Creativity: The Essence of Mathematics

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For the gifted mathematics student, early mastery of concepts and skills in the mathematics curriculum usually results in getting more of the same work and/or moving through the curriculum at a faster pace. Testing, grades, and pacing overshadow the essential role of creativity involved in doing mathematics. Talent development requires creative applications in the exploration of mathematics problems. Traditional teaching methods involving demonstration and practice using closed problems with predetermined answers insufficiently prepare students in mathematics. Students leave school with adequate computational skills but lack the ability to apply these skills in meaningful ways. Teaching mathematics without providing for creativity denies all students, especially gifted and talented students, the opportunity to appreciate the beauty of mathematics and fails to provide the gifted student an opportunity to fully develop his or her talents. In this article, a review of literature defines mathematical creativity, develops an understanding of the creative student of mathematics, and discusses the issues and implications for the teaching of mathematics.

"The moving power of mathematical invention is not reasoning but imagination."—Augustus de Morgan (1866, p. 132)

Background

In 1980, the National Council of Teachers of Mathematics (NCTM) identified gifted students of mathematics as the most neglected segment of students challenged to reach their full potential. In 1995, the NCTM task force on the mathematically promising found little had changed in the subsequent 15 years (Sheffield, Bennett, Beriozábal, DeArmond, & Wertheimer, 1999). The definition of mathematical giftedness varies depending on the identification tools used and the program offered. Regardless of the definition used, finding students

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Journal for the Education of the Gifted. Vol. 30, No. 2, 2006, pp. 236–260.

with mathematical giftedness is a challenge for both educators and society. Often giftedness in mathematics is identified through classroom performance, test scores, and recommendations. Yet, research suggests that a high level of achievement in school mathematics is not a necessary ingredient for high levels of accomplishment in mathematics (Hong & Aqui, 2004; Mayer & Hegarty, 1996; Pehkonen, 1997; Sternberg, 1996). This apparent detachment between school mathematics and mathematical accomplishments indicates that some talented students are overlooked by current practices in school.

Polya (1962) defined mathematical knowledge as information and know-how. Of the two, he regarded know-how as the more important, defining it as the ability to solve problems requiring independence, judgment, originality, and creativity. A gifted student of mathematics possesses all of these characteristics and needs the opportunity to use them when solving challenging problems. Calls for renewed emphasis on mathematics education accompany each new round of published test results, but often such efforts stress remediation or additional practice rather than the development of a mathematical frame of mind. Frequently, all students, including those who met or exceeded the test goals, simply receive more of the same methods of instruction that yielded the results under examination. If mathematical talent is to be discovered and developed, changes in classroom practices and curricular materials are necessary. These changes will only be effective if creativity in mathematics is allowed to be part of the educational experience.

The visionary classrooms described by leaders in the NCTM enable students to

> confidently engage in complex mathematical tasks . . . draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until they find methods that enable them to make progress. (NCTM, 2000, p. 3)

For many adults, this vision is unlike the math classrooms they remember from their youth. Time was spent *learning from the master* where the teacher demonstrated a method with examples, and the students practiced with similar problems (Pehkonen, 1997). For these adults, the concept of mathematics is of "a digestive process rather than a creative one" (Dreyfus & Eisenberg, 1996, p. 258). However, mathematics is not a fixed body of knowledge to be mastered but rather a fluid domain, the essence of which is the creative applications of mathematical knowledge in the solving of problems (Poincaré, 1913; Whitcombe, 1988).

Defining and Measuring Mathematical Creativity

An examination of the research that has attempted to define mathematical creativity found that the lack of an accepted definition for mathematical creativity has hindered research efforts (Ford & Harris 1992; Treffinger, Renzulli, & Feldhusen, 1971). Treffinger, Young, Shelby, and Shepardson (2002) acknowledged that there are numerous ways to express creativity and identified more than 100 contemporary definitions. Runco (1993) described creativity as a multifaceted construct involving "divergent and convergent thinking, problem finding and problem solving, self-expression, intrinsic motivation, a questioning attitude, and self-confidence" (p. ix). Haylock (1987) summarized many of the attempts to define creativity. One view "includes the ability to see new relationships between techniques and areas of application and to make associations between possibly unrelated ideas" (Tammadge, as cited in Haylock, 1987, p. 60). The Russian psychologist Krutetskii characterized creativity in the context of problem formation (problem finding), invention, independence, and originality (Haylock, 1987; Krutetskii, 1976). Others have applied the concepts of fluency, flexibility, and originality to mathematics (Haylock, 1997; Jensen, 1973; Kim, Cho, & Ahn, 2003; Tuli, 1980). In addition to these concepts, Holland (as cited in Imai, 2000) added elaboration (extending or improving methods) and sensitivity (constructive criticism of standard methods). Torrance (1966) offered the following definition of creativity:

