A Comparative Analysis of Number Sense Instruction in Reform-Based and Traditional Mathematics Textbooks

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This study compared number sense instruction in three first-grade traditional mathematics textbooks and one reform-based textbook (*Everyday Mathematics* [EM]). Textbooks were evaluated with regard to their adherence to principles of effective instruction (e.g., big ideas, conspicuous instruction). The results indicated that traditional textbooks included more opportunities for number relationship tasks than did EM; in contrast, EM emphasized more real-world connections than did traditional textbooks. However, EM did better than traditional textbooks in (a) promoting relational understanding and (b) integrating spatial relationship tasks with other more complex skills. Whereas instruction was more direct and explicit and feedback was more common in traditional textbooks than it was in EM, there were differences among traditional textbooks with respect to these two criteria. Although EM excelled in scaffolding instruction by devoting more lessons to concrete and semiconcrete activities, traditional textbooks provided more opportunities for engaging in all three representations. However, EM emphasized (a) a variety of models to develop number sense concepts, (b) a concrete, or semiconcrete, to symbolic representational sequence, and (c) hands-on activities using real-world objects to enhance learner engagement. Finally, even though traditional textbooks excelled over EM in providing more opportunities to practice number sense skills, this finding may be an artifact of the worksheet format employed in traditional textbooks. At the same time, adequate distribution of review in subsequent lessons was evident in EM and in only one of the traditional textbooks. Implications for practice in accessing the general education curriculum for students with learning problems are discussed.

Assessments of mathematics achievement of students in the United States have evoked "both a sense of despair and of hope" (National Research Council [NRC], 2001, p. 55). Although recent data from the National Assessment of Educational Progress (National Center for Education Statistics [NCES], 2006) indicate small gains in mathematics achievement among public school students, achievement gaps are wide, with low levels of achievement among minority students (i.e., African American, Native American, Latino), limited English proficient students, students with disabilities, and students from low socioeconomic status. Further, the low mathematical proficiency of U.S. students when compared to students in most developed countries is well documented (Lemke et al., 2004; Mullis, Martin, Gonzalez, & Chrostowski, 2004; NCES, 2003; Schmidt, 2002).

Several factors may explain the relatively poor performance of U.S. students. For one, increasing numbers of children with diverse learning and curricular needs receive instruction in general education classrooms (McLeskey, Henry, & Axelrod, 1999; Morocco, 2001). A potential barrier to these children's access to the general curriculum is poorly designed textbooks and educational materials that fail to provide experiences to develop critical mathematical ideas (Jones, Langrall, Thornton, & Nisbet, 2002; Suter, 2000). Evidently, mathematics content organization and instruction influence student achievement (Schmidt, Jakwerth, & McKnight, 1998). Traditional mathematics curricula have been criticized for being "relatively repetitive, unfocused, and undemanding" (Hiebert, 1999, p. 11). In addition, U.S. textbooks, when compared to textbooks from other countries, seem to lack "focus and coherence" and fail to provide "meaningful connections between the big ideas of mathematics" (Valverde & Schmidt, 1997/1998, p. 63). Converging evidence suggests that "differences in the quality and quantity of mathematics instruction" account for cross-national differences in mathematics achievement (e.g., Mayer, Sims, & Tajika, 1995, p. 444).

The increasing discontent with traditional mathematics textbook instruction has led to a new way of conceptualizing the teaching and learning of mathematics. With the publication of the National Council of Teachers for Mathematics's (NCTM's) *Curriculum and Evaluation Standards for School Mathematics* in 1989 and *Principles and Standards for School*

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Mathematics (*Principles and Standards*) in 2000, the emphasis has shifted from procedural knowledge and rote driven computation to conceptual knowledge (Schoenfeld, 2002). An interesting finding was that traditional textbooks emphasizing teacher-directed instruction "account for well over 80 percent of the textbooks used in schools" (Van de Walle, 2007, p. 8). However, reform-based mathematics textbooks, designed to develop conceptual understanding by engaging students in problem-solving opportunities that emphasize reasoning and thinking using open-ended approaches, are increasingly adopted in many schools across the nation (Findell, 1991; Fraivilling, Murphy, & Fuson, 1999; Greenes, 1996; Remillard, 2005).

The increased complexities of a diverse population in the U.S. educational system, along with the less than positive school outcomes in mathematics, clearly call for the need to analyze the adequacy of information presented in textbooks. Textbooks are considered a de facto national curriculum and are the primary means of imparting new information to students (Britton, Woodward, & Binkley, 1993; Chandler & Brosnan, 1994; Garner, 1992; Mayer et al., 1995; Osborne, Jones, & Stein, 1985; Porter, 1989; Valverde & Schmidt, 1997/1998). Garner (1992) noted, "Textbooks serve as critical vehicles for knowledge acquisition in school" and can "replace teacher talk as the primary source of information" (p. 53). Therefore, examining what is taught is critical in light of recent calls for challenging learning standards and school accountability (Chatterji, 2002; Nolet & McLaughlin, 2005).

Although many educators recognize that the quality and adequacy of mathematics textbooks are important factors in promoting student learning, few content analyses have focused on the implications of instructional design for learners at risk for mathematics disabilities (Carnine, Jitendra, & Silbert, 1997; Jitendra, Carnine, & Silbert, 1996; Jitendra, Deatline-Buchman, & Sczesniak, 2005; Jitendra, Griffin, et al., 2005; Jitendra, Salmento, & Haydt, 1999). None of these studies, however, have compared instruction presented in reform-based and traditional textbooks or have focused on beginning mathematics skills such as number sense, which is a prerequisite for the development of higher-level mathematics skills (Isaacs & Carroll, 1999; Markovits & Sowder, 1994; McIntosh, Reys, & Reys, 1992; NCTM, 1989, 2000; NRC, 1989). The purpose of our study was to compare how number sense was taught in reform-based and traditional mathematics textbooks. In particular, we examined the quality of the instructional design features of mathematics programs with an emphasis on number sense instruction for learners at risk for mathematics disabilities.

