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Learning through Teaching: A Case Study on the Development of a Mathematics Teacher's Proficiency in Managing an Inquiry-Based Classroom

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This study examined the development of a mathematics teacher's proficiency in managing whole-class discussion in the context of an inquiry-based classroom. We analysed three lessons taught with the same class by a teacher-researcher. The first and second lessons were 10 months apart, the second and third lessons were 6 months apart. For each of the three lessons the analysis was carried out at two levels: macro-level analysis was applied to the general organisation of the inquiry-based lesson and micro-level analysis was applied to both the teacher's structure of the discussion and to the quality of the discussion. Based on the two-level analysis we formulate criteria to define teacher proficiency and demonstrate the enhancement of proficiency through teaching.

The learning through teaching (LTT) phenomenon is often addressed in research literature. LTT has been shown to be a critical component of teachers' life-long learning (Day, 2001), and under the circumstances of educational reform it has a special significance for new and experienced teachers alike. Ma (1999) noted its significance and Cobb and McClain (2001) "were surprised at the extent to which the classroom constituted a learning environment for [a] teacher" (p. 208) during a year-long experiment. In his analysis of mathematics teachers' awareness, Mason (1998) reported that many people have discovered that "it is only when you come to teach something that you really come to understand it" (p. 260). Lampert and Ball (1999) maintained that teachers in their first years of teaching generally believed that they learned to teach from experience. For Franke, Carpenter, Levi, and Fennema (2001), teaching is a 'generative' process in the course of which teachers accumulate knowledge.

Although experience and expertise are not equivalent terms, teachers' expertise is usually considered a function of their experience (Darling-Hammond & Youngs, 2002). Research that compares and contrasts the knowledge and skills of experienced and novice teachers (e.g., Berliner, 1987; Leinhardt, 1993; Roth, 1998) supports this connection and provides additional evidence of LTT. From observation of cyclic models of teaching (e.g., Artzt & Armour-Thomas, 2002; Simon, 1995; Steinbring, 1998), a teacher's ongoing improvement may be anticipated and reflective mechanisms put in place for such an improvement. Teachers' ability to learn from reflection is amply documented in action research studies (Edwards & Hensien, 1999; Jaworski, 1994; 1998; Roth, Masciotra, & Boyd, 1999; Schifter, 1996; Wood, 2005).

LTT research concentrates primarily on teachers' continual inquiry into students' thinking and learning (e.g., Franke et al., 2001; Schifter, 1998). However, Schifter (1998) points out that "there has been insufficient appreciation of how the analysis of student thinking can, itself, become a powerful site for teachers' further mathematical development" (p. 57).

Despite the significance of LTT, the educational community has relatively limited understanding of changes in teachers' knowledge and skills and of how these develop in authentic classroom situations. For example, Mason (2002) proposed that something more than experience is needed in order to learn from experience. In an effort to investigate the nature of that 'something', the first author focused her research on understanding how and what teachers learn from teaching their students (Leikin, 2005a, 2005b, 2006; Leikin & Kawass, 2005). In particular, Leikin (2006) analysed the transformations of teachers' intuitions into knowledge and beliefs in the field of mathematics and of pedagogy and, in another study, Leikin (2005a) analysed mathematics LTT using the case of an experienced secondary school mathematics teacher. She explained this learning by the interactive processes that took place in the classroom, and by the teacher's flexibility, attentiveness, and critical thinking.

The present case study is part of this series of practitioner-oriented studies. It draws on the teaching of the second author of this paper. We begin with the understanding that teachers' knowledge, intuitions, beliefs, and skills are the main prerequisites for teaching proficiency and focus our attention on the teacher's management of a whole-class mathematics discussion. We aimed to address Nelson's (2001) concern about "the need to develop more detailed and refined descriptions of pedagogy" (p. 267). In particular, we were interested in extending our understanding of teachers' pedagogy in a whole-class discussion. In response to a claim by Lampert and Ball (1999) that mathematics education lacks a practical and theoretical language for communicating about teachers' activity, this study proposes a vocabulary for connecting theory and practice in mathematics teaching.

Inquiry-based Instruction and Teacher Proficiency

Facilitative teaching, in general, and the inquiry-based classroom, in particular, are central to many reform-oriented curricula that establish, strengthen, and broaden conjecturing, exploration, and investigation procedures in the mathematics class (e.g., Ball, 1992; Jaworski, 2003; Lampert, 1990, 2001; Lampert & Ball, 1999; Schifter, 1996; Wood, 1999; Yerushalmy, Chazan, & Gordon, 1990). Inquiry-based mathematics classes are consistent with a constructivist view of knowledge and learning, as they offer challenges to stimulate mathematical thinking and create opportunities for critical reflection on mathematical understanding (Cobb, Wood, & Yackel, 1990; Jaworski, 1994, 2003; von Glasersfeld, 1996; Wells, 1999). In inquiry-based whole-group discussion, students and their teacher share their roles (Forman & Ansell, 2002; Lampert, 2001; Nelson, 2001), and the success of this practice depends heavily on the teacher's knowledge and skills.

We focus our investigation on LTT in an inquiry-based classroom. In this challenging context mathematics teachers are expected to be flexible, ready, and able to change their hypothetical learning trajectories in order to connect lesson development to their students' conjectures (Leikin 2005a; Leikin & Dinur, 2003; Simon, 1997; Wood, 1999, 1998). Classroom discussion, as an integral part of an inquiry-based environment, requires the development of particular teaching skills. In a longitudinal study on the development of mathematical argumentation in elementary school classroom Wood (1999) pointed out that a productive mathematics discussion was characterised intellectually and socially by the "manner in which the teacher created an environment in which children were expected to participate in the examination, critique, and validation of their mathematical knowledge through reasoned discourse" (p.188). To manage productive mathematics discourse the teacher must create a social context that supports students' conceptual development (Cobb, Wood, & Yackel, 1990; Lampert, 2001; Wood, 1999, 2001). The creation of this context provides teachers with opportunities for learning in the classroom (Wood, 2001).

In this study, lesson organisation is one of the indicators of the teacher's proficiency in the classroom. Its importance for the learning process is well known (e.g., Artzt & Armour-Thomas, 2002; Lampert, 2001; Leinhardt & Greeno, 1986; Stigler & Hiebert, 1999). Lesson organisation may be considered as consisting of several phases usually constituting a sequence of teaching-learning experiences (Artzt & Armour-Thomas, 2002; Wood, 1999). As the nature of the mathematical task shifts toward exploration, the organisation of mathematics lessons, and the structure of the whole-class discussion, in particular, being to change (Forman & Ansell, 2002; Lemke, 1985, 1990; Mehan, 1979; Wells, 1999). Therefore, in this study, discussion structure constitutes an additional indicator for teacher skilfulness. We probed the development of one teacher's proficiency in an inquiry-based class.