> Creativity is a process of becoming sensitive to problems, deficiencies, gaps in knowledge, missing elements, disharmonies, and so on; identifying the difficult; searching for solutions, making guesses or formulating hypotheses about

> the deficiencies; testing and re-testing these hypotheses and possibly modifying and re-testing them; and finally communicating the results. (p. 8)

In developing an operational definition of mathematical creativity, Singh (1988) applied Torrance's definition of creativity to the formulation of cause and effect hypotheses in mathematical situations.

Balka (1974) introduced criteria for measuring mathematical creative ability. He addressed both convergent thinking, characterized by determining patterns and breaking from established mindsets, and divergent thinking, defined as formulating mathematical hypotheses, evaluating unusual mathematical ideas, sensing what is missing from a problem, and splitting general problems into specific subproblems. In reviewing Balka's criteria, breaking from established mindsets was a defining feature in the efforts of others to understand the creative mathematician. Haylock (1997) and Krutetskii (1976) believed that overcoming fixations was necessary for creativity to emerge. Both, like Balka, focused on the breaking of a mental set that places limits on the problem solver's creativity. Limits are also established when creativity and systematic applications are confused. In an earlier work, Haylock (1985) discussed the difference between creativity and being systematic in mathematical problem solving. By applying learned strategies, a student can systematically apply multiple methods to solve a problem but never diverge into a creative strategy, never exploring areas outside the individual's known content-universe. To encourage the development of mathematical creativity, educators need to enable creative exploration and reward students who seek to expand their content-universe.

The Essence of Mathematics

The essence of mathematics is thinking creatively, not simply arriving at the right answer (Dreyfus & Eisenberg, 1966; Ginsburg, 1996). In seeking to facilitate the development of talented young mathematicians, neglecting to recognize creativity may drive the creatively talented underground or, worse yet, cause them to give up the study of mathematics altogether. Hong and Aqui (2004) studied the differences between academically gifted students who achieved high grades in school math and the creatively talented in mathematics, students with a high interest, who were active and accomplished in math but not necessarily high achieving in school math. As they were examining differences, their study did not include students with strengths in both areas. Hong and Aqui found significant differences in cognitive strategies used by the two groups with the creatively talented being more cognitively resourceful. Neither group of students should be neglected, yet Ching (1997), a supervisor of teaching practices, found hidden talent to be rarely identified by typical classroom practices. Traditional tests to identify the mathematically gifted, such as the commercially available achievement tests or state assessments, do not identify or measure creativity (Kim et al., 2003) but often reward accuracy and speed. These tests identify students who do well in school mathematics and are computationally fluent (Hong and Aqui's academically talented), but neglect the creatively talented in mathematics. Brody and Mills (2005) report that the talent search models of identification have been proven to be valid as a predictor of academic achievement, as well as achievement later in life for those students who qualify. Services and opportunities provided to these students match the identification system used and are appropriate for their needs. These students typically have supportive families and advantaged homes (Brody & Mills) and are invited to participate in the search based on their academic performance. Hong and Aqui's research supports the need to look deeper for mathematical talent. Encouraging mathematical creativity in addition to computational fluency is essential for children to have a productive and enjoyable journey while developing a deep conceptual understanding of mathematics. For the development of the mathematical talent, creativity is essential.