Number sense is "an awareness and understanding about what numbers are, their relationships, their magnitude, the relative effect of operating on numbers, including the use of mental mathematics and estimation" (Fennell & Landis, 1994, p. 187).

It is a way of thinking about numbers in terms of their various uses and interpretations that is deemed critical to all aspects of mathematics and also is the foundation for the development of students' learning and understanding of complex problems (Case, 1998; Griffin, 2003, 2004a, 2004b; Sowder & Schappelle, 1994; Yang, 2002). Number sense develops gradually over time as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms (Thornton & Tucker, 1989; Van de Walle, 2007). Early number development is intimately related to other mathematics content (e.g., measurement, operations, data, basic facts, place value, computation). For example, knowledge of number relations is directly related to the measurement of length, height, weight, or size, which, in turn, requires knowing how to count, compare, and connect numbers to real-world objects. Also, computational fluency and flexibility with numbers can be traced back to students' knowledge of base ten and part–part–whole relationships (Griffin, 2003; Griffin & Case, 1997; Griffin, Case, & Sielger, 1994; Van de Walle, 2007). In sum, proficiency in number sense may be related to later achievement in mathematics. The present study is the first content analysis to examine the extent to which reform-based and traditional mathematics programs at the first-grade level adhered to instructional design criteria.

Method

Materials

Three first-grade traditional mathematics textbooks from three publishers, Harcourt Brace (HB; Maletsky et al., 2004), Houghton Mifflin (HM; Greenes, Leiva, & Vogeli, 2005), and Scott Foresman (SF; Charles et al., 2004), and one reform-based textbook, *Everyday Mathematics* (EM; Bell et al., 2004), were evaluated. Currently, there are three commercial standards-based elementary curricula. We selected EM only, because unlike the other two standards-based curricula (*Math Trailblazers* and *Investigations in Numbers, Data, and Space*), the organization of its content is similar to that of traditional programs (e.g., each lesson incorporates various segments such as "Teaching the Lesson" and "Ongoing Learning and Practice") and is a stand-alone mathematics program. Based on consultations with mathematics educators, teachers, and school administrators, the textbooks selected were deemed representative of mathematics textbooks typically adopted in the United States. The rationale for focusing on first grade was the importance of a strong mathematical foundation for later school success. Also, if educators are to assist students with mathematics disabilities to access the general education curriculum, promote later learning, and reduce subsequent gaps between them and their typically achieving peers, it is important to attend to math content in early grades.

The data source consisted of all lessons on number sense in the four textbooks. The number of lessons in the traditional textbooks ranged from 6 to 11 (mean = 8.67) and included a total of 8 lessons in EM.

Data Analysis Procedures

We scrutinized teacher manuals of the four textbooks, as well as all lessons within each textbook that specified number sense objectives. In each lesson, items or examples related to number sense were tallied and evaluated to determine the extent to which they adhered to principles of effective instruction. The first author rated all lessons independently. As a further check on reliability, the first author trained a doctoral student in special education to independently rate 30% of the lessons.

Coding Criteria

The criteria for evaluating number sense instruction focused on the principles of effective instruction for students at risk for mathematics failure (e.g., Kame'enui, Carnine, Dixon, Simmons, & Coyne, 2002). They included teaching big ideas, conspicuous instruction, mediated scaffolding, and judicious review. A brief description of each criterion and of our coding procedures follows.

Big Ideas

Big ideas are "networks of interrelated concepts" that facilitate and enhance the broadest attainment of skills and knowledge (Carnine, 1997; Kame'enui et al., 2002; Ritchhart, 1999; Van de Walle, 2007, p. 27). Organizing information around key concepts maximizes student learning (Carnine, 1997; Prawat, 1989). The three big ideas in number sense are (1) "counting tells how many things are in a set," (2) "numbers are related to each other through a variety of number relationships," and (3) "number concepts are intimately tied to the world around us" (Van de Walle, 2007, p. 120).

An examination of the scope and sequence, as well as the content (i.e., lesson objectives), of the four textbooks revealed that the big idea "counting tells how many things are in a set" was introduced in kindergarten and only reviewed in first grade. As such, this big idea was not evaluated in this study. To code the remaining two big ideas of number sense (i.e., number relationships and real-world connections), we examined the lesson objectives specified at the beginning of each lesson for the presence of these big ideas. Next, we identified and tallied the number of items or examples in the entire lesson that were related to these two big ideas. A description of the dimensions within each of the two big idea categories and of our coding procedures are provided in the following section.

Number Relationships. The four types of number relationships that "must be created for children to develop number sense" include *spatial relationships; one and two more, one and two less; anchors or benchmarks of 5 and 10;* and *part–part–whole relationships* (Van de Walle, 2007, pp. 124–125).

Spatial relationships. This requires recognizing sets of objects arranged in different configurations and specifying "how many without counting" (Van de Walle, 2007, p. 125). Items were tallied when they involved instantly recognizing a pattern (e.g., ) for a number quantity (3) less than 10 without direct counting.

One and two more, and one and two less. This refers to knowledge about how one number is related to another. For example, 8 is 1 more than 7 or 2 less than 10. Activities that emphasize the basic relations (i.e., more, less, same) between numbers involve more than just the ability to count. Instead, the focus is on understanding that all numbers are related to one another in a variety of ways (Van de Walle, 2007). Items related to the concepts of more, less, or same were tallied for this relationship.

