The Mathematical Knowledge and Understanding Young Children Bring to School

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As part of the Victorian Early Numeracy Research Project, over 1400 Victorian children in the first (Preparatory) year of school were assessed in mathematics by their classroom teachers. Using a task-based, one-to-one interview, administered during the first and last month of the school year, a picture emerged of the mathematical knowledge and understanding that young children bring to school, and the changes in this knowledge and understanding during the first year of school. A major feature of this research was that high quality, robust information on young children's mathematical understanding was collected for so many children. An important finding was that much of what has traditionally formed the mathematics curriculum for the first year of school was already understood clearly by many children on arrival at school. In this article, data on children's understanding are shared, and some implications for classroom practice are discussed.

Background

Many children have well developed informal or intuitive mathematical competence before they start formal education (Clements & Sarama, 2004; Ginsburg, 2002; Ginsburg, Inoue, & Seo, 1999; Kilpatrick, Swafford, & Findell, 2001; Pepper & Hunting, 1998; Urbanska, 1993; Young-Loveridge, 1989). Children engage in all kinds of everyday activities that involve mathematics (Anderson, 1997), and consequently develop a wide range of informal knowledge (Baroody & Wilkins, 1999; Perry & Dockett, 2004). From infancy to preschool, children develop a base of skills, concepts and understandings about numbers and mathematics. Perry and Dockett (2002) noted that:

> much of this learning has been accomplished without the 'assistance' of formal lessons and with the interest and excitement of the children intact. This is a result that teachers would do well to emulate in our children's school mathematics learning. (p. 96)

In addition, research overseas and in Australia has highlighted the critical role that parents play in fostering children's mathematical development (Henderson & Berla, 1994; Liedtke, 2000; Sharpe, 1998). Of course, some parents are in a better position than others to support the mathematical development of their children, and many adults think of mathematics for young children as counting and adding numbers (Coates & Thompson, 1999). There is considerable evidence (e.g., Siegler, 2003; Tymms, Merrell, & Jones, 2004) that children from low-income or other disadvantaged backgrounds demonstrate lower achievement on arrival at school than other students.

Several scholars have noted that national, state, and territory syllabi and classroom programs may not be reflecting the capabilities of many children on entry to school (e.g., Aubrey, 1993; Kilpatrick, Swafford, & Findell, 2001; Wright, 1994; Young-Loveridge, 1989). For example, 11% of a sample of 859 New South Wales kindergarten students at the beginning of the school year "were performing beyond the expectation of the Kindergarten syllabus, ... [with] numeral knowledge beyond ten" (Stewart, Wright, & Gould, 1998, p. 562). This provides evidence that for this percentage of children at least, the curriculum may not be providing appropriate challenges. Bobis (2002) described the use of textbooks as a potential obstacle to the recognition of prior knowledge.

Young-Loveridge (1988) interviewed 81 children from 18 primary schools in Christchurch in their first month of schooling. Fourteen number tasks were presented to the children. The tasks on which children were most successful on entry to school involved identifying the ordinal position "first", forming a set of two, and identifying a numerical difference of one. The lowest success rates were achieved on tasks involving rote counting to 30, subtraction with imaginary objects of 2 from 5, and addition with imaginary objects of 4 and 3. Her study noted substantial improvements by the end of the first year of school. However, an important finding was the following:

> Children who entered school with relatively little knowledge about numbers made greater learning gains than did their more knowledgeable peers. … The reason appeared to be that the school mathematics programme they were getting was well matched to the existing skills of the less knowledgeable, but was not well matched to the skills of the children who already knew a lot about numbers. (p. 2)

In the last twenty years, state and national curricula (e.g., Board of Studies New South Wales, 2002; Education Department of Victoria, 1975; National Council of Teachers of Mathematics, 2000) appear to have given less emphasis to the traditional "logical operations" approach (Piaget, 1953) in the first year of schooling, with increasing consensus emerging that a greater emphasis on counting may be preferred:

> There is more likelihood of young children developing an implicit understanding of a concept such as one-to-one correspondence by actually indulging in the counting process itself, rather than by joining the members of a set of four cups to the members of a set of four saucers—a pre-number activity common to many commercial mathematics schemes. (Thompson, 1997, p. 160)

Clements (1983) compared teaching approaches in early number with four-year olds. His teaching experiment involved three groups: one taught classifying and ordering skills, one taught various counting strategies, and a control group. The students were then assessed with a “number concepts” and a “logical operations” test. Both experimental groups outperformed the control group on both tests, and the “number skills” group outperformed the “logical operations” group on the number test. Of greater interest was that there was no significant difference in performance between the two experimental groups on the “logical operations” test. Clements concluded that logical operations do not necessarily constitute a prerequisite for the learning of early number concepts. Perry and Dockett (2002) noted that:

> the Piagetian notion that classification, conservation, and ordering of number were foundational aspects on which many other aspects of number had to wait may have acted as a deterrent to the recognition and development of the extensive number repertoire of many children. (p. 93)

In this article we look at data from a broadly representative sample of 34 Victorian primary schools (27 government, 4 Catholic, and 3 independent schools) from a task-based interview developed to provide teachers with a picture of the mathematical knowledge and understandings that children bring to school.