Mathematical creativity is difficult to develop if one is limited to rule-based applications without recognizing the essence of the problem to be solved. Köhler (1997) discussed an experiment by Hollenstein in which one group of children worked on a math exercise presented in the traditional method. These problems were complete or closed in that they were constructed so that a single correct answer existed (Shimada, 1997). A second group was given the conditions on which the first group's exercise were based and was asked to develop and answer problems that could be solved using calculations. The open-ended nature of the task given to the second group did not limit them to a set number of problems. This group created and answered more questions than were posed to the first group, calculated more accurately, and arrived at more correct results. Researchers at Japan's National Institute for Educational Research conducted a 6-year research study that evaluated higher order mathematical thinking using open-ended problems (problems with multiple correct answers). In a round-table review of the study, Sugiyama, from Tokyo Gakugei University, affirmed this approach as a means to allow students to experience the first stages of mathematical creativity (Becker & Shimada, 1997).

Doing What Mathematicians Do

Doing what mathematicians do as a means of developing mathematical talent (as opposed to replication and practice) is consistent with the work at The National Research Center for the Gifted and Talented (Reis, Gentry, & Maxfield, 1998; Renzulli, 1997; Renzulli, Gentry, & Reis, 2003, 2004). Emphasis is placed on creating authentic learning situations where students are thinking, feeling, and doing what practicing professionals do (Renzulli, Leppien & Hays, 2000). The fundamental nature of such authentic high-end learning creates an environment in which students apply relevant knowledge and skills to the solving of real problems (Renzulli et al., 2004).

Solutions to real problems also entails problem finding, as well as problem solving. Kilpatrick (1987) described problem formulation as a neglected but essential means of mathematical instruction. Real-world problems are not presented in a textbook or by a teacher. They are ill-formed and require one to employ a variety of methods and skills. In addition to equations to solve and problems designed to converge on one right answer, students need the opportunity to design and answer their own problems. In his Creative Mathematical Ability Test, Balka (1974) provided the participants with mathematical situations from which they were to develop problems. Mathematical creativity was measured by the flexibility, fluency, and originality of the problems the participants constructed. By working with these types of mathematical situations, students can be encouraged to use their knowledge flexibly in new applications. Flexible applications of knowledge require more than knowledge of acquaintance, or a simple *knowledge of*, which is the entry level of knowledge identified by William James (as cited in Renzulli et al., 2000; Taylor & Wozniak, 1996). James considered conceptual knowledge to be a higher level of knowledge. He referred to this level as *knowledge about*, a way of knowing that is based on a continuity of experiences (Taylor & Wozniak). Conceptual knowledge lies within authentic mathematical experiences provided to students rather than simple replication of demonstrated methods.

Factual and procedural knowledge is necessary to develop proficiency in mathematics, but research suggests that conceptual understanding is equally as important (Bransford, Brown, & Cocking as cited in NCTM, 2000, p. 21; Schoenfeld, 1988). Davis, Maher, and Noddings (1990) listed four types of mathematical experiences that children need in the classroom. The first experience is characterized by teacher demonstration, followed by student drill and practice. This type of experience is the most prevalent in the classroom but has limited usefulness in developing deep mathematical understanding. In the report, *All Students Reaching the Top*, the National Study Group for the Affirmative Development of Academic Ability found that drill and practice may actually be working against the transfer of learning to applications not replicated by the drills (Learning Point Associates, 2004). Schoenfeld reported similar findings in his research in which he found that students fail to connect drill-and-practice learning with real-world problems.

To allow for a transfer of learning, students need more than drill and practice; they need to understand the mathematical concepts beyond the practice exercises (Davis et al., 1990). Bassok and Holyoak (1989) conducted an experiment with 12 high-ability ninth-grade students in an accelerated scientific program to investigate the transfer of learning between algebra and physics problems with the same underlying structure. Their results showed that 90% of the students who were trained to solve the physics problems were unable to make the transfer of learning to solve similar but unfamiliar algebra problems. However, 72% of the algebra students success-fully transferred their understanding of the mathematical concepts to solve the physics problems. The difference in transferability may have been compromised by the context of the problem-solving situations in which the students were trained (Learning Point Associates, 2004), with the algebra students being able to recognize similarities in problem structure more readily. Davis et al. (1990) also described general readiness-building experiences designed to prepare students to recognize mathematical situations. Open-ended experiences also provide students opportunities to demonstrate their conceptual understanding. Both of these types of experiences lend themselves to the transfer of learning from classroom to real-word situations. Learning mathematics, therefore, involves much more than memorizing arithmetical facts and mastering computational algorithms; it entails incorporating experiences and conceptual understanding to solving authentic mathematical problems