Anchors or benchmarks of 5 and 10. Relating any given number to a criterion or benchmark (5 or 10) is known to enhance students' acquisition of mathematical concepts (Sowder & Schappelle, 1994). The benchmark number 10, for example, plays an important role in the numeration system, and because two 5s make up 10, both 5 and 10 are useful anchors to consider when developing relationships between numbers less than 10. This relationship not only allows students to think about various combinations of numbers (e.g., 7 is "5 and 2 more" and "3 away from 10"), but helps in the development of mental computation of larger numbers (Van de Walle, 2007). Items that related a given number to these two anchors were tallied for this relationship.

Part–part–whole relationships. The interpretation of numbers in terms of *part–part–whole relationships* is the most important conceptual achievement of early school mathematical understanding (Fischer, 1990; Resnik, 1989). This relationship requires understanding that a number is made up of two or more parts. For example, 5 can be thought of as a set of 3 and a set of 2 or a set of 1 and a set of 4. Any items that focused on a number (or number parts) and involved manipulating, reading, or writing the number parts (or whole) as a means to reflect on the *part–part–whole relationship* were tallied.

Real-World Connections. Understanding the relationship of numbers to real-world quantities and measures helps students develop flexible and intuitive ideas about numbers that are most desired (Van de Walle, 2007). Making personal connections with real situations allows students to attach meaning to numbers and make sense of the world in a mathematical manner (Sowder & Schappelle, 1994; Van de Walle, 2007). For example, an activity that requires estimating the length of a desk followed by actually measuring the desk would serve to connect numbers to a real situation. For this criterion, we tallied all items that involved personalized contexts to quantify real-world connections.

Conspicuous Instruction

"Conspicuous instruction is direct and explicit" (Santoro, Coyne, & Simmons, 2006, p. 125). Number sense instruction should be explicit and not left to natural development or incidental learning (Gersten & Chard, 1999). A firm understanding of concepts and skills is facilitated by consistent, logical, and complete explanations (Leinhardt, 1989; Leinhardt & Putnam, 1987). For this criterion, the "Teach" or "Teaching the Lesson" section of each number sense lesson was reviewed, and the coding procedure focused on teacher directions or instruction to explicate the process. When instruction was present, it was coded as either *conspicuous* if it included teacher modeling (e.g., demonstrating, thinking aloud) and/or explanations prior to students applying the skills independently or *not conspicuous* if it did not provide an explicit teacher model (e.g., posing questions only) or required students to make inferences.

Mediated Scaffolding

Instruction must be not only conspicuous but adequately scaffolded to support learners with intensive needs (Carnine, 1997). Scaffolding is defined as the provision of temporary supports and can be teacher mediated, materials mediated, or task mediated (Kame'enui et al., 2001; Smith et al., 1998). Teacher-mediated scaffolds include high-quality instructional feedback as learners apply newly acquired skills and strategies (Santoro et al., 2006). Task or content scaffolds control task difficulty by introducing "concepts and skills systematically in increasing levels of difficulty," carefully selecting and sequencing examples to reinforce learned material, and providing a range of examples (Santoro et al., 2006, p. 125; Swanson & Hoskyn, 1998; Vaughn, Gersten, & Chard, 2000). Material scaffolds may include graphic organizers, procedural facilitators, visual prompts, and representations (Vaughn et al., 2000). For the purpose of this study, we focused on scaffolds related to the type of feedback or representations (e.g., concrete, semiconcrete, and symbolic) provided in the textbooks.

Feedback. Providing immediate and explanatory feedback to correct student errors is a key element in an instructional system and is known to enhance student performance (Brosvic, Dihoff, Epstein, & Cook, 2006; Lhyle, & Kulhavy, 1987; Moreno & Mayer, 2005). All lessons were analyzed for the presence of feedback in sections titled, "Check," "Common Error Alert," "Common Error," or "Ongoing Assessment." When present, feedback was coded as either instructive (i.e., specified explicit error correction procedures) or not instructive (i.e., alerted the teacher to a potential error, but did not specify any correction strategies).

Representations. Scaffolds that "support learners as they internalize skills and strategies" include representations that promote learning of critical concepts (Larkin, 2001; Santoro et al., 2006, p. 125; van Garderen, 2006). Researchers who have explored the use of multiple representations (concrete, visual, verbal) in mathematics have noted that they are useful in developing rich understandings of new and difficult concepts (Arcavi, 2003; Brenner et al., 1997; Flevares & Perry, 2001; Lesh, Post, & Behr, 1987; McCarty, 1998; McCoy, Baker, & Little, 1996; Sowder & Schappelle, 1994). For this criterion, number development items in the entire lesson that incorporated concrete (e.g., manipulatives such as blocks, counters), semiconcrete (e.g., drawings, pictures), and symbolic representations (e.g., 10 is 8 and 2) were tallied. It must be noted that when a number task required the use of more than one representation, each was counted separately.

Judicious Review

Review must be sufficient and provide students with ongoing opportunities to apply previously learned knowledge (e.g., a concept) until they are able to demonstrate mastery of the concept or skill (Chard & Kameenui, 1995; King-Sears, 2001). A critical feature of judicious review is scheduling consistent and systematic practice opportunities to acquire newly introduced skills (Carnine, 1997; Dempster, 1991; Kame'enui et al., 2002). At the same time, review should be distributed over time, because spaced review promotes long-term retention and automaticity (Ambridge, Theakston, Lieven, & Tomasello, 2006; Carnine, 1997; Carnine, Dixon, & Silbert, 1998; Dempster & Farris, 1990; Seabrook, Brown, & Solity, 2005).