There have been several major Australian and New Zealand projects in early numeracy in recent years. Bobis et al. (2005) highlighted the common features of these: the development and use of research-based frameworks; the use of task-based, one-to-one assessment interviews; and ongoing, reflective professional development. Bobis et al. (2005) documented the changes in student achievement and teacher practice that resulted from these initiatives. The focus of the present article is on one of these projects, with particular reference to student achievement at the beginning of school and at the end of the first year of school.

The Early Numeracy Research Project (ENRP) in Victoria, Australia involved 353 Preparatory to Grade 2 teachers (the first three years of school in Victoria) who participated in a three-year research and professional development project which explored the most effective approaches to the teaching of mathematics in the early years of schooling. The key components and findings of this project have been discussed in a range of publications (e.g., Cheeseman & Clarke, 2005; Clarke, 2001; Clarke, Cheeseman, McDonough, & Clarke, 2003; Clarke & Clarke, 2004; Sullivan et al., 2000), and will therefore not be elaborated here. This paper focuses on interview data on student achievement at the beginning and end of the first year of school which have not previously been reported in detail.

Methodology

Participants

The 34 Victorian schools for which student data are reported in this article were selected from around 400 that applied to be part of the ENRP. They were chosen to be representative of the diversity of Victorian schools in respect of school size, location, percentage of children from non-English speaking backgrounds, and socio-economic status. There were 1438 students in the Preparatory [Prep] interviewed in February or early March, and 1450 were interviewed in November 2001.

Children in Victoria are allowed to begin school if they turn five before April in the year they are hoping to attend, however most children are at least five years of age when they begin school, 86% in the case of the sample discussed in this paper.

Interview

A one-to-one interactive, task-based interview was the chosen form of student assessment. The limitations and disadvantages of pen and paper tests in gathering accurate data on children's knowledge were well established by Clements and Ellerton (1995). They contrasted the quality of information about students gained from written tests (both multiple-choice and short-answer) with that gained through one-to-one interviews, and observed that children may have a strong conceptual knowledge of a topic, revealed in a one-to-one interview, but be unable to demonstrate that during a written assessment. The appropriateness of the use of pen and paper tests is a particular issue with young children, where reading issues are of great significance.

For the past fifteen years, it has become increasingly common for teachers of literacy to devote considerable time to assessing students individually, and using the knowledge gained to teach specific skills and strategies in reading (Clay, 1993; Hill & Crevola, 1999). Although clinical (one-to-one) interviews in mathematics have had a long history in research, these were usually conducted by researchers with relatively small sample sizes. It is now increasingly accepted that the use of such interviews in mathematics can enhance many aspects of teacher knowledge, with consequent benefits to students.

Although the full ENRP interview involved assessment of counting, place value, addition and subtraction, multiplication and division, measurement (time, length, and mass), and space (properties of shape, and visualisation and orientation), only tasks and data from the First Year of School Mathematics Interview (hereafter, “FYSMI”) are discussed in detail in this article.

In consultation with project teachers during the first and second years of the project about the kinds of additional information they wished to gather about children starting in Prep, and in light of the research literature on children of this age, the FYSMI was developed and refined, ready for use in 2001. The decision was made that this would be used with all children in their first year of school.

The FYSMI covered concepts such as simple counting, one-to-one correspondence, “more” and “less”, patterning, ordinal number, part-part-whole reasoning, the language of location, conservation, subitising, numeral recognition, and ordering objects by length. We do not claim that the FYSMI is completely comprehensive but represents a selection of topics generally recognised in the research literature as important in the early years of schooling. The broader ENRP interview (Victorian Department of Education and Training, 2001) provided the opportunity for students to move to much more difficult tasks if they were able, within the nine mathematical domains mentioned earlier.

It needs to be acknowledged that as well as providing teachers with a detailed picture of what individuals and whole grades understood and could do in mathematics, the intention was that by using the interview in exactly the same way across 34 schools, a picture would emerge more broadly of children's mathematical understanding, at the beginning and end of the school year. The teachers found that the interview provided rich and useful information on individuals and children in general, which informed their ongoing planning in a very useful way (see Clarke, 2001).

The ENRP interview, including the FYSMI, was administered by the child's own teacher in the first few weeks of the school year. The teachers were trained to administer the tasks and were required, for research purposes, to follow a clearly outlined script, and complete a provided "record sheet." The FYSMI took approximately 15 minutes out of a total interview of around 45 minutes.

A major focus of the teacher's role during the interview was to listen to children (Copley, 1999; Paley, 1986), noting their response, focusing on the strategies used, and the ways in which they explained their thinking, as well as asking (scripted) questions of the kind, "How did you work that out?", and "Could you do that a different way?"

