Students' conceptual understanding and critical thinking

A case for concept maps and vee-diagrams in mathematics problem solving

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Introduction

Student teachers' usual feedback after teaching practicum revolves around issues of what to teach, and how to teach it. While the former emphasises knowledge and skills to be developed, the latter is a pedagogical issue of teaching strategies and student activities. Of equal importance are assessment questions which typically follow such class discussions such as, "How do you assess what you teach?" and, "How would you know students are really getting it?" often asked with an air of perplexity especially when describing classroom experiences in which previous lessons were perceived to be "reasonably well-taught." Later they find, much to their disbelief, that some students are still not "getting it." Subsequent discussions eventually converge onto another equally relevant question about lesson objectives. For example, if one of the objectives is developing students' understanding of concepts, how do you assess this conceptual understanding? Certainly, there are well-established ways that experienced teachers have refined over the years (Ollerton, 2003; Zevenbergen, Dole & Wright, 2004). One example is through appropriately designed problems and investigative activities that challenge students' knowledge of the conceptual ideas (conceptual understanding) as well as test their ability to critically analyse and apply ideas in the context of problems (critical thinking).

Traditionally, to assess students' understanding of concepts (language), students express the meanings in their own words perhaps with some illustrative examples or initiate a class discussion around the concepts. Relevant terminology for topics are usually lists in syllabus documents such as those in the New South Wales Board of Studies syllabus (2002). In this paper, I present an argument for the potential use of two meta-cognitive tools, concept maps and vee-diagrams, as viable means of assessing students' conceptual understanding, fluency with the language of mathe-

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matics and critical thinking in problem solving. While students' understanding of the topic's mathematical language may be illustrated on a concept map in which nodes are concept names with linking words describing interconnections, student's higher-order problem solving skills such as critical thinking may be assessed using vee-diagrams. Supplementary to established methods (Ollerton, 2003; Zevenbergen, Dole & Wright, 2004), I propose that vee-diagrams provide a systematic guide to scaffold students' reasoning and conjecturing as they contemplate ways of solving a problem. However, before presenting examples from research conducted with mathematics students and teachers, I will describe the two meta-cognitive tools.

Concept maps and vee-diagrams

The literature refers to different types and uses of concept maps (Liyanage & Thomas, 2002; Williams, 1998; Ruiz-Primo & Shavelson, 1996), however this paper focusses on the Novak-type concept maps in which concepts are arranged in a hierarchical order of generality with respect to the main topic and including linking words (Novak, 1998; 2002; 2004; Novak & Gowin, 1984; Minztes, Wandersee & Novak, 1998; 2000). Novak defines a proposition as a statement formed by a (node – linking words – node) triad or strings of triads. By joining selected nodes, and describing links, students demonstrate their knowledge and understanding of concepts embedded hierarchically within the network of interconnecting concepts. This constructive activity provides the teacher with an idea of the state and level of students' understanding of the mathematics involved.

A vee-diagram, on the other hand, is a heuristic for analysing the knowledge structure of a problem (adapted from Gowin's epistemological vee (Novak & Gowin, 1984); Novak, 2002, 1998) in terms of its conceptual framework (left-hand side) and methodological information (right-hand side). Figure 1 shows the vee-diagram structure with its telling questions to guide the reasoning and thinking process as students analyse a mathematics problem (Afamasaga-Fuata'i, 1998; 2004). The curved arrow indicates the constant interplay between the two sides as students reflect upon given information and critically analyse the knowledge structure of the problem and relevant mathematics while simultaneously searching for