The Role of Risk Taking in Mathematics

The new open-ended assessments used by many state Department of Education officials often place little value on creative solutions. Problems with test scoring in Connecticut's 2003–2004 mastery tests illustrate some of the issues where strict guidelines focusing on accuracy are the norm. "There is an art to scoring . . . there is subjectivity . . . our work is to remove as much of that variable as possible" according to Hall, CTB/McGraw-Hill's director of hand-scoring (Frahm, 2004, p. A1). While accuracy is important, strict emphasis on accuracy when assessing a child's conceptual understanding of mathematics discourages the risk taker who applies her or his knowledge and creativity to develop original applications in solving a problem (Haylock, 1985). Such an individual would be in the company of Poincaré, Hadamard, and Einstein, all eminent scientists and mathematicians who confessed to having problems with calculations (Hadamard, 1945).

Mayer and Hegarty (1996) reported converging evidence that students leave high school with adequate skills to accurately carry out arithmetic and algebraic procedures but inadequate problem-solving skills to understand the meaning of word problems. A good mathematical mind is capable of flexible thought and can manipulate and investigate a problem from many different aspects (Dreyfus & Eisenberg, 1996). Procedural skills without the necessary higher order mathematical thinking skills, however, are of limited use in our society. There is little use for individuals trained to solve problems mechanically as technology is rapidly replacing tedious computational tasks (Köhler, 1997; Sternberg, 1996). Often the difference between the errors made by eminent mathematicians and students of mathematics is a function of their insight into and appreciation of mathematics, not their computational skills (Hadamard, 1945).

With the increased emphasis on accountability from the No Child Left Behind Act of 2001, teachers are under even more pressure to teach to the test rather than to work toward developing in their students a conceptual understanding of mathematics. Encouraging students to take risks and look for creative applications reintroduces variability in scoring that assessment teams are working to eliminate. Discouraging risk taking limits student exposure to genuine mathematical activity and dampens the development of mathematical creativity (Silver, 1997). For substantial and permanent progress in a child's understanding of mathematics, an appreciation of "the difficult-beautiful-rewarding-creative view of mathematics" (Whitcombe, 1988, p. 14) must be developed. However, rather than developing an appreciation for mathematics by focusing on qualities of mathematical giftedness, teachers who only emphasize algorithms, speed, and accuracy provide the creative student negative reinforcement. Thus, many talented students do not envision themselves as future mathematicians or in other professions that require a strong foundation in mathematics (Usiskin, 1999). Failing to encourage creativity in the mathematics classroom denies all children the opportunity to fully develop their mathematical understanding. For the mathematically talented, lack of creativity may delay, or worse yet, prevent the realization of their potential to contribute to new understandings of the world around us through the advancement of mathematical theory.

The Enjoyment Factor

In a recent undergraduate course on the teaching of mathematics, future elementary school teachers were asked to describe their most memorable childhood experience in school mathematics. The overwhelming majority described an unpleasant experience (Mann, 2003). "We have known for some years now . . . that most children's mathematical journeys are in vain because they never arrive anywhere, and what is perhaps worse is that they do not even enjoy the journey" (Whitcombe, 1988, p. 14). It is difficult to develop an understanding of mathematics if the effort to inspire this knowledge is uninteresting. Confirming this, Csikszentmihalyi, Rathunde, and Whalen (1993) found that enjoyment is central to capturing a child's interest and developing his or her talent. Using Amabile's (1989) ingredients of creativity, Starko (2001) also discussed the role of interest in intrinsic motivation for the development of creativity. The greater a child's intrinsic motivation, the greater the likelihood of creative applications and discoveries. Yet, intrinsic motivation is highly dependent on social environment (Amabile, 1989) and the social environment of a classroom is dependent on the teacher. When teachers do not look beyond the wrong answer, they convey the belief that math is divided into right and wrong answers (Balka, 1974; Ginsburg, 1996) and may reject creative applications, fostering a classroom environment that discourages developing creativity.