In the remainder of the article, the different mathematical aspects assessed will be discussed in turn, looking at the specific components of the FYSMI, the data which emerged from it, and discussion of the results.

Tasks and Results

The full script for the FYSMI is provided within the text of this section. Extensive use is made of small plastic teddy bears—referred to as "teddies" in the interview script. The words in italics are instructions for the interviewer, and in normal text are the words said to the child by the teacher.

In the discussion of the data, it is important to note that the students are from schools that were involved in an extensive professional development program. However these children were broadly representative of Victorian Prep children at the start of the school year, and so the impact of the professional development program would be evident in the *end of year achievement*, when any teaching effects would be relevant.

Prior to being taken through the FYSMI, the children had been asked to use a cup and take a large scoop of teddies from a bucket of teddies (at least 20), estimate the number, and then count them.

To reduce considerable repetition, in what follows it was decided to discuss the results for a given group of tasks at the point of presentation of the results, relating the findings to other relevant research data as appropriate. Comparing data with that of other researchers was not always straightforward, as slightly different numbers or contexts were used in tasks. Also, other research reports did not always include detailed descriptions of the tasks which had led to the stated conclusions of these authors. General findings and issues arising from these data will then be discussed.

Simple counting tasks

Simpler Counting Tasks / More or Less / Conservation

Place a pile of 20 teddies in front of the child in a scattered pattern, made up of exactly 4 yellow teddies, 5 red teddies, 3 green teddies, and 8 blue ones.

Please put the yellow teddies together.

How many yellow teddies are there?

Put a group of 3 green teddies together near the 4 yellow teddies (giving two different small groups).

Are there more green teddies or more yellow teddies?

Push the yellow and green teddies aside.

Please get five blue teddies…

Now put them in a line. *(If the child has already put them in a line, ask the child to "move them together now".)* … Tell me how many blue teddies there are.

These tasks were designed to provide evidence of whether the children could sort by colour, count a small collection, determine more or less by comparison, make a set of five, and conserve number. Table 1 presents the results for these tasks. In all tables, percentages are rounded to the nearest whole number.

Table 1
Percentage Success on Tasks with Small Sets

Item	February / March Beginning of first year of school (n = 1438)	November End of first year of school (n = 1450)
Sort by colour	98	100
Count a collection of 4	93	99
Identify one of two groups as "more"	84	99
Make a set, cardinal number 5	85	98
Conserve number	58	88

With a success rate of 84%, the data on "more" and "less" support the findings of Baroody and Wilkins (1999) that most children entering school can determine more and less for small collections. Bertelli, Joanni, and Martlew (1998) found that even three year-olds could answer questions about more and less, even before mastering counting.

There is some question as to the value of tasks involving one-to-one matching between the objects of one collection and the objects of another to compare their sizes (Thompson, 1997). Brainerd (1997) suggested that using matching in such a way is a relatively late development, and that young children are more likely to use counting to answer the "Which set has more?" question. These data do not record the solution strategy used by the children, however we can say that while 93% can count the numbers in each small set, only 84% can say successfully which is more.

The final question in this set focused on conservation of number. The value of such tasks continues to be debated in the literature as, contrary to earlier views, many children are successful counters while not performing successfully on tasks involving conservation, seriation, and classification (Baroody & White, 1983; Donaldson, 1978; Hughes, 1986; Young-Loveridge, 1989). Indeed, Pennington, Wallach, & Wallach (1980) found that over 70% of the 5- and 6-year olds in their study who had "failed" a conservation of number test were able to make accurate judgments of equivalence when they used counting. Thompson (1997) offered the conjecture that "the ability to use counting competently shows an *implicit* understanding of one-to-one correspondence, whereas number conservation tasks assess *explicit* knowledge of the concept" (p. 156).

The conservation question in the ENRP interview was changed during the project in response to concerns expressed by teachers and colleagues. One such concern was that the child will assume that the quantity must have changed, or the teacher would not have asked the question. The initial version of the task that was used in 1999 involved *the interviewer* moving the teddies around, after the child had successfully counted them, and it was later agreed that a child may interpret this action of the teacher as necessarily changing the number, possibly seeing it as a trick (Donaldson, 1978; Hughes, 1986). Having the child move the teddies around was considered preferable. On a similar task involving five counters, Clements, Sarama, and Gerber (2005) found that 40% of Texas and New York students in preschools serving mostly low socio-economic status [SES] children could tell how many were present without recounting.

It is clear that most children come to school with an awareness of colour, and the capacity to make and count small sets. Understanding that rearranging a set doesn't change its cardinal number is less clear, and 12% of children still had difficulty with this task at the end of the year.

Location language

Location / Pattern / Ordinal Number

Please put out a yellow teddy… Now put a blue one beside it… Now put a green one *behind* the blue teddy… Now put the green teddy in front of the blue teddy…

Many teachers requested the inclusion of questions relating to location language. Such concepts are important indicators of children's spatial development, although the link with literacy understanding must also be acknowledged. The data are shown in Table 2.