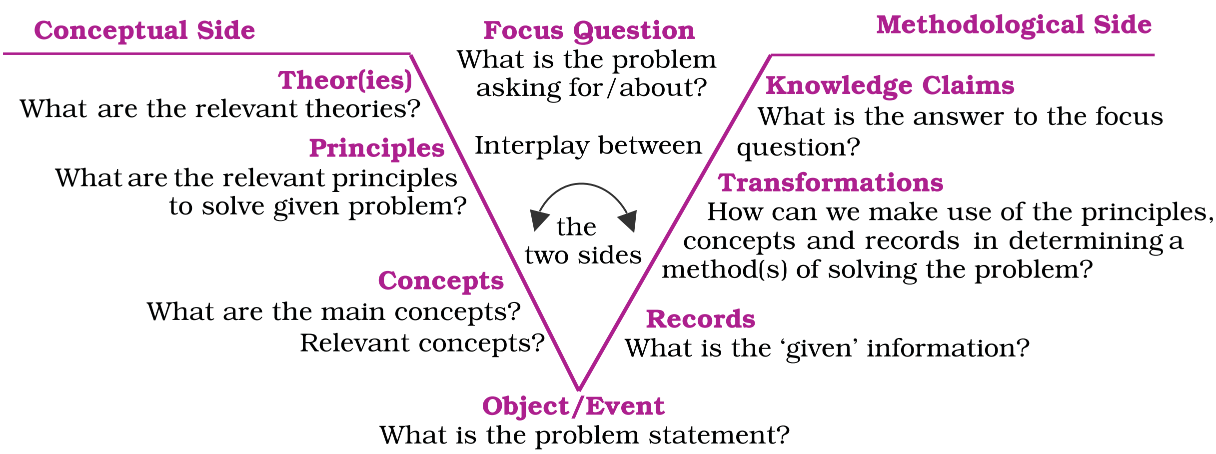


Figure 1. Problem solving vee diagram of mathematics problems (Afamasaga-Fuatai, 2004, 1998).

suitable mathematical principles that suggest methods of transforming given information to generate potential solutions. If, instead, students have already obtained a solution then the challenge is to think in the reverse direction, in identifying principles underpinning their methods. Specifically, students should be encouraged to provide conceptual justifications for their solution's main steps to overtly make connections between procedures and concepts. This is similar to asking students to explain "why" a problem is solved a particular way as, "[l]earning to ask why is discovering that there are reasons not just facts, that statements can be justified, not just asserted loudly and slowly in order to persuade through intimidation" (Mason, 2001, p. 8). For that reason, establishing classroom practices of students justifying solution steps in terms of mathematical principles and displaying the conceptual and methodological information side by side on a vee-diagram overtly focus students' attention on the dynamic interplay between concepts and procedures. Mason (2002) further argues that, "[b]y supporting learners in developing and refining their powers to think mathematically it is possible to go some way to, if not guarantee, at least make more likely that learners will construe through doing, know through construing, and know to act (to do) through knowing to use their developing powers to think mathematically (p. 6)." If sustained over time, students can begin to raise their awareness of their own powers to reason, make connections, and think analytically and mathematically (Mason, 2002). Spending sufficient time on a problem to ensure that students are not only learning about methods of solving problems but are routinely providing justifications, posing and solving challenging problems, have been identified as significant features of Japanese mathematics classrooms (Hollingsworth, 2003; Stigler & Hiebert, 1999, as cited in Anderson, 2003).

In the following sections, examples of concept maps and vee-diagrams are presented to illustrate their potential as tools to assess, monitor, teach and develop students' conceptual understanding, fluency with the language of mathematics and scaffold their critical thinking and reasoning in problem solving.

Concept map examples

Dora, a mathematics teacher who participated in the study, was asked to construct a concept map to illustrate functions. The recommended procedure is compiling an initial list of 8 to 10 relevant concepts, ranking concepts from most general to most specific and then arranging them in a meaningful hierarchy. After ranking and positioning her concepts hierarchically, interconnecting nodes and describing links, Dora produced her first concept map as redrawn in Figure 2. Choosing to place the main concept "Functions or Mappings" at Level 1, the rest were strategically placed to facilitate valid interconnections distributed over five more levels. Reading vertically from top to bottom, it is possible to identify about 22 meaningful, complete propositions as listed in Table 1. The first column (Table 1) indicates level connections. For example, Proposition 7 is formed from relevant nodes at Levels 1, 3, 4, and 5 whilst Proposition 22 is a crosslink from Level 6 back to Level 1. By inspection, Table 1 indicates that the propositions are typical statements students make when articulating their understanding of functions.

