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Schematising Activities as a Means for Encouraging Young Children to Think Abstractly

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One of the missions of education is to prepare children for complex tasks that occur in their cultural environment. By means of abstracting, the effects of this complexity can be reduced. Recent research and theoretical development show us that young children already seem to be able to think abstractly. The acknowledgement of this potential in young children may provide a vehicle for the promotion of more mathematical thinking. In this article, we describe an approach to abstraction that focuses on schematising activities. On the basis of our research, we conclude that young children are able to improve the quality of their abstract mathematical thinking by these means.

Being able to think abstractly is not an explicitly defined goal of the Dutch educational system. However, education has the potential to contribute to the development of abstract thinking by bringing children in contact with theoretical knowledge that cannot be derived directly from concrete observation. For example, mathematical thinking is generally acknowledged to be abstract, but grammatical parsing is also an abstract accomplishment which requires that students extricate themselves from concrete linguistic usage and think of language as a theoretical system in which, by means of abstract rules, they can construct new knowledge.

The goals of primary education, however, consist to a great extent of such abstract knowledge, as well as skills or procedures to handle that knowledge. Nevertheless, many children have difficulties with dealing with abstractions during their school career: They do not seem to understand the abstract theoretical notions in their textbooks, and they cannot explain them or employ them in new situations. For instance, young children's problems with the number concept are well documented (Brissiaud, 1989; Gelman & Gallistel, 1978; Piaget, 1952). The same goes for many other conceptions such as — to name just a few famous examples — the numeration system (Nunes & Bryant, 1996), fractions (Keijzer, 2003), and functions (Sfard, 1992). It is widely assumed that these problems have to do with the abstract nature of the concepts.

Despite the variety of difficulties, there may be a common cause underlying these problems in the area of mathematics. In the 1970s, Russian psychologists such as Davydov (1972/1990) pointed out that empirical approaches to mathematics learning, which are based on direct experience with objects, develop the idea that concepts refer to concrete experiences or objects. In his view, however, this idea distorts the view of mathematics as a relational system. According to Davydov, mathematics (as well as other scientific disciplines) are abstract because they are based on relations that systematise concrete or mental objects despite their differences. The concept of number is abstract, according to Davydov, because it does not refer to the things to be counted or calculated but to an invisible relationship between a quantity and the unit that is used for the measurement of that quantity. For example, the array:

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*   *   *
*   *   *
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represents the number three when we take {*, *} as a unit, but represents the number two when we take {*, *, *} as a unit. The problem in the development of number, according to Davydov, is rooted in the fact that children do not appropriate this relational (and abstract) view of quantity and number.

Davydov may be criticised for reducing the number concept to this one particular conceptualisation (Freudenthal, 1979). According to Freudenthal, Davydov imposed just one structure onto children's thinking, while the core business of mathematics is not in the structures but in the *structuring*, that is, in the way people organise experience in functional ways. Nevertheless, both Freudenthal and Davydov concentrate on the construction and use of relationships in the process of mathematising. These relationships and their formal symbolisations belong to the heart of mathematical thinking. It is because of the symbolised references to such relationships that mathematics can be called abstract.

The transition to abstract relational thinking is often problematic in the mathematics of young children (Hughes, 1986). According to Hughes, young children do not usually make this transition when they learn mathematics on the basis of symbolic manipulations with numbers. Children are often introduced to mathematics as a formal discipline by drill and practice, without a proper understanding of the relation between their experiences and concrete actions on the one hand and the formal-symbolic operations that they learn to perform on the other. Hughes claims that the cause of many problems in mathematics is a lack of understanding of the process of translating concrete experience into abstract forms.

Problems in the development of abstract thinking are a serious impediment for pupils' future schooling, as much theoretical knowledge that has to be learned in further education is also abstract. If developing abstract thinking is an implicit task of education, we have to explore how it develops and how pupils can be optimally assisted to appropriate abilities for dealing with abstractions. What is abstract thinking, can young children develop it, and if so, how does that develop and can it be supported?