Table 2

Percentage Success for Language of Location (Space) Tasks

Item	February / March (n = 1438)	November (n = 1450)
"Beside":	88	97
"Behind":	87	97
"In front of":	83	96

These examples from the language of location shown in Table 2 are well understood at the start of school by most children. The data presented represent students from a very diverse range of schools and are not discussed here at an individual school level. However, an analysis was completed for these location tasks, comparing two schools with a very high percentage of children from non-English speaking background (NESB) students with the children in other schools. These two schools averaged 82% NESB children, with many recent arrivals to Australia. Not surprisingly, the data for these children at the beginning of the school year showed a much lower rate of success on the three tasks: 48%, 67%, and 46%, respectively. However, as the children's language facility increased during the Prep year, these percentages increased considerably, with the percentages being 89%, 88%, and 85% respectively, much closer to the overall performance shown in Table 2.

Patterning

> Now watch what I do with the teddies.
> *Make a pattern with the teddies (G, Y, B, B, G, Y, B, B) in front of the child.*
> I've made a pattern with the teddies. Please say the colours for me as I point.
> *Hand the container of teddies to the child.*
> Please make the same pattern.
> *(If the child's pattern is a correct copy, point to it. If not, point to your pattern.)*
> Please make the pattern go on a bit more.
> How did you decide what came next in the pattern each time?

The inclusion of tasks relating to patterning was strongly encouraged by the teachers. The tasks involved making, copying, extending, and explaining a pattern. It is interesting to reflect on the value placed on this by teachers especially in preschool settings (Economopoulos, 1998). Pattern is acknowledged as a vital component of mathematics—mathematics is often defined as the study of pattern. However, there is not yet consensus on those aspects of patterning which should be included in preschool experiences. How much of what is currently included is "taken for granted" practice? What thinking is being evidenced when children engage in making patterns? What makes a pattern complex for a child?

One of the limitations of the large-scale assessment is that it was not possible due to time, consistency, recording, and interpretation issues—to include a task for which children would be asked to create their own patterns and explain them. Of course, this would be a most appropriate task for a teacher to use during normal classroom interactions. Ginsburg (2002), for example, observed that much of the everyday mathematics in which children engaged during "free play" was in enumeration, magnitude, and pattern. Lin and Ness (2000) found that four- and five-year old preschoolers in New York City and Taiwan were involved in pattern and shape activities during free play time more than any other mathematical activities. Data for the patterning tasks are shown in Table 3.

Table 3
Percentage Success on Pattern Tasks

Item	February / March (n = 1438)	November (n = 1450)
Name colours in pattern	94	99
Match pattern	76	97
Continue pattern	31	87
Explain pattern	31	87

The patterning task provided further evidence of the children's facility with naming colours and most were able to match the given pattern, but the notions of continuing and explaining patterns were more challenging. These data were similar to Klein, Starkey, and Wakeley (1999), who found that 68% of preschoolers could duplicate a pattern and 19% extend it (n = 41). Although the percentages of success for continuing and explaining the pattern were identical, some children could continue the pattern but not explain their reasoning, while others could justify well a strictly incorrect pattern. These data are not surprising, as this task is clearly related to experience with patterns and to the complexity of the pattern. By the end of the Prep year, most had this understanding. We could conclude that patterning is very much a school-learned task, although Ginsburg's (2002) work suggested that patterning is not confined to school experience. Pattern work would appear to be vital in many areas of mathematics from geometric understanding to algebraic thinking, but the relationship between its early foundations and middle and high school mathematics has not yet been established, despite many examples of apparently worthwhile activity in the early years (e.g., McClain & Cobb, 1999).

What should be looked for when young children work with patterns? What sort of markers are there to identify development? What is really known about young children's engagement with patterns and the thinking that is being evidenced? Are opportunities being provided for reasoning and justification that take the activity beyond "what comes next?" (Economopoulos, 1998).

Ordinal number

> *Point to the green teddy in 1st position.* The green one is the 1st teddy in my pattern. You point to the 3rd one. What colour is the 3rd teddy? You point to the 5th teddy. What colour is the 5th teddy?

Given the high success rate for colour recognition (see Tables 1 and 3), it seems clear that knowledge of colour names was not a hindrance in these tasks. The requirement to both name and point to the relevant teddies meant that the interviewer could be certain which teddies the student had chosen. Data for these tasks are shown in Table 4.

Table 4
Percentage Success for Ordinal Number (3rd and 5th) Tasks

Item	February / March (n = 1438)	November (n = 1450)
Nominating colour of 3rd teddy in a line of teddies	29	85
Nominating colour of 5th teddy in a line of teddies	20	76

Ordinal number (in this case, nominating the third and fifth items in a set) proved very difficult on arrival to school (only 29% and 20% could do the respective tasks), with three-quarters succeeding on both by the end of the year. This again may reflect that this is a school-based task with which children have little informal experience prior to school. The success rates of Prep children from the two schools with high percentages of NESB students were 9% and 8% on the two tasks respectively, with considerable growth to 68% and 62% respectively by the end of the year.

