Meeting the Curricular Needs of Academically Low-Achieving Students in Middle Grade Mathematics

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An important component of the National Council of Teachers of Mathematics Standards is the equity principle: All students should have access to a coherent, challenging mathematics curriculum. Many in the mathematics reform community have maintained that this principle can be achieved through one well-designed curriculum. However, the extant research on equity—which focuses on either ethnic diversity or academic achievement—suggests that this principle is illusive. The current study compares the effectiveness of two curricula in teaching a range of math concepts to 53 (28 male; 25 female) middle school students at risk for special education services in math. The yearlong, quasi-experimental study involved achievement and attitudinal measures. Results indicated that students in the intervention group who used materials designed according to instructional principles described in the special education literature achieved higher academic outcomes ($p < .05$, $p < .001$) and had more positive attitudes toward math ($p < .001$) than did students in the comparison group.

A central tenet of the National Council of Teachers of Mathematics (NCTM) Standards (1989, 2000) has been that *all students* can succeed in complex mathematics. This tenet is commonly referred to as the *equity principle,* and while it is acknowledged that students do not need to be treated in the same way, the NCTM Standards and other influential policy documents (e.g., *Measuring What Counts,* National Academy of Sciences, 1993) nonetheless advocate that all children should have access to a coherent, challenging mathematics curriculum. Since the early 1990s, some critics have been understandably skeptical of this principle, particularly given the NCTM Standards' considerable emphasis on conceptual understanding, problem solving, and constructivist pedagogy (Harris & Graham, 1996; Woodward & Montague, 2002).

Some of the more widely cited studies of equity in the mathematics literature have focused on low-income, ethnically diverse students. Results from the Algebra Project (Moses, 2001), QUASAR (Quantitative Understanding Amplifying Student Achievement and Reasoning; NCTM, 1999; Silver, Smith, & Nelson, 1995), and Cognitively Guided Instruction (Fennema, Franke, Carpenter, & Carey, 1993) indicate that challenging mathematics programs—which emphasize conceptual understanding, problem solving, and communication—have promise for minority students. These findings are important because there is a historical tendency to place far too many poor students from ethnically diverse backgrounds in low-track, skills-based classrooms (Schoenfeld, 2002; Secada, Fennema, & Adajain, 1995).

However, efforts to craft one rigorous curriculum for all students has proven more difficult than many in the mathematics reform community had anticipated (Carey et al., 1995). Research in the late 1990s indicates that naturalistic efforts to raise the achievement of low-ability, low-income secondary students produces mixed results. Gamoran and his colleagues (Gamoran, Porter, Smithson, & White, 1997; Gamoran & Weinstein, 1998) reported that efforts to eliminate general track math classes and to replace them with transition courses (i.e., courses that allow students to keep up with students who enroll in college prep classes) have been only partially successful. Achievement levels tend to be somewhere between students in low-track, skills-based classrooms and students in college prep courses. Increasing the rigor of the class alone is insufficient, and tenets such as the equity principle often underestimate the technical complexities of teaching a mixed ability group of students, particularly at the secondary level. Nonetheless, the findings generally reinforce the view that low-track students should not be placed in "dead end" general-track math classes with little access to challenging content.

One of the most difficult arenas for the equity principle involves students at risk for special education or students with learning disabilities. The beneficial effects of challenging mathematics on these students are often anecdotal (e.g., Silver et al., 1995; Fennema, Franke, Carpenter, & Carey, 1993). In-depth examinations of this population indicate that without substantive modifications, these students do not exhibit high levels of success on either academic measures or everyday activities (e.g., small group work and whole class discussions; Baxter, Woodward, & Olson, 2001; Baxter, Woodward, Wong, & Voorhies, 2002; Woodward & Baxter, 1997).

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Special education intervention research in mathematics has articulated instructional principles potentially beneficial to this population of students. Research supports the use of visual models or manipulatives (Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Cass, Cates, & Smith, 2003; Jitendra, Hoff, & Beck, 1999; van Garderen & Montague, 2003; Witzel, Mercer, & Miller, 2003; Woodward, Baxter, & Robinson, 1999), carefully distributing practice on key concepts and skills (Kameenui, Carnine, Dixon, Simmons, & Coyne, 2002), as well as controlled pacing and high expectations (Fuchs & Fuchs, 2001).

