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Constructing Coordinate Graphs: Representing Corresponding Ordered Values with Variation in Two-Dimensional Space

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Coordinate graphs of time-series data have been significant in the history of statistical graphing and in recent school mathematics curricula. A survey task to construct a graph to represent data about temperature change over time was administered to 133 students in Grades 3, 5, 7, and 9. Four response levels described the degree to which students transformed a table of data into a coordinate graph. *Nonstatistical* responses did not display the data, showing either the context or a graph form only. *Single Aspect* responses showed data along a single dimension, either in a table of corresponding values, or a graph of a single variable. *Inadequate Coordinate* responses showed bivariate data in two-dimensional space but inadequately showed either spatial variation or correspondence of values. *Appropriate Coordinate* graphs displayed both correspondence and variation of values along ordered axes, either as a bar graph of discrete values or as a line graph of continuous variation. These levels of coordinate graph production were then related to levels of response obtained by the same students on two other survey tasks: one involving speculative data generation from a verbal statement of covariation, and the other involving verbal and numerical graph interpretation from a coordinate scattergraph. Features of graphical representations that may prompt student development at different levels are discussed.

As an introduction to this study of how students construct coordinate graphs, the historical development of coordinate graphing, the place of graphing in the school curriculum, and previous research on the construction of coordinate graphs are considered. Background research to the current study is then presented.

Historical Development of Coordinate Graphing

Various accounts of the historical development of graphing (Beniger & Robyn, 1978; Biderman, 1990; Funkhouser, 1937; Tilling, 1975; Wainer & Velleman, 2001) illustrate some of the sources, stages, difficulties, and the long time involved in the evolution of coordinate graphs—graphs that today many of us take for granted. The significance of this historical development for the current study is that students may encounter similar issues in learning to design graphs.

Since the times of ancient Egyptian surveyors, coordinate systems have been features of maps in which the space of the map corresponds to physical space (Funkhouser, 1937). Coordinate systems have been a part of abstract mathematics since about 1390 when Oresme drew proto-bar graphs of idealized curve functions (Biderman, 1990) with the insight that "everything measurable can be represented by a line" (Clement, 1989, p. 84). Descartes linked the coordinate system to formal equations in 1637. This form of graphing was aligned with a rational philosophy that was not readily adopted for recording empirical data and that may have hindered the development of statistical graphing until Playfair's creative developments in 1786 more than a century later (Biderman; Wainer & Velleman, 2001).

Time-series data featured heavily in early graphs of empirical data. From 1663 mechanical devices were used to record meteorological data in time-series graphs, for example temperature change over time recorded "on a moving chart by means of pen attached to a float on the surface of a thermometer" (Tilling, 1975, p. 195), although these graphs were often copied into tabular form (Beniger & Robyn, 1978). Playfair's 1786 publication, *The Commercial and Political Atlas,* made popular the use of graphs of empirical data. The atlas included 44 charts, most of which were line graphs of monetary values over time. The significance of Playfair's work was based in his insight that

> anything that could be expressed in numbers could be represented as well by lines ... [T]hese uses of 'lineal arithmetic' he learned in his boyhood ... keeping a register over time of the readings of the thermometer by drawing lines on a divided scale. Here, lengths of the thermometer column were literally what was observed. (Biderman, 1990, p. 9)

Playfair argued that tables of values recorded precise numerical values, whereas graphs had an alternate purpose for conveying a global perspective "of the gradual progress and comparative amounts, at different periods" (Playfair, 1801, p. xi, cited by Funkhouser, 1937, p. 281).

Today, coordinate graphs are taught as part of school algebra, and are often known as Cartesian graphs from Descartes' work. More generally, graphs are commonly used for conveying data; for example, many weather reports in Australia include a line graph showing temperature change over times of the day, as shown in Figure 1. Maps and time-series graphs are the most common uses of coordinate representations; less than 10% of newspaper and magazine graphs surveyed by Tufte (1983) used coordinates to represent something other than time or position. This significance of time-series data for providing a context for studying coordinate graphs is recognised in recent school mathematics curricula (e.g., National Council of Teachers of Mathematics [NCTM], 2000).

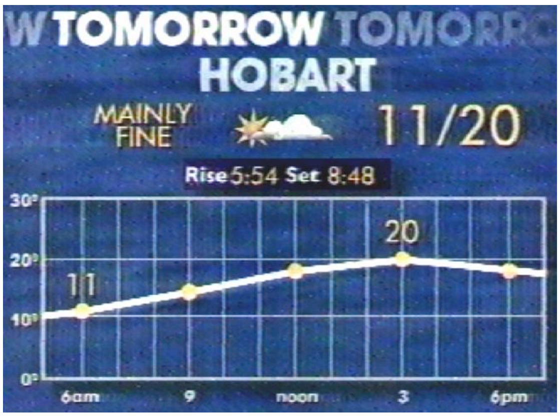


Figure 1. Weather graph in the media
(from WIN television news, 19 January, 2002).

Graphing in School Curricula

Constructing graphs and interpreting graphs appear in school mathematics curricula for both primary and secondary school years in Australia (Australian Education Council [AEC], 1991, 1994), New Zealand (Ministry of Education, 1992), and the United States (NCTM, 2000) as parts of both algebra and statistics. The topic of understanding scales and number lines also appears within measurement and number strands, and coordinates appear as part of the space strand, for example in reference to maps (AEC, 1991). As part of algebra, qualitative change over time is recommended to introduce functions. Possible activities for Australian primary students include, "Sketch informal graphs to model familiar events such as variations in hunger through the day" (AEC, 1991, p. 193).

As part of the statistics curriculum, these documents recommend that primary students represent univariate data in pictographs and bar graphs, employing baselines and consistent scales. Bivariate data are recommended for secondary school students, for example "time series data in line graphs" (AEC, 1994, p. 109). The New Zealand curriculum (Ministry of Education, 1992) gives particular attention to the representation and interpretation of time-series data. In upper primary grades, for example, students should be "collecting and graphing simple time-series data such as the height of a classroom-grown bean plant at midday each day" (p. 179). Secondary students should be "devising ways to display data showing variations of variables over time and using conventional time-series displays" (p. 187). This attention to time-series graphs is prior to the mention of coordinate graphing of scatterplots to assess bivariate association, which is suggested for senior secondary school years.

Previous Research on Constructing Coordinate Graphs

Coordinate graphs employ a general graphic feature that "position denotes value" and apply this feature in two-dimensional space to represent values of two variables. They represent both (a) the data points, emphasizing the *correspondence* of values of two variables, and (b) general trends, emphasizing *variation* of the two variables due to the ordination of the values along each axis. Wavering (1989) suggested that two Piagetian schema are developed in reasoning to create bivariate graphs: (a) one-to-one correspondence of bivariate data values, and (b) seriation of values of a variable, necessary for scaling of graphs to produce a coordinate system. Similar dichotomies have been described as pointwise and variational approaches (Nemirovsky, 1996), static and dynamic data models (Clement, 1989), and local and global aspects (Ben-Zvi & Arcavi, 2001). Graph interpretation has also been described by Curcio (2001) as having various purposes including reading data values, reading beyond the data by predicting based on global trends, and an intermediate level of reading between the data, such as comparing values. When constructing graphs, students' responses may be influenced by their beliefs about these purposes for the representation more than by their ability to represent a given graph form (Roth & McGinn, 1997). Students are often asked to plot points (e.g., Kerslake, 1977) and to read and compare data values (e.g., Curcio, 1987); thus it is not surprising that researchers have often found that students construct and read graphs pointwise (e.g., Kerslake, 1977; Bell, Brekke, & Swan, 1987; Brasell & Rowe, 1993).