Pedagogically, a concept map such as in Figure 2 can be used as a focus for a discussion or a means of implementing the working mathematically

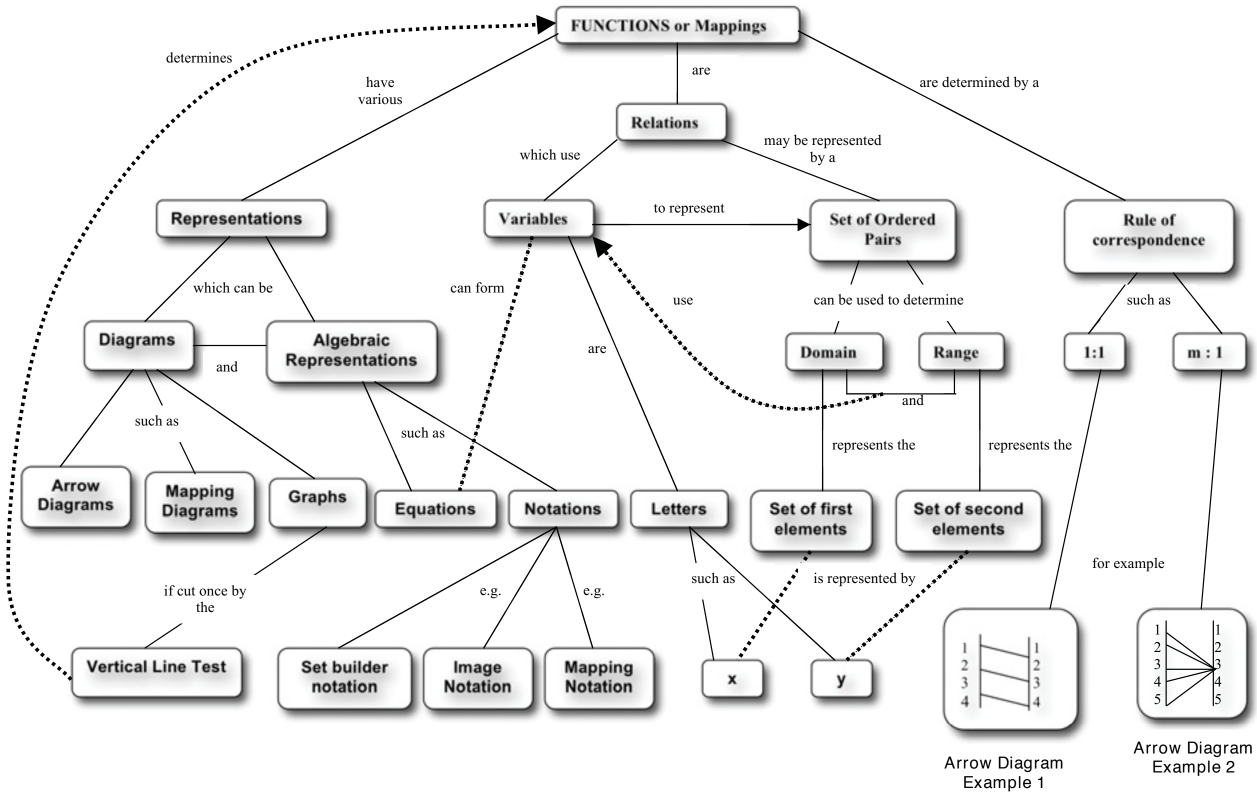


Figure 2. Dora's first concept map.

Table 1. Proposition list from the first concept map.