For this criterion, all sections within a number sense lesson titled "Ongoing Learning and Practice," "Guided Practice," "Try It Out," "Practice," or "Independent Practice" were examined. Review was considered adequate if there were 10 or more items related to number sense within the same lesson in which the new skill was introduced; otherwise, it was deemed insufficient. This decision was based on previous content analysis research (Jitendra, Deatline-Buchman, et al., 2005; Jitendra, Griffin, et al., 2005). Review across lessons involved scrutinizing each subsequent lesson of the program to determine whether the lesson reviewed the two big ideas. Next, we counted the number of lessons that included these big ideas and tallied the number of items within those lessons related to number sense. A minimum of four review items per subsequent lesson was considered sufficient, which was based on findings from previous studies (Jitendra, Deatline-Buchman, et al., 2005; Jitendra, Griffin, et al., 2005; Jitendra et al., 1999).

Results and Discussion

Reliability

Reliability was computed as the number of agreements divided by the number of agreements and disagreements multiplied by 100. The mean interrater agreement for coding of the instructional design principles was 96% (range = 94%–100%). For big ideas and conspicuous instruction, the mean interrater agreement was 100%; this was 94% (range = 78%–100%) for mediated scaffolding, and 95% (range = 82%–100%) for judicious review.

Instructional Design Criteria

Table 1 presents the frequency, percentage, and proportion of number sense items in reform-based (EM) and traditional mathematics textbooks (HB, HM, SF). Table 2 presents the summary of findings regarding number sense items for each of the three traditional textbooks. Results indicated that EM and traditional textbooks differed with respect to their adherence to the principles of effective instruction. In addition, there were variations and similarities across the three traditional textbooks.

Big Ideas. Although the mean number of lessons that addressed number sense instruction was similar across reform-based (8.00) and traditional (8.67) textbooks, the programs differed with respect to their adherence to the two big ideas: number relationships and real-world connections. Overall, traditional textbooks (mean proportion = 10.27) excelled in providing more opportunities for number relationship tasks than did EM (7.00), whereas EM (1.38) was rated better with regard to its emphasis on real-world connections. No such connections were present in traditional textbooks. Research indicates that teaching big ideas in mathematics promotes relational understanding that, in turn, makes it easier to retrieve information (Hiebert et al., 1996; Prawat, 1989). An examination of the types of number relationships and real-world connections revealed that the percentage of lessons that addressed them was higher in EM than it was in traditional textbooks (see Table 1). However, the mean proportion of tasks related to *spatial relationships*, *anchors and benchmarks of 5 and 10,* and *part–part–whole relationships* was higher in traditional textbooks than it was in EM (see Table 1). Across the three traditional textbooks, the mean proportion of number relationship tasks was the highest for SF (13.11), followed by HM (9.33) and HB (8.45; see Table 2).

TABLE 1. Mean Number of Activities, Percentage of Lessons, and Proportion of Activities in Reform-Based and Traditional Mathematics Textbooks by Instructional Design Criteria

	Reform-based (EM)			Traditional (HB, HM, SF)		
Instructional design criteria	***n***	**%**	**Proportion**	***n***	**%**	**Proportion**
Number sense lessons	(8.00)			(8.67)		
Big ideas						
Number relationships	56.00		7.00	89.00		10.27
Spatial relations	18.00	62.50	2.25	27.67	34.62	3.19
One & two more, one & two less	22.00	100.00	2.75	18.33	34.62	2.11
Anchors & benchmarks of 5 and 10	4.00	37.50	0.50	25.67	30.77	2.96
Part–part–whole relationships	12.00	50.00	1.50	17.33	34.62	2.00
Real-world connections	11.00	87.50	1.38	0.00	—	0.00
Instruction						
Present		100.00			100.00	
Conspicuous		0.00			43.67	
Not conspicuous		100.00			56.33	
Mediated scaffolding						
Feedback						
Present		50.00			77.27	
Instructive		50.00			92.59	
Not instructive		50.00			7.41	
Representations						
Concrete	23.00	100.00	2.87	49.33	73.08	5.69
Semiconcrete	9.00	87.50	1.13	28.67	38.46	3.31
Symbolic	2.00	25.00	0.25	36.00	88.46	4.15
Judicious review						
Within lessons						
Present	18.00	100.00	2.25	87.67	100.00	10.38
Sufficient	0.00	0.00	0.00	49.33	42.31	13.45
Insufficient	18.00	100.00	2.25	38.33	57.69	7.67
Across lessons	(21.00)			(12.00)		
Present	23.00	100.00	1.10	48.00	100.00	4.00
Sufficient	0.00	0.00	0.00	35.00	59.00	5.53
Insufficient	23.00	100.00	1.09	13.00	41.00	2.29

Note. N = Number of activities/items. Numbers in parentheses refer to number of lessons. EM = *Everyday Mathematics;* HB = Hartcourt Brace; HM = Houghton Mifflin; SF = Scott Foresman. Proportion was calculated by dividing the number of activities/items by the total number of number sense lessons.