It is also interesting to note that even at the end of Prep, quite a few children were not yet able to succeed on this task. In the authors' opinion, the 76% figure was one of the more surprisingly low rates of success reported in this article, although it finds support in the comment of Ginsburg (2002) that teachers have not had great success in teaching ordinality. It is also of interest that Klein, Starkey, and Wakeley (1999) found that of 41 Californian preschool children, the percentages of children understanding the ordinal number terms were: "first" (76%), "second" (63%), "third" (27%), and "fourth" (20%), quite similar results to those found in the ENRP data on entry to school. Similarly, on the identification of "fifth," possibly the only task in which Young-Loveridge's (1988) data can be compared reasonably with the ENRP data, 30% of students were successful in Young-Loveridge's study, compared to 20% for ENRP students.

Subitising

Subitising

I'm going to show you some cards quite quickly. Tell me how many dots you see.

Show each pink flashcard for 2 seconds only, in the following order and orientation:

Subitising, "instant recognition of the numerosity of small collections" (Clements, Sarama, & Gerber, 2005, p. 10) is acknowledged as an important skill in early number development (Bobis, 1996; Young-Loveridge, 1988). Data for the subitising tasks are shown in Table 5, with the order reflecting increasing level of difficulty at the start of the school year.

Table 5
Percentage Success in Subitising Tasks

Item	February / March (*n* = 1438)	November (*n* = 1450)
Recognise 2 without counting	95	100
Recognise 3 without counting	84	98
Recognise 0 without counting	82	97
Recognise 4 without counting	71	96
Recognise 5 without counting	43	75
Recognise 9 without counting	9	44

Subitising tasks, in this case instant recognition of regular dot patterns, were accomplished well, with larger sets (5 and 9) being, not surprisingly, more difficult. Clements, Surama, and Gerber (2005) found a similar pattern of order of difficulty for subitising with preschool students, with 3, 4, 5, 10, and 8 being increasingly difficult.

The five dots were presented to the child in a row of two dots above a row of three dots. This arrangement was chosen so as not to favour children with experience with dice. This less common way of presenting five may partly explain the low success rate of 43%.

The 9 arrangement (a 3 x 3 array) was chosen for the interview as it corresponded to the logo used for the local free to air television station where the word, the formation of dots, and the numeral 9 are presented together repeatedly. It had been suggested accordingly, that the children would recognise this arrangement more easily than for other numbers. The authors anticipated that students would make the connections between experiences outside and inside the classroom, but in this case there was little evidence of a link being made between the logo and the number “9.”

It is well known that most children can identify the cardinality of a small set of objects before they begin school (Bobis, 1996). Gelman and Gallistel (1978) found that most children by the age of four were capable of *instantaneously* recognizing groups of four objects. Baroody (1987) nominated subitising as a fundamental skill in the development of children’s number understanding. As Young-Loveridge (1988) pointed out, board games, card games, and dominoes provide enjoyable contexts for learning about quantities in this way.

Matching numerals to dots

Now put the dot cards all down in the order shown here.

Spread out the pink 0 – 9 cards randomly, face up, in front of the child, between the child and the dot cards.

b) Find the number to match the dots. *(If the child seems puzzled that there are more numeral cards than sets of dots, explain that “you won’t need to use all the numbers.”)*

The introduction of symbols is often seen as the school’s responsibility, however children are exposed to numerical symbols in a large range of contexts prior to starting school. The linking of the symbol with its corresponding cardinal set is an important component of early number development. Data for these tasks are found in Table 6, presented in order of increasing difficulty at the beginning of the school year.

Table 6
Percentage Success in Matching Numerals to Dots Tasks

Item	February / March (n = 1438)	November (n = 1450)
Match numeral to 2	86	100
Match numeral to 3	79	99
Match numeral to 4	77	98
Match numeral to 5	67	94
Match numeral to 0	63	97
Match numeral to 9	41	82

Once again, it was clear that, in general, the larger the number, the more difficult the task. Many teachers commented that children were more comfortable with the number “0” than they may have thought previously. It is sometimes claimed by teachers that zero is better left out in the first year of school, as children have some difficulty with the notion of zero. These data do not support this opinion and, as Ginsburg (2002), Greenes (1999), and others have found, the concept of zero is understood readily by many children in the first year of school. With the exception of “9”, virtually all children in our sample were matching successfully by the end of the first year of school.

Data from four year-olds in the United Kingdom, reported by Tymms, Merrell, and Jones (2004), related to students’ capacity to identify digits. The order of increasing difficulty was 4, 1, 3, 2, 5, 6, 7, 8, 9. As with our matching numerals to dots task, there was a general pattern of larger digits being harder to identify, but the relative position of the digit “4” is probably explained by the age of the pupils and recent experience with cards, birthday cakes and so on. Interestingly, Swedish three year-olds who were regularly in a mathematically-rich environment, had considerable success in choosing dot cards with specified numbers of dots during an interview situation, although there was no matching to written numerals required in that study (Doverborg & Samuelson, 2000).