Various studies have identified difficulties or alternative graphing methods of students such as drawing a graph as a picture (Brasell & Rowe, 1993; Sherin, 2000), including irrelevant contextual elements (Nemirovsky & Tierney, 2001), omitting variables (Mevarech & Kramarsky, 1997), omitting constant values (Nemirovsky & Tierney), and reversing axes (Brasell & Rowe). Brasell and Rowe asked 84 Grade 12 physics students to construct a graph of five paired data values indicating heights from which a ball was dropped and to which it rebounded. Reversal of axes and inadequate labeling were each a problem for approximately 50% of students. Mevarech and Kramarsky asked Grade 8 students to represent verbal statements of functional relationships, and found three alternative conceptions: representing a single pair of corresponding values, representing a single variable, and representing an increasing function despite the task specifying a constant or negative function. Konold (2002) noted that as an alternative to scatterplots, some students produce ordered case-value bars. He observed students creating two bar graphs of teeth-brushing times and of plaque levels, both across cases of student names, which were ordered by brushing times. As each univariate bar graph had corresponding positions for student names, scanning the plaque graph for evidence of a trend corresponded to detecting covariation with brushing time.

At the primary level, Ainley (1995) found that intuitions about the context of height growth allowed most primary school students to construct bivariate graphs and correct plotting errors that did not fit the trend of growth. Moritz (2000) found that about 90% of upper primary students could draw a graph to show that people grow taller as they get older, albeit unconventionally, and most adopted the natural mapping of height as represented vertically. More than 30% of students, however, had difficulties graphing the constant function "when you are 20 years old, you stop growing". The constant function was identified as a significant task feature requiring students to display two independent axes, one for each variable.

A number of other studies have proposed frameworks of levels of graph construction and interpretation. Some have involved generic levels that may be applied to univariate or bivariate graphs (e.g., Jones et al., 2000). Chick and Watson (2001) gave Grades 5 and 6 students an open-ended multivariate exploration task, and they identified three levels of interpretation and three levels of representation. Representation levels included (1) lists of data values with no attempt at aggregation, (2) univariate bar graphs, and (3) bivariate scattergraphs. Notably, levels of representation concerned the ways students engaged the data themselves, and as the open nature of the task allowed students to graph the complexity of data they chose, most students drew bar graphs. Wavering (1989) asked students in Grades 6 to 12 to construct graphs and then to identify the relationship in the data sets provided for each of three items, involving a positive slope, a negative slope, and an exponential curve. Nine hierarchical categories of response were distinguished. Categories 1 to 3 ranged from no response to pre-Cartesian graphs in which one-to-one correspondence was evident but the data were not ordered. Successive improvements in the use of scale were observed in responses in categories 4 to 7, so that each data point was represented not only by its numerical value, but also by its position along the axis. The final categories concerned verbal graph interpretation. The current research aimed to build upon these previous frameworks (e.g., Wavering, 1989), but with the view that correspondence and variation are complementary aspects involved in constructing coordinate graphs, rather than successively developed skills.

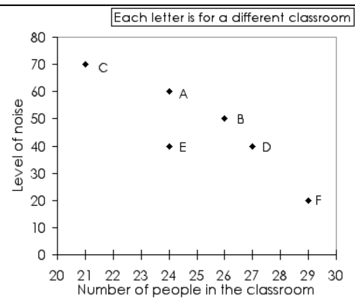
Background to the Current Study

The current study is part of a larger investigation into developing representations and understandings of statistical covariation. Other studies—in some cases involving the same students as those in the current study—have reported on students' numerical and verbal interpretations of graphs (Moritz, 2003, in press), and on students' graphical responses to illustrate verbal statements of an association between two variables (Moritz, 2000, 2002). Moritz (2002, in press) postulated that graphing verbal statements depends on two skills: *speculative data generation*, that is, suggesting data that illustrate a verbal statement; and *graph production*, that is transforming numerical data into graphical form. Moritz (2000) analysed graphs about heights and ages without distinguishing the two skills, and Moritz (in press) considered speculative data generation, but not graph production, for students' graphs about study times and test scores. The current study aimed to investigate coordinate graph production skills in isolation. The two aspects of (a) correspondence of values between two variables and (b) variation of values within each variable (Clement, 1989; Nemirovsky, 1996; Wavering 1989), evident in student responses for graphing verbal statements (Moritz, 2003, in press), are aspects relevant also to the construction of a coordinate graph from given data.

For the first task shown in Figure 2, asking students to graph six unspecified values to show that people who studied for more time got lower scores, four levels of speculative data generation have been identified previously (Moritz, 2002, in press) concerning the presence of correspondence and/or variation in responses. The Level 0 or *Nonstatistical* responses did not represent data. *Single Aspect* responses (Level 1) represented either correspondence or variation but not both. At Level 2, *Inadequate Covariation* responses involved both aspects but were not integrated appropriately. Finally, *Appropriate Covariation* responses (Level 3) integrated the correspondence and the variation of values. Of 130 students in Grades 3, 5, 7, and 9, 74 represented appropriate values at Level 3, although not always in a coordinate system: 13 used data tables, and 11 used graph forms described as series comparisons, in which both test scores and study times were aligned along the same axis across six cases, such as in two bar graphs (cf. Konold, 2002). It was argued by Moritz that development towards conventional coordinate graphs of bivariate data involved (a) ordering of data cases by one variable, and (b) maintaining correspondence of pairs of values in a direct graphic manner, such as by a single position in coordinate space as a cross-reference on two axes, rather than by indirect methods requiring the cross-referencing of two graphs.

Anna and Cara were doing a project on study habits.
They asked some students two questions:
"What time did you spend studying for the spelling test?"
"What score did you get on the test?"
Anna asked 6 students. She used the numbers to draw a graph.
She said, "People who studied for more time got lower scores."

Q2. Draw a graph to show what Anna is saying for her 6 students. Label the graph.



Some students were doing a project on noise.
They visited 6 different classrooms.
They measured the level of noise in the class with a sound meter.
They counted the number of people in the class.
They used the numbers to draw this graph.

Q6a. Pretend you are talking to someone who cannot see the graph. Write a sentence to tell them what the graph shows. "The graph shows...

Q6b. How many people are in Class D?

Q6c. If the students went to another class with 23 people, how much noise do you think they would measure? (Even if you are not sure, please estimate or guess.) Please explain your answer.

Q6d. Jill said, "The graph shows that classrooms with more people make less noise". Do you think the graph is a good reason to say this?
❑ YES or ❑ NO Please explain your answer.

Figure 2. Tasks to assess speculative data generation (Q2), verbal graph interpretation (Q6a, Q6d), and numerical graph interpretation (Q6b, Q6c) (adapted from Moritz, in press).

The second of the tasks in Figure 2 involved a data-gathering context and a scattergraph showing six data points configured such that noise level was negatively correlated with number of people in the classroom. Moritz (2003, in press) observed four levels of student responses for numerical graph interpretation and for verbal graph interpretation, including Level 0, or no relevant response, for both types of interpretation. For numerical graph interpretation, most secondary school students could read data values (Level 1) from the scales of either axis of the coordinate system, as could many primary students. Interpolation was more challenging, involving working with multiple data points (Level 2) or the whole data set (Level 3). Similarly, for verbal graph interpretation, many students provided a verbal statement that the graph showed a single variable (Level 1, e.g., "noise levels"), but fewer specified both variables (Level 2) and the appropriate relationship between them (Level 3).