Time and Experience

Creativity needs time to develop and thrives on experience. Drawing from contemporary research, Silver (1997) suggested, "creativity is closely related to deep, flexible knowledge in content domains; is often associated with long periods of work and reflection rather than rapid, exceptional insight; and is susceptible to instructional and experiential influences" (p. 75). Poincaré's (1913) essay on mathematical creation also discussed the need for reflection. He described his discovery of the solution to a problem on which he had worked for a considerable amount of time arriving as a sudden illumination as he stepped onto a bus on a geologic excursion. This illumination was "a manifest sign of long, unconscious prior work . . . which is only fruitful, if it is on the one hand preceded and on the other hand followed by a period of conscious work" (p. 389). This period of incubation appears to be an essential aspect of creativity requiring inquiry-oriented, creativity-enriched mathematics curriculum and instruction (Silver, 1997). Whitcombe (1988) described an impoverished mathematics experience as one in which instruction only focuses on utilitarian aspects of mathematics and is without appropriate interest-stimulating material and time to reflect needed by the student. Such experiences deny creativity the time and opportunities needed to develop.

Hong and Aqui's (2004) division of mathematical talent into the academically gifted and creatively talented is critical in the consideration of talent development. The academically gifted student may excel in the classroom by demonstrating high achievement, or "schoolhouse giftedness," that is valued in traditional educational settings. These students' abilities remain relatively stable over time (Renzulli, 1998). Those academically gifted in mathematics are able to acquire the skills and methodologies taught often at a much more rapid pace than for less able students and perform well on standardized testing. The academically gifted usually demonstrate their mastery of the utilitarian aspects of mathematics, but neither speed nor accuracy in computation or the analytical ability to apply known strategies to identified problems are measures of creative mathematical talent. Hadamard (1945) described individuals he labeled "numerical calculators" as "prodigious calculators—frequently quite uneducated men—who can very rapidly make very complicated numerical calculations . . . such talent is, in reality, distinct from mathematical ability" (p. 58). Thus, it is possible to be considered academically gifted in mathematics but lack creative mathematical talent.

While speed of information processing is important in testing situations in which students' mathematical thinking is assessed using standardized tests, it is less important when a mathematician spends months or even years trying to work out a proof (Sternberg, 1996). Although current tests of number or numerical facility emphasize speed with stress imposed by severe time limits and accountability on the accuracy of the solutions (Carroll, 1996), the next generation of mathematicians must be shown the "wellsprings of mathematics; creativity, imagination, and an appreciation of the beauty of the subject" (Whitcombe, 1988, p. 14). In an analysis of cognitive ability theory and the supporting psychological tests and factor analysis, Carroll noted that despite six to seven decades of work, the relationships between the discrete abilities measured by psychometric tests and performance in mathematics remains unclear. Restricting the search for mathematical talent to the academically gifted who perform well on timed standards-based assessments denies opportunities to the creatively talented that go undiscovered because of lower levels of classroom achievement or limited educational experiences.

Understanding the Creative Student of Mathematics

NCTM's task force on the mathematically promising (Sheffield et al., 1999) characterized our promising young mathematics students in light of their ability, motivation, belief (self-efficacy), and opportunity/experience, all considered variables that must be maximized in order to fully develop a student's mathematical talent. Davis (1969) considered developing creativity in students of mathematics in terms of three major parameters: attitudes, abilities, and techniques (methods of preparing and manipulating information). While 26 years separate these efforts, they offer similar recommendations. Skills with the techniques of the discipline develop only through opportunity and belief in one's ability. Renzulli's (1978, 1998) model of giftedness defines three learner attributes (above-average ability, task commitment, and creativity) in three overlapping rings signifying an interdependence of these qualities to produce giftedness. Of these three qualities, there are two (task commitment and above-average ability) that mirror Davis' parameters. Overlaying the two models yields a conceptual framework in which mathematical creativity can be considered. The child possesses his or her innate ability that remains dormant if not developed with the appropriate challenges and experience. Teachers and parents must help develop task commitment by creating opportunities for purposeful and meaningful experiences, by fostering an understanding of techniques through instruction and modeling, and by establishing a creative environment that encourages risk taking and curiosity.

If any of these elements are missing, creativity in mathematics may not develop. Classrooms in which teachers do not accept alternative views and in which the rote application of skills is valued will provide the world with students that only have the capability to apply techniques in known situations. These students will struggle when they encounter unknown situations in which originality, creativity, and problem solving are necessary.