Level#	*Level#*	*Propositions from Dora's concept map 1*
1 → 2	1	Functions or mappings are relations.
1 → 3	2	Functions or mappings are determined by a rule of correspondence.
1 → 3	3	Functions or mappings have various representations.
3 → 4	4	Representations which can be diagrams and algebraic representations.
1 → 2 → 3	5	Functions or mappings are relations which use variables.
1 → 2 → 3 → 3	6	Functions or mappings are relations which use variables to represent set of ordered pairs.
1 → 3 → 4 → 5	7	Functions or mappings have various representations which can be diagrams such as arrow diagrams, mapping diagrams, graphs.
3→4→5→6→1	8	Representations which can be diagrams such as graphs if cut once by the vertical line test determines functions or mappings.
3 → 4 → 5	9	Representations which can be algebraic representations such as equations, notations.
3 → 4 → 5 → 6	10	Representations which can be algebraic representations such as notations; for example, set builder notation, image notation, mapping notation.
3 → 5	11	Variables can form equations.
3 → 5 → 6	12	Variables are letters such as x, y.
3 → 4 → 4	13	Set of ordered pairs can be used to determine domain and range.
4 → 5	14	Domain represents the set of first elements.
4 → 5	15	Range represents the set of second elements.
5 → 6	16	Set of first elements is represented by x.
5 → 6	17	Set of second elements is represented by y.
1 → 3 → 4	18	Functions or mappings are determined by a rule of correspondence such as 1:1, m:1.
1 → 3 → 4 → 6	19	Functions or mappings are determined by a rule of correspondence such as 1:1; for example, arrow diagram example 1.
1 → 3 → 4 → 6	20	Functions or mappings are determined by a rule of correspondence such as m:1; for example, arrow diagram example 2.
4 → 4 → 3	21	Domain and range use variables.
6 → 1	22	Vertical line test determines functions or mappings.

process strand particularly the processes of questioning, applying strategies, communicating, reasoning and reflecting (NSW, 2002). If propositions are mathematically incorrect or vague then it provides an opportunity for the teacher and student to negotiate for an acceptable re-statement of linking words and/or possible re-organisation of the hierarchy. This teacher–student interaction can also take place between students themselves if working in pairs or collaboratively in small groups. Alternatively, the teacher can design more effective tasks that specifically redress the misconceptions. Thirdly, if propositions are all correct, the teacher can re-assign it as an enrichment task to be extended as new concepts are learnt over subsequent lessons. A variation would be for the teacher to delete links and linking words and ask students to construct individualised concept maps (or work in pairs or small groups). An exploratory option is to give students the opportunity to examine their own conceptual understandings in-depth at the completion of a topic, then construct individual concept maps using their own lists (or if preferred, one given by the teacher such as those in syllabus documents). Since student-constructed concept maps indicate their level of conceptual understanding and fluency with the topic's mathematical language, they can be presented in class to initiate mathematical dialogues, communications and discussions amongst the students as they learn collaboratively from each others' work and share ideas.

Whereas Figure 2 visually depicts Dora's perception of the integrated, hierarchical, network of interconnecting nodes, these extra dimensions are not easily discernible in the linear list of Table 1. I propose in this paper that the multi-dimensional aspects of the diagrammatic concept map "offer a wider scope for multiplicity of interpretation" (Mason, 2001), and organisation particularly as the task of concept mapping demands much cognitive processing of information and reflective critical thinking whilst arranging the same concepts in a meaningful hierarchy, linking and describing interconnections. In practice, constructing a hierarchical concept map explicitly pushes students to a higher level of thinking and reflection, which are desirable skills to cultivate and develop for effective problem solving and an essential part of working and thinking mathematically. In contrast, solving a problem by simply executing a procedure or applying a formulas such as finding the derivative using the power rule without fully comprehending the meanings of underlying concepts is to miss out on an aesthetic appreciation of calculus and indicative of a procedural, limited view of derivatives.

After presenting her first attempt in class (consisting of myself and her peers), Dora continued to revise her concept map for the second and third time by adding nodes, revising some labels and including illustrative examples. Her peers also took turns in presenting their concept maps for critique. Subsequent discussions and social interactions (student–student and teacher–student) focussed on critiquing whether or not displayed interconnections and linking words were mathematically sound and correct. As an extension, all of them were asked to expand their first maps to include more relevant nodes and illustrative examples. Dora's fourth attempt is redrawn in Figure 3 with a proposition list in Table 2. A comparison of Figures 2 and 3, and Tables 1 and 2 shows an increase in meaningful propositions (from 22 to 30) with the inclusion of more nodes (from 28 to 39) and links (from 34 to 51), an extra hierarchical level (from 6 to 7) and 5 more illustrative examples and 3 graphs. The final concept map had evidently expanded, becoming more complex with more integration and differentiation between concepts resulting in more meaningful propositions. Dora had also reversed the ranking of concepts "Functions" and "Relations"