Less prominent in the special education literature—but of considerable importance—is the issue of motivation. Middleton & Spanias's (1999) synthesis of research on motivation in mathematics suggests that when a student fails repeatedly in math, he or she tends to attribute that failure to a stable belief. Specifically, the poor performance is a function of low ability instead of what are considered more desirable attributions (e.g., the failure was due to a lack of effort, the task was too difficult) (Dweck, 2000; Weiner, 1986). Over time, students tend to avoid academic challenges, often by adopting self-handicapping strategies for coping with failure (Covington, 1992). This issue becomes paramount in the context of the NCTM Standards, as well as a nationwide testing movement that advocates for more challenging mathematics at the secondary level.

Creating opportunities for success—and hence the belief that it is possible to succeed in mathematics—can be complicated. Offering students a series of relatively easy tasks can lead to a false sense of self-efficacy, and this practice is at odds with the intent to give students access to challenging mathematics. Ironically, students need to experience periodic challenge and even momentary failure to develop higher levels of self-efficacy and task persistence (Bandura, 1986; Middleton & Spanias, 1999; Shunk & Pajares, 2001). Research indicates that achieving a balance between sufficient opportunities for success and tasks that require considerable effort and that may need to be solved through small-group efforts, rather than individually, requires carefully designed curricular materials and instructional practices (Woodward, 1999).

Purpose of the Study

The purpose of this study was to examine the effects of two kinds of curricula on middle school students at risk for special education in mathematics. Curricular research in mathematics has become increasingly important because of the need to document the effectiveness of standards-based curricula (e.g., Reys, Reys, Lapan, Holliday, & Wasman, 2003; Riordan, & Noyce, 2001; Senk & Thompson, 2002), as well as an increased emphasis on curriculum validation by the U.S. Department of Education, Institute for Educational Sciences. There is limited empirical research on the effectiveness of standards-based curricula for this population of students, and even less research involving yearlong, naturalistic interventions. This study contrasts materials that were designed according to principles identified in the special education literature with one of the more prominent middle school mathematics programs originally funded by the National Science Foundation. This quasi-experimental study focused on two main questions:

1. Would intervention students (i.e., those taught with a curricular approach based on principles identified in the special education literature) achieve higher levels of success on standardized and criterion measures than students in the comparison condition?
2. Would intervention students achieve more positive attitudes and beliefs about mathematics than students in the comparison condition?

Method

Participants

Teachers. The participants in this study were 6 middle grade teachers and their 53 students in two middle schools located in medium-sized, suburban school districts. The two schools, located in nearby districts, were comparable along many variables. Both were lower middle-class, suburban schools with similar socio-economic status (determined by an average of 50% of students on free or reduced lunch), as well as other demographic information provided by the districts (e.g., percentage of students in special education; absentee rate; percentage of classes taught by teachers meeting the No Child Left Behind (NCLB) Act (Bush, 2001) highly qualified definition; students who passed seventh-grade statewide assessments in the previous 3 years). Furthermore, all teachers in the study were veteran teachers with at least 5 years experience teaching math to low-achieving students at the middle grade level. Two of the teachers were in the intervention school and four were in the comparison school. All had received systematic in-service training in their respective curricula prior to the beginning of the study.

Students. At the intervention school, 25 sixth-grade students participated in this yearlong study. At the comparison school, 28 sixth-graders participated, for a total of 53 student participants. (The original sample included 76 students, but 6 students from the intervention school and 8 students from the comparison school were excluded from the data analysis because they were not present for either the pretesting or posttesting.) All of the students in both schools had been identified for intense, remedial instruction in mathematics based on recommendations from elementary teachers, as well as academic test score data from the end of elementary school. When the study began, these students were members of intact classrooms in their respective schools. Further demographic

TABLE 1. Participant Characteristics

Variable	Intervention group	Comparison group
n	25	28
Students with learning disabilities		
Math	0	0
Reading	21	14
Other	4	0
CTB Terra Nova test	12.36 (2.78)	13.36 (2.26)
Attitude survey	50.92 (4.02)	53.32 (6.36)
Gender ratio	11F:14M	15F:13M

Note. All scores in parentheses represent standard deviations. F = females; M = males.

information on the participants in this study is presented in Table 1.