The Study

The Purpose and the Questions

Shelly, the teacher-researcher, initiated facilitative teaching (Nelson, 2001) in the third year of her teaching and introduced her students to an inquiry-based learning environment. The purpose of the current study was to analyse outcomes of LTT that Shelly experienced in two years of managing inquiry-based mathematics. As noted above, lesson organisation and discussion structure served as two main indicators for the analysing Shelly's proficiency. We based our exploration around two research questions:

(1) How does the organisation of inquiry-based lessons change over time?
(2) How does the structure and the quality of discussion change over time?

Methodology

The case study considered the process of Shelly's LTT as a continuous process of solving pedagogical and mathematical problems (for the use of case study as an investigation of learning, see Patton, 1990). According to Stake (2000), a case study incorporates observations and analyses of human activity in a certain place and time. Our study investigated a particular period in the development of Shelly's career.

In addressing the first research question we relied on the categories of lesson organisation described in the literature. We consider introduction, investigation, and whole-class discussion as the main stages of the inquiry-based lesson (Artzt & Armour-Thomas, 2002, Wood, 1999). To answer the second research question we described categories named "discussion actions" (see below for details) following a qualitative research paradigm rooted in the grounded theory approach (Strauss & Corbin, 1990). We analysed Shelly's classroom discourse and sought to qualify discussion actions as they emerged from the analysis of the three lessons. The categories were formulated initially when we analysed the first lesson. We then refined the categories when they were applied to the second and the third lessons. The fact that one of the researchers was also the teacher whose practice was being investigated called for special sensitivity to her roles as both teacher and researcher. Her role as a teacher strengthened the reflective elements of the analysis. At the same time, we tried to minimise the effect of her experience as a mathematics teacher on her views as a researcher. In developing categories of discussion actions we were mindful of the extensive research that addresses whole-class discussion (Forman & Ansell, 2002; Lampert, 2001; Sfard, Nesher, Streefland, Cobb, & Mason, 1998).

The Teacher

Shelly had been a beginning primary school mathematics teacher during the three years preceding the experiment we analyse. During these years, she was dissatisfied with several features of her communication with the students. Class routines were not always sufficiently crisp; questions and needs of some of the students were not always answered or addressed; lessons were not sufficiently structured; the objectives of the lessons, usually well defined, often remained unachieved. Moreover, Shelly was disappointed with the learning outcomes: the answers, achievements, and the pace of progress of her students.

In the second year of her teaching she contacted the mathematics educator who coordinated the mathematics program at her school and together they organised a development team specializing in materials for an inquiry-based mathematics class (Friedlander, 1997; Friedlander & Rota, 1996a, 1996b, 1997). When the first version of the materials was developed, at the end of the third year of her teaching, Shelly started experimental implementation of these materials. While developing the materials, team members discussed the mathematics of student activities and the importance of different types of class organisation: a small-group approach to tasks and a whole-class discussion were clearly integral parts of the activity. Our study explored the development of Shelly's proficiency in the course of the 15-months teaching experiment (following Cobb, 2000; Cobb et al., 1990), which was intermittently videotaped. We subsequently carried out the analysis of the videotaped lessons.

The Data

Overall, 21 of Shelly's lessons were videotaped for purposes of formative evaluation of the teaching materials. She appeared to be at ease in front of the camera. At that time she was not aware of the possibility of using these data for the current study. This paper reports on three of the 21 lessons with the same group of students based on the following criteria: the lessons had similar intended structure and videotapes were sufficiently clear to perform content analysis. The first of the selected lessons was videotaped when the students were in the middle of 2nd grade, the second when they were at the beginning of 3rd grade, and the third when they were at the end of 3rd grade.

Intended lesson organisation. Lesson organisation included three main types of activities: (a) introduction, consisting of teacher actions aimed at establishing the students' readiness for investigation; (b) investigation, during which the students tackled a task individually or in small groups, recognising relationships in the object under investigation, conjecturing, and constructing new meanings; (c) whole-class discussion, which enabled students to present their ideas generated at the stage of investigation, to integrate their ideas, and to attempt to reach a shared meaning. At the introduction stage Shelly presented students with the problem. During the second stage students were expected to work in small groups and Shelly helped them by answering questions and by giving clues. During the last stage students were required to present the results of their investigations to the class, discuss them, compare different solutions of the problem, and to negotiate the meaning of the solution. Shelly's role at this stage was to help students make progress in their presentations, generalise the results, and end the lesson with a shared mathematical meaning. Mathematical tasks for all three lessons included a series of questions to be investigated by the students. Below are the mathematical tasks given to students in the three lessons considered in this paper.

First lesson. The task was to investigate the relationship between a graph of discrete points and their common property (from *Dice Activity*: Friedlander & Rota, 1996a, see Figure 1). The investigation was carried out in two ways: first, by drawing graph points on a coordinate plane, given a common property of the points obtained by throwing a pair of dice, real or imaginary. Examples of these properties include 'one of the dice always shows the same number', and 'the sum of the two dice is always the same." The investigation was also carried out by identifying the common property of the points from observing the graph. In this paper we refer to the discussion surrounding this latter activity, after the class had discussed the former.

Write down the results marked by the points on the boards in figure 1.
Try to describe all the marked points.