TABLE 2. Mean Number of Activities, Percentage of Lessons, and Proportion of Activities in the Three Traditional Mathematics Textbooks by Instructional Design Criteria

	Harcourt Brace (HB)			Houghton Mifflin (HM)			Scott Foresman (SF)		
Instructional design criteria	*n*	%	Proportion	*n*	%	Proportion	*n*	%	Proportion
Number sense lessons	(11.00)			(6.00)			(9.00)		
Big ideas									
Number relationships	93.00		8.45	56.00		9.33	118.00	—	13.11
Spatial relations	48.00	54.55	4.36	35.00	50.00	5.83	0.00	0.00	0.00
One & two more, one & two less	18.00	45.45	1.64	10.00	33.33	1.66	27.00	22.22	3.00
Anchors & benchmarks of 5 and 10	12.00	36.36	1.09	4.00	16.67	0.67	61.00	33.33	6.78
Part–part–whole relationships	15.00	18.18	1.36	7.00	16.67	1.17	30.00	66.67	3.33
Real-world connections	0.00	—	0.00	0.00	—	0.00	0.00	—	0.00
Instruction									
Present		100.00			100.00			100.00	
Conspicuous		63.64			66.67			0.00	
Not conspicuous		36.36			33.33			100.00	
Mediated scaffolding									
Feedback									
Present		81.82			50.00			100.00	
Instructive		77.78			100.00			100.00	
Not instructive		22.22			0.00			0.00	
Representations									
Concrete	39.00	54.55	3.55	15.00	66.67	2.50	94.00	100.00	10.44
Semiconcrete	59.00	45.45	5.36	26.00	66.67	4.33	1.00	11.11	0.11
Symbolic	21.00	81.82	1.91	13.00	100.00	2.17	74.00	88.89	8.22
Judicious review									
Within lessons									
Present	114.00	100.00	10.36	76.00	100.00	12.67	73.00	100.00	8.11
Sufficient	76.00	63.64	10.86	50.00	33.00	25.00	22.00	22.22	11.00
Insufficient	38.00	36.36	9.50	26.00	66.67	6.50	51.00	77.78	7.29
Across lessons	(24.00)			(7.00)			(5.00)		
Present	82.00	100.00	3.42	29.00	100.00	4.14	33.00	100.00	6.60
Sufficient	54.00	43.83	4.91	23.00	71.43	4.60	28.00	60.00	9.33
Insufficient	28.00	54.17	2.15	6.00	28.57	3.00	5.00	40.00	2.50

Note. N = Number of activities/items. Numbers in parentheses refer to number of lessons. Proportion was calculated by dividing the number of activities/items by the total number of number sense lessons.

The mean proportion of tasks related to number relations of *one or two more* and *one or two less* was higher in EM than it was in traditional textbooks. This is an important big idea of number sense development, because understanding that numbers are related to one another contributes to "the overall concept of number" (Van de Walle, 2007, p. 121). Yet, only EM and SF included a lesson objective that required "finding the number that is 1 more or 1 less than a given number" (Bell et al., 2004, p. 30). An interesting finding was that the textbooks were not consistent with the vocabulary used to refer to the critical comparison concepts. For example, EM used the terms *more*, *less*, and *same*. HB referred to them as *more*, *fewer*, and *equal*, whereas HM and SF addressed them as *more*, *fewer*, and *same*.

With regard to *spatial relationships*, *anchors and benchmarks of 5 and 10,* and *part–part–whole relationships,* the less than positive findings for EM when compared to traditional textbooks must be qualified. Specifically, we examined the types of activities/tasks used in textbooks to address the development of number relationships. Typically, EM included a teacher-guided whole-class activity followed by partner activity and used hands-on tasks using real-world materials (e.g., pennies, dominoes). For example, an instructional objective on *part–part–whole relationships* in EM had students determine combinations of 10 during a whole-class activity. Using 10 pennies as an initial quantity, children were to "grab a handful of pennies with one hand and to pick up the rest with the other hand" (Bell et al., 2004, p. 98). Next, volunteers stated the number of pennies in each hand, and the teacher used language to indicate what each number represented (i.e., a count of real objects) as she recorded them on the board. Although the whole-class activity provided several opportunities, it was coded as one example given that the textbook did not specify the exact number of examples. The above-mentioned activity, which used real-world objects and explicitly connected the concrete to the written symbol using direct language, is likely to promote student understanding of the relation between the concepts parts and whole. In contrast, traditional textbooks included worksheet-type exercises that students completed individually following teacher modeling of similar tasks. An activity on *part–part–whole relationships* in HM, for example, had students complete several items that required adding the number parts to find how many in all, using counters and a workmat that included space for parts and the whole. The emphasis in this activity is the product (i.e., how many in all?) rather than understanding the mathematical process. That is, the lesson did not use explicit language to connect and communicate that parts make up the whole.

Also, we examined how textbooks integrated number sense with other content. Unlike traditional textbooks, EM first introduced *spatial relationship* tasks and later integrated them with other more complex skills (e.g., addition) in subsequent lessons. Evidently, this integration of background knowledge with newly learned skills to solve complex items is critical for promoting mathematical thinking (Chard & Kameenui, 1995). With regard to *part–part–whole relationships*, it is encouraging that all four textbooks reviewed this relationship when introducing addition and subtraction concepts. Integration of this number relationship is critical, because research indicates that interpreting numbers in terms of their different parts is important for later mathematical development (Resnik, 1988; Van de Walle, 2007).

Regarding *real-world connections*, EM included several activities that were personalized when compared to no such connections in traditional textbooks. For example, one of the activities tallied as personal in EM required students to complete a chart on the board to illustrate whether they had a dog, a cat, any other pet, or no pets. This chart was later used to examine whether the number of pet dogs was *more* than*, less* than, or the *same* as the number of pet cats (Bell et al., 2004). Another personalized context in EM was one in which parents had to help children look for numbers in a newspaper advertisement for grocery items, a calendar page, or a picture of a clock. The purpose of the activity was to expand the child's awareness of numbers in the everyday world (Bell et al., 2004).