None of the students in the intervention or comparison schools had Individualized Education Programs (IEPs) in mathematics; however, all of the students in the intervention group had IEPs in one subject area, predominantly reading (21 of 25 students). Half of the 28 students in the comparison school had IEPs in reading. All of the special education students who participated in the study had been identified through district and statewide criteria for learning disabilities. They were of normal intelligence, and a discrepancy model was used to determine placement in special education. All students were given the mathematics portion of the CTB Terra Nova (McGraw-Hill, 2002) in October. Mean performance on this test for the 53 student participants was the 20th percentile. There were nonsignificant differences between groups on this measure, $t(1, 51) = 1.62$, $p = .11$).

Materials

Intervention Group. Teachers in the intervention group taught students a conceptual foundation of whole number operations, number theory concepts, data analysis, measurement, and geometric concepts using *Transitional Mathematics, Level 1* (Sopris West, 2004) curricular materials. These resources reflect the 2000 NCTM Standards, and they have been adapted to the needs of students at risk for academic failure, as well as students with disabilities, using many of the instructional principles validated through special education research (e.g., distributed practice, extensive use of visual models, high expectations). Grants from the U.S. Department of Education, Office of Special Education Programs provided the funding for the development of these materials. Development, piloting, field-testing, and experimental research for the curriculum occurred over an 8-year period. Throughout the curriculum, there is considerable emphasis on conceptual understanding, as well as extended opportunities for problem solving and for whole class discussions.

As an example of the curriculum's conceptual orientation, students reviewed operations on whole numbers with an emphasis on the role of place value. Alternative "expanded algorithms" and visual models (as shown in Figure 1) helped underscore the concept of regrouping.

The intervention group's materials were also designed to show the explicit connections between topical areas. For example, when students studied area and perimeter, they used graph paper to draw various rectangles with a fixed perimeter (e.g., 30 units). In a subsequent lesson, they applied their knowledge of bar graphs to represent the systematic relationship between the unit-by-unit decrease in the longest side and the subsequent increase in the area of the rectangle. This activity provided an explicit connection between measurement, geometry, and graphing data. These relationships are shown in Figure 2.

The curriculum also contained distributed practice on key skills and concepts. For example, students practiced factoring numbers over multiple lessons through a variety of representations such as arrays, tables, and what are often called *factor rainbows*. Students frequently practiced math facts, particularly if they were relevant to a lesson in the near future. These distributed practice activities both allowed students to master important skills and concepts once they had been taught and prepared students for more complex concepts (e.g., greatest common factor and least common multiple).

The curriculum's problem solving and application exercises were designed so that teachers needed to introduce and explain the context for exercise and to actively monitor students as they worked in pairs or in small groups. The problems used real-world scenarios (e.g., musical groups, movie sales, the Internet, sports) and were written in such a way as to minimize the amount of independent reading required of students. Problems often included drawings or diagrams, and teachers generally read problems to students before they worked on them. Frequently students used rulers, calculators, graph paper, and colored pens and worked from diagrams or data tables to solve problems.