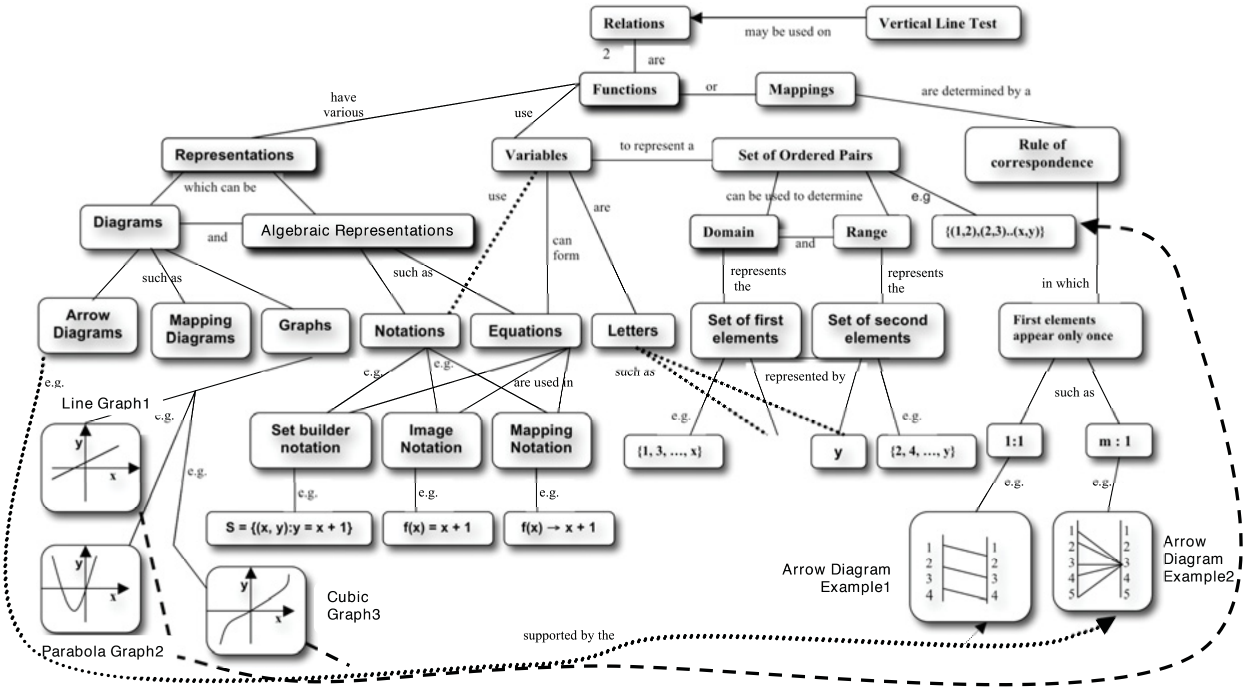


Figure 3. Dora's final concept map.

Table 2. Proposition list from the final concept map.