Conspicuous Instruction. Instruction was more direct and explicit in traditional textbooks than it was in EM. Research is unequivocal about the role of extensive teacher modeling rather than unguided, discovery instructional methods in promoting learning for students at risk for mathematics disabilities (Baker, Gersten, & Lee, 2002; Baxter, Woodward, & Olson, 2001; Kroesbergen & Van Luit, 2003; Kroesbergen, Van Luit, & Maas, 2004; Woodward & Baxter, 1997). All lessons (100%) in EM and traditional textbooks included teacher directions. However, 44% of the lessons in traditional textbooks provided direct and explicit instruction, compared to 0% in EM. In EM, the emphasis was on guided learning, with the teacher questioning and students inferring the concept and/or skill. For example, in a lesson on *one more, one less* in EM, students had to pull out two cards from a stack of number cards, infer which card was larger, and then check their answers using the number line (Bell et al., 2004). Despite the fact that explicit instruction was more common in traditional textbooks, there were variations across the three programs in meeting this criterion. That is, none of the lessons in SF included explicit explanations, whereas most lessons in both HB (64%) and HM (67%) included conspicuous instruction characterized by teacher modeling. The following example in HM exemplifies instruction that is direct and explicit using models, explanations, and process questions. When teaching the concept of addition as *part–part–whole,* the teacher models adding 4 and 2 by placing "4 counters on one part of the part–part–whole transparency and 2 counters on the other part" followed by the teacher questioning, "how many," for each part and recording it in the space for each part (Greenes et al., 2005, p. 37). Next, the teacher "moves the counters from both parts to the whole section" and asks, "How many counters in all?" (p. 37). The teacher then points to each part on the mat and summarizes that "each number is part of the whole. You add when you put two parts together to find the whole" (p. 37).

Scaffolding

Feedback. Feedback was more common in traditional textbooks than in EM. Research on instructional feedback indicates that providing immediate, ongoing, and individualized feedback to students with mathematics disabilities is an effective practice to support the learning of critical skills (Baker et al., 2002; Brosvic et al., 2006). The majority of lessons (77%) in traditional textbooks included feedback, whereas only 50% of the lessons in EM included it. At the same time, the presence of feedback differed across the three traditional programs. That is, 50%, 82%, and 100% of the lessons in HM, HB, and SF, respectively, included feedback. When feedback was present, traditional textbooks included instructive or elaborated feedback most often (93%), when compared to 50% in EM. An example of instructive feedback in a lesson in EM on *part–part–whole relationships* was as follows:

> Watch for children whose number pairs do not add up to 10. Ask them to count aloud the number of pennies in each hand. Some children may observe that the numbers in a person's two hands always add up to 10 pennies, particularly since each person started out with 10 pennies—but do not expect all children to understand this fact at this time. (Bell et al., 2004, p. 98)

Similarly, when children made numerical errors in their math statements in a lesson on *part–part–whole relationships* in SF, instructive feedback entailed reminding students to check the total number of counters before placing them on the part–part–whole mat (Charles et al., 2004). In HM, explanatory feedback involved the teacher reviewing definitions of *fewer* and *more* using vocabulary cards, in the event that children confused the two terms when drawing lines to match sets in tasks to determine the set with fewer objects (Greenes et al., 2005). In HB, feedback in a lesson on *part–part–whole relationships* emphasized that changes in the order of the numbers did not change the sum and that students say the numbers in the order in which they were presented in each exercise (Maletsky et al., 2004).

It must be noted that although feedback was less explicit in EM than in traditional textbooks, factors that contribute to classroom learning, such as social interaction with other students and the teacher, should be considered. In EM, lessons were designed to provide careful attention to cognitive development of the mathematical process by having students share their answers with the class and potentially receive feedback from the teacher and peers (Woodward & Baxter, 1997). For example, a lesson on *part–part–whole relationships* in EM had the teacher display two parts of a given number on a projector and required students to estimate the whole of the two parts. Students had to not only write their estimate, but share it with the class (Bell et al., 2004). This activity illustrates the feedback inherent in the instructional system, even though it is not explicitly stated in the teacher's manual.

Representations. EM excelled in devoting more lessons to concrete and semiconcrete activities than did traditional textbooks, whereas traditional textbooks excelled in providing more opportunities for engaging in all three representations. Research suggests the importance of representations in developing understanding of critical mathematical concepts (Lesh et al., 1987; Rittle-Johnson & Koedinger, 2005; Thompson, 1992; Yang & Huang, 2004). The percentage of lessons that included concrete and semiconcrete activities was higher in EM than it was in the traditional textbooks (see Table 1). In contrast, symbolic activities were present in only 25% of number sense lessons in EM, as compared to 88% of the lessons in traditional textbooks. An examination of the mean proportion of items related to the three representations was higher in traditional textbooks than it was in EM. That is, EM included 2.87, 1.13, and 0.25 concrete, semiconcrete, and symbolic activities, respectively, whereas traditional textbooks included 5.69, 3.31, and 4.15 concrete, semiconcrete, and symbolic activities, respectively. However, there were large variations across and within the three traditional textbooks with regard to the presence of the three types of representations (see Table 2). For example, SF showed the largest mean proportion of concrete (10.44) and symbolic (8.22) representations. It included the lowest proportion of semiconcrete representations (0.11) when compared to HM and HB (range from 4.33 to 5.36).