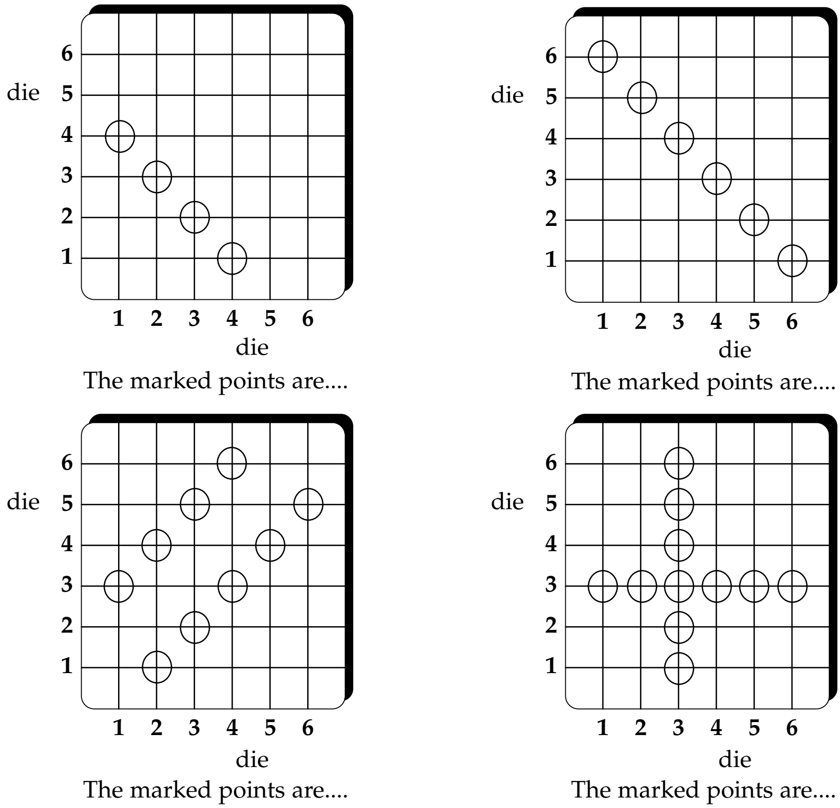


Figure 1. Task for the first lesson.

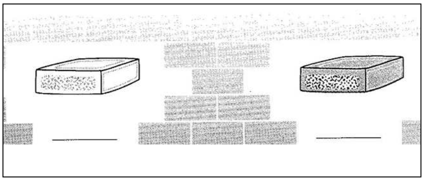
Second lesson. The children were asked to find mathematical connections (patterns and rules) in a calendar and other tables of numbers (from *A Calendar:* Friedlander & Rota, 1997, See Figure 2).

4	**3**	**2**
11	**10**	**9**
18	**17**	**16**

1. Divide the 9 numbers into 3 equal groups that have an equal sum.
2. Can you divide the 9 numbers in a different way?
3. Choose another square with 9 dates on the calendar. Check whether you obtain equal sums in this square if you divide it the same way you did before.

Figure 2. Task for the second lesson.

A. There are 21 matches in the two boxes. How many matches are hiding in each box in each of the following cases?



A1. In the white box there are 5 matches more than in the dark box.

A2. In the white box there are 3 matches fewer than in the dark box.

A3. In the white box there are half as many matches as in the dark box.

B. There is a total of 16 matches in several boxes. Each box of a given colour (white or dark) contains the same number of matches. How many matches are hiding in each box in each of the following cases? Find several answers for each case.

B1: There are 2 white boxes and 2 black boxes.

B2: There are 2 white boxes and 3 black boxes.

Figure 3. Task for the third lesson.

Third lesson. Students were asked to find an unknown number of matches in two boxes based on certain conditions (from *Matches Activity*: Friedlander & Rota, 1997. See Figure 3).

Method

To code the videos we moved beyond our individual impressions in identifying discussion actions (Stigler & Hiebert, 1999). The data of the three lessons were transcribed, watched multiple times, and discussed to refine the categories and the role of the interpretations, explanations, and meanings that were elicited. Because teaching is an interactive process we found it necessary to analyse the development of the students' discussion actions in order to obtain an indication of the teacher's professional development (e.g., Edwards & Hensien, 1999).

A set of categories emerged to describe the development of Shelly's proficiency in the course of the teaching experiment (see below). The two authors of this paper and an independent researcher coded ten minutes of the transcripts, with inter-coder agreement of 84%. We discussed the remaining 16% of the utterances to reach agreement on the coding.

We provide an analysis of the data on two levels: (1) a macro-level analysis addresses lesson organisation; (2) a micro-level analysis considers the teacher's and students' discussion actions in the discussion phase of the lesson. Analysis of the students' discussion actions served both as the setting for the development of the teacher's proficiency through practice and as an indication of it. We used the metaphor of teaching as problem-solving and applied Schoenfeld's (1985) time-line presentation of protocol analysis in problem-solving research to both macro-level and micro-level analyses (see Figures 4 and 6). Using these diagrams we analysed the distribution of discussion actions between teacher and students, the distribution of the teacher's discussion actions among different categories, and the frequency of actions. Additionally, the second level of analysis addressed the quality of Shelly's discussion actions, and focused particularly on her questioning. We used "questioning quality" as an indication for teacher proficiency (see Roth, 1998).

Results: LTT

Changes in Lesson Organisation

Figure 4 depicts the actual lesson organisation for each of the lessons. The three types of class activities (introduction, investigation, and whole-class discussion) are presented on the vertical axis; their occurrence during the three lessons is shown along the time line (1minute as a time unit on the horizontal axis). The organisation for all three lessons is presented in the same figure for ease of comparison between the lessons.

Figure 4 shows changes both in the sequence in which class activities were performed and in their time distribution. The main change occurred between the first and second lessons. The second and the third lessons were better organised, and revealed sharp boundaries between different activities and a clear correspondence between the actual and the intended lesson organisation. Note that the task presented to the students in the second lesson was longer than the task in the third lesson. In the second lesson Shelly summarised the discussion twice, once to ascertain the students' progress in the investigation and once again to conclude work on the tasks. In the third lesson students worked on two shorter tasks, and hence the diagram representing the organisation shows the introduction-inquiry-discussion pattern twice.

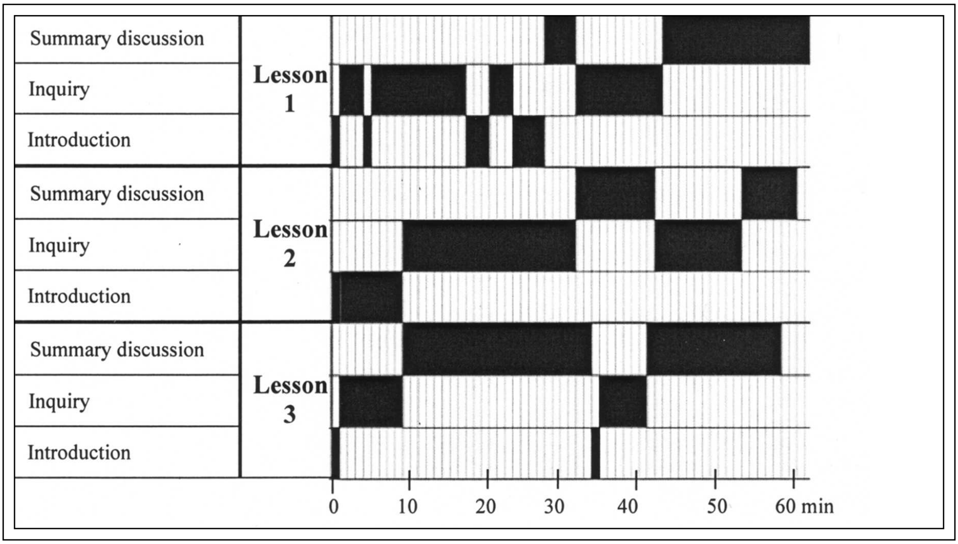


Figure 4. Organisation of the three lessons.