Underdeveloped Talent

One may wonder how many potential creative mathematicians are lost when weak analytical skills prevented them from progressing through the levels of mathematics offered in our educational system. Limiting use of creativity in the classroom reduces mathematics to a set of skills to master and rules to memorize. Doing so causes many children's natural curiosity and enthusiasm for mathematics to disappear as they get older, creating a tremendous problem for the mathematics educators who are trying to instill these very qualities (Meissner, 2000). Sternberg (1996) referred to comments from mathematicians who suggest that:

> . . . performance in mathematics courses, up to the college and even early graduate levels does not effectively predict who will succeed as a mathematician. The prediction failure is due to the fact that in math, as in most other fields, one can get away with good analytic but weak creative thinking until one reaches the highest levels of education. . . . . However, it is creative mathematical thinking that is the most important . . . (p. 313)

It is important that teachers work to develop mathematical creativity as the child begins his or her educational journey. Haylock (1997) suggests that the pupil's mathematical experience and techniques limit their creative development. Yet, Hashimoto (1997) found that, in general, most classroom teachers think there is only one correct answer and only one correct method to solve a mathematics problem. If taught that there is only one right answer, only one correct method, a student's concept of mathematics as an application of mathematic techniques is reinforced. Köhler (1997) illustrated this point in a discussion with an elementary classroom teacher about a student who had arrived at the correct answer in an unexpected way:

> While going through the classroom, that pupil asked me [the teacher] whether or not his solution was correct. *I was forced* [italics added] to admit that it was. That is what you get *when you don't tell the pupils exactly what to do* [italics added]. . . . The teacher now reproaches himself for *not having prevented this solution* [italics added]. He is obviously influenced by an insufficient understanding of what is mathematics, by the image of school as an institution for stuffing of brains . . . (p. 88)

Teachers often encourage students to explore, question, interpret, and employ creativity in their studies of other disciplines such as language arts, science, or the social sciences; yet, this example illustrates many teachers focus on the use of rules-based instruction for mathematics. If the instruction focuses on rote memorization rather than meaning, then the student will correctly learn how to follow a procedure, and will view the procedure as a symbol-pushing operation that obeys arbitrary constraints. Without a conceptual understanding of the underlying concepts and principles necessary for creative applications, students may overgeneralize from "bits and pieces" of prior knowledge and apply procedures correctly in inappropriate problem situations arriving at computationally correct "wrong" answers (Ben-Zeev, 1996). Students may gain computational skills, yet have little or incomplete understandings of the applications for which these are appropriate. Pehkonen (1997) suggested that the constant emphasis on sequential rules and algorithms may prevent the development of creativity, problem-solving skills, and spatial ability. The development of mathematical creativity deserves the same emphasis offered to the creative development in other disciplines.

Incorporating Creativity in the Teaching of Mathematics

Teachers must be prepared to appreciate the beauty and creativity of mathematics. They must explore the world of mathematics before they can help their students discover it. It is easy for teachers to forget the value of the struggle they may have encountered as they learned mathematics as children and fall into a teaching practice that involves demonstration by teacher and replication by the student (Pehkonen, 1997). Yet, for students to experience the true work of mathematicians, the struggle is necessary as they discover and apply mathematical theory to solve problems. Poincaré (1913) believed that true mathematicians had an intuitive sense that guided them in creative applications of mathematics. Jensen (1973) referenced Bruner in describing a creative act as one that produces surprise only recognizable by those prepared to see it and explained "creativity demands readiness and understanding of the problem both by the producer [the student] and by those who would use and appreciate the creative act [the teacher]" (p. 22). It is therefore necessary for teachers of mathematics to have Poincaré's appreciation for mathematics and a prepared mathematical mind to help students develop an understanding of the beauty of mathematics. It is also necessary for teachers to encourage the development of mathematical creativity.

There is significant power in learning conceptually. This power comes from the ability to recombine and relate concepts in a variety of settings, as opposed to factual learning, which has applications within the circumstances as they exist (Skemp, 1987). Mathematics is a powerful tool that can be used at varying levels of complexity in almost every occupation. Yet, many students leave school disliking mathematics and with the belief that they just cannot do math. Whitcombe's (1988) model of the mathematical mind is based on his belief that mathematical minds function efficiently when three aspects of mathematics are involved: algorithms (logical), creativity (intuitive), and beauty (speculative). Intuition and speculation function at a conceptual level, while algorithms are a rule-based application of mathematics. Yet, "algorithms constitute the majority of the mathematical diet of many of our children, . . . [they are] the least important as machines can do it better and faster, . . . [they] are the least important because they are boring," (Whitcombe, p. 15) and they are the least important as they offer the student no sense of the structure of mathematics. Accuracy is important, but accuracy without understanding is of minimal use. The right answer to the wrong problem is as potentially harmful on a construction site or in a hospital ward as the wrong answer to the right problem.