Ordering numbers

Remove the dot cards and the zero card. Shuffle the numeral cards and spread them out, face up randomly on the table.

Please put the number cards in order from smallest to largest.

If the child is successful, hand across the zero card.

Where would this one go?

The interviewer introduced the zero card only given complete success with ordering the numbers from 1 to 9. Data for the ordering tasks are shown in Table 7.

Table 7
Percentage Success with Ordering Numbers Tasks

Item	February / March (*n* = 1438)	November (*n* = 1450)
Order numeral cards 1–9	46	91
Order numeral cards 0–9	38	88

In the place value domain of the broader ENRP interview, the ordering of numbers was a more challenging task than the reading and writing tasks for the same number of places. It is clear from these data that fewer than half of the students entering school could read and order single-digit numbers successfully, that the inclusion of the number zero added some difficulty on arrival at school, and that ordering the full set 0-9 remained difficult for 12% of students at the end of the school year. It should be noted that the 38% and 88% refer to percentages of the total sample respectively. Students who were not successful in ordering numbers from 1 to 9 were not given the opportunity to order numbers from 0 to 9. However it seems highly unlikely that these students would have been successful in the latter task.

Part-part-whole

> Please show me 6 fingers… *(if correct* … Can you show me 6 fingers another way? Another way?)

It is widely acknowledged that notions of “part-part-whole” are important for young children, if they are to work flexibly with numbers in a range of situations (Resnick, 1983; Shane, 1999; Young-Loveridge, 1988). Part-part-whole refers to the idea that a given number can be potentially partitioned in a variety of different ways. In this task, we were interested to see whether the children could “see” the number six as more than just part of the counting sequence—the number after five. Data for the part-part-whole task are shown in Table 8.

Table 8
Percentage Success with Part-Part-Whole Tasks

Item	February / March (n = 1438)	November (n = 1450)
Show 6 fingers (usually 5 & 1)	78	99
6 fingers-2nd way	20	73
6 fingers-3rd way	8	51

This task provided considerable discussion among teachers and researchers in relation to its inclusion in the interview. It is clear that when a child is able to show six in other ways than five and one, they are providing evidence of a richer notion of six. However, there is some anecdotal evidence of children with apparently well-developed number sense who had some difficulties interpreting the instructions. Some people have suggested that providing an example might be helpful, but it is not clear if this may become a counting task for some children rather than providing insight into whether they mentally see six in flexible ways. This is one reason for the use of fingers in this task rather than teddies, which may have been more likely to evoke a counting strategy.

It is important to note that it was agreed by the research team, that for the purposes of these data, five fingers on the left hand and one finger on the right would be considered "the same" as when the hands are reversed, or upside down. As the data show, there is a sharp decline in performance at the beginning and end of the school year, when children were required to produce a second and third way.

Flexible, visual images of quantities seem to be quite important in number learning. Bobis (1996) found that activities with five-year-olds that focused on the visual identification of groups of numbers rather than counting one-by-one, helped children to develop part-whole relationships, especially the decomposition of ten, a key understanding in developing addition facts. Fischer (1990) found that instruction that emphasised part-whole number relationships aided the development of basic number concepts and children's ability to solve addition and subtraction word problems and to deal with place value, even though these applications were not specifically the focus of instruction.

Many project teachers were not aware of the value and importance of these kinds of activities to young children's mathematical development. As a result of their engagement with the interview and associated professional development, they increased their focus on tools such as tens frames and expanded their views on what constitutes the understanding of a number, taking it beyond the word, the symbol, and the quantity, to the components of the quantity and other ways of representing it. It is interesting that Nelson

(1999) claimed that "there is no greater evidence that young children are developing true number sense than their emergent awareness that numbers are made up of other numbers" (p. 137).

Numbers before and after

> When you are counting by ones, what is the number after 4? *(if successful* … after 10? *if successful* … after 15?)
> What is the number before 3? (*if successful* … before 12? *if successful* … before 20?)

In the broader ENRP interview, children were invited to carry out a series of "rote counting" exercises, to see if they could count forwards and backwards from a variety of starting points, thereby being required to break the counting sequence. Fuson (1988) described the stage at which children can count up from an arbitrary number in the sequence, "the breakable chain level" (p. 51). These tasks included counting from 53 to 62 and 84 to 113. Not surprisingly, the size of the numbers made these tasks very difficult for most Prep children. It was therefore decided to include tasks involving breaking the number sequence, but involving much smaller numbers. Table 9 provides the data for these tasks.