The four-level hierarchies identified from these studies are all consistent with the SOLO taxonomy (Biggs & Collis, 1982), an assessment framework and general theory of development. The response levels include (1) prestructural responses, which do not involve relevant elements; (2) unistructural responses, which use one relevant element; (3) multistructural responses, which use multiple relevant elements without integration; and (4) relational responses, which appropriately relate relevant elements for the task. The generic nature of these descriptors allows them to be applied in the current study with respect to the relevant elements, particularly correspondence and variation, involved in constructing coordinate graphs when given data and a narrative context.

The Current Study

This study aimed to explore the different representations students draw to show change in temperature values over time. A number of issues were of interest. What levels of student response are observed? What other aspects distinguish categories of response? Do levels of response relate to levels of response observed on tasks related to speculative data generation and interpretation of a coordinate scattergraph? These questions and the issues arising from historical perspectives and previous research—such as tabular emphasis, reduction to a single variable, continuity, and axis allocation—were considered in formulating the following research questions.

1. What proportions of students in Grades 3, 5, 7, and 9 draw coordinate graphs when asked to graph bivariate data?
2. What structures do students construct as alternatives to appropriate coordinate graphs?
3. In what ways do students draw appropriate coordinate graphs of temperature change over time?
4. How do students' response levels for constructing coordinate graphs relate to the same students' levels for (a) speculative data generation in response to a verbal statement of covariation, and (b) interpretation of a coordinate scattergraph, when interpreting both verbally and numerically?

These questions were explored by examination of student-drawn graphs. Analyses of responses gave particular attention to how students structured relevant elements in their attempts to construct coordinate graphs.

Method

Participants

Students were surveyed from two Tasmanian private schools, one a boys' school and the other a girls' school. Both schools would be expected to draw students from a higher socio-economic group than the general school population in Tasmania. At each school, one class from each of Grades 3, 5, 7, and 9 was surveyed. A total of 133 students responded to the task in this study; no Grade 9 females responded to this task due to time constraints. Females described as fifth grade were from a composite class of Grade 4 and 5 students, with 13 students at each grade level. The surveys were administered near the middle of the school year, thus students within each class were likely to have had a common teaching experience for half a school year. Classes were selected by principals or other senior staff. Grade 9 students were from the highest ability class in the boys' school, based on availability of this class to undertake the survey with minimal interruption to the normal mathematics curriculum in the middle of the year.

Discussions with teachers indicated that teaching was in accord with Australian curriculum guidelines (AEC, 1991); in general, the primary students engaged with univariate graphs, and secondary students engaged with coordinate graphs. The Grade 3 females' classroom featured a large poster with a block graph of the students' birth months. The teacher of the Grade 3 males expressed surprise that her students demonstrated skills in graphing relations between variables that she imagined would be beyond their ability. Informal conversations with Grade 9 males indicated they had undertaken graphing in science classes as well as mathematics classes. Further evidence of the teaching experiences of the 133 students was provided by the responses of 80 of the students to a later question concerning whether they had drawn, seen, or not seen a graph like the coordinate scattergraph shown in Figure 2. Of 44 primary students, 17 claimed not to have seen such graphs, 18 to have seen them, and nine to have drawn them; whereas for 36 secondary students, only one claimed not to have seen such graphs, 12 to have seen them, and 23 to have drawn them. It should be noted that responses may not be an accurate measure of student exposure to coordinate graphs because (a) responses were self-reports; (b) it is not clear whether students were responding about a general coordinate structure, a scattergraph more specifically, or a graph about noise and number of people; and (c) students had drawn certain graphs in preceding tasks, such as the graph about temperature change over time, that may have influenced responses for their graph exposure.

Procedure

The task shown in Figure 3 was among a total of six or seven tasks in a written survey administered to students during class time. Prior to engaging with the task in Figure 3, introductory instructions and preceding questions provided a context of the general purposes of graphing that is likely to have influenced expectations about how to respond to the task. First, the author gave initial classroom instructions to draw graphs to show, both accurately and clearly, information requested in the question. Students were encouraged to represent data graphically, as might be shown in a poster, with passing reference to any posters in the classroom, but that students should not spend excessive time on colourful presentation. Second, two or three graphing tasks preceded that in Figure 3. Question 1 concerned graphing three statements related to height growth with age (Moritz, 2000), Question 2 concerned graphing a verbal statement about study time being related to test scores for six students (as shown in the first part of Figure 2) (Moritz, 2002), and Question 3 (for secondary students only) concerned graphing a verbal statement concerning motor vehicle use and heart death incidence (Watson, 2000). The impact of these preceding questions on responses to the current task was not assessed, however it is likely that most students were disposed to represent data graphically, aware of graphical potential to show general trends as requested in preceding tasks. Subsequent tasks included interpreting a scattergraph (as shown in the latter part of Figure 2) (Moritz, 2003, in press). Graphing tasks were placed before interpretation tasks to ensure exposure to the printed graphs in the interpretation tasks did not suggest a graphing method.

A science class was studying temperature. They used a thermometer to measure the room temperature every 5 minutes for 30 minutes.

First they turned a heater on for 15 minutes.
Next they turned the heater off for 10 minutes.
Lastly they opened the window for 5 minutes.

They wrote down these numbers.

Time (Minutes)	5	10	15	20	25	30
Temperature (°C)	15	20	25	25	25	15

Draw a graph to show how the temperature changed over time.

Figure 3. Task used to examine students' approaches to translating from numerical data to a graph.

For the task shown in Figure 3, students were presented with a context involving temperature in relation to various events, and with a data table including six numeric values of temperature and time. Students were asked to draw a graph to show how the temperature changed over time. The context of temperature was chosen as likely to be familiar from viewing television graphs (cf. Figure 1). The narrative context, which described turning the heater on and off and opening the window, was used not only to aid interpretation of the task, but also to allow students to decide which features of the narrative or the data table were most important to represent in a graph to show how the temperature changed over time. The data table involved discrete data points, but the context indicated these data were sampled from measures that were likely to be recognized as continuous, providing clues about intermediate values to represent gradual change. Six data points were chosen to match other questions as seen in Figure 2. The data included repeated temperatures for different times to ensure students were required to distinguish two variables, following studies reporting student difficulties with the constant function (Moritz, 2000; Nemirovsky & Tierney, 2001; Sherin, 2000).

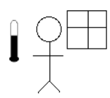
Student's responses to the task in Figure 3 form the primary data for this study. Student responses from the task involving graphing a verbal statement relating test scores and study times for six students (Figure 2) were categorized in four levels of speculative data generation (Moritz, 2002, in press). A subset of 98 students also attempted the task that involved interpreting, verbally and numerically, a scattergraph relating noise level and number of people (Figure 2) (Moritz, 2003, in press). This subset excluded many Grade 3 females and Grade 5 and Grade 7 males due to lack of time to complete this task. The data from this task—the levels of both numerical and verbal graph interpretation—were also used for the current study.

Coding and Analysis

Qualitative analysis of responses followed iterative clustering processes (Miles & Huberman, 1994). Survey responses were first clustered according to visual aspects alone. Many responses were clearly recognisable as pictures, tables, coordinate bar graphs, or coordinate line graphs. About 50 remaining responses were in forms that modified, combined, or omitted features from one of these basic forms, and these responses involved the greatest consideration for coding. The coding framework concerned identifying features to provide a lens to make sense of students' responses, as evidence of their understandings. Elements of the responses identified as relevant to the task included (a) conveying clearly and accurately the values of each data set, and (b) consistent use of spatial position to show variation of values, including ordering and scaling axes. Correspondence and variation emerged as overarching themes for coding, and sub-coding involved more specific issues such as graph form or axes allocation.