We will start with some brief notes on the development of abstract thinking and continue with an explanation of our notion of abstraction. Then we will focus on the concept of schematising activities as a possible answer to young children's need to deal with abstractions, particularly of a mathematical kind. During participation in schematising activities children learn to represent thoughts and ideas by means of their self-invented symbolism and schemes. This participation is valuable because in this way children learn to understand the need and the function of symbolism. After all, this understanding is very important in later mathematics education because mathematics is full of symbolism and schematic representations (Freudenthal, 1973). By means of schematising, young children can develop abilities which are in themselves not mathematical but are prerequisites for mathematical thinking.

In this article, we will describe part of our research on the development of abstract thinking through schematising in young children as preparation for later mathematics learning. It is our aim to demonstrate that the use of schematising activities can improve children's mathematical thinking.

Theories on the Development of Abstract Thinking

Piaget's theory on the development of abstract thinking has been influential for many years in psychology and educational theory. In his theory, abstract thinking is about discovering schemas in action (*abstraction empirique*) and about reflexively developing these action schemas into formal operations which are flexible and can be accessed on a mental level (*abstraction réfléchissante*). Young children are able to pour water from one glass into another. However, as soon as they find out that this action in fact is reversible and nothing has changed regarding the water, a start has been made with abstract thinking. According to Piaget, the child not only pays attention to the action itself (pouring water), but also to the action schema that underlies it. This abstract thinking, for Piaget, has a close connection with formal thinking, which he states develops later (at about age 12). So, according to Piaget, abstract thinking can only fully develop in the stage of formal operations.

Piaget's theory was questioned by the Dutch developmental psychologist Philip Kohnstamm (1948). Kohnstamm was one of the first researchers who believed that abstract thinking might occur in young children. According to Kohnstamm, abstract thinking is a structured way of viewing the world, and children can do that if they are provided with the means with which to structure the world. The trainability of the Piagetian abstract operations like inclusion was empirically proved about 20 years later by another Kohnstamm (1967) as well as other researchers (Sheppard, 1973). These investigations show that at least some abstractions can be taught to children at an earlier age than Piaget supposed.

Davydov (1972/1990) was also able to demonstrate empirically that children at the age of 8-10 years are able to think abstractly — if we teach them theoretical models with which to analyse the world. The abstract is a means to structure the concrete, according to Davydov, and in this sense the young child can ascend from the abstract to the concrete if he or she gets help from more knowledgeable others in viewing situations from the perspective of theoretical models. For instance, if children have learned to count and to structure quantities, they initially do that on the basis of experience. But we can also give these children theoretical means (like a number line or a formula) by which they can operate deliberately to structure quantities and numbers. According to Davydov, abstract thinking is bound to the use of theoretical models that structure human concrete experiences.

Egan (2002) explicitly wanted to dispel the fallacy that young children first think concretely and only later learn to think abstractly. According to him, young children's ability to think abstractly comes with the appropriation of language (Egan, 1997, p. 47). However, for the employment of this ability it is important that the child is emotionally involved in the act of understanding a concrete situation. A fundamental frame for abstract thinking for young children is an engaging story. Within the frame of narratives or stories, children can get carried away in series of events which are far from concrete. In such a story, children see reality from a certain perspective. Of course, young children know that puppets, machines and toy animals like in Bob the Builder or Kermit the Frog cannot talk and cannot build houses in one day, but if children are able to get emotionally involved in the story of these figures, they can imagine themselves a world which is not based on the concrete reality with which they are familiar. The imagined world is based on only a couple of dimensions or perspectives. According to Egan, these perspectives are mostly expressed in terms of contrasts like love-hate, good-evil and so on. Children are very well able to deal with these kinds of abstractions, as long as they are meaningful to them.

Both Davydov's and Egan's conceptions of abstraction share the idea that abstracting is a way of perceiving and conceiving of reality that can be taught to young children .