Table 9
Percentage Success with Numbers Before and After Tasks

Item	February / March (*n* = 1438)	November (*n* = 1450)
Number after 4	82	97
Number after 10	60	93
Number after 15	30	95
Number before 3	53	88
Number before 12	29	81
Number before 20	15	72

As expected, the number *before* was more difficult than the number *after*, in the same way that counting backwards is more difficult for most children than counting forwards. Similarly, there was a rapid decrease in performance as the numbers involved increased. There was a considerable difference between the data for "number after 4" and that of preschool students reported by Clements, Sarama, and Gerber (2005), where 27% of low and middle SES New York students, and only 12% of Texas low SES students, were successful. Although age differences need to be taken into account, the between-country differences here contrast with the similarity of the Victorian and US data for the conservation task. It is possible that student performance on conservation tasks is less influenced by the home and community environment than tasks involving counting.

One-to-one correspondence

Place 5 cups out in a line. Hand the child 9 straws.
Please put one straw in each cup.

In the same way as was discussed with conservation, finding an appropriate task for assessing one-to-one correspondence is difficult. In the broader ENRP interview, in the domain of multiplication and division, the children were shown four matchboxes ("teddy cars"), and asked to put two teddies in each car. This proved to be surprisingly difficult, with around 67% of Preps at the beginning of the year and 25% at the end of the year, unable to do so. The research team and teachers discussed why this might be the case, and concluded that perhaps it was because children had to attend to two different notions—"two teddies" and "each car."

The difficulties with this "many to one" task prompted the research team to include a more straightforward one-to-one correspondence task in the FYSMI. Data for this task are shown in Table 10.

Table 10
Percentage Success with One-to-one Correspondence Task

Item	February / March (n = 1438)	November (n = 1450)
One-to-one correspondence (straws to cups)	92	99

The data shown in Table 10 indicate that this is clearly a much more straightforward task than the teddies in the car task. In hindsight, the context may not have been the best one. Is the process of putting straws in a glass so commonplace that it is not sufficiently problematic? Children can be observed in daily life doing something which clearly evidences one-to-one correspondence, but creating tasks that can be given to all children and be confidently interpreted is more difficult. These are of course ongoing issues with any sort of assessment. It has been suggested that a less familiar context (e.g., given a collection of teddies and counters, "please put one teddy on each counter") may have been an improvement. That said, the absence of similar tasks in the assessment protocols of other recent researchers may indicate that they use counting tasks such as the counting teddies task (see Table 12) as appropriate indicators of understanding and use of one-to-one correspondence.

Measurement ordering tasks

Spread out three candles (20 cm, 5 cm, and 10 cm in that order from left to right).
Please put these candles in order from smallest to largest … Please point to the largest … Please point to the smallest.

If successful, add in the 15 cm candle. This time, place the candles like this: 10 cm, 20 cm, 5 cm, and 15 cm, in that order from left to right.
Now put these candles in order from smallest to largest … Please point to the largest … Please point to the smallest.

This task was included in the FYSMI in response to a finding in the first year of the project that ordering numbers was considerably more difficult for many students than reading and writing numbers. Some project teachers believed that this was due to the language used, "please put these numbers in order from smallest to largest." It was decided to use similar language in a different context, that of length. Table 11 shows the percentages of children who were successful on these two ordering tasks.

Table 11
Percentage Success with Measurement Ordering Tasks

Item	February / March (*n* = 1438)	November (*n* = 1450)
Ordering candles smallest to largest (3)	61	94
Ordering candles smallest to largest (4)	50	91

Williams and Shuard (1982), in discussing the complexity of the thinking required to put objects in order, noted that "it can be seen that this is a complicated judgement to make and few children can deal with three things in this way before the age of five" (p. 10). Not surprisingly, comparing three objects is considerably more difficult than comparing two. Moving from the notion of "big" and "little" to deciding where a third object "fits" in relation to the other two is more complex. Adding a fourth candle makes the task more difficult again. The results for these tasks are probably what the authors might have expected, with 11% of the cohort being able to order three candles, but not four. As with many of the tasks however, the percentage success by the end of the year was considerably greater.

On the issue of language, these data appear to show that it was the difficulty of ordering numbers per se, rather than the language, which made the ordering numbers task difficult in the broader ENRP interview.

Further evidence of mathematical knowledge and understanding

The data from the FYSMI were only part of the interview data that were available to teachers within the project. Many of the children beginning Prep were able to go well beyond the tasks outlined above. For example, Table 12 shows the performance of Prep children on some counting tasks from the broader ENRP interview.

Table 12
Percentage Success of Prep Children on Particular Counting and Place Value Tasks

Item	March (n = 1438)	November (n = 1450)
Rote counting to 20	57	96
Counting a collection of at least 20	39	90
Counting by 1s (forward / back / number before / after)	3	33
Counting from 0 by 2s, 5s, 10s	0	18
Counting from x ($x > 0$) by 2s, 5s, 10s	0	2
Extending and applying counting skills (counting money, giving change, etc.)	0	0
Make a collection of size 7 (shown the numeral 7)	67	97

By the end of Prep, 90% of the children were able to count a collection of at least 20 teddy bears successfully, with 2% even able to count from 23 by tens, and 24 by fives. The vast majority of children arrived at school with the capacity to read one-digit numbers, and virtually all could do so by the end of the year.