Clusters were successively refined, particularly for responses in which elements of conventional coordinate graphing were mixed with inconsistencies or omissions, such as poor labelling. A few graphs (e.g., Figures 7c and 7d) were initially clustered with coordinate graph responses, until the deficiencies of correspondence between the variables in these graphs emerged as common issues for other students, and they were assigned to a separate cluster. Common characteristics of clusters were identified to define categories of response.

Responses were assigned to one of four levels according to the degree to which they represented the information by making effective use of spatial features, based on the four levels of the SOLO taxonomy (Biggs & Collis, 1982), and closely related to those previously described for data representation (e.g., Chick & Watson, 2001; Jones et al., 2000), and for graph interpretation and speculative data generation of covariation between variables (Moritz, 2002, 2003, in press). The framework for analysis of responses is shown in Figure 4. *Nonstatistical* responses (Level 0) did not display the data, but included either elements of the narrative context (Category 0A) or a graph format (Category 0B). *Single Aspect* responses (Level 1) represented at least one data series, either as a table of corresponding values (Category 1C) or as a graph of a single data series (Category 1V). *Inadequate Coordinate* representations (Level 2) displayed both sets of values but did not use position in two dimensions to denote values of the two variables, either showing correspondence but lacking an ordered scale to show variation (Category 2C), or showing variation but lacking a direct correspondence of variables (Category 2V). *Appropriate Coordinate* representations (Level 3) used position in two dimensions to denote values of the two variables in bar graphs (Category 3C) or line graphs (Category 3V).

	Aspect of Covariation	
Response Level	Correspondence (C)	Variation (V)
0. Nonstatistical Given data values are not represented	0A. Context Picture or narrative shows the context  0B. Graph Form Axes or Lines drawn; some show irrelevant values	

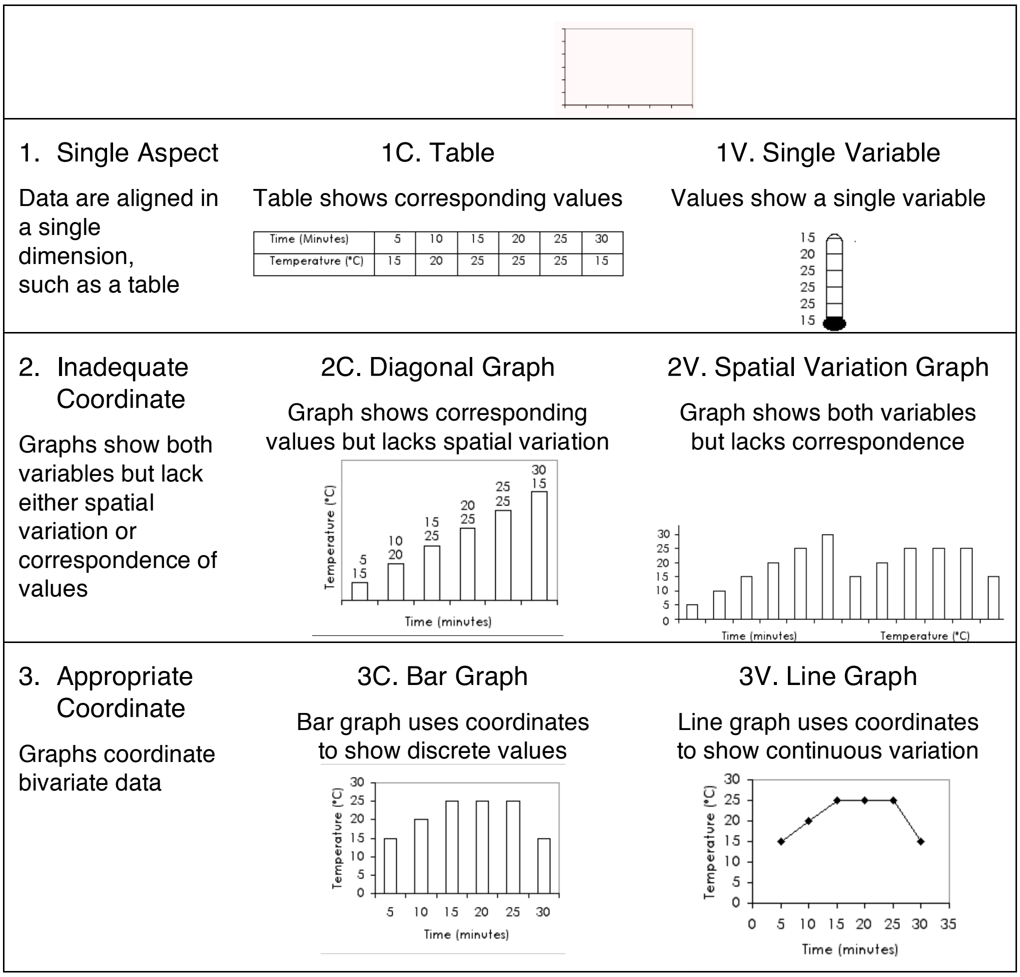


Figure 4. Categories of coordinate graph production according to response level and aspect of covariation. (Pictures are illustrative, and are not representative of all responses in the category.)

The sample involved a single class from each gender and year level, hence statistical tests between groups were inappropriate. Frequencies of responses at various levels and categories are reported for each class, so that each class can be considered as a whole with common teaching for the preceding six months. The associations of graph production response levels with levels of speculative data generation, numerical graph interpretation, and verbal graph interpretation, were considered by cross-tabulating frequencies for the available response samples and by calculating the Pearson product-moment correlation coefficient.

Results

Results are presented for the four research questions. Frequencies of student responses at various response levels and categories are provided. Examples illustrate each response category, with annotations to specify the grade and gender of respondents; for example "G9m" denotes a Grade 9 male.

Research Question 1: Proportions of Students Drawing Appropriate Coordinate Graphs

All Grade 9 students (26 students) offered coordinate graphs, most as line graphs (24) as shown in Table 1. Most Grade 7 students (22/33) drew coordinate graphs, either as bar graphs (13) or line graphs (9). Most Grade 5 students responded at Levels 1 (9/35) or 2 (15/35), and most Grade 3 students responded at Levels 0 (21/40) or 1 (14/40).

Table 1
Numbers of Student Responses at Coordinate Graph Production Levels and Categories by Gender and by Grade

Graph Production	Female Grade			Male Grade				Total
Response Level	3	4/5	7	3	5	7	9	
0. Nonstatistical	15	5	2	6	3	2	0	33
1. Single Aspect	4	9	0	10	0	3	0	26
2. Inadequate Coordinate	2	11	1	1	4	2	0	21
3. Appropriate Coordinate	0	1	8	2	2	14	26	53
Response Category (within response levels above)								
0A, 0B	9, 6	0, 5	0, 2	1, 5	2, 1	0, 2	0, 0	12, 21
1C, 1V	4, 0	4, 5	0, 0	10, 0	0, 0	0, 3	0, 0	18, 8
2C, 2V	0, 2	6, 5	1, 0	0, 1	0, 4	1, 1	0, 0	8, 13
3C, 3V	0, 0	1, 0	6, 2	2, 0	1, 1	7, 7	2, 24	19, 34
Total	21	26	11	19	9	21	26	133

Research Question 2: Students' Alternatives to Appropriate Coordinate Graphs

Students' alternatives to appropriate coordinate graphs appeared at the first three levels of the hierarchical framework. The character of these responses is illustrated in the following results, with examples.