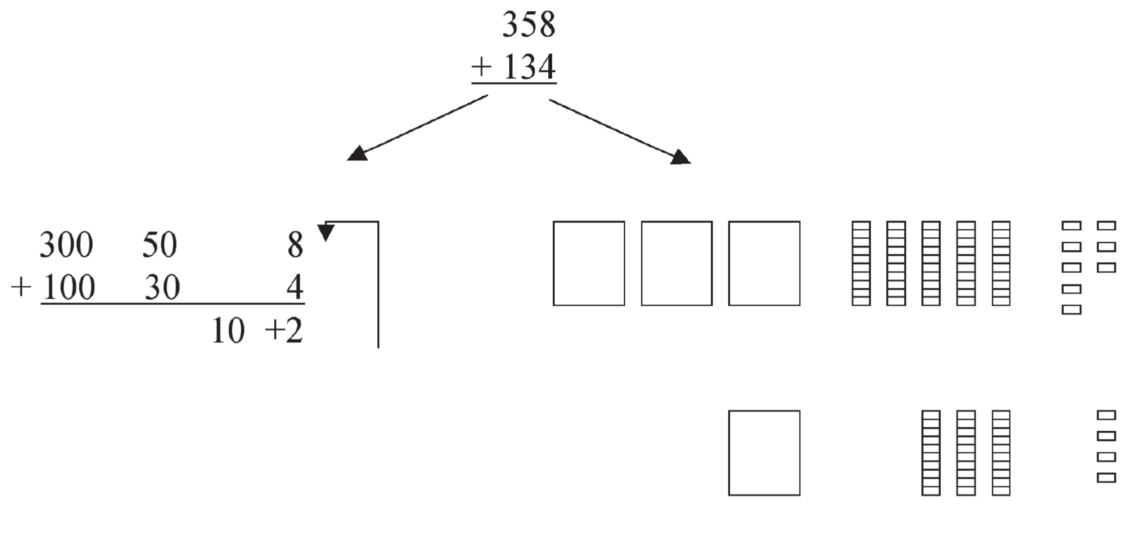

FIGURE 1. Expanded addition for teaching place value.

Comparison Group. Students in the comparison group used modules from the first level of the *Connected Mathematics Program* (Prentice Hall, 2002). This curriculum was funded by the National Science Foundation (NSF), and it fully reflects the 2000 NCTM Standards. Development, field testing, pilot testing, and research for this curriculum occurred over a 5-year period (Ridgway, Zawaojewski, Hoover, & Lambdin, 2003). The program has been used with low-income minority students as part of comparative research (Schoenfeld, 2002) and as a way of addressing the equity principle.

This program emphasizes core NCTM (2000) strands (e.g., numbers and operations, measurement, geometry, data analysis, and probability). A common emphasis throughout all of the strands is problem solving, and students are often required to read lengthy descriptions of problems as an integral part of the activity.

Real-world application activities are a regular part of the curriculum, and students are encouraged throughout the program to explain their thinking verbally or in writing. For example, the program uses an extended example of baking brownies as the context for helping students understand multiplication of fractions. The *Connected Mathematics Program* (Prentice Hall, 2002) reflects recent efforts in mathematics to provoke deeper mathematical thinking by having students work on challenging problems for a significant portion of the instructional period. These investigations usually entail paired or small-group work and subsequent whole class discussions. Each investigation includes follow-up problems, known as *applications, connections, and extensions,* or *ACE*. The program also stresses the importance of visual models and hands-on activities as aides to conceptual understanding.

Over the preceding 2 years, administrators and teachers in the cooperating district using the *Connected Mathematics Program* (Prentice Hall, 2002) had been concerned about the lack of systematic skills development in the program. Consequently, the middle school, like all of the other schools in the comparison district, had added supplemental basic skills practice. This segment extended the mathematics period for an additional 25 min. Typically, students completed worksheets that contained math facts, operations on whole numbers, and operations on fractions. These worksheets were not integrated with particular *Connected Mathematics* modules.

Procedure

Prior to the intervention, researchers used Secada's (1997) *Instructional Environment Observation Scale* to document the classroom environments and instructional practices. The scale, which has been used in previous research on low-achieving and special education students (e.g., Baxter et al., 2002), helped us to determine the comparability of the intervention and the comparison classrooms. The scale measures classroom routines, climate, cross-disciplinary connections, social support for student learning, and student engagement. After reviewing the observation scales and notes, we indicated that no major differences were detectable between the intervention and the comparison classrooms. Furthermore, class size for both conditions was approximately 15 students.

