent search may have decreased the observed magnitude of the relationship. Further study should be conducted among populations having a more evenly distributed range of parental education to clarify this relationship.

Research Question 4: What Is the Effect of Being Identified as Gifted on Algebra I Achievement?

Interestingly, whether a student is identified as gifted has a moderate effect on mathematical reasoning, and mathematical reasoning has a moderate effect on algebra achievement, the direct effect of gifted status on algebra achievement is not statistically different from zero. This indicates that while giftedness influences performance on the SAT Mathematics, it does not impact the end-of-course measure of Algebra I performance after controlling for SAT Math scores. Rather, all of the impact of giftedness on math achievement is explained by math SAT scores. Based on the available data, it is not known if students receiving gifted services for Algebra I performed differently than those who did not. Conversely, this finding also suggests that being identified as gifted does reflect real differences in mathematical ability within this study population. Future research should explore possible influences that might explain the differential impact of being identified as gifted on mathematical ability (SAT Mathematics scores) versus mathematics achievement (algebra score results). Potential influences outside the scope of the current study included classroom environment, teaching styles, availability and types of gifted education services provided, and students' motivation. In addition, future studies should attempt to replicate the effects found in this study with other talent search populations and should examine how these relationships may vary with gender, ethnicity, and socioeconomic status.

Educational Implications

The results of this study have educational implications for Algebra I instruction. The most important educational implication is the need for differentiated instruction for gifted learners. Being identified as gifted implies higher mathematics ability in this sample of students. Further, the differences in strengths in the path coefficients among gifted status and mathematical reasoning and gifted status and Algebra I achievement indicate there are differences in achievement among gifted students. Therefore teachers should ensure that gifted learners receive appropriately advanced or differentiated instruction so that these students' abilities are reflected in their academic achievement.

Another educational implication of this study is the need for increased use of class discussion in algebra courses. Rather than reliance on traditional lecture-based instruction, teachers should incorporate more classroom discussion on mathematical topics. This increased discussion appears to support increased time spent on homework, as well as increased academic achievement in Algebra I. The researchers are careful to caution teachers that simply adding more homework to their courses may not yield increased achievement, but it may be the case that the type of homework assigned (e.g., engaging, higher level thinking activities) produces the desired result.

Limitations

The reader should be aware of several important limitations that apply to the present study. First, data for some of the variables were self-reported by students. Therefore, data regarding the amount of time spent on class discussions, lectures, and homework may have unknown biases.

Another limitation is posed by the nature of the variables. Categorical variables had to be coded to complete the analysis. This process can make it difficult to interpret the results, especially when determining the degree of relationship among variables. In addition, for latent variables having a single indicator, the error terms had to be fixed to avoid model identification problems. These fixed errors of necessity influence the remaining factors in the model, which may lead to an inaccurate estimate of the model relationships if our error estimates were inaccurate.

Finally, while the sample size was sufficient, it was limited to students who participated in a single program (the Duke TIP talent search) during either of 2 years, and who resided in just 1 of the 16 states served by this program. The unknown nature of selection biases that may influence talent search participation (Matthews, 2007) does not permit these findings to be generalized beyond students who participate in TIP, who are an estimated half of identified gifted seventh-grade students in North Carolina public schools (Matthews, 2006) but only 5% of seventh-grade students overall in this state. However, because similar educational, familial, and socioeconomic situations exist for talent search participants outside of North Carolina, it may be appropriate to generalize these results to students from other Southeastern states who self-select to participate in talent search testing through the Duke TIP program.

Finally, the relatively low *r*-squared values we observed for mathematical reasoning and Algebra I achievement indicate that additional, unexplored variables likely influence Algebra I performance. Future research should revisit this proposed model with other representative populations of highly able students and should incorporate additional variables, including direct observation when possible, and perhaps should also incorporate multilevel modeling techniques. Such research also should consider incorporating skill-level components of Algebra I achievement that might extend our understanding beyond the holistic evaluation provided by standardized testing.

References

Allison, P. D. (2003). Missing data techniques for structural equation modeling. *Journal of Abnormal Psychology, 112,* 545–557.

Benbow, C. P. (1988). Sex-related differences in precocious mathematical reasoning ability: Not illusory, not easily explained. *Behavioral and Brain Sciences, 11,* 217–232.

Benbow, C. P., & Arjmand, O. (1990). Predictors of high academic achievement in mathematics and science by mathematically talented students: A longitudinal study. *Journal of Educational Psychology, 82,* 430–441.

Center for Child and Family Policy. (2007, June). *Data housed in the North Carolina Education Research Data Center*. Retrieved August 9, 2007, from http://www.pubpol.duke.edu/centers/child/ep/nceddatacenter/data.html

Cooper, H., Lindsay, J. J., Nye, B., & Greathouse, S. (1998). Relationships among attitudes about homework, amount of homework assigned and completed, and student achievement. *Journal of Educational Psychology, 90,* 70–83.

Cooper, H. M., & Valentine, J. C. (2001). Using research to answer practical questions about homework. *Educational Psychologist, 36,*143–154.

Davis, G. A., & Rimm, S. B. (2004). *Education of the gifted and talented* (5th ed.). Boston: Pearson.

Garson, G. D. (2008). *Structural equation modeling*. Retrieved January 15, 2008, from http://www2.chass.ncsu.edu/garson/pa765/structur.htm

Gavin, M. K., Casa, T., Adelson, J. L., Carroll, S. R. Sheffield, L. J., & Spinelli, A. M. (2007). Project M3: Mentoring Mathematical Minds—A research-based curriculum for talented elementary students. *Journal of Advanced Academics, 18,* 566–585.

Gerber, S. B. (1996). Extracurricular activities and academic achievement. *Journal of Research and Development in Education, 30,* 42–50.