Level 0: Nonstatistical. Thirty-three students gave Nonstatistical responses, in which the given numerical data were not displayed (see Figure 4). Twelve students offered *Context* responses, 10 in the form of a picture (e.g., Figure 5a) and two as a narrative. Twenty-one students drew *Graph Form* responses in which a graph or table format was represented but not with the given data. Of these 21, five drew axes or a graph frame without values (e.g., Figure 5b) and six included unlabeled inappropriate values. The remaining ten Graph Form responses included (a) five bar graph variants with each bar related to the on/off status of the heater and bar height implying temperature values that were not specified or were inappropriate (e.g., Figure 5c), (b) three attempts at coordinate graphs but with inappropriate or uninterpretable values, and (c) two tables of inappropriate values (e.g., Figure 5d). Graph Form responses did not show the data, but did show evidence of adopting the mechanics of graphing that are foundational for representing statistical data. As shown in Figure 4, Level 0 response categories did not emphasize either correspondence or variation, although some responses showed weak evidence of these emphases, as illustrated in the confused correspondence of two temperatures for given times in Figure 5d, or the visual impression of variation shown in Figure 5c.

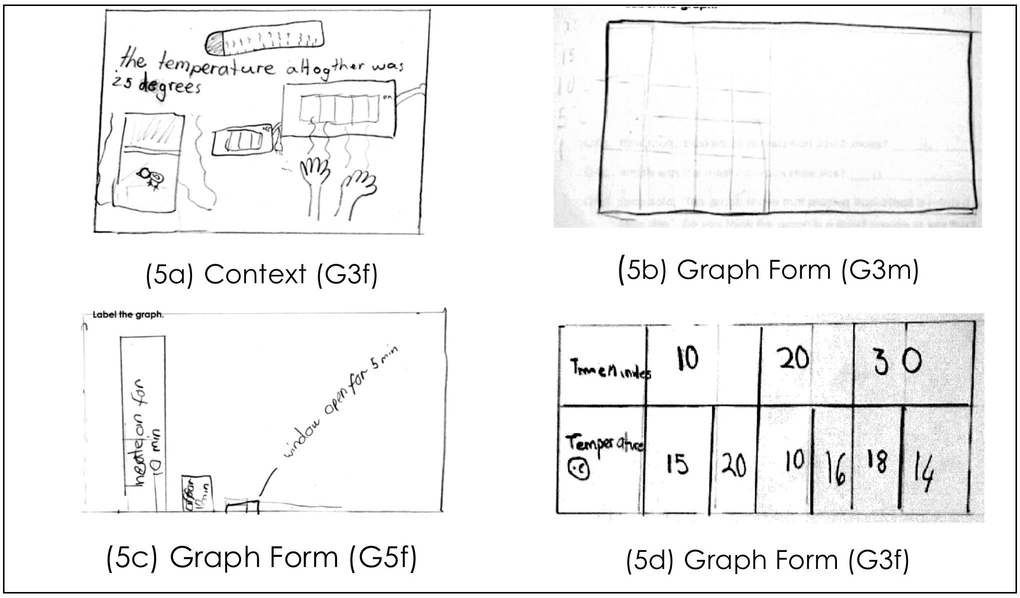


Figure 5. Student responses at Level 0: Nonstatistical.

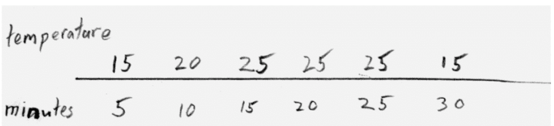
Level 1: Single Aspect. Twenty-six students represented numeric values of temperature, time, or both, but written values were aligned along a single dimension (see Figure 4). In all these responses, values were written numbers; that is, position was not used to indicate value.

Eighteen students produced a *Table* of the given data, even though a graph was requested. Eight responses involved minimal change from the table given in the task, six of which represented only the six data cases (e.g., Figure 6a), and two of which annotated information (e.g., Figure 6b). Ten students transposed the table such that it was aligned vertically rather than horizontally. Of these ten students, (a) five aligned the corresponding values on each side of a centre line appearing as an axis (e.g., Figure 6c); (b) two wrote the values on the same side of a vertical axis, one interleaving corresponding times and temperatures (Figure 6b) and the other listing all times then all temperatures; (c) two wrote the values within a grid structure like the table but aligned vertically; and (d) one wrote the values on two separate vertical axes but without bars, lines, or points to show the series of data values in the graph space.

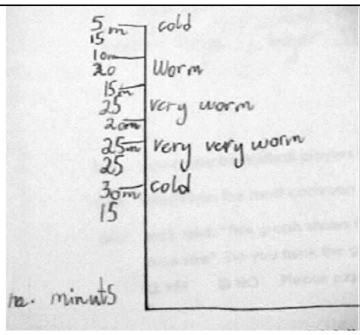
Eight students represented a *Single Variable*, seven showing temperature values (e.g., Figure 6d), and one showing time values. Of these eight students, (a) three drew rectangular axes but showed values for temperature only without accuracy in spatial positioning; (b) two drew rectangular axes with the values of one variable written along one axis without any values on the other axis or values represented in the graph space; (c) two drew thermometers (e.g., Figure 6d), one showing only distinct values of 15, 20, and 25; and (d) one drew a pie graph of distinct sector sizes with distinct temperature values colour coded.

Level 2: Inadequate Coordinate. Twenty-one students drew a two-axis system for representing the data, but lacking either consistent use of space to denote ordinal variation or clear correspondence between variables (see Figure 4). Because given time values were ordered and equally spaced, it could not be determined whether some responses were using position to denote time values, or merely as convenient space in which to write a list of categorically labelled values.

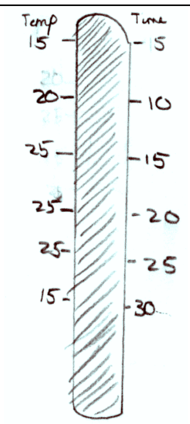
Eight students drew a *Diagonal Graph* in which bivariate data points were shown as a series of increasing bars with written values. These students preserved the one-one correspondence with a notion of two dimensions, however at least one dimension was not ordered, and thus written values rather than position were used to denote value. Two students wrote the values for both variables on bars in the graph space (e.g., Figure 7a); that is, they reproduced the table data in a diagonal layout, but within a graphical framework of two axes. One student wrote the values for times on the vertical axis, and the values for temperatures on the bars. The remaining four students wrote the values on the axes with increasing bars and values of temperature not in value order but in the order they appeared in the table; two placed temperature on the vertical axis (e.g., Figure 7b), and two on the horizontal axis.



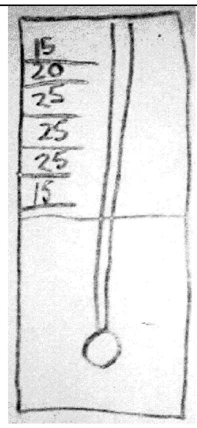
(6a) Table (G3m)



(6b) Table (G3m)



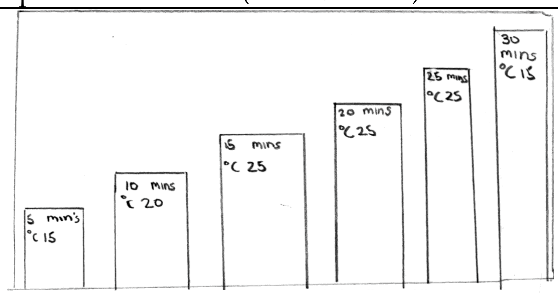
(6c) Table (G5f)



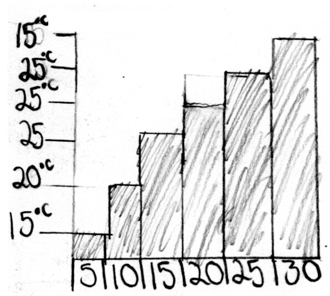
(6d) Single Variable (G5f)

Figure 6. Student responses at Level 1: Single Aspect.