Below are our observations about the differences between the intended and the actual organisation of the lessons.

Introduction. In the first lesson Shelly introduced the task to the students four times because apparently it was not clear to them. Our analysis shows that the introduction of the new task was not connected with the students' previous activities and their existing knowledge. By contrast, in the second and the third lessons new tasks were presented to the students with a clear connection to relevant learning materials studied earlier. In the first lesson students did not feel they could cope with the task on their own and asked the teacher for help several times. In the first lesson Shelly talked about the preceding activity without asking the students to look in their notebooks or showing them the activity.

> Shelly: Last week I asked you to do something. Do you remember what I asked? To mark with an "_" all the points... Remember? Now it is exactly the opposite. They give you a board with the points and ask you to give a description of all the points marked. [see Task 1]

This introduction was unclear to the students. They did not understand the meaning of "it is exactly the opposite" or how the task they had performed the previous week might help them to deal with the new one. They were not able to connect the mathematics of the two lessons and felt that the task confused them. They raised their hands and asked questions like:

> Asaf: What should we do here?

Or

> Ben: Here I can't find a sum, so what is the answer?

The videotapes show that in the second and the third lessons the students started to solve the problem immediately after its introduction; they asked few verifying questions and did not ask for Shelly's help (Figure 4). The following excerpts demonstrate changes in the introduction of the tasks. The way in which Shelly reminded her students about the activities performed in preceding lessons was clearly different from the first lesson. This time she connected the tasks to the previous activities by repeating the instructions of the preceding task and asking the students to look in their notebooks to check how they did it.

> Shelly: Please open at the pages we worked on in the last lesson. Look at the first page. Let's try to remember what we did here. On the first page there is a picture of a calendar page with a square of nine numbers marked [see Task 2]. You were asked to mark some other square of nine numbers and find interesting ideas about the numbers in the squares. Who would like to remind us what he or she found?

Shelly allowed her students recall what they did during the previous lesson. The connection was clear and the students approached the new activity as a continuation of the earlier one.

The Inquiry. The inquiry in small groups during the first lesson was interrupted several times because Shelly recognized the students' confusion and decided that they needed an additional introduction of the task. The video shows that all through the inquiry Shelly did not allow students to do independent work and often interrupted their work in small groups. Her questions and comments were intended to direct the students' investigation in the direction of the initial plan, as seen in the following dialogue between Shelly and Tomer.

Shelly: [looking in Tomer's notebook and pointing to his written notes] This is wrong.

Tomer: What's wrong?

Shelly: This is not what you were asked to do. You must write something like "All the numbers are smaller than or greater than."

Throughout the inquiry stage of the first lesson Shelly remained close to her lesson plan and led her students toward ground that was familiar to her. In the second and third lessons Shelly granted students much more autonomy, showing greater confidence that her students were able to work independently. The video shows her walking around the classroom without interfering with the students' inquiry work.

We argue that changes in lesson structure were not task dependent because the three tasks were designed for similar lesson organisation. We found two interrelated reasons for changes in lesson structure in general and for Shelly's ability to rely on her students' work in the second and third lessons. First, as Shelly became aware of the students' progress she grew more confident about granting them independence. Second, Shelly's growing expertise in managing inquiry-based lessons accelerated the students' progress and active learning. That is, students' ability to cope with the inquiry tasks improved as Shelly's proficiency in managing inquiry-based lessons developed. Detailed analysis of the students' performance in the three lessons revealed major changes in their behaviour between the second and the third lessons. Here, however, we focus on changes in teacher actions. The major change in Shelly's behaviour occurred between the first and second lessons. We concluded therefore that the development of teacher proficiency is a precursor of student progress.

Changes in Discussion Actions

We used microanalysis to examine the structure of whole-class discussions in the three lessons. To characterise changes in the discussion structure we first analysed the transcripts to identify various types of teacher discussion actions. At more advanced stages we realised that the same categories of discussion actions also describe student participation in mathematical discussion. In analysing changes in the discussion structure as an indication for changes in teacher proficiency, we address distribution, frequency, and the quality of Shelly's discussion actions.

Main Types of Discussion Actions

The students' learning activities were focused mainly on solving mathematical problems and constructing their personal mathematical meaning. Shelly's activity, by contrast, was focused primarily on solving teaching problems and supporting the students' construction of mathematical meaning. Following Forman and Ansell's (2002) statement that teacher and students alike are "involved in revoicing each other and listening to, reflecting on, clarifying, expanding, translating, evaluating and integrating each other's explanations" (p. 272), we attempted to look closely at the discussion transcripts and identify the main teacher and student discussion actions.

First, based on the transcripts of the three lessons, we identified three themes of discussion actions: *stimulating initiation*, consisting of actions that begin the discussion of a new mathematical question; *stimulating reply*, referring to actions that stimulate the continuation of a discussion and are connected with prior utterances; and *summary reply*, consisting of actions that conclude the discussion of a particular question. We added the "listening and watching" theme as complementary to the three other themes. Next, within the first three themes we defined categories of discussion actions: questioning, translating a representation, exact repetition of (other) students' utterances, constructing a logical chain, stating a fact, and providing feedback. Figure 5 suggests a model of teacher discussion actions by systematising the relationships between different actions. As reflected in Figure 5, we maintain that discussion actions have a dual nature: as with problem-solving algorithms and strategies, they can be taught effectively; and as with heuristics, it is difficult to explain and predict when to apply them (for this distinction between algorithms and heuristics in mathematics problem solving, see Schoenfeld, 1985). We use the categories listed in the model in our analysis of changes in discussion structure.

Questioning. This category includes utterances that presented students with a problematic situation asking them to find a solution for it, for example, posing a problem, starting a new stage in the discussion, or changing the problem. Teacher and students may ask a question that was planned or may refer to a student's conjecture, statement, or difficulty. Thus, the questioning category was subdivided into sub-categories according to the purposes of the utterance (e.g., 'opening' questions such as "Who is ready to present a solution?" 'promoting' questions like "And then what did you do?" and 'clarification' questions like "Why did you take [these numbers] one and two?" or "What can you tell us about the points on this diagonal line?" or "Let's check whether this result is correct"). Note that teacher and the student utterances were categorised as questioning without relation to their semantic structure. Questioning appeared primarily during the two stimulating stages, and *stimulating initiation* appeared only in question form.