Crosswhite (1987) defined the process of "bottom line problem solving" and "bottom line teaching" as the unwritten game students and teachers play. The students patiently wait through the teacher's lesson presentation knowing that, in the end, the preferred method of solving the problem will be presented. The student then becomes accountable only for a replication of the process with similar problems. Creativity plays no role in this kind of teaching and learning. To fully implement the changes needed in instruction, teacher education programs must change. This is a long process, as the teachers of teachers must also make the shift. For current teachers, an understanding of the role of creativity in mathematics is an important first step, but curricular materials, classroom and administrative support, and training are all needed for progress to continue.

The development of mathematical communications skills is necessary for creativity to be recognized, appreciated, and shared. The impact on a creative student who thinks symbolically but is consistently asked to explain in written or oral language may be significant because thoughts and understanding may be lost in the translation. When describing a characteristic of mental activity associated with creative mathematical thought, Sir Francis Galton said

> It is a serious drawback to me in writing, and still more in explaining myself, that I do not so easily think in words as otherwise. It often happens that after being hard at work, and having arrived at results that are perfectly clear and satisfactory to myself, when I try to express them in language I feel that I must begin by putting myself upon quite another intellectual plane. I have to translate my thoughts into a language that does not run very evenly with them. I therefore waste a vast deal of time in seeking for appropriate words and phrases, and am conscious, when required on a sudden, of being often very obscure through mere verbal maladroitness,

> and not through want of clearness of perception. That is one of the small annoyances of my life. (as cited in Hadamard, 1945, p. 69)

Teachers who require each step of the problem to be written out, each answer justified both in spoken or written language, as well as in mathematical symbols, may not understand the processes used by creative, intuitive students. However, there are many stories of mathematical discoveries lost or delayed because the mathematician's work was not effectively communicated. After Riemann's death in 1866, a brief note found in his papers dealing with the distribution of prime numbers has become the focus of the careers of many mathematicians. Riemann's note simply stated "These properties of ζ(s) are deduced from an expression of it which, however, I did not succeed in simplifying enough to publish it" (as cited in Hadamard, p. 118). The lack of any other references to his findings underscores the need for effective communication skills. While the ability to explain, justify, and defend one's work is important, for most it is a learned skill. In language arts, children are taught how to write persuasive essays or creative short stories and how to evaluate the writing of others; a similar investment to develop mathematical communication skills is necessary for the expression of mathematics.

Teaching practices need to shift to a more balanced application of Whitcombe's (1988) model of the mathematical mind that recognizes creativity and the beauty of mathematics, as well as the rule-based algorithms that dominate most mathematics classrooms. Rather than overemphasize rules, algorithms, and convergent thinking to produce a single right answer, instruction should center on mathematical thought. Dreyfus and Eisenberg (1996) found that mathematical thought is more than absorbing some piece of mathematics or solving some mathematical problem: "it is closely associated with an assessment of elegance" (p. 255). Terms such as *beauty* and *elegance* are as difficult to define as *creativity*, yet each creates a vision of mathematics as extending beyond algorithms. "In mathematics, facts are less important than in other domains; on the other hand, relationships between facts, relationships between relationships and thus structure, are more important than in other domains" (Dreyfus & Eisenberg, p. 265). Dienes (2004) compared the work of a mathematician with that of an artist. Both labor to construct in an effort to communicate their understanding with others. Both play with ideas, combining them in various ways and as different structures until something emerges that the individual finds satisfying. Without the opportunity for creative play with a problem, students' problem-solving skills are limited to the recall of methods created by others (Bronsan & Fitzsimmons, 2001).