Defining Abstract Thinking

Various theoretical points of view have been developed with regard to abstract thinking. Since Aristotle many philosophers and psychologists have tried to clarify the notion of "the Abstract" (Bolton, 1972). In our conception of abstraction, we start out from the ideas of the German neo-Kantian philosopher Ernst Cassirer (1874–1945). Cassirer is probably the first scholar to underscore the idea that abstracting is basically an act of taking a partial point of view (Cassirer, 1923, 1957). He argues that "the general is not the end result of an abstraction process; rather, some general principle is always the beginning. An abstract concept then is not so much a reproduction of reality, but actually establishes a point of view that guides our thinking" (van Oers, 2001, p. 284). Hence, in Cassirer's view, the abstract is not the recognition of a new, previously unnoticed general characteristic of the objects involved but an attribute added to the object in our thinking. It is the interrelatedness we conceive when objects are viewed from a particular point of view. Following Cassirer, we can now provisionally define abstracting as *a process of constructing relationships between objects from a particular point of view*. So the points with coordinates (0, 0), (4, -2) and (9, 3) can be viewed as three arbitrary points, but they can also be seen as connected when we look at them from the perspective of the parabolic structure $x = y^2$. It is this latter point of view that relates the points and that makes them into an abstraction. Abstractions are the products of human creativity, not intrinsic features of the objects.

Cassirer (1923) pointed out that abstraction always includes taking a point of view from which the concrete can be seen as meaningfully related (van Oers, 2001). However, finding an appropriate point of view depends on the contextualisation of one's actions. Therefore abstraction is seen here as a dialectical process between the concretely given objects and the abstract representations of them. For example, abstracting the colour red from a collection of objects is based on the assumption of "redness" as an initial point of view, and subsequently seeing the similarity of these objects from the point of view chosen. The whole dialectic process includes both the process of going from the concrete to the abstract and the process of ascending from the abstract to the concrete.

Translated into psychological terms, the process of abstraction should be conceived as a process of focusing attention on relationships between objects, regardless of the different qualities of the objects themselves. This process of focussing can be explained as the construction of a mental object that can be expressed as an abstract symbolic model. Abstraction therefore is not something theoretical which has no relation with the concrete reality. Quite the contrary, "abstraction can never produce meaningful insights in the concrete world, unless there is some inner relationship between the concrete and the abstract" (van Oers, 2001, p. 287).

As explained elsewhere (van Oers, 2001, p. 288):

> We must conceive of abstract thinking as a new type of activity emerging from a concrete situation. Abstract thinking is a state of being highly involved in a theoretically construed world, based on explicitly used relations, logical rules, and strict norms of negotiation. This new activity is not a detached way of acting but a new cultural activity driven by both cultural contents and human desires.

In this way, the abstract can also be seen as an incomplete, not yet developed, and impoverished description of reality. "It is a partial point of view from which the concrete can be understood in its systemacity, as well as a point of view that is not particularized in all its details, and that is uncomplicated by deforming influences" (van Oers, 2001, p. 287).

Schematising Activities

One important form of abstraction, both in sciences and in schooling, is the construction of models or schematic representations. In schematic representations, people represent part of their situation from a structural point of view, basically representing the main relationships between the objects while neglecting details. This is true for the planetary model of our solar system, for the algebraic representation of the elliptic orbit of the planets, for the representation of hurricanes and for the maps that pupils make of their playground. All these and other examples give partial representations of reality from a particular point of view. In our view it is the partiality of the representation, related to the point of view taken, that really makes a representation abstract.