Translating a representation. Teacher and student actions of symbolic representation or graphic representations of other students' utterances (re-wording, writing, or drawing on the board) were considered translations of representations. This category and the construction of a logical chain usually appeared together.

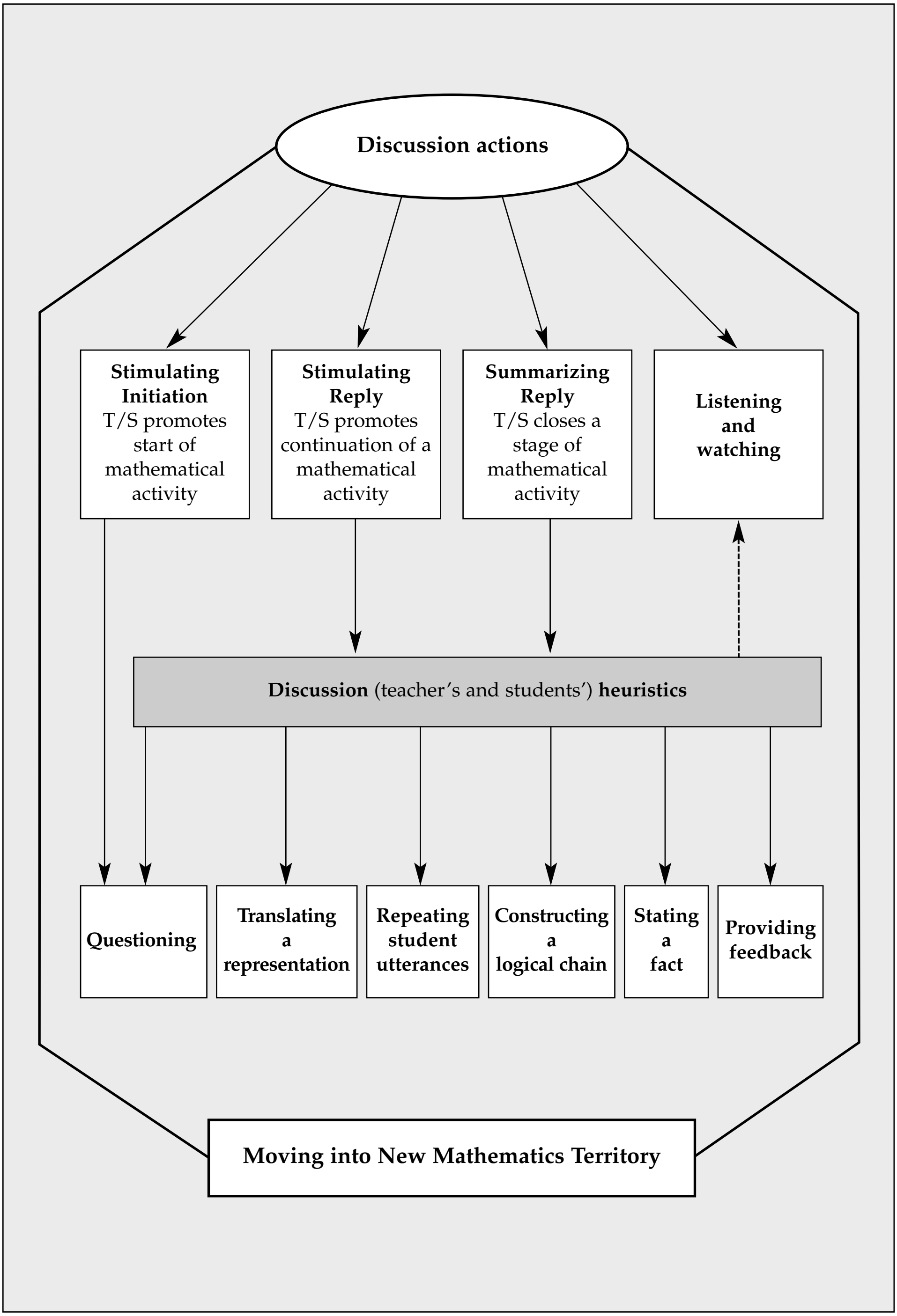


Figure 5. Teachers' discussion actions.

Exact repetition of (other) students' utterances. This action by the teacher or a student was employed for various purposes, e.g., to continue the discussion, to stress some student's ideas, to encourage reasoning about the correctness of conjectures raised during the discussion, to clarify what had been said, or to verify that ideas were clear and properly understood.

Constructing a logical chain. Chains were of the "*if-then*" type. Initially, constructing a logical chain and translating a representation was considered a complex explanation, but later we subdivided complex explanations into two categories because often translations were either performed without the construction of a logical chain or were not part of the construction of the logical chain

Stating a fact. This category included teacher and student statements of mathematical or meta-mathematical facts. Mathematical facts were usually not stated in a logical chain, but verification of a fact usually resulted in constructing a chain or in translating a representation.

Providing feedback. These discussion actions included reflective evaluation of other students' solutions. Figure 6 shows the coded transcript of the discussion segment in the third lesson.

Changes in the Structure and Quality of the Discussion

Figure 7 shows a time-line diagram of the discussion structure of each lesson. The vertical axis represents the teacher's discussion actions. We analysed the 10 concluding minutes of each discussion. The 10 minutes were divided into 300 units of two seconds each (Figure 7). Similar diagrams were constructed for the students' discussion actions, as we zoomed in on the teacher's listening to the students, but within the space constraints of this paper we present only the diagrams charting the teacher actions. However, we use some of the findings derived from analysis of the students' discussion actions as supplementary data to support our arguments about the development of the teacher's proficiency in managing the whole-class discussion.

We used several criteria to analyse the structure of the teacher's discussion actions in a way that enabled a comparison of the diagrams of the three lessons (Figure 7): (1) the distribution of discussion actions between teacher and students; (2) the distribution of the teacher's discussion actions among different categories along the time-line; (3) frequency of the actions; (4) quality of the actions. The first three criteria are depicted in the diagrams (see Figure 7); the fourth is a qualitative criterion which we address throughout the data analysis without using the diagrams.