Two researchers on the project observed classes throughout the study to determine fidelity of implementation. These two researchers who were knowledgeable of both sets of curricular materials observed monthly throughout the yearlong intervention to determine if the materials were being used as designed. Observers used a checklist of key instructional techniques for each intervention, as well as field notes, to record fidelity. Observers discussed this information with the 6 participating teachers throughout the course of the study. These discussions helped to ensure that each curriculum was imple-

Width	Length	Area
1	**14**	**14**
2	**13**	**26**
3	**12**	**36**
4	**11**	**44**
5	**10**	**50**
6	**9**	**54**
7	**8**	**56**

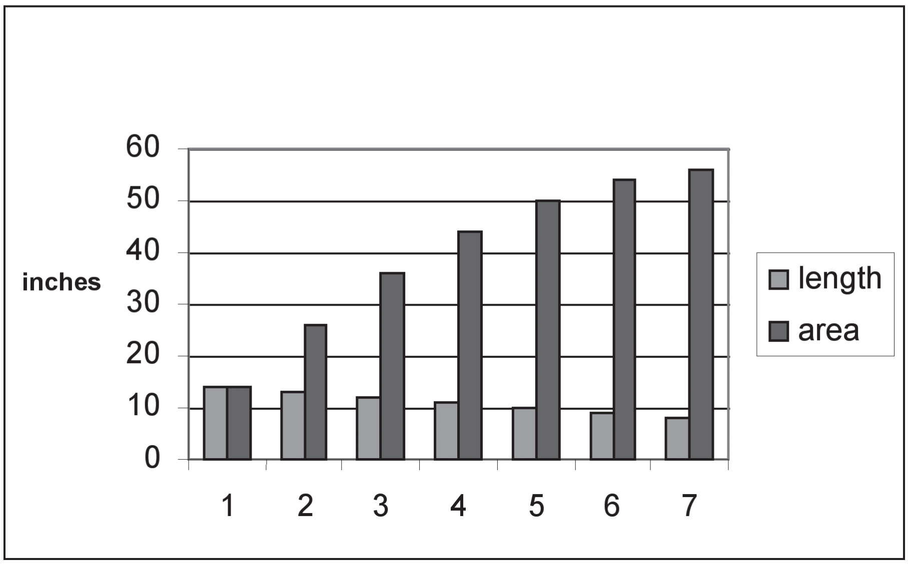

FIGURE 2. Graphic relationship between the perimeter and the area of a rectangle.

mented consistently and correctly. The checklists and field notes indicated no significant deviations from the prescribed guidelines for each curriculum. Typical patterns of instructional practice for each group are described below.

Intervention Group. The daily 55-min instruction was divided into three basic components: (a) math warm-up, (b) guided practice on new concepts, and (c) problem solving or application of concepts. Warm-up activities provided distributed practice on skills and concepts (e.g., facts, basic algorithms). These structured activities occurred for approximately 5–7 min. The guided practice phase of the lesson involved the introduction of a new concept followed by typically no more than five practice problems. In addition to traditional conception of guided practice, which emphasizes a clear introduction of a new concept, along with the opportunity to complete a brief set of problems, the 2 intervention teachers probed students on their understanding of concepts. There was a significant emphasis on detecting student misconceptions as well. This phase of the lesson lasted approximately 20 min.

The remaining 30 min of the lesson involved paired or small-group problem solving, along with a subsequent classroom discussion. Teachers encouraged students to share their answers and emphasized problem-solving strategies used to derive answers to problems or exercises. They also helped students develop longer, more descriptive verbal explanations of how they solved problems. This latter focus was particularly important because of the tendency of many low-achieving students to not participate in whole class discussions.

Comparison Group. The comparison group had 80 min of instruction. The additional 25 min of daily instruction over the intervention group was simply an artifact of the way the middle school structured mathematics for all of its academically low-achieving students. The daily 80-min instruction was divided into four components: (a) launch, (b) explore, and (c) summarize. The first three components involved the specified structure in the *Connected Mathematics Program* (Prentice Hall, 2002). The teachers began by "launching" or setting the context for the daily problem. The purpose of this segment of the lesson was to make sure that both the context for the problem and what made it challenging was clear to the students. It was also an opportunity to provide background information and to review relevant concepts. This section typically lasted 10 min.