Level#	***Level#***	***Propositions from Dora's concept map 1***
1 → 2	1	Relations are functions.
1 → 1	2	Vertical line test may be used on relations.
2 → 2 → 3	3	Functions or mappings are determined by a rule of correspondence.
2 → 3 → 3	4	Functions use variables to represent a set of ordered pairs.
2 → 3	5	Functions have various representations.
3→ 4 → 4	6	Representations which can be diagrams and algebraic representations
2 → 3 → 4→ 5	7	Functions have various representations which can be diagrams such as arrow diagrams, mapping diagrams, graphs.
2 → 3 → 4→ 5 → 6	8	Functions have various representations which can be diagrams such as graphs; for example, linear graph1, parabola graph 2, cubic graph 3.
2 → 3 → 4→ 5 → 7	9	Functions have various representations which can be diagrams such as arrow diagrams; for example, arrow diagram example1 , arrow diagram example 2.
2 → 3 → 4→ 5	10	Functions have various representations which can be algebraic representations such as notations, equations.
5 → 6	11	Notations; for example, set builder notation, image notation, mapping notation.
5 → 6	12	Equations are used in set builder notation, image notation, mapping notation.
6 → 7	13	Set builder notation; for example, $S = \{(x, y): y = x + 1\}$.
6 → 7	14	Image notation, for example, $f(x) = x + 1$.
6 → 7	15	Mapping notation; for example, $f(x) \to x + 1$.
3 → 5 → 6	16	Variables are letters such as x, y.
3 → 5	17	Variables can form equations.
3 → 5	18	Variables use notations.
3 → 4	19	Set of ordered pairs can be used to determine domain and range.
3 → 4	20	Set of ordered pairs for example $\{(1,2), (2, 3), \ldots (x, y)\}$.
4 → 5	21	Domain represents the set of first elements.
4 → 5	22	Range represents the set of second elements.
5 → 6	23	Set of first elements is represented by x.
5 → 6	24	Set of first elements for example $\{1, 3, \ldots x\}$.
5 → 6	25	Set of second elements is represented by y.
5 → 6	26	Set of second elements for example $\{2, 4, \ldots y\}$.
3 → 5 → 6	27	Rule of correspondence in which first elements appear only once such as 1:1, m:1.
5 → 6 → 7	28	First elements appear only once such as 1:1; for example, arrow diagram example 1.
5 → 6 → 7	29	First elements appear only once such as m:1; for example, arrow diagram example 2.
6 → 1	30	Linear graph 1, parabola graph 2, cubic graph 3 supported by the vertical line test.

and included concept “First elements appear only once.” Given her background as a teacher, her final concept map attempted to capture the typical “functions” terminology at early secondary level.

Vee-diagram examples

To introduce vee-diagrams in problem solving to a Year 10 class, I used the following problem:

> Find the equation(s) of the line(s) which pass through (3,-3) and forms with the coordinate axes a triangle of area 6 square units. Find equation(s) of the line(s) in general form.

Guided by the telling questions in Figure 1, I explained how the problem statement can be used to complete the sections: Object/Event, Focus Question, Records and Concepts by asking questions such as, “What are the mathematical concepts used in stating the problem?”, “What are you asked to find?”, “What is the given information?” which is also consistent with Polya’s first principle for problem solving namely “understanding the

CONCEPTUAL SIDE

METHODOLOGICAL SIDE

FOCUS QUESTION

What are the equations of the lines passing through (3, -3) in general form?

THEORIES

Set Theory
- relations & functions
- coordinate geometry

PRINCIPLES

1. The general form of equations of a straight line is:

   Ax + By + C = 0.

2. The intercept form of equations of straight lines is:

   $$\frac{x}{a} + \frac{y}{b} = 1,$$

   where

   a = x-intercept, and
   b = y- intercept.

3. Area of triangle is: $\frac{1}{2}$(base x height).

CONCEPTS

Equation, coordinate axes, lines, point, area, triangle, base, height, square units

KNOWLEDGE CLAIMS

Lines passing through point (3, -3) and making triangles of area 6square units with the coordinate axes have equations:
x + 3y + 6 = 0 & 3x + y - 6 = 0.

TRANSFORMATIONS

Area of triangle: = base x height
= ab = 6 sq. units;

$$b = \frac{12}{a} \ldots\ldots\ldots\ldots\text{(i)}$$

Using Principle 2; equation of the line is

$$\frac{x}{a} + \frac{y}{b} = 1 \ldots\ldots\ldots\ldots\text{(ii)}$$

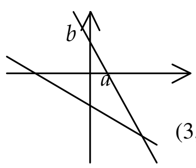
gives: $$\frac{x}{a} + \frac{ay}{12} = 1 \ldots\ldots\ldots\text{(iii).}$$

But lines pass through (3, -3), therefore substituting for the values of x and y in (iii) should give

$$\frac{3}{a} - \frac{3a}{12} = 1 \ldots\ldots\ldots\text{(iv).}$$

Rearranging and simplifying (iv) gives a = -6 and a = 2. Substituting the values of a in (i) and (ii) gives the equations of the lines:
x + 3y + 6 = 0 and 3x + y - 6 = 0.

RECORDS



(3,-3)

Area = 6 square units

EVENT/OBJECT

Find the equation(s) of the line(s) which pass through (3,-3) and forms with the coordinate axes a triangle of area 6 square units. Find equation(s) of the line(s).