A comparison of the diagram of lesson 1 with those of lessons 2 and 3 (Figure 7) reveals that in the last two lessons Shelly provided students with more opportunities to talk and to present and explain their findings. She spent more time listening to the students. We found students not only answering Shelly's questions but also asking questions, translating representations, and rewording other students' answers. Student participation in the discussions in lessons 2 and 3 was more active and Shelly's participation became more responsive. We

Time	**Name**	**Utterance**	**Coding**
0:16:05	Shelly	O.K. [Ben is raising his hand] Ben, how did you do it?"	Summary reply: *Feedback* Stimulating initiation *Questioning*
0:16:07	Ben	I took all the twenty one matches. I put two on one side and after every two on one side I put two on the other side, two on one side and one on the other. I did sountil I had seven in one box and fourteen in the other.	Listening to students
0:16:23	Shelly	Why did you take two and one, two and one?	Stimulating reply: *Questioning*
0:16:27	Ben	Because two divided by one equals one and half of two is one.	Listening to students
0:16:33	Shelly	Okay. Half of two is one. Did you understand what Ben did?	Stimulating reply: *Feedback* Stimulating reply: *Repeating* Stimulating reply: *Questioning*
0:16:37	Asaf	Two divided by one equals one.	Listening to students
0:16:39	Shelly	Two divided by one equals two [Shellydraws on the board while talking to the class]. What Ben said was that he took half of two is one, right? So every time he took two and put them here he put one there, add two here and one there until he had no matches left. How many did you have here? [Shelly is pointing to the writing on the board]. **2+2+. . .** + **1+1+. . .** **=2** **Matches**	Stimulating reply: *Stating a fact* Stimulating reply: *Translating a representation* Stimulating reply: *Questioning*

Figure 6. Example of a coded transcript (from the discussion in Lesson 3).

observed a decrease in her initiating and summarising actions. A decrease in teacher initiation is associated with the teacher's reflectiveness and ability to manage the discussion based on student initiatives and responses. In this case, Shelly learned to trust her students' ability to summarise the discussion and to ascribe greater value to their summarising contributions.

In all three lessons *stimulating initiation* always took the form of a question asked by Shelly. In no lesson did students raise questions that initiated the discussion of a new mathematical topic or concept. Stimulating initiation actions were relatively minor in all three discussions. In the ten concluding minutes of the discussion Shelly initiated new topics of the discussion only three times in the first lesson, and twice in the second and third lessons. The main difference between lessons in this category of discussion actions was in the quality of the questions Shelly asked. In the first lesson Shelly asked 'closed' questions requiring short answers regarding computation results and mathematical facts. Questions asked to initiate discussion in the second and third lessons were more open, requiring students to analyse and explain the manner in which they found their answers and to reflect on the reasoning involved in the small-group investigation procedures. A typical question for the first lesson was: "What is the answer for this question?" In lessons 2 and 3 Shelly asked questions of the type "How did you find the answer?"

As shown in Figure 7, in all three lessons most of the teacher's discussion actions were categorised as *stimulating reply*. Shelly reacted to student actions and tried to manage the discussion based on their ideas. As expected for the stimulation category, the most common discussion action was asking a question. The diagrams (Figure 7) show a clear difference in the time spent on this kind of action in the different lessons. In lesson 1 she spent 37% of the discussion time asking questions. In lessons 2 and 3 she asked questions for 16% and 19% of the discussion time. We attribute this change to the development of Shelly's trust in her students' ability to make progress in the discussion without her continual direction.

Not only did the number of questions Shelly asked to stimulate continuation of the discussion change, but so did the quality of the questions improve. The change proceeded through the two periods of investigation, from lesson 1 to lesson 2 and from lesson 2 to lesson 3 in a spiral, rather than linear, fashion. In the first lesson the questions asked directed students with precision to particular "correct" answers, mostly very short, so that every single step of the discussion was under the teacher's control. For example, Shelly asked:

Shelly: What is common to all the points in the third board?

She may have started with a rather general question, but she immediately narrowed it by focusing the students' attentions on the coordinates of one particular point:

Shelly: Who can tell us what are the coordinates of the first point?

Student: Two and one.