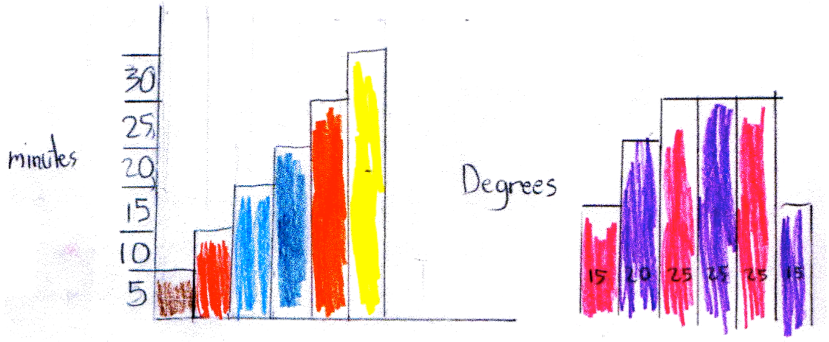
Thirteen students offered a *Spatial Variation Graph* in which temperature variations were shown spatially in the vertical dimension as a series of data values, and time was represented but not directly corresponding with the temperatures. Six students drew series-comparison graphs in which each variable was graphed as a series of data values (e.g., Figure 7c); the display of temperature variation gave the spatial appearance of a coordinate representation, however the correspondence with time was weak, relying on cross-referencing the ordinal position of the bars in two graph frames. Seven other students drew graphs resembling coordinate axis systems, but involving serious errors of (a) inconsistent spacing such as baseline violations, or (b) labelling problems that meant that time values could not be read with corresponding temperatures. An example of the latter is shown in Figure 7d: repeated scales resembling thermometers were used to represent temperatures, however one data value was missing and times were poorly represented by sequential references ("next 5 mins") rather than values (e.g., "15 minutes").



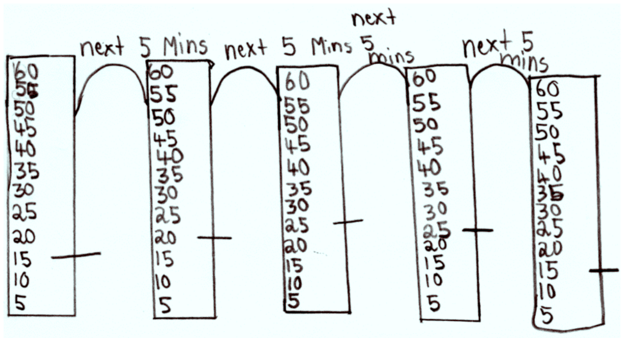
(7a) Diagonal Graph (G5f)



(7b) Diagonal Graph (G5f)



(7c) Spatial Variation Graph (G5f)



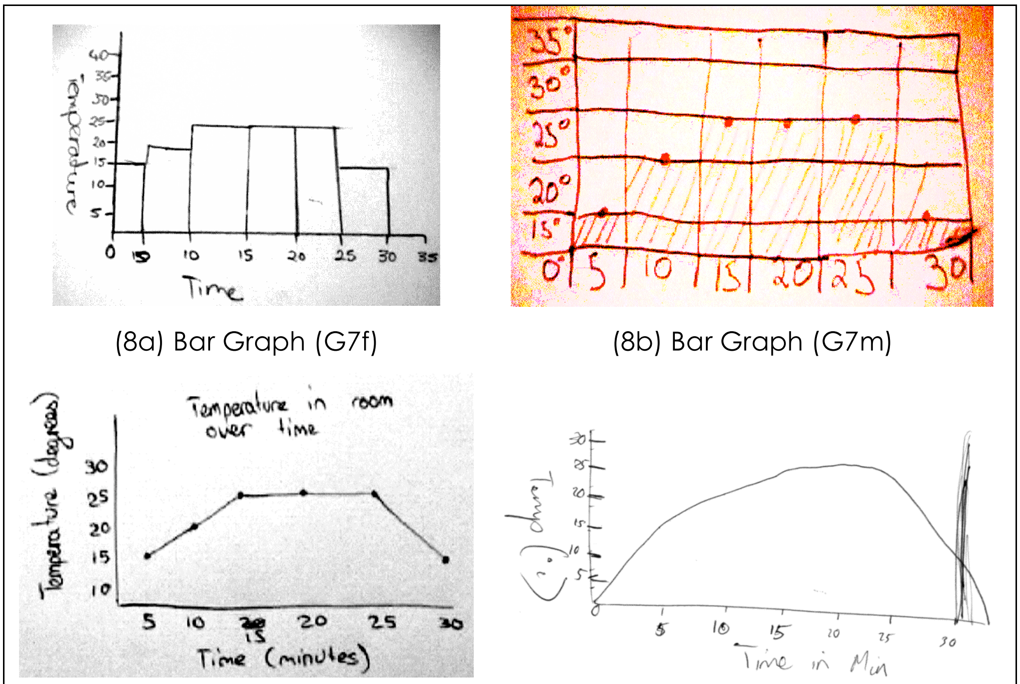
(7d) Spatial Variation Graph (G3f)

Figure 7. Student responses at Level 2: Inadequate Coordinate.

Research Question 3: Students Drawing Appropriate Coordinate Graphs

Level 3: Appropriate Coordinate. Students whose responses were classified at Level 3 were the ones who showed an appropriate coordinate graph to demonstrate the association of time and temperature. Fifty-three students employed a coordinate system to represent the bivariate data, in which each variable was ordered along an axis for which position was consistently ordered to denote value. Bar graphs showing discrete data emphasized correspondence of values whereas line graphs emphasized continuous variation of temperature with passing time (see Figure 4).

Nineteen students drew a *Bar Graph* within a Cartesian coordinate system, in which six discrete bivariate values were represented. All except one represented temperature on the vertical axis and time on the horizontal axis. Fourteen labelled "Time" and "Temperature" as well as data values (e.g., Figure 8a), whereas five had minor labelling omissions (e.g., Figure 8b), but the values or units distinguished the variables. Bars were adjoining for thirteen responses, whereas the remaining six had a small gap between bars. Fifteen students labelled times in the middle of bars (e.g., Figure 8b), and four on the right hand edge of the bar (e.g., Figure 8a).



(8a) Bar Graph (G7f)

(8b) Bar Graph (G7m)

(8c) Line Graph (G9m)

(8d) Line Graph (G9m)

Figure 8. Student responses at Level 3: Appropriate Coordinate.

Thirty-four students drew a *Line Graph* within a coordinate system, in which the six bivariate data values were shown, as well as intermediate values along the line. All thirty-four placed time on the horizontal axis. Twenty-nine of these emphasized the data points along the line (e.g., Figure 8c), whereas five drew lines without evidence of points (e.g., Figure 8d). Eleven responses used a line to indicate appropriate temperatures from time 5 minutes to 30 minutes, that is without making assumptions of temperatures outside of these times (e.g., Figure 8c). Fourteen responses assumed the line should start at the origin, six of which also made assumptions about the temperature after time 30 minutes (e.g., Figure 8d). Nine other students made another assumption about the temperature of time 0, five indicating a temperature of 10°, three 15°, and one 5°.