We conclude that abstract thinking is not a purely theoretical thing or something far from concrete reality. This view is radically different from Piaget and extends the conceptions of Davydov and Egan. In this view we see schematising and modelling as abstract accomplishments, since they represent situations consistently from a structural point of view and tend to regard different aspects of a situation as connected. In the next section, we will argue and illustrate that even young children can structure situations schematically and therefore think abstractly. In our research, we were most interested in so-called dynamic schematisations, that is, representations of change and transformation. In our view, mathematising is also an activity of consistently building, elaborating and using dynamic schematisations. Basically, even simple equations like $y = 2x + a$, functions like $F(x) = x^2$, and matrices are schematic representations of dynamic processes. Therefore we hypothesised that the early appropriation of schematising abilities could be a productive starting point for later mathematical thinking. In the next section we will describe our approach to schematising, as a form of abstract thinking, in early childhood education.

Valuable tools to structure and organise our ideas and thoughts and improve abstract thinking are schematisations. However, it is pointless or even impossible to train young children directly in the construction and use of schematisations. It is essential that children acknowledge the value of these schematic representations for their current activities. In previous case studies, van Oers (1994, 1996) was able to demonstrate that schematising as an activity is accessible for young children (from the age of 5) when this schematising is a meaningful and integral part of their play activities. When children are playing with a railway track, it is not too difficult to encourage them to make a drawing of the track so that it can be rebuilt later on. By doing so, they soon discover that it is unhelpful to include everything in the drawing; so they make a selection of what to draw and what to leave out. In order words, they make representations of the railway track from the perspective of rebuilding it. By use of such schematisations during play activities, children are able to represent what they think or what they mean; they can represent their view of reality in drawings, in graphs, by symbols, and so on. They can draw what they see and invent their own symbolism to point out what they think is important.

As we have already mentioned, we conceive of schematisations as symbolic representations of a part of the physical or sociocultural reality that serve as tools for the organisation of a person's field of experience. This field of experience can result from empirical perception or from the awareness of theoretical notions, concepts, or ideas. Fields of experience can be organised by thinking with the help of symbolic representations such as diagrams, graphs, tables and narratives. In our research, we distinguish different types of schematisations depending on their value for representing different aspects of reality. We speak of *static schematisations* when the stable parts of reality are represented. These schematisations represent a status quo, in the way that a map represents a local situation or a construction plan represents a building. On the other hand, we have schematisations that represent change, movement, or transformation, such as exploded view images, flow schemes, functional graphs, musical scores, and route descriptions. We call these representations *dynamic schematisations*. As we pointed out above, we believe they are particularly important for the development of academic (e.g., mathematical) thinking.

Abstractions serve as tools for handling the complexity of the concrete reality, by allowing a consistent view from one particular perspective. The choice of the perspective is a psychological process that can have different bases. Sometimes the perspective is imposed by the context (e.g., a mathematics lesson), sometimes it is transmitted by tradition (e.g., interpreting weather in terms of rain and temperature), and sometimes it is a creative act of invention (e.g., Newton's perspective on gravity in terms of attracting masses).

The potentially complexity-reducing power of abstractions is what makes them a valuable means for children to learn while mastering their current activities. One way to organise the concrete reality around us is to look at it from the point of view of structure. Children can learn to see a structure in reality (e.g., mathematical tasks) and represent that structure graphically. Young children seem able to do that, certainly if we help them and offer them the right schematisation tools. Below we will demonstrate how young children can become involved in schematising activities. A *schematising activity* is any cognitive activity aimed at the construction or the improvement of a symbolic representation of a part of the physical or the sociocultural reality. The resulting schematisation can be seen as a structured representation of reality, by which one can make statements about that reality, and which thereby helps us to organise our views and thoughts. We shall illustrate this approach with an observation from our experimental classrooms in which we investigated the effects of a schematising program in early childhood (at age 5-6) on children's later mathematical learning outcomes (at age 7).

Schematising Activities in Young Children

Our research is done from a sociocultural perspective based on the ideas of Vygotsky (Wertsch, 1987). Central to Vygotsky's theory is the idea that human beings develop towards intellectually autonomous cultural identities by participating in cultural activities and subsequently appropriating the constituting meanings of that particular activity.