The second portion of the lesson (i.e., explore) allowed students to explore problems in pairs or small groups. Students worked at tables, and teachers actively monitored student work, often clarifying the task and assisting students with problem-solving strategies. The third portion of the lesson (i.e., summarize) involved a classroom discussion in which students were given the opportunity to summarize their findings, present data, or describe a strategy used to solve a problem. The teacher's main role in this portion of the lesson was to rephrase and refine what students were saying and to reinforce the main concepts of the lesson. The length of time for the second and third portions of the lesson often varied based on the day's topic. However, it was typical for students to spend approximately 20 min working in groups and the remaining 25 min on classroom discussion.

The final 25 min of the lesson involved structured basic skills practice described earlier. These materials were independent of the *Connected Mathematics Program* (Prentice Hall, 2002), and they provided the kinds of basic skills development and review commonly found in remedial and special education classrooms. These teacher-made materials were developed because of the past performance of low-achieving students on district and statewide tests. Students typically used traditional algorithms to complete worksheet computational problems.

Measures

CTB Terra Nova. The sixth-grade level of the CTB Terra Nova (McGraw-Hill, 2002) is a standardized achievement test that is aligned with the National Assessment of Educational Progress (NAEP) achievement levels. Norms for the entire K–12 CTB Terra Nova are based on 275,000 students throughout the United States, and the sampling includes students with disabilities. The math portion of the CTB Terra Nova contains select and construct response formats, each reflecting different Item Response Theory (IRT) models. The select response items use the following three-parameter logistic model: item difficulty, item discrimination, and probability of a correct response by a low-scoring student. The construct response items use two parameters: item discrimination and a difficulty parameter for each score point assigned to the item.

We used the CTB Terra Nova (McGraw-Hill, 2002) in this study because it enabled us to determine comparability of the two groups on a range of grade-level skills and concepts. It was also an independent measure of growth over time. The test contained computational problems (e.g., operations on whole numbers, fractions, decimals), short word problems, measurement, and geometry concepts. Some items required students to use tools (e.g., rulers) and then explain their answers in a construct response format. This measure was administered in late October and again in early June.

Core Concepts Test. This criterion test was a 30-item test that we developed to measure the concepts that were common to both the intervention and the comparison groups. We constructed items based on the extent to which they were central topics or concepts covered during the year in both curricula (i.e., *Transitional Mathematics,* Sopris West, 2004, and *Connected Mathematics Program,* Prentice Hall, 2002). Only three of the items on the Core Concepts test——the ones pertaining to whole number operations and place value—addressed concepts found only in the intervention curriculum. However, these concepts were addressed in the supplemental basic skills materials used with the comparison group throughout the year. This was a cumulative measure of core math concepts that had been taught across the entire year, and it was administered in early June. The test assessed understanding of place value, estimation, prime and composite numbers, prime factorization, greatest common factor, least common multiple, exponents, area and perimeter, geometry, and interpreting data from graphs and tables.

Half of the items on the test required select response. The remaining half of the items involved construct responses (e.g., writing appropriate information based on a table). Items in this section of the test also contained multiple responses (e.g., identifying all of the prime numbers in a list of numbers).The maximum score on this test was 36. A test–retest reliability was performed on this test with an independent sample of 34 students, yielding a reliability of .87.

Attitude Measure. To determine attitudes toward math, students were given a 20-item survey entitled "Attitudes Toward Math." This survey assessed general attitudes toward the subject and the extent to which students thought they (a) were good at problem solving, (b) worked well with numbers, and (c) believed that working hard in mathematics led to doing well in the subject. Each question involved a 4-point Likert scale, yielding a maximum score of 80 points. This pretest and posttest measure was group-administered in late October and again in early June. The attitude survey provided a global indication of student attitudes toward math. Format for items used on this survey were based on the Piers-Harris Children's Self Concept Scale (Piers, Harris, & Herzberg, 2002). The measure has been used in previous research (e.g., Woodward, 1999, 2005), and test–retest reliability on an independent sample of 42 students yielded a reliability of .91.

Results

Analyses of the data are grouped according to the two academic measures of math achievement and the "Attitudes Toward Math" survey. Cohen's d was used as a measure of effect size for all between-group comparisons that were statistically significant.