Research Question 4: Relationship of Responses to Those for Other Tasks

Students' response levels for speculative data generation for the task shown in Figure 2 were related to the levels of coordinate graph production for the task in Figure 3 ($r(131) = 0.46$, $p < 0.01$), as shown in Table 2. Of 133 students, speculative data generation response levels were higher, the same, and lower than coordinate graph production levels for 55, 62, and 16 students respectively, providing some evidence that high-level speculative data generation develops prior to high-level coordinate graph production. This can be observed further by collapsing across the low response levels (0 and 1) and the high response levels (2 and 3) for both variables: 37 students responded with low-level graph production and high-level speculative data generation, whereas only 7 students responded with high-level graph production and low-level speculative data generation.

There was a statistically significant association between verbal graph interpretation level for the scattergraph task in Figure 2 and coordinate graph production level for the task in Figure 3 ($r(96) = 0.56$, $p < 0.01$), as shown in Table 2. Of 98 students, verbal graph interpretation response levels were higher, the same, and lower than coordinate graph production levels for 25, 53, and 20 students respectively, indicating these skills may often be developed concurrently. There also was a statistically significant association between numerical graph interpretation level for the scattergraph task in Figure 2 and coordinate graph production level for the task in Figure 3 ($r(96) = 0.56$, $p < 0.01$), as shown in Table 2. Of 98 students, numerical graph interpretation response levels were higher, the same, and lower than coordinate graph production levels for 23, 40, and 35 students respectively, providing some evidence that higher levels of coordinate graph production are developed prior to higher levels of numerical graph interpretation. Notably, of 23 students who numerically interpreted the graph at Level 3 (interpolation using the trend), all except one drew coordinate graphs at Level 3 of graph production, whereas 18 students drew coordinate graphs (Level 3) but numerically interpreted at lower levels. In this sense, the skill to produce coordinate graphs appears often to be developed prior to (and possibly enables) the skill to interpret numerical trends on a graph, likely due to the common need to understand the coordinate system.

Table 2
Numbers of Student Response Levels for Other Tasks by Coordinate Graph Production Response Level

Response Level For Other Tasks	Coordinate Graph Production Response Level				Total
	0	1	2	3	
Speculative Data Generation (N = 133)					
0. Nonstatistical	8	3	2	1	14
1. Single Aspect	5	6	3	1	15
2. Inadequate Covariation	10	8	3	6	27
3. Appropriate Covariation	10	9	13	45	77
Verbal Graph Interpretation (N = 98)					
0. Nonstatistical	5	4	1	1	11
1. Single Aspect	7	7	4	3	21
2. Inadequate Covariation	6	5	12	7	30
3. Appropriate Covariation	3	2	2	29	36
Numerical Graph Interpretation (N = 98)					
0. Nonstatistical	5	4	2	0	11
1. Single Aspect	9	7	11	6	33
2. Inadequate Covariation	6	7	6	12	31
3. Appropriate Covariation	1	0	0	22	23

Discussion

The results for each research question are discussed with reference to curriculum recommendations and previous research. Implications arising for teaching and future research are also discussed.

Research Question 1: Proportions of Students Drawing Coordinate Graphs

Coordinate graphs were drawn by most secondary students and a few primary students, as shown in Table 1. This result is in line with curriculum expectations that recommend the teaching of coordinate graphing in upper primary school or junior secondary school (AEC, 1991; NCTM, 2000). The proportions of students drawing appropriate coordinate graphs were similar to those for drawing test scores versus study times (Moritz, 2002, in press), slightly lower for the primary students than comparable students observed for the context of height versus age (Ainley, 1995; Moritz, 2000), and slightly higher for the secondary students than comparable students observed for plotting height of a ball's rebound versus height of drop (Brasell & Rowe, 1993). It is speculated that the variables to be graphed are likely to affect success rates. Students in this study also came from an aboveaverage socio-economic group, although there is little evidence the primary teachers had provided students any learning experiences about graphs beyond normal curriculum expectations, and they gave some indication of underestimating their students' abilities. The diversity of levels and categories of responses offered within each class group for Grades 3, 5, and 7 suggests such tasks are accessible and challenging for these grades. If teachers offer students opportunities to graph in their own ways, the diversity of responses may provide a rich resource for classroom discussion.

Research Question 2: Students' Alternatives to Appropriate Coordinate Graphs

Students constructed a range of graph structures other than using coordinates appropriately. These responses were assigned to Levels 0, 1, and 2 of the framework in Figure 4. Thirty-three students offered nonstatistical responses (Level 0), in that they did not display the given data. Twelve students represented the context in a picture or narrative (Category 0A), illustrating students' attempts to communicate visually, and perhaps indicating confusion between graphs and pictures (cf. Brasell & Rowe, 1993; Sherin, 2000). Some represented the events involving the heater and window, attempting to show all they knew about the narrative context (Nemirovsky & Tierney, 2001) rather than focussing on the data. A further twenty-one students drew a graph form (Category 0B). The use of a graph form provides evidence that students are aware graphic elements may be used to show data. The reasons such responses did not display the given data could be discussed in the classroom to determine whether students were attempting to represent the given data but encountered a difficulty in doing so, or whether they had no intention of graphing the data but merely are supplying "a graph" as a response to the task. In some cases, students displayed data that appeared fictitious, with little resemblance to the data values given. It is likely this was a consequence of preceding questions in which students were provided verbal statements and asked to represent speculative data (Moritz, 2002, in press).

Responses that showed a single aspect (Level 1) displayed either correspondence of values in a table with no use of graphic position to denote value, or variation of values within a single variable graph. A table of data (Category 1C) was drawn by 18 students, similar to the lists of values observed by Chick and Watson (2001). Although it is not possible to discern with confidence students' intentions or abilities from their survey responses alone, this result would appear to suggest that these students did not see a purpose in graphically representing data when a table would suffice to show the values (Roth & McGinn, 1997). A preference for tabular data to show values has a strong tradition in the early history of graphing (Biderman, 1990) and persists today in academic circles—for example, the presentation of results in this paper, as seen in Tables 1 and 2. Students should be encouraged to discover the value of graphs for showing change, or as Playfair put it, "gradual progress and comparative amounts, at different periods" (Funkhouser, 1937, p. 281). It may be, however, that the notion of "gradual progress" of trends is not simple to master; students may be able to read discrete data points in a coordinate scatterplot, but fail to identify a trend when asked to verbalise it or interpolate (Moritz, 2003, in press). The idea of "comparative amounts," however, may be accessible from iconic forms such as two thermometers side-by-side, used by some students. Notably, in his youth, Playfair also represented temperatures on a divided scale according to the literal height on a thermometer, forming the basis for his "lineal arithmetic" and development of graphing (Biderman, 1990). Eight students drew a single variable (Category 1V), a phenomenon observed previously (Chick & Watson, 2001; Mevarech & Kramarsky, 1997). Seven of these represented only temperature; apparently this was considered to be the measured variable of interest, whereas time was implicit or irrelevant. In the classroom, asking students what a bar or point shows, and importantly asking for the corresponding time value, may alert students to the missing variable.