Figure 4. Vee map of problem (method 1).

problem." The other three principles are: (2) devising a plan; (3) carrying out the plan; and (4) looking back (Polya, 1973). Polya's four principles provide an overview of the process of completing a vee-diagram with more specific-section questions in Figure 1. Of fundamental importance is the enculturation of students to the "thinking part of the problem solving process (which) is typically suppressed" in textbooks by "embedding conceptualisation directly in the flow of the problem solving process" (McAllister, 1994). Accordingly, the teacher should initiate a brainstorming session in which students are asked to make suggestions, conjectures and pose questions to "crack the code," interpret and analyse the problem statement for the intended meaning, relevant concepts and principles as they explore potential solutions whilst simultaneously slotting emerging information into the relevant sections of the vee-diagram. By overtly drawing students' attention to the different sections, connections between displayed conceptual and methodological information are reinforced and consolidated. Shown in Figures 4 and 5 are vee-diagrams illustrating two methods of solution. Subsequent discussions can focus on clarifying, confirming and articulating connections between sections. Extension work may include explorations for more methods and relevant underlying principles with students using vee-diagrams to record findings and in subsequent class

CONCEPTUAL SIDE | **FOCUS QUESTION** | **METHODOLOGICAL SIDE**

FOCUS QUESTION

What are the equations of the lines passing through (3,-3) in general form?

CONCEPTUAL SIDE

THEORIES

Set theory, Number theory
---relations & functions
---coordinate geometry

PRINCIPLES

1. The general form of equations of a straight line is: Ax + By + C = 0.
2. The point-point form equations of straight lines is: $\frac{(y-y_1)}{(x-x_1)} = \frac{(y_2-y_1)}{(x_2-x_1)}$ where (x_1, y_1) and (x_2, y_2) are points on the line.
3. Area of triangle is: $\frac{1}{2}$(base x height).

CONCEPTS

equations, coordinate axes, point, area, triangle, base, height, square units,

METHODOLOGICAL SIDE

KNOWLEDGE CLAIM

The same answers as in Method 1

TRANSFORMATIONS

From the diagram, a and b are the intercepts, giving the points $(a, 0)$ & $(0, b)$ where

$x_1 = a,\ y_1 = 0,\ x_2 = 0,\ y_2 = b$

$\therefore$ equation of line is:

$$\frac{(y-0)}{(x-a)} = \frac{(b-0)}{(0-a)}$$

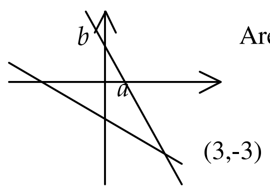
$$\frac{y}{(x-a)} = \frac{-b}{a} \quad \ldots\ldots\text{(i)}$$

Using Principle 3, $b = \frac{12}{a}$,

Substituting b in (i) gives: $\frac{y}{(x-a)} = \frac{-12}{a^2}$ ....(ii)

Substituting the values of x and y from point (3, -3) in (ii) will give the same values of a as in Method 1 above and consequently the same answers.

RECORDS



Area = 6 square units

EVENT/OBJECT

Find the equation(s) of the line(s) which pass through (3,-3) and forms with the coordinate axes a triangle of area 6 square units. Find equation(s) of the line(s).

Figure 5. Vee map of problem (method 2)