Grigg, W., Donahue, P., & Dion, G. (2007). *The Nation's Report Card: 12th-grade reading and mathematics 2005* (Report No. NCES 2007-468). Washington, DC: U.S. Government Printing Office.

Hoffer, T. B. (1992). Middle school ability grouping and student achievement in science and mathematics. *Educational Evaluation & Policy Analysis, 14,* 205–227.

House, J. D. (2005). Motivational qualities of instructional strategies and computer use for mathematics teaching in Japan and the United States: Results from the TIMSS 1999 Assessment. *International Journal of Instructional Media, 32,* 89–104.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academic Press.

Lee, S.-Y., Matthews, M. S., & Olszewski-Kubilius, P. (2008). A national picture of talent search and talent search educational programs. *Gifted Child Quarterly, 52,* 55–69.

Lubinski, D., & Benbow, C. P. (2006). Study of Mathematically Precocious Youth after 35 years: Uncovering antecedents for the development of math-science expertise. *Perspectives on Psychological Science, 1,* 316–345.

Ma, X. (1999). Dropping out of advanced mathematics: The effects of parental involvement. *Teachers College Record, 101,* 60–81.

Ma, X. (2002). Early acceleration of mathematics students and its effect on growth in self-esteem: A longitudinal study. *International Review of Education, 48,* 443–468.

Ma, X. (2003). Effects of early acceleration of students in mathematics on attitudes toward mathematics and mathematics anxiety. *Teachers College Record, 105,* 438–464.

Matthews, M. S. (2006). Gifted students dropping out: New evidence from a talent search population. *Roeper Review, 28,* 216–223.

Matthews, M. S. (2007). Talent search programs. In J. A. Plucker & C. M. Callahan (Eds.), *Critical issues and practices in gifted education* (pp. 641–653). Waco, TX: Prufrock Press.

Matthews, M. S., & McBee, M. T. (2007). School factors and the underachievement of gifted students in a talent search summer program. *Gifted Child Quarterly, 51,* 167–181.

Matthews, M. S., & Dodge, K. A. (2008). *Ethnic variation in the prediction of Algebra I achievement among academically able students*. Manuscript in preparation.

Mayer, R. E. (1995). Mathematical ability. In R. J. Sternberg (Ed.), *Encyclopedia of human intelligence: Vol. 2* (pp. 688–691). New York: Simon & Schuster Macmillan.

Musil, C. M., Jones, S. L., & Warner, C. D. (1998). Structural equation modeling and its relationship to multiple regression and factor analysis. *Research in Nursing and Health, 21,* 271–281.

Nokelainen, P., Tirri, K., & Meremti-Välimäki, H. (2004, April). *Self-attributions and parental influence as predictors of mathematical tal-*

ent. Paper presented at the annual conference of the American Educational Research Association, San Diego, CA.

Raykov, T., & Marcoulides, G. (2006). *A first course in structural equation modeling* (2nd ed.). Mahwah, NJ: Lawrence Erlbaum.

Reis, S. M., & Diaz, E. I. (1999). Economically disadvantaged urban female students who achieve in schools. *The Urban Review, 31,* 31–54.

Schreiber, J. B. (2002). Institutional and student factors and their influence on advanced mathematics achievements. *Journal of Educational Research, 95,* 274–286.

Siegle, D., & McCoach, B. (2007). Increasing student mathematics self-efficacy through teacher training. *Journal of Advanced Academics, 18,* 278–312.

Stanley, J. C. (1988). Some characteristics of SMPY's "700–800 on SAT-M before age 13 group": Youths who reason extremely well mathematically. *Gifted Child Quarterly, 32,* 205–209.

Stanley, J. C., & Benbow, C. P. (1982). Educating mathematically precocious youths: Twelve policy recommendations. *Educational Researcher, 11,* 4–9.

Stanley, J. C., Keating, D. P., & Fox, F. H. (1974). *Mathematical talent: Discovery, description, and development*. Baltimore: Johns Hopkins University Press.

Subotnik, R. F., & Steiner, C. L. (1994). Adult manifestations of adolescent talent in science: A longitudinal study of 1983 Westinghouse Science Talent Search winners. In R. F. Subotnik & K. D. Arnold (Eds.), *Beyond Terman: Contemporary longitudinal studies of giftedness and talent* (pp. 52–76). Norwood, NJ: Ablex.

Talent Identification Program. (2007). *Duke University Talent Identification Program*. Retrieved August 8, 2007, from http://www.tip.duke.edu

Test characteristics of the SAT Reasoning Test™: Reliability, difficulty levels, completion rates. (2006). Retrieved July 27, 2007, from http://www.collegeboard.com/prod_downloads/highered/ra/sat/sat-test-characteristics.pdf

U.S. Department of Education. (1997). *Mathematics equals opportunity* [White Paper prepared for U.S. Secretary of Education, Richard W. Riley]. Washington, DC: Author. (ERIC Document Reproduction Service No. ED415119)

Wagner, H., & Zimmermann, B. (1986). Identification and fostering of mathematically gifted students: Rationale of a pilot study. *Educational Studies in Mathematics, 17,* 243–259.

Walker, S., & Senger, E. (2007). Using technology to teach developmental African-American Algebra students. *Journal of Computers in Mathematics and Science Teaching, 26,* 217–231.

Wang, J., & Goldschmidt, P. (2003). Importance of middle school mathematics on high school students' mathematics achievement. *Journal of Educational Research, 97,* 3–19.

Zarch, M. K., & Kadivar, P. (2006). The role of mathematics self-efficacy and mathematics ability in the structural model of mathematics performance. *WSEAS Transactions on Mathematics, 6,* 713–720.

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