Difficulties that students encountered in representing conventional coordinate graphs were evident in the 21 responses offered at Level 2, most of which were drawn by Grade 5 students (see Table 1). Eight students offered diagonal graphs that showed the correspondence of bivariate values within two-dimensional space, but they failed to use the graph space to show variation in values. In these responses, values have to be read from labels rather than inferred by graph position, in some cases more closely resembling a diagonally-arranged table than a graph. Similar responses have been reported by others (Mevarech & Kramarsky, 1997) who observed students representing increasing functions irrespective of task descriptions. In this case, however, the discrete data values represented indicate students attended to representing a series of bivariate points rather than planning the structure of the graph to show spatial variation.

Thirteen responses, also at Level 2, displayed spatial variation but violated some aspect of correspondence; all thirteen showed a form of the inverted-U shape of the series of temperature values, but some omitted some data values or labels that could have clarified the correspondence of the variables. In particular, six of these students represented the temperature and time values in separate series of bars (e.g., Figure 7c), previously described as series-comparison graphs (Moritz, in press) or ordered case-value bar graphs (Konold, 2002). In this study, such responses were considered inadequate for showing coordinates as they failed to show the correspondence of the *n*th bar in one series with the *n*th bar in the other series. In other tasks, a linking case identifier was available or invented that established the correspondence, such as using students' names to identify cases (Konold, 2002; Moritz, in press), or calendar year to link the incidence of heart deaths with data about the use of motor vehicles (Watson, 2000).

An important result of the analysis framework at Level 2 was the identification of correspondence and variation as important features in constructing a coordinate system, aspects identified by others researchers (e.g., Clement, 1989; Nemirovsky, 1996; Wavering, 1989). In this respect, the framework more closely resembled frameworks from studies of interpreting and generating data-based covariation (Moritz, 2002, in press) than other frameworks of graph construction (Chick & Watson, 2001; Jones et al., 2000). The importance of both correspondence and variation was also used to assign both tables of data (Category 1C) and single variable responses (Category 1V) to level 1, in contrast to Chick and Watson (2001) who considered univariate graphs at a higher level of graphing than lists of values.

Research Question 3: Students Drawing Appropriate Coordinate Graphs

Of 53 students who drew coordinate graphs (Level 3), 19 represented discrete values in bar graphs, whereas 34 drew line graphs. The reasons that students drew bar graphs or line graphs are not evident from their responses alone: for example, perhaps each student was familiar with only the one graph form of the response.

The clear majority of Grade 9 males who responded with line graphs, however, probably indicates the effects of a common teaching experience in mathematics and science classes. Brasell and Rowe (1993) asked physics students to graph discrete data about the height of bouncing balls and sought lines of best fit rather than the line graphs connected point-to-point that many students provided. In the current study, however, temperature was a function of the gradual passing of time. In this sense, the graphing task resembled those used in algebra teaching of functions as much as tasks of discrete data set representation often used in school statistics teaching (AEC, 1991; Ministry of Education, 1992; NCTM, 2000). This context meant that representation of intermediate values between known points using a line was reasonable. Teachers must be alert, however, to challenge assumptions that might be associated with graph forms, such as lines beginning at the origin. Equipped with examples of various bar graphs and line graphs as observed in this study, classroom discussion could address the assumptions and implications conveyed in each format.

Of the 53 students who drew coordinate graphs, 52 represented temperature on the vertical axis and time on the horizontal axis. The overwhelming preference for conventional axis allocation may indicate exposure to this allocation, particularly with time on the horizontal axis, in news reports (e.g., Figure 1) or other experiences such as from science classes. The inversion of axes by students observed in previous studies may have involved variables that did not lend themselves to natural mapping; for example, in the study of Brasell and Rowe (1993), both variables concerned heights of the ball, before and after bouncing. In contrast, Ainley (1995) and Moritz (2000) observed most students showed height and age of a person on the vertical and horizontal axes respectively. In the current study, a few responses at various levels showed temperature in a thermometer form (e.g., Figures 5c, 5d, and 6d), giving some indication of one reason for assigning temperature to the vertical axis.

Another reason for the preference in axis allocation may be that the students drawing coordinate graphs appreciated the nature of the covariation evident in the constant temperature segment or in the implied dependence of temperature on time. This possibility is supported by the observation that this preference was less evident at lower levels where covariation was lacking. Of students who drew inadequate coordinate graphs (Level 2), many created spatial variation graphs that assigned both variables to the vertical dimension, some drew diagonal graphs that assigned variables to the values within the graph space rather than axes, and the few who did assign one variable to each axis were evenly split in their allocation. Diagonal graphs at Level 2 notably did not acknowledge visually the constant segment, and spatial variation graphs did not make clear the dependence between the variables. It may be that for students who drew coordinate graphs, the constant segment or other repeated temperature value cued students to represent the repetition horizontally rather than vertically. Horizontal repetition retains the common visual property of line graphs involving no vertical repetition, that is temperature as a function of time such that each time had only one corresponding temperature value. In this way, the constant function may help not only to identify student difficulties in coordinate graphing (Mevarech & Kramarsky, 1997; Moritz, 2000), but also to assist in drawing attention to the covariational nature of the bivariate data to be represented in coordinates. The reasons for axis allocation could be explored in future research using tasks contrasting those with and without (a) natural mappings of height or time, (b) variables with self-evident dependency, and (c) data sets with constant segments or repeated values.

Research Question 4: Relationship of Responses to Those for Other Tasks

Students' response levels for graph production were moderately related to their levels for speculative data generation, verbal graph interpretation, and numerical graph interpretation (see Table 2). There was some indication in these data that the skill of speculative data generation—involving data handling with a sense of global covariation—may develop prior to the skill to produce a coordinate graph to represent such covariation. This finding follows Chick and Watson (2001) in suggesting that the ways students structure data are the bases for classifying levels of representations. Verbal graph interpretation was observed to develop roughly concurrently with coordinate graph production, suggesting that the global sense needed to produce structured coordinate graphs also underlies global interpretation in verbal statements. Coordinate graph production, however, appears in some cases to develop prior to the skill of numerical graph interpretation to interpolate values based on a trend shown in a coordinate graph, with reference to the numerical scales. This suggests it may be most effective to begin teaching of coordinate graphing with hypothesizing relationships between variables, which introduces a clear purpose to undertake data collection and exploratory data analysis. Experiences could also include production and interpretation of qualitative graphs without specific numbers (e.g., Mevarech & Kramarsky, 1997; Moritz, 2000). Students should be encouraged to engage and represent data in ways meaningful to them rather than, or at least prior to, emphasizing the mechanics of conventional graph construction, such as scaling axes. When graphs are to be interpreted by others, however, there may be good reason to adopt (or adapt) conventions, such as coordinates. Interviews conducted with a subset of students of the current study indicated that some students could not interpret *their own* graphs a few weeks after drawing them, emphasizing that their graphs were not self-evident. In classrooms, discussion with students about interpreting others' graphs could explore the need for certain graph features or conventions in order to communicate effectively.

Conclusion

The issues involved in graphing data include both cognitive aspects concerning ways of structuring the complexity of series of data values for multiple variables, and purpose-driven aspects concerning decisions about what is relevant to represent and for which audience. In some cases, students may judge the contextual narrative more significant to represent than the data values, and in other cases a data table may be preferred to show the exact values. Future research and classroom teaching, involving discussions in interviews or in classroom settings, may further distinguish students' reasons for drawing graphs for tasks concerning time-based functional relationships or discrete value covariation. By engaging in graphing tasks that involve consideration of both values and trends, and by discussion of a variety of graphic methods, students should learn the power of coordinate graphs to represent both correspondence of values and variation trends. The levels and categories identified in this study should help teachers to identify the issues students are encountering and the components of data handling for coordinate graphs that may be further developed.

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