Mathematics Measures

CTB Terra Nova. A 2×2 analysis of variance (ANOVA) was performed on the CTB Terra Nova (McGraw-Hill, 2002) measure, which was administered in late October

TABLE 2. Means and Standard Deviations for the CTB Terra Nova

		Pretest		Posttest	
Group	*n*	*M*	*SD*	*M*	*SD*
Intervention	25	12.36	2.78	17.88	4.06
Comparison	28	13.36	2.26	15.11	4.26

TABLE 3. Means, Standard Deviations, and Percentage Correct for the Core Concepts Test

Group	*n*	*M*	*SD*	Mean % correct
Intervention	25	21.02	5.64	58
Comparison	28	13.07	4.09	36

TABLE 4. Means and Standard Deviations for the "Attitude Towards Math" Survey

		Pretest		Posttest	
Group	*n*	*M*	*SD*	*M*	*SD*
Intervention	25	50.92	4.02	65.36	7.40
Comparison	28	53.32	6.36	51.86	6.53

and again in early June. Descriptive statistics for students by group and time (i.e., each of the repeated measures) can be found in Table 2. Results of the 2 × 2 ANOVA indicate nonsignificant differences between groups, $F(1, 51) = 1.55$, $p = .22$), but significant differences for time, $F(1, 51) = 35.60$, $p < .001$, as well as significant interactions for groups and time, $F(1, 51) = 8.58$, $p = .01$.

Due to the significant interaction, *t* tests were performed on the pretest and posttests. There were non-significant differences between groups on the pretest, $t(1, 51) = 1.29$, $p = .21$, but significant differences between groups on the posttest, $t(1, 51) = 2.35$, $p < .05$, $d = 1.23$, favoring the intervention students.

Core Concepts Measure. An analysis of covariance (ANCOVA) was performed on the criterion measure, which was administered in early June. Descriptive statistics for students by group and time (i.e., each of the repeated measures) can be found in Table 3. The October CTB Terra Nova (McGraw-Hill, 2002) was used as the covariate in this analysis. Results indicate significant differences between groups, $F(1, 50) = 37.52$, $p < .001$, $d = 1.61$, again favoring the intervention students.

Attitudes Toward Math

A 2 × 2 ANCOVA was performed on the "Attitudes Toward Math" measure. The October CTB Terra Nova (McGraw-Hill, 2002) score was used as the covariate in this analysis. Descriptive statistics for students by group and time (i.e., each of the repeated measures) can be found in Table 4. Results of the 2 × 2 ANCOVA indicate significant differences between groups, $F(1, 50) = 11.46$, $p = .001$, and time, $F(1, 50) = 178.01$, $p < .001$, as well as significant interactions for groups and time, $F(1, 50) = 267.42$, $p < .001$.

We performed *t*-tests on the pretest and posttest. There were nonsignificant differences between groups on the pretest, $t(1, 51) = 1.62$, $p = .11$, but significant differences between groups on the post test, $t(1, 51) = 7.06$, $p < .001$, $d = 1.95$, favoring the intervention students.

Discussion

In this study we contrasted two curricula for teaching challenging mathematics to students who were at risk for special education in mathematics. We conducted our research as part of a wider effort to determine if the equity principle found in the 2000 NCTM Standards was applicable for students at risk for special education services in mathematics. The pattern of results indicates that the curriculum that used research-based principles found in the special education literature tended to lead to superior achievement and attitudinal results by the end of the year. These results occurred in spite of the fact that there was an additional 25 min of skills instruction per day for the comparison students.

Results on the CTB Terra Nova (McGraw-Hill, 2002) are noteworthy insofar as the significant interaction between condition and time. As Table 1 indicates, in late October, intervention students scored, on average, lower than comparison students did. This situation reversed by early June, and there were significant differences between the two groups. Cohen's *d* indicates relatively large effect sizes for the June testing.