In addition, research in mathematics education emphasizes the importance of multiple representations (manipulative models, pictures, written symbols, oral language, and real-world situations), including the ability to move between and among representations (Cunningham, 2005; Lesh et al., 1987; Noss, Healy, & Hoyles, 1997), and representations that are meaningful (Hegarty & Kozhevnikov, 1999; Hiebert, 1986; Lubinksi & Otto, 2002; Van Garderen & Montague, 2003). Further, a concrete to semiconcrete to symbolic representational sequence is known to promote learning of critical concepts for students at risk for mathematics disabilities (Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Miller & Mercer, 1993). To address these aspects of representations, we examined the ways that textbooks used representations to develop conceptual or relational knowledge. In particular, EM emphasized using a variety of models to develop number sense. When comparing and ordering numbers, for example, EM used multiple representations, such as a deck of number cards, counters, and a number line, to develop the concept of *more than* or *larger*. The activities also directly linked the different representations in that students learned to compare numbers using one representation (number cards) and check their answer using a different representation (number line). Further, a lesson on complements of 10 illustrates how EM systematically progressed from concrete or semiconcrete to symbolic representations within a lesson. The lesson had children complete the task of finding the parts that made up 10 using pennies (Bell et al., 2004, p. 98). Following individual work with several examples of complements of 10, students were required to work on similar items with partners and record the number of pennies (i.e., written symbols) on their slates (Bell et al., 2004).

In contrast, traditional textbooks provided few opportunities for students to complete number tasks using more than one representation. For example, students were limited to the use of pictures for an activity that required comparing numbers in HM. That is, students had to examine the pictures (e.g., 5 flowers, 3 pencils) in each row of a worksheet and circle the row with more items (5 flowers). Although a lesson on *part–part–whole relationships* in SF had students move from one representation (i.e., counters to represent the parts and whole) to another (i.e., written number parts that made up the given number), the lesson did not use explicit language to scaffold the connection between the two representations. Overall, traditional textbooks provided several concrete, semiconcrete, and symbolic representations. However, most tasks required students to use concrete/semiconcrete and symbolic representations concurrently rather than progress from one to another representation in a systematic fashion.

In addition, EM included hands-on activities using real-world objects that not only reflected the use of meaningful representations, but also enhanced learner engagement as students worked with partners. For example, when teaching the concepts of *more*, *less*, and *same,* a lesson in EM presented students with pennies and had them work with a partner, wherein one student started by adding or taking away pennies and the other student determined whether they had *more* pennies than, *less* pennies than, or the *same* number of pennies as the original set. The activity ended with the second partner checking the answer by counting the pennies to develop reasoning skills (Bell et al., 2004). In contrast, traditional textbooks presented representations in worksheet-type exercises that are more conducive to promoting procedural rather than conceptual knowledge. For example, activities related to the concept of *fewer* in HM had students draw lines to match two sets and circle the set that had fewer objects (picture illustrations; Greenes et al., 2005). Unlike the partner activity in EM, this exercise does little to engage the student beyond completing the items. In sum, differences between EM and traditional with regard to the proportion of representations may be attributed to the type of activities (e.g., hands-on vs. worksheet) employed.

Judicious Review. Traditional textbooks excelled in providing more opportunities than EM did for students to practice the newly introduced skills and concepts, both within a lesson and in subsequent lessons. Research has highlighted the importance of adequate practice opportunities to promote acquisition and retention of learned skills and strategies for students at risk for mathematics disabilities (Carnine, 1997; Dempster, 1991; Kame'enui et al., 2002). Although results suggested that both programs provided opportunities for practice within lessons, the mean proportion of practice items per lesson was only 2.25 in EM when compared to 10.38 in traditional textbooks. Further, none of the lessons in EM provided sufficient practice opportunities, as compared to 49% of the lessons in traditional textbooks. Across the three traditional textbooks, there were variations with HB including the highest percentage (64%) of lessons that provided adequate practice opportunities, when compared to 33% in HM and 22% in SF (see Table 2).

Research also emphasizes the importance of distributed practice over massed practice for promoting retention of learned skills (Ambridge et al., 2006; Dempster & Farris, 1990; Seabrook et al., 2005). On average, review of the newly introduced concepts and/or skills was distributed across 12 subsequent lessons in traditional textbooks and 21 subsequent lessons in EM. The adequately spaced review in EM ensures that students maintain the learned skills (Montague, 2005). However, review in EM was deemed insufficient (mean proportion = 1.10), whereas it was adequate in traditional textbooks (mean proportion = 4.00). Further, 42% of the lessons in traditional textbooks included sufficient review, as compared to 0% of the lessons in EM. Across the three traditional textbooks, there were variations in that HB, HM, and SF included review in 24, 7, and 5 subsequent lessons, respectively. The mean proportion of review items was highest in SF (6.60), followed by HM (4.14), and HB (3.42); whereas, the percentage of lessons that included sufficient review was highest in HM (71%), followed by SF (60%), and HB (44%; see Table 2).

Overall, traditional textbooks provided more opportunities for students to practice the newly introduced skills and concepts. However, this is an artifact of the worksheet format employed in traditional textbooks, which makes it feasible to count the precise number of opportunities based on the number of items in a given worksheet. For example, a lesson on *part–part–whole relationships* in SF provided students with a worksheet consisting of seven items. Because the exact number of items was known, it was counted as seven practice items. This was also the case when teaching the task and maybe an important factor to consider when examining results quantitatively only. In contrast, EM provided opportunities for students to go beyond routine exercises to demonstrate their ability to explore and understand numbers. Most practice tasks included partner activities that were to be repeated a number of times according to the teacher's manual. However, each of these partner activities was counted as one activity, because the lesson did not specify the exact number of times the activity was to be repeated. For example, a lesson in EM on *part–part–whole relationships* had students work with a partner. The activity began with placing dominoes facedown on a table and having each player turn over a domino to determine the total number of dots. Both dominoes went to the player with the larger total. The game then ended when all dominoes had been played, and the player with the most dominoes won the game (Bell et al., 2004). This activity provided students with a number of opportunities to practice the skill, but was counted as one activity given that the program did not specify the number of dominoes involved.