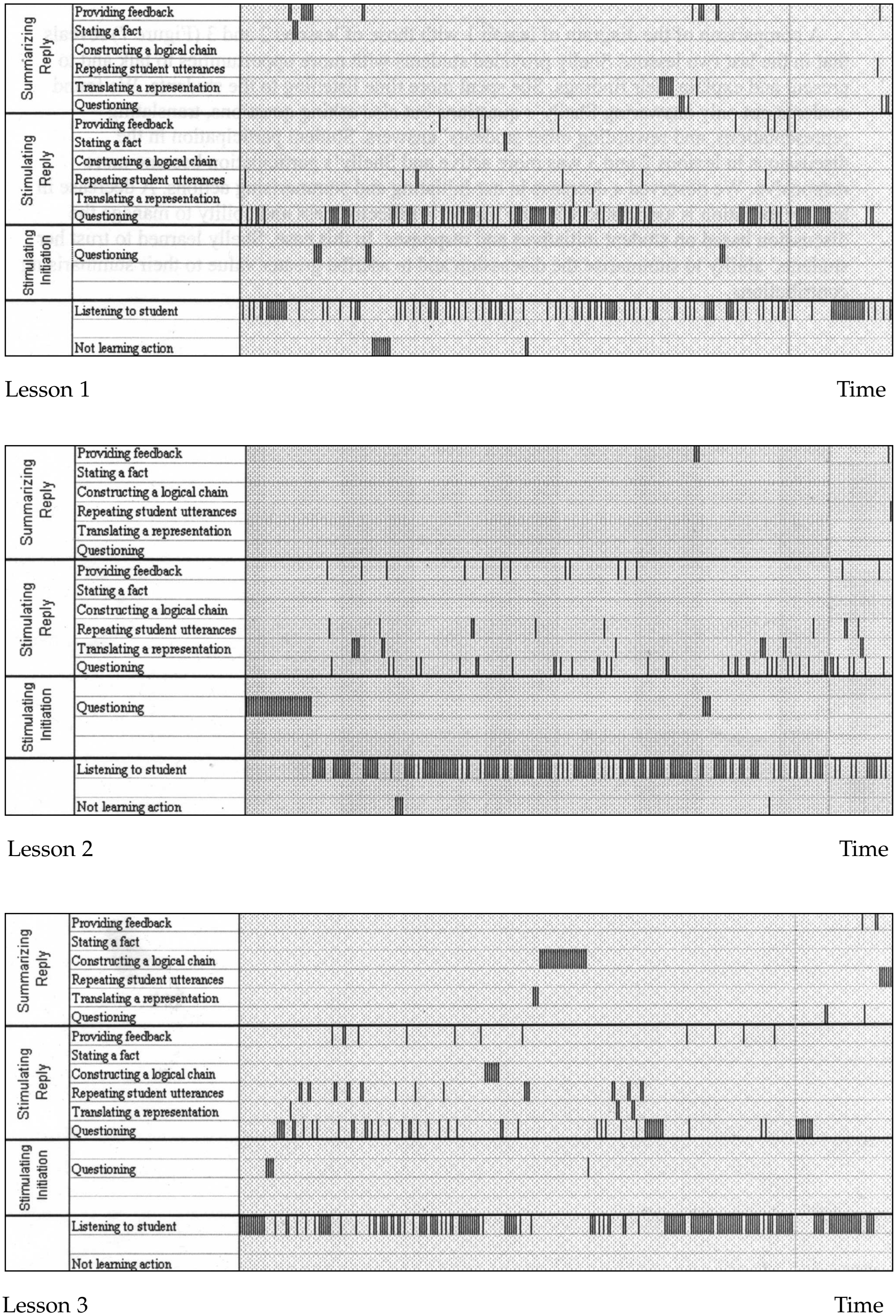


Figure 7. Structure of the teacher's discussion actions over a 10 minute interval.

When one of the students suggested a rather general answer, instead of asking him to explain how he got the answer, she continued asking him narrowing questions:

Larry: If you divide one number by the other you will always get 2.

Shelly: 4 divided by 2 is...

Student: 2.

Shelly: 6 divided by 3 is...

Student: 2.

In the discussion in the lesson 2 Shelly's questions led students to reason about how they arrived at their solutions. They were of the type: "How did you do it?" "How did you find it?" "If this is the way you found the sum, then how did you find each of the numbers?" These questions required students to describe the procedures they performed to obtain the answer. In lesson 3 Shelly began asking questions that provided students with the opportunity and freedom to ask their own questions and express their opinions about other students' answers, and to perform meta-level analysis of their solutions. For example, during lesson 3 Shelly gave Ron the opportunity to share his answer with the class (see Figure 3 for the task):

Shelly: Ron, please [what did you do?].

Ron: Am... ah... I took from the eight, eight, zero, zero and zero I didn't take one from each eight because I knew it fit only two digits so I took off from each eight three so I took overall six. Then I divided six by three and I got two, two, and two.

We can describe the progress in the quality of Shelly's questions during the discussions as a chain: *answer-directed* questions→*procedure-directed* questions→*reasoning-directed* questions.

Changes observed in the other categories of discussion actions also demonstrate growth in Shelly's skills in managing the whole-class discussion. For example, the diagrams (Figure 7) show that *exact repetition of other students' utterances* was more frequent in lesson 2 than in lesson 1, and in lesson 3 more than in lesson 2. We attribute this rise in frequency to Shelly's improved ability to use student utterances in designing the discussion storyline. Use of students' ideas led to teacher-student cooperation. In the following excerpt Omer explains to the class how he found his answer to the task in Figure 3.

Omer: I knew that it couldn't be a one-digit number plus a one-digit number and it also couldn't be a two-digit number plus a two-digit, so I tried to find a one-digit number and two-digit number.

Shelly: Why couldn't it be a two-digit number plus a two-digit number?

Omer: A two-digit number plus a two-digit number would be too big and a one-digit number plus a one-digit number would be too small.

Larry: But ten plus ten is twenty. Isn't it too big?

Omer: But the difference between them [ten and ten] is not 5 [this does not fit the conditions].

The frequency of *translating a representation* in Shelly's discussion actions increased from lesson 1 to lesson 2. It was a rare activity in lesson 1, more common in the discussion in lesson 2, when Shelly wrote on the board student ideas and oral presentations. Analysis of this discussion action demonstrates growth in Shelly's ability to connect the discussion to her students' ideas. In lesson 3 the frequency of translating representations decreased. To explain this observation we turned to the analysis of the students' discussion actions and found that they performed translation of representations mainly in lesson 3. Thus, the actual discussion trajectory became linked to the students' ideas by means of the autonomy granted to students during the discussion. We account for our finding that Shelly *stated a fact* only in lesson 1 and *constructed a logical chain only* in lesson 3 through the changes in the quality of mathematics during the discussion.

Shelly's actions categorised as *summary reply* also altered during the investigation period. In lesson 1 her summary replies included asking questions, translating representations, stating facts, and providing feedback; in lessons 2 and 3 these actions were mainly providing feedback to student actions and repeating student utterances, mostly in the form of summarising student ideas or constructing them into a logical chain. This attests to an improvement in the quality of the mathematics in the students' replies to each other, especially in lesson 3. For example, when Ben finished explaining his answer to task 3B he said that he knew that the number of matches in each black box must be bigger then five and that the number of matches in each white box must be smaller then five. This statement fits two of the three possible answers. Ran's comment led Ben to a more specific answer and allowed him to correct his statement:

Ran: I have to say something to Ben. He said that one side of the equation must be bigger than five, and it's wrong because.

Ben: Yea, you're right except the four, four, four [in the white boxes] and two and two [in the black boxes].

We have described changes in Shelly's discussion actions, which were distributed differently between and within the three main categories of actions and were of different frequency. The observed changes in lesson organisation, in discussion structure, and in the quality of the discussion actions indicate the development of Shelly's proficiency in managing the inquiry-based lesson. In her reflective analysis of the lessons Shelly pointed out that in lesson 1 she felt she needed to proceed according to her initial plan, but that in lesson 2, and even more in lesson 3, she felt more secure in basing the discussion on the students' conjectures. Shelly became more flexible (i.e., changed her initial plans), more cooperative (i.e., more inclusive of students' ideas), and more able to show trust (i.e., more trusting in the students' ability to make progress in mathematical investigation and in whole-class discussion).

Changes in Students' Discussion Actions

We address changes in student discussion actions only briefly, assuming that student progress is an indicator of development in teacher proficiency (Edwards & Hensien, 1999). In our analysis of student discussion actions, using the same categorisation as in teacher actions, we observed modifications both in the distribution and frequency of the student discussion actions and in their quality. In the first lesson students took an active part in whole-class discussion 36% of the time, in the second lesson in 56% of the time, and in the third lesson 50% of the time.