The even larger statistical differences between the intervention and comparison groups, as well as effect sizes on our Core Concepts Test, merit further analysis. One might have hypothesized that the differences would not have been as large given the additional 25 min of skills practice that the comparison students had each day. However, the items on the Core Concepts Test were not designed to assess basic skills (e.g., facts or proficiency in operations on whole numbers using traditional algorithms). Thus, the direct effects of this kind of practice were less likely to be evident on the test. Furthermore, interviews with teachers in the study and in other research (e.g., Woodward & Howard, 1994) indicate that this kind of added, and often unrelated, skills practice is often less effective than it might appear. Students tend to be unengaged in these activities because of their lengthy and highly routine nature. They also afford the unintended opportunity to practice errors and misconceptions, particularly when the skills are multidigit multiplication or long-division computation problems.

Another possible explanation for the differences on the Core Concepts Test is rooted in the way concepts were presented in the two curricula. The *Connected Mathematics Program* (Prentice Hall, 2002) often embeds concepts in application activities and does not explicitly teach them to students. It is usually the teacher's decision whether to present more practice on an algorithm or to offer other explanations of a concept. The program also does not contain the kind of distributed practice contained in *Transitional Mathematics* (Sopris West, 2004). These two design characteristics may explain differences in conceptual understanding and/or retention.

One other result on the Core Concepts Test merits discussion. Even though the intervention group averaged 58% correct, it must be remembered that this test was a cumulative measure that assessed concepts taught in late October and early November. Ideally, we should have assessed concepts in a way that was more proximal to the time the concepts were taught. However, the two instructional programs often presented the concepts assessed on the Core Concepts Test in a different manner and sequence and, at times, for a different duration. These variations are an uncontrolled byproduct of large-scale, naturalistic comparison studies of curricula. Therefore, we developed a cumulative measure that would assess all of the common concepts at the end of the school year (i.e., early June).

Results on the "Attitudes Toward Math" survey are encouraging in that they reflect the need to modulate instructional activities so that students have frequent opportunities for success and other occasions when problems cannot be solved immediately or even individually. That is, paired or small-group instruction, in which teachers scaffold understanding and assist in the completion of the task, can be an effective way of increasing student motivation. As Middleton and Spanias (1999) suggest, students need occasional opportunities in which they are not immediately successful on tasks so that they develop a more reasonable sense of self-efficacy and task persistence.

An intentional feature of the program was the mix of relatively easy tasks, particularly distributed practice activities, and more challenging application and problem-solving activities designed students at risk for academic failure. This consideration evolved from previous research on NSF-sponsored math programs (Baxter et al., 2001; Harris & Graham, 1996; Woodward, 1999; Woodward & Baxter, 1997), and classroom observations and interviews conducted as part of this study. Because NSF-sponsored mathematics programs are effectively written for the modal student, the cognitive load of the materials can be excessive for students at risk for academic failure, thus leading to too many opportunities to fail at tasks.

It must be remembered that this study involved students with learning disabilities who did not have IEPs in mathematics. Interviews with school administrators, the special education staff, and participating teachers confirmed that these students were at risk for special education services in math, but for various reasons, they did not qualify at the beginning of middle school. However, one benefit of focusing on this portion of the academically low-achieving population is that it helps control for the high variability in student performance associated with students who have learning disabilities and IEPs in mathematics (see Fuchs & Fuchs, 2001).

Two other limitations to the research reported here should be noted. First, it was a quasi-experimental study involving intact groups. The duration of study, as well as district policies for using specifically adopted curricula, made the quasi-experimental design unavoidable. Nonetheless, future research in which interventions have a shorter duration would allow students to be randomly assigned to conditions.

An equally if not greater limitation to this study was the inability to determine which of the research-based principles had the largest effect on students in the intervention group. This kind of problem has occurred in past research (e.g., Woodward & Gersten, 1992), and it is a function of the way curricular materials integrate different principles, as well as instructional practices, over the length of time for a study such as this one. Determining the relative effects of different principles (e.g., the use of visual models or manipulatives) are best suited to experimental studies of shorter durations.

In conclusion, the findings from this study suggest that many instructional principles now discussed in the special education math literature are applicable to students who do not have IEPs in the subject. Nonetheless, more research needs to be conducted with students who have disabilities to determine to what extent the equity principle described in the 2000 NCTM Standards is applicable to all students. As Semmel and Gerber (1990) have observed, instructional solutions for students with disabilities is a recursive process. Interventions need to be continually refined to address the increasingly unique characteristics of students in special education.

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