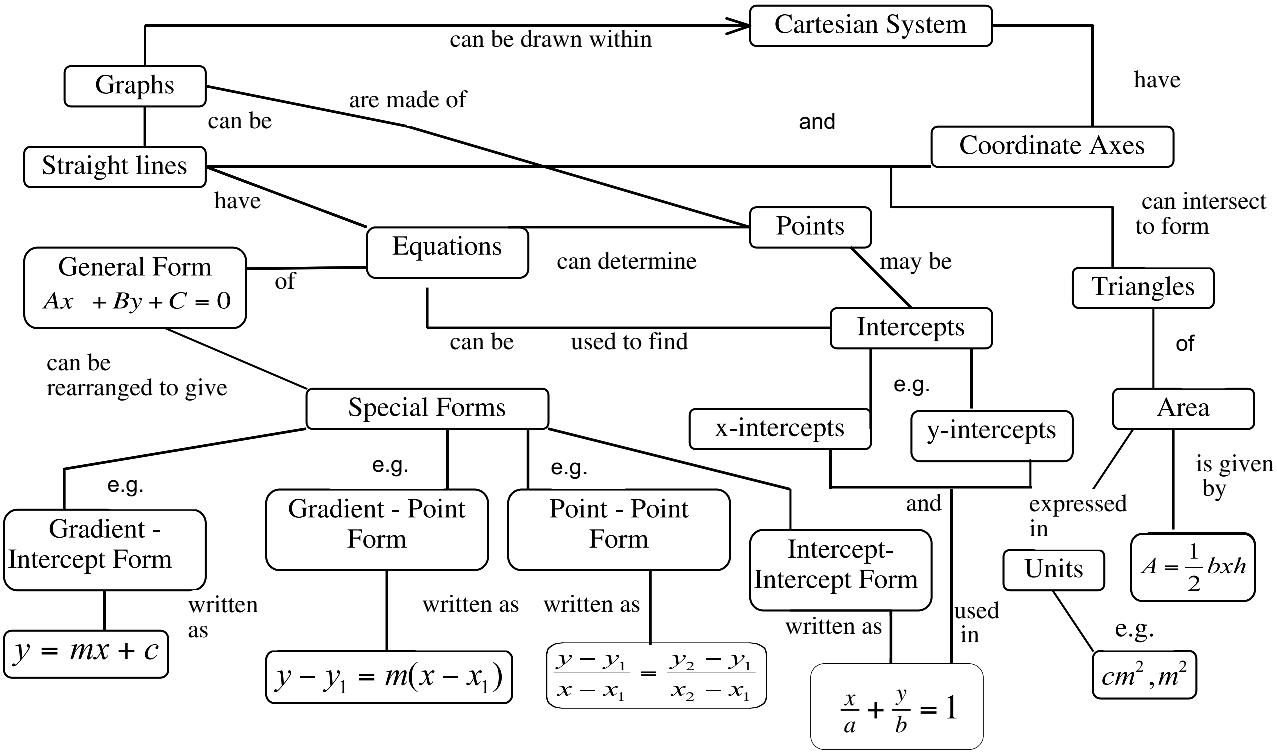


Figure 6. Concept map of the problem.

presentations, effectively communicate their ideas. Alternatively, students may be asked to identify other relevant principles missing from the "Principles" lists of Figures 4 and 5. Another option will be to ask students to pose their own problems and then construct vee-diagrams to display relevant conceptual and methodological information.

Earlier on, I presented two examples of concept maps to illustrate the knowledge structure of some concepts on "functions." Another use of concept maps in association with problems is to display the conceptual framework on the LHS of vee-diagrams to effectively communicate the connections between concepts, mathematical language, and procedures as shown in Figure 6. This problem concept map can be extended further to incorporate students' findings from their own investigative and exploratory activities as suggested above. Resulting concept maps will reveal connections between concepts and procedures that cross multiple topics.

Summary

With the current focus in mathematics education on the importance of developing students' conceptual understanding, fluency with the language of mathematics, critical thinking, and working mathematically, teachers are constantly expected to design challenging and investigative tasks that can engage and motivate students in their learning of mathematics. An integral part of creating exemplary and conducive learning environments in mathematics classrooms is for teachers to be innovative and creative in the ways they teach and assess students. In this paper I have demonstrated how the Novak-type concept maps and Gowin's vee-diagrams can be used in mathematics classrooms as learning, teaching and assessment tools as they have been found to be quite effective in many international classrooms across many disciplinary areas as evident by the number of presentations (approximately 150 papers and posters) accepted for the First International Conference on Concept Mapping held on 14–17 September 2004 in Spain, see http://www.cmc.ihmc.us.

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From Helen Prochazka's Scrapbook

I do regret leaving school so early, especially when I can't do my kid's homework. The other day my daughter was looking for the square root of the hypotenuse. Hell. I didn't even know it was lost!
Kathy Lette, author (2005)