In lesson 1 *stating a fact* was the students' most frequent discussion action in answering questions. In lesson 2 stating facts was less frequent, and in lesson 3 we observed changes in the nature of the action: students presented the conditions set out in the problem and linked them to their solutions. In lesson 3 stating a fact was used by the students both to describe the solution and to justify the answer. This change is obvious when comparing the following two excerpts. Excerpt from lesson 1 (Task 1 discussion):

Shelly: Four divided by two?

Noam: Two.

Shelly: So what is between this number and this number [points to two and four]?

Excerpt from lesson 3 (Task 3b discussion):

Shelly: OK, the first possibility you said is four, four, four, and four. Is that right?

Gilad: Yes.

Shelly: Why?

Gilad: Because four plus four is...

Leyal: Four times four is sixteen.

Together with changes in the nature of *stating a fact* actions, in lesson 3 there was an improvement in the quality of mathematics in the students' statements. We observed students *constructing a logical chain* only in the discussion in lesson 3:

Ben: Besides the possibility with the four matches I knew that in the other possibilities on one side there must be more then five [in each black box] and on the other side must be less then five [in each white box].

Shelly: Why is that right?

Ben: Because if we put five in the two dark boxes, and into the other three also five, then the sum would be greater than sixteen. In this case even if we put in four it would be greater than sixteen (see figure 3B).

The other advance in the students' discussion actions was translating representations, which they performed mostly in lesson 3. The nature of the feedback that students provided to each other also changed from statements like "the answer is right/wrong" to explanations of why it was so. We also found changes in the duration of student discussion actions. In lesson 1 their actions were very short; in lessons 2 and 3 they become longer and more elaborated.

Summary and Discussion

We have presented a microanalysis of the structure of inquiry-based lessons and of whole-class discussions in three lessons by the same teacher with the same students, ten months and six months apart. In the first lesson the teacher was at the beginning of her experience managing inquiry-based classes. Her proficiency developed in the course of her teaching experience without any professional development intervention. We consider our findings an indication that the act of teaching presents an opportunity for teachers to learn.

To perform a precise analysis of LTT, we narrowed our definition of teacher proficiency from the range that appears in the research literature (e.g., Berliner, 1987; Leinhardt, 1993; Noddings, 1992) to lesson organisation and discussion structure. Precise lesson organisation, discussion structure that reflects teacher flexibility (more responsive than active behaviour), and the quality of teacher discussion actions served as indicators of teacher proficiency. The changes in lesson structure show that Shelly became more proficient in lesson structuring, which according to the research literature (e.g., Artzt & Armour-Thomas, 2002; Stigler & Hiebert, 1999), affects the quality of mathematics teaching.

Our findings were presented by means of time-line diagrams similar to those suggested by Schoenfeld (1985) for analysis of problem-solving strategies and to those used by Bromme and Steinbring (1994) for analysis of interactive development of subject matter in the mathematics classroom. We constructed diagrams of the lesson structure based on the intentional components of the inquiry-based lesson – introduction, inquiry, discussion – as defined in the research literature (Artzt & Armour-Thomas, 2002; Wood, 1999). We took a closer look at the discussion structure by means of the grounded theory approach.

The Discussion Actions Model

We suggest considering the system of discussion actions developed in this study as a model for teacher discussion actions. We defined four themes for discussion actions. Two of the themes were subdivided into six categories of teacher actions (Figure 5). Our categories are consistent with Lampert's (2001) description of management of whole-class discussions, where lessons are divided into episodes according to the teacher's roles in the discussion, and with Forman's and Ansell's (2001) social participation structure of the classroom. In Lampert's terms, all the teacher actions were intended to lead students into new mathematics territory (Figure 5). As noted earlier, we consider teacher and student roles in the inquiry-based class to be almost symmetrical. For example, in Figure 5 all the actions are attributed to both teacher and students.

We suggest that the model is useful for diagnostic purposes. By addressing the distribution of actions between the teacher and the students and between the classes of actions, as well as the frequency and duration of the actions in the course of the discussion it is possible to analyse teacher proficiency. The model includes quantitative and qualitative characteristics of the teacher's and the students' discussion actions. For example, the model reveals that the more proficient teacher performs more actions of the stimulated reply type, connected to student conjectures, designing the actual learning trajectory based on student ideas. Teachers' discussion actions should support and stimulate those of the students. Teachers should avoid stating mathematical facts, asking questions that require short unelaborated answers, and providing immediate summarising replies that impede discussion of students' ideas.

This model can be elaborated and refined. For example, in all three lessons in this study the teacher performed stimulated initiation in the form of asking a question. This may have been a result of the students' age or a consequence of the nature of the mathematical tasks. For example, Leikin and Dinur (2003) found that stimulated initiation might be performed by students in the form of stating a fact containing their conjectures that differ from the teacher's expectations. Our definition of teacher proficiency, which embraces distribution, frequency, and quality of the discussion actions, can also be refined by implementing the model in various frameworks for the analysis of teacher lessons.

Applications of the Research to Teachers' Professional Development and the Heuristic Nature of Teacher Actions

Our model was based on analysis of the work of one teacher who taught inquiry-based lessons. Whether it is applicable in the context of other teachers' work and other teaching approaches remains to be determined. We used the metaphor of teaching as problem solving in developing the research tools and model for the current study (e.g., Artzt & Armour-Thomas, 2002; Lampert, 2001). Teachers in their everyday practice solve mathematical and pedagogical problems. Going a step further, we suggest that teaching experience develops teachers' heuristics (Figure 5). As Schoenfeld (1985) has reported, one can effectively teach algorithms but it is difficult to teach the heuristic of when to apply the proper algorithm. Heuristic reasoning, and the behaviours that reflect it, are generally considered characteristics of problem-solvers, whereas heuristic processes may be viewed as inherent in the problems (McClintock, 1979). In mathematics the nature and degree of the inherence varies from problem to problem and from solver to solver, but we assume that most pedagogical problems that teachers solve when teaching are heuristic in nature. We assume that teaching actions are teachable, but it may be difficult to explain when to apply a particular teaching action. In this sense we would like to speculate that teachers' actions are of a heuristic nature, that is, they are not describable by algorithm.

The model suggested in this paper served as a benchmark in the development of instructional interactions (Leikin, 2005a). It complements studies on the development of teachers' reflective learning (Wood 2005), and of teachers' learning of mathematics from teaching (Leikin, 2005b; Leikin, 2006). It also provides additional information about the ways in which the teachers perform learning management (Jaworski, 1994). We suggest that our categories can serve as benchmarks in planning and analysing discussions, and may be useful in different professional development programs for mathematics teachers that focus on the issue of whole-class inquiry-based discussion.

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