When students begin to explore the structure of mathematics, they begin to explore the beauty of the domain and develop a sense of mathematics. Poincaré (1913) described this mathematical sense as mathematical intuition or the ability to see the whole and to find harmony and relationships gained through study and experience. These experiences can lead to students' mathematical growth. The pupil's insight can only be facilitated by a challenging problem that is sufficiently demanding, as well as sufficiently accessible. The emphasis in teaching mathematics shifts from replication of demonstrated methods to allowing the student the right to make mistakes and explore alternative routes, thereby opening new perspectives (Köhler, 1997). Rather than closed problems with a single solution, students should be provided open-ended problems with a range of alternative-solution methods (Fouche, 1993). Some methods to solve the problems may be too simple, some may be out of reach of the child, while still others are within the child's grasp. Encouraging a child to reach beyond the familiar and probe deeper into the relationships and structures of a problem is the essence of teaching mathematics creatively.

Summary

A child's growth in mathematics involves more than just mastering computational skills. Identification of mathematical talent using only speed and accuracy of computation would qualify hand-held calculators to be called talented mathematicians. Mathematical talent requires creative applications of mathematics in the exploration of problems, not replication of the work of others. Problem solving is the heart of genuine mathematical activity, yet the supply of curricular materials designed to support a problem-solving approach to mathematical instruction is small in comparison to the materials aligned with a procedural, mechanical point of view (Silver, 1997).

In the United States, the combined efforts of the NCTM and the National Science Foundation have begun to yield appropriate curricular materials to develop more creative talents in mathematics. Yet, the journey is only beginning. Methods of assessing mathematical creativity, teacher accountability, and greater emphasis within teacher education programs on teaching for conceptual understanding and pedagogically relevant content knowledge are needed. Without such a shift, the education community perpetuates the accepted practices of the childhood classroom experiences for our next generation of teachers. School board members and administrators need to encourage and support innovative methods of teaching mathematics. This effort needs to be more than a policy statement; funding for materials and training, promoting mathematics as a creative endeavor within the community, providing specialists in mathematics and gifted education, and support and encouragement for classroom teachers are all needed. Classroom teachers should examine their teaching practices and seek out appropriate curricular materials to develop mathematical creativity. The challenge is to provide an environment of practice and problem solving that stimulates creativity, while avoiding the imposition of problem-solving heuristic strategies (Pehkonen, 1997) that will enable the development of mathematically talented students who can think creatively and introspectively (Ginsburg, 1996).

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Media Themes and Threads

I am pleased to announce the return of "Media Themes and Threads" to the *Journal for the Education of the Gifted*. My name is Kristie Speirs Neumeister, and I will be the editor for this section. I received my doctorate in educational psychology, with an emphasis on gifted and creative education, from the University of Georgia in 2002. I am currently an assistant professor at Ball State University, where I teach undergraduate courses in educational psychology and graduate courses in gifted education. I am excited about the opportunity to edit this section of the journal.

Veteran readers of the journal will be familiar with "Media Themes and Threads," as it appeared in past volumes of *JEG*; however, for newer readers, I wanted to provide a description of the section. The purpose of the section is to feature reviews of recent media products pertaining to gifted education, including current publications of books, videos, and instructional DVDs and CD-ROMs. Three reviews are published in this current issue.

Rebecca Nordin has written a thorough review of Gary Davis's new book, *Gifted Children and Gifted Education: A Handbook for Teachers and Parents*. This book provides a foundational overview of the most salient issues in gifted education and would be a great resources for parents and teachers of gifted children alike.

Middle school has been referred to as the "black hole" of gifted education due to a lack of programming and options for students. Susan Rakow's book, *Educating Gifted Students in Middle School: A Practical Guide*, reviewed by Jamie MacDougall, expertly responds to this assertion by providing a description of effective programming options for gifted middle school students and also emphasizing the need for strong guidance and counseling components at this level.

Finally, Felicia Dixon reviewed Laurence Coleman's recent addition to the field, *Nurturing Talent in High School: Life in the Fast Lane*, in which he accounts the experiences of high school students attending a residential academy for gifted learners. Coleman offers a unique contribution to the field with this insider's perspective on lived experiences of gifted adolescents.

I hope you will find these reviews informative as you continue to search for new information on parenting and educating gifted students. Should you be interested in reviewing a new media publication for *JEG* or if you would like to have a specific current publication reviewed, please send your inquiries directly to me at the address below.

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