We conducted our research in six schools that base their practice on a Vygotskian approach to educational development and learning. This approach, called *Developmental Education*, has been developed in the Netherlands since the 1970s. One characteristic is the assumption that the leading activity in the age group 5-8 should be play. Play activities create enormous space for fantasies and experimentations with meanings and symbols (Vygotsky, 1978, Chapter 7). In the period of our research intervention, we created an experimental group of pupils who learned to schematise during play activities. During one school year, a teacher trainer assisted the teachers in the three experimental schools with teaching their pupils to participate in schematising activities as an enrichment of their play activities. There were 75 children in these three classes and 58 in the control classes.

An example of a schematising activity is representing a song. We can represent a song in several ways: We can invent symbols to indicate the loudness or pitch of certain words, or symbols to illustrate that you have to make a certain movement for a word. We invent a system to represent something in a particular way from a particular perspective. These representations are definitely abstract, for the reasons explained above. Our data include an example of children schematising a song. A couple of children in the experimental condition were interested in dinosaurs. The teacher taught them a song about dinosaurs and after singing the song a couple of times, the teacher and the children became curious as to how songs are represented in sheet music. After studying some songbooks, the children started to invent their own notation systems. They drew a staff (musical score), added lines sloping upwards and downwards below the words to show whether the voice should go up or down, and drew dots below words for which they had to clap hands. After creating this schematisation, the students started to use this notation system for other songs and were able to reproduce the dinosaur song again later.

Making drawings of familiar objects is another schematising activity. For example, a child (age 5) has been reading about St. Nicholas. St. Nicholas has a ship and the child wants to rebuild this ship. The teacher trainer teaches the teacher to encourage the child to make a drawing of the construction, because then the child will be able to show his work to his younger brother or his father. The child starts to draw his construction. Furthermore, he starts to make a legend: He draws one small block and writes a 20 next to it, which means he used 20 small blocks. He also draws one big block and writes a 15 next to it: He used 15 big blocks. In this way, he is making a symbolic representation of his reality and starts looking at it from the perspective of number.

We have many more examples from our experimental classrooms which show children thinking abstractly by means of schematising. Larissa built a ship and made a drawing of it. She drew the several blocks and the sequence in which she built the ship (see Figure 1). This drawing is abstract because it is drawn from the perspective of reconstruction. It focuses on the relevant relationships (structure), leaves out irrelevant traits (like colour), and represents only one side of the ship. Larissa made the drawing because she wanted to show her brother at home how she built the ship and enable him to make such a boat himself. For

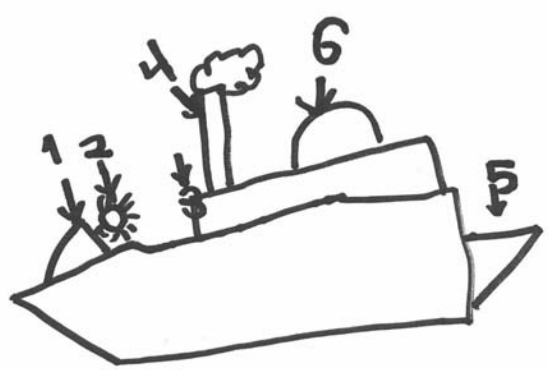

Figure 1. Larissa's ship.

this purpose, it was very useful and meaningful to represent the boat only schematically. The abstraction was based on the perspective of re-constructing, and all features were left out that did not contribute to this reconstruction.

Schematising becomes even more obvious in situations in which children intentionally do not represent repetition. For example, a group of Grade 2 children were drawing a trip. The teacher had asked them to think about places they wanted to visit and then to draw these places on a sheet of drawing paper. The children drew five different places, but did not think about the actual distance between these places. The teacher asked the children to think about how they would get from one place to another. The children thought that you needed to go from Turkey to Paris by plane. They decided to draw a little plane with a couple of arrows to represent the direction the plane had to go. The plane had only one window (representing all the windows of planes) and only one child (representing the six children). The children assumed it would be obvious that this one child represented all the children. Similarly, the children thought it would not be necessary to draw the airports, because everybody would understand that an airplane flies from airport to airport. This schematisation is an example of deliberately reducing a recurring phenomenon to just one element, omitting the exact number of objects. As has been reported in the literature several times, this kind of representation of a recurring movement through one of its constituents is a basic form of abstraction (van Oers, in press).