Summary and Conclusions

In summary, findings indicate variations across traditional and reform-based textbooks, as well as among traditional textbooks, in meeting the principles of effective instruction. EM and traditional textbooks differed with respect to their adherence to the two big ideas in number sense. Traditional textbooks included more opportunities for number relationship tasks than did EM; in contrast, EM emphasized more real-world connections. At the same time, tasks related to *one or two more* and *one or two less* only were higher in EM and SF than in the other two traditional textbooks. However, an examination of the types of activities/tasks used in textbooks to develop number relationships revealed that EM did better than traditional textbooks did in (a) promoting relational understanding and (b) integrating spatial relationship tasks with other more complex skills. An interesting finding was that both programs reviewed the *part–part–whole* relationship when introducing addition and subtraction concepts.

Instruction was more direct and explicit and feedback was more common in traditional textbooks than it was in EM. However, there were differences among traditional textbooks with respect to these two criteria. Similar to the reform-based program, SF did not include conspicuous instruction, and feedback was present in only 50% of the lessons in HM, even though the feedback was (100%) instructive when compared to that in EM (50%). EM excelled in devoting more lessons to concrete and semiconcrete activities, but traditional textbooks provided more opportunities for using all three representations. However, there were large discrepancies across and within the three traditional textbooks with regard to opportunities for representations. Unlike traditional textbooks, EM emphasized (a) a variety of models to develop number sense concepts, (b) concrete, or semiconcrete, to symbolic representational sequence within number sense lessons, and (c) hands-on activities using real-world objects to enhance learner engagement and learning. Finally, traditional textbooks excelled in providing more opportunities for students to practice the newly introduced skills and concepts both within a lesson and in subsequent lessons. However, this is an artifact of the worksheet format employed in traditional textbooks. At the same time, adequate distribution of review in subsequent lessons was evident in EM and HB only (21 and 24 lessons, respectively).

In this study, we focused on comparing and evaluating mathematics curricula in terms of what is taught in commercially prepared teaching materials. However, it must be noted that although textbooks play a critical role in shaping teachers' instructional practices and students' learning (Nathan, Long, & Alibali, 2002), the implemented curriculum (i.e., what happens in the classroom) may play an even greater role in facilitating student achievement. Within the constraints of this limitation, we discuss implications of our study findings.

First, results from this study add to the emerging body of literature on mathematics textbook analysis (e.g., Jitendra, Deatline-Buchman, et al., 2005; Mayer et al., 1995). However, findings must be interpreted with caution given the various limitations. First, the sample was limited to four first-grade mathematics textbooks: one reform-based and three traditional mathematics textbooks. Although the textbooks selected were commonly used, we did not review and compare all mathematics textbooks and supplemental materials (e.g., workbooks). Moreover, our analysis of the *instructional design principles* was limited to number sense content. However, this analysis is more extensive than previous studies (Jitendra et al., 1999; Mayer et al., 1995) in that we evaluated all number sense lessons in each textbook. Finally, while we focused on dimensions of the lessons related to number sense instruction, we did not examine other aspects of these texts, such as readability and illustrations.

Second, the reported findings indicate the need to improve mathematics textbook instruction, both reform-based and traditional, to enhance the learning of students with disabilities. Clearly, meaningful development of number sense and accessing the general curriculum for students with mathematics disabilities means designing textbook lessons and materials that emphasize real-world activities/tasks to promote relational understanding and ultimately meaningful development of number sense (Carnine, 1997; Kame'enui et al., 2002; Van de Walle, 2007). An instructional sequence that includes multiple models and makes explicit the connections between and among representations, as well as provides meaningful representations, is necessary if we are to expect students with learning difficulties to understand the task and the mathematical process (Van Garderen, 2006). Further, instruction that is direct and explicit and that provides unambiguous explanations and strategic application of newly learned skills is critical to meet the needs of students at risk for mathematics disabilities (Gersten & Baker, 1998; Santoro et al., 2006). Another essential feature of a good instructional sequence is incorporating feedback within lessons. To promote skilled error-free performance, it is necessary that students analyze their performance in relation to the feedback provided. Finally, students must be provided with sufficient and consistent opportunities to master newly acquired skills (Santoro et al., 2006).

Most textbooks analyzed (i.e., traditional) provided little support for teachers, who may not have a deep understanding of the content to adequately prepare their students or accommodate individual differences (Ma, 1999). Because "no curriculum teaches itself," it is imperative that teachers know mathematics well enough to use such instructional materials wisely to improve student learning (Ball, 2003, p. 1). For example, teachers must be able to create learning environments in which students engage in classroom discourse that research has shown promotes mathematical thinking and reasoning (Baxter, Woodward, Voorhies, & Wong, 2002).

Finally, the difficulty of interpreting quantitative analyses of textbook materials is clear from this study. Because simple counts of the frequency of instances may lead to faulty conclusions, the importance of qualitative analysis is critical to consider in future textbook analyses studies.

In conclusion, because number sense is a way of thinking that permeates all aspects of learning mathematics, planning for teaching number sense concepts must be carefully designed to enable diverse learners to make sense of numbers by creating a learning environment that nurtures number sense (Gersten & Chard, 1999; Sowder & Schappelle, 1994). This would include focusing on teaching the big ideas in number sense using explicit instruction (e.g., modeling, posing process questions), scaffolding instruction during initial learning, and providing judicious review to acquire and retain information.

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