Implications of these data for mathematics teaching in the first year of schooling

Some implications of the data have already been discussed in the previous section, but some further comments are relevant, including the responses of the actual interviewers, the classroom teachers, to the experiences of the interview and the resulting data.

At a project professional development session in March 2001, following the first use of the FYSMI, Prep teachers were asked to “suggest one action that you might take as a result of what you have seen in the data or heard in the discussion with colleagues.” The most common responses were more work on patterning, greater emphasis on mathematical language, and more emphasis on notions of part-part-whole, ordinal number, and number recognition. The data indicate that these strategies appear to have had positive effects, although of course it is always difficult to separate the effect of classroom experiences from home, maturation, and other effects.

Clearly most Prep children arrive at school with considerable skills and understandings in areas that have been traditional mathematics content for that age. As acknowledged by many ENRP Prep teachers, this means that expectations could be raised considerably in terms of what can be achieved in that first year.

One of the most powerful anecdotes from the first year of the project was when a Prep teacher, invited to comment on highlights / surprises from her first use of the interview, wrote: "discovering in the first few weeks of school that I have a child who can read an eight digit number on the calculator, and tell me that it's 36 million, 285 thousand, …" The teacher commented that "I have usually spent the first year of school focusing on the numbers from 1 to 20. I might need to reconsider my curriculum!" This teacher might concur with Ginsburg, Inoue, and Seo (1999) who observed the sophistication of mathematical activity of children in free play situations, and concluded that "young children engage in a variety of mathematical explorations and applications, some of which appear to involve surprisingly 'advanced' content and might even be considered developmentally inappropriate for a preschool or kindergarten curriculum, at least by conventional standards" (p. 89). Clearly, this research supports the recommendation of Aubrey (1993) in regard to offering children opportunities to use the problem-solving skills they already possess on arrival at school.

One measure of changed expectations on the part of Prep teachers over the course of the ENRP was teachers' responses to one questionnaire item. Teachers who taught Prep for the three years of the project were asked at the beginning and end of the project whether "all", "most", "some" or "none" of their children would know that "78 is 7 tens and 8 ones" by the end of Prep. At the start of the project, 9% of teachers nominated "some" and 91% "none", while at the end of the project, the respective percentages were 12% most, 70% some, and 18% none (n = 34). A general pattern in the changed expectations was that teachers were far more likely to indicate some or most, with all or none being far less common. It can be claimed reasonably that as teachers have increased knowledge about the mathematical understanding of individuals and groups through the use of a carefully-developed assessment instrument, they are more aware of the range of children in their grades, and can plan accordingly.

It is also important to note that students produced impressive growth over the year. As the data in several tables indicate, even for tasks for which few children were successful at the beginning of Prep, the vast majority succeeded in November.

The use of the interview, embedded within a professional development program, enhanced teacher knowledge and informed planning. As Clements (2004) noted, "knowledge of what young children can do and learn, as well as specific learning goals, are necessary for teachers to realize any vision of high-quality early childhood education" (p. 9).

The instrument described in this article was developed for use in a large-scale project, but we would argue that it provided teachers with useful information about what might be possible in their own classrooms. During the ENRP, the FYSMI was of major use in a special school context (a school for low-functioning students). The data for this school are not included in any of the data above. The teachers in the *special school* found it to be a very valuable tool that was easily used and interpreted in their context (see Clarke & Faraghar, 2004). The FYSMI has also been used by researchers in preschool settings with considerable levels of engagement (Clarke & Robbins, 2004). We would strongly urge the use of the FYSMI and also the broader ENRP interview, so that the opportunities for young children to show what they know and can do are not limited.

When children start school, they bring with them a myriad of experiences. Everyday family life, playgroup, day care, and preschools all provide informal opportunities for the development of mathematical concepts and skills. The data presented in this article can provide some indicators in relation to the nature of the mathematical knowledge and understandings that children bring to school. However a note of caution is necessary in relation to how these data are used. It is tempting to lose the individual in the context of overall data. Gilmore (1998) noted in the New Zealand context that “the diversity of results reported in this analysis demonstrates the importance of thinking of new entrants as individuals, not as a ‘cohort’” (p. 7). The diversity of individuals is not obvious in the data presented here, with the exception of the data related to NESB students. However, in the ENRP professional development setting, where teachers of students from relatively advantaged backgrounds sat around the table with teachers of students from relatively disadvantaged backgrounds, the sharing of data brought the considerable diversity of performance into sharp focus. Such diversity reaffirmed the power of the one-to-one interview in providing the teacher with rich data on what students knew and could do, and a basis for classroom planning for individuals and the groups.

In other contexts where data have been presented from the ENRP, we have always presented them in the form of graphs and tables that represent the full range of understandings evidenced by children. When we start to look at what is typical, we can be lulled into thinking that instruction should be aimed at a specific level for a grade of children rather than providing rich, varied, and open activities that provide a level of engagement for all children. The mathematical thinking of the child should be the starting point.

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