We give one last example of a schematising activity. Children were playing at being pirates, searching for treasure. The teacher asked them how they would know where to find the treasure. The children assumed they would need a treasure map and decided to make one themselves. They drew a map of their classroom, since their classroom was the "treasure island" (see Figure 2). One child hid the treasure and, using arrows, drew on the map the route to be followed in order to find the treasure. The other children had to read this abstract representation of reality. After finding out where to walk and arriving at the end of the route, the children expected to find the treasure. However, another sign was drawn on the map: a ladder. The treasure was hidden on top of a cupboard, and that was what the ladder represented.

In the examples we have given, children were representing the structure of dynamic situations by symbolising changes in place and time. If Larisa had drawn the ship without indicating the sequence in which she had built it, the drawing would have been a static schematisation representing the structure of a stable situation. Representing change in dynamic schemes is more complex because it is impossible to represent all the intermediate states in a changing situation. Our data shows conclusively that young children (age 5-8) are able to develop dynamic schematisations.

In the control groups, these kinds of activities rarely took place. In these schools, the teacher trainer only gave general support for the implementation of the Developmental Education concept and did not pay attention to schematising.

In our research, we have been able to collect many fruitful examples of schematising activities which show abstract thinking in young children (Poland,

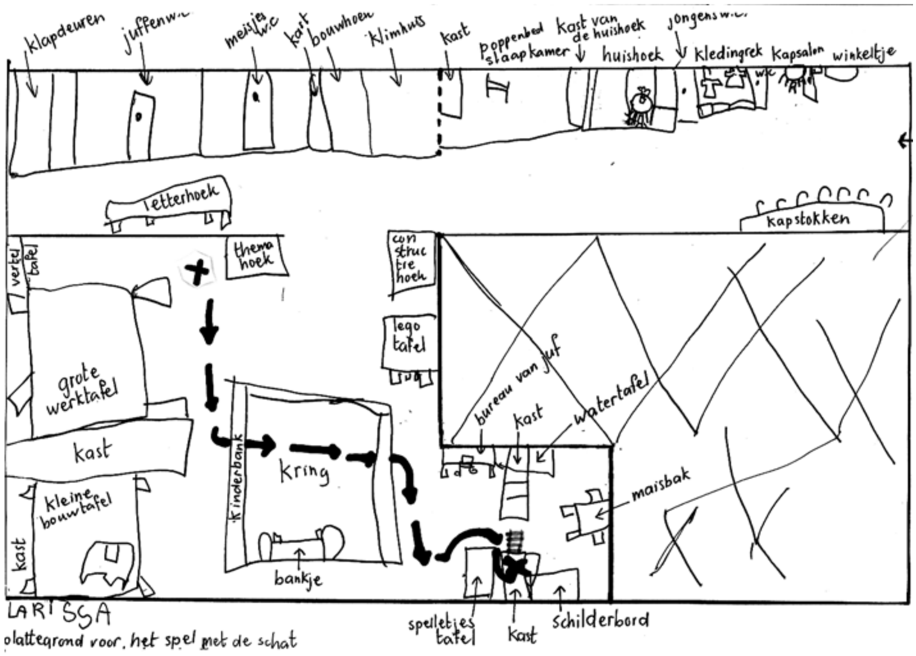

Figure 2. The treasure map.

van Oers, & Terwel, 2006). In a longitudinal study, we were able to empirically demonstrate the value of early schematising for later mathematical thinking. We demonstrated that the children who were repeatedly involved in meaningful schematising activities in Grade 2 (5-6 year olds) outperformed pupils from the control group at the beginning of Grade 3 (6-7 year olds) on a standardised test which included counting, ordering, and arithmetical tasks like adding and subtracting. Although the advantage disappeared after half a year in Grade 3, it can be concluded that schematising in the context of play positively influences early mathematics education (Poland, in preparation). We believe that schematising encouraged children to take a structural perspective on play activities and situations, and to handle dynamic relationships in this situation. We assume that this brought children into a better position to become successfully involved in their mathematical activities in Grade 3.

Significance of Schematising

The use of schematisations in young children's mathematics education is getting more and more attention (Carruthers & Worthington, 2003). Learning about and working with schematisations can be very important, allowing children to bridge the gap between concrete, practical thinking in early childhood and logical-symbolical thinking in later mathematical development (Dijk, van Oers, &

Terwel, 2004). In early childhood (ages 5-6) children engage in much play and fantasy, but in later development they are confronted with several school tasks the meaning of which may not evident to them. Unfortunately, many children have to bridge this gap from play activity to real mathematics without training, experience or the appropriate tools. It seems that education switches from being more or less child-centred in early childhood to method-centred in later years. Children are allowed to play a lot more in the first two grades than in third grade, where they have to learn to write, read and participate in mathematical activities. They suddenly have to learn two different, strangely formalised notation systems: writing and arithmetic. In addition, children have to learn to switch between the concrete and the abstract. Teachers teach children to switch between the concrete and the abstract by emphasising mathematical procedures. Children learn mathematical notation from these procedures instead of from their play activities, motivations or interests. There is a tendency for teaching to become more procedurally oriented as a child gets older, on the assumption that children can deal with these procedures without any difficulty. This tendency neglects the fact that the gap between informal and formal learning, and between concrete and abstract thinking, is very large (Hughes, 1986). We need to find a way to help children bridge this gap, starting from early childhood.

Problems arise when children have difficulty connecting the abstract and the concrete. According to Siegler (2003, p. 221), children often "fail to grasp the concepts and principles that underlie ... mathematical procedures". Our argument is that the source of these difficulties lies in children's inability to organise and structure ideas and thoughts from a mathematical perspective (Freudenthal, 1980; Ruijssenaars, 1997). Many children lack appropriate action plans or strategies and are often not able to reflect on their own cognitive processes, strategies and problem solving. For structuring our thoughts is like having a dialogue with ourselves. By talking to ourselves, we organise what we think; and after this structuring process, we can re-create the procedure.

Children often have the opportunity to learn new strategies by observing other people around them using strategies. Crowly and Siegler (1999, p. 305) found that explanations about the logic of strategies "may provide a framework linking the subgoals within a strategy and thus making it easier for children to assess what they have already done and what they must do next to continue successful execution." Thus, children struggling with mathematical problems need structuring tools such as strategies to organise their thoughts in order to solve mathematical problems (Siegler, 2003).

Conclusion

The examples explained above demonstrate that young children are able to think abstractly, and that this skill can be developed by means of schematising activities, provided these schematisations fulfil a meaningful role in children's (play) activities. Indeed, abstract thinking by means of schematising activities can be very useful and meaningful in children's own activities, for example when abstractions have the function of remembering, imagining, or communicating (like the drawing of the ship for Larissa's brother). Through involvement in schematising activities, children are encouraged to invent schemes, models, symbols, and inscriptions in order to represent their point of view of a concrete situation or their current activity. During schematising, they reflect on several perspectives they could take in order to represent an event or an object. It is evident from our data that abstract thinking, as we have defined it, is definitely accessible to young children and significantly supports children in the process of learning to mathematise (Poland, in preparation). Our longitudinal research provides empirical evidence that the emphasis on schematising as a type of abstract thinking supports children's mathematical learning processes. For later development of mathematical thinking, an early introduction to abstract thinking turns out to be a basic prerequisite that helps children bridge the gap between concrete activity and formal thinking, and prepares them for future learning in which abstract formulae and models will probably play a major role.

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