A Typology of Mathematics Teachers' Beliefs about Teaching and Learning Mathematics and Instructional Practices

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The primary purposes of the study reported here were to investigate Greek mathematics teachers' beliefs regarding the nature of mathematics and mathematics teaching and learning, and to explore the various links between these beliefs and instructional practice. Two orientations that are characteristic of secondary mathematics teachers' beliefs were identified: A contemporary – constructivist orientation, and a traditional – transmission – information processing orientation. A case study of a veteran teacher demonstrated that classrooms can be complex sites of political, historical, social and cultural influences, and that the teacher's beliefs about mathematics learning and teaching were less traditional than her actual teaching practice.

Introduction

Attitudes and beliefs about teaching and learning have attracted considerable interest in relatively recent research studies (Leder, 1993; Leder, Pehkonen, & Törner, 2002; McLeod, 1989, 1992). Pajares (1992) argued that knowledge and beliefs are inextricably interwoven and that beliefs strongly affect an individual's behaviour. Hollingworth (1989) reported that the way teachers implement new methods or programs in their classrooms relates to whether teachers' beliefs are congruent with the proposed new methods or programs.

Various other researchers have suggested a number of definitions or descriptions of the term beliefs over the past two decades. The term has been a particularly difficult one to define in the educational and psychological literature. McLeod's description (1992) was considered adequate for this research study:

> Beliefs are largely cognitive in nature, and are developed over a relatively long period of time. Emotions, on the other hand, may involve little cognitive appraisal and may appear and disappear rather quickly … Therefore we can think of beliefs, attitudes and emotions as representing increasing levels of affective involvement, decreasing levels of cognitive involvement, increasing levels intensity of response, and decreasing levels of response stability. (p. 579)

Other studies have investigated various aspects of teachers' beliefs – for instance how teachers' beliefs impact on teachers' daily practices (Buzeika, 1996); how to encourage teachers to reflect on their teaching practices (Malone, 1995); alternative models for mathematics teaching (Malone, 1995; Malone, Thornton, Langrall & Jones, 1997); espoused primary and secondary teachers' beliefs about mathematics and the learning and teaching of mathematics (Howard, Perry & Lindsay, 1997; Perry, Howard, & Conroy, 1996; Perry, Howard, & Tracey, 1999). In the next section, important aspects of the research literature on beliefs and conceptions of mathematics teachers are examined.

Mathematics teachers' beliefs about teaching and learning

One obstacle to progress in this domain has been the non-alignment of terminology used by mathematics education researchers. Schoenfeld (1992), in an attempt to develop a theory of mathematical thinking and problem solving, listed beliefs and affects as one of five aspects of cognition. Leder (1993) provided a definition of "affect" as a term used to: "Denote a wide range of concepts and phenomena including feelings, emotions, moods, motivation and certain drives and instincts" (Leder, 1993, p. I–46). Despite a lack of consensus among mathematics education researchers on the use of terminology, the affective domain is generally regarded as referring to constructs that, according to McLeod (1992), go beyond the cognitive domain, and that beliefs, attitudes and emotions can be considered as subsets of affect.

Pajares (1992) cited a number of constructs which can be considered as subsets of the broadly defined "educational beliefs" term, the most commonly used being: teacher efficacy, epistemological beliefs, attributions, anxiety, self-concept, self-esteem, self-efficacy, and specific subject-matter beliefs. The clarification needed here is, according to Thompson (1992), a careful consideration of the concept of beliefs from both a philosophical and a psychological perspective. Green (1971) proposed a multidimensional perspective on the structure of beliefs – one that incorporated both philosophical and psychological constructs. He claimed that there are three dimensions of belief structures: the quasi-logical relation between beliefs, the central-peripheral dimension, and the premise that beliefs are held in clusters.

Rokeach (1968, cited in Pajares, 1992) pioneered the introduction of the notion of *belief systems* and his analysis included three assumptions:

> Beliefs differ in intensity and power; beliefs vary along a central-peripheral dimension; and, the more central a belief, the more it will resist change… Rokeach defined centrality in terms of connectedness: the more a given belief is functionally connected or in communication with other beliefs, the more central the belief… He proposed four assumptions for connectedness that form a set of priorities for the perceived importance of the self. Beliefs on an individual's identity or self are more connected, as are beliefs one shares with others. Derived beliefs are learned from others; underived beliefs are learned by direct encounter with the belief object. Underived beliefs have more functional connections, partly because the: "I saw with

> my own eyes" phenomenon is existential and connected to one's sense of self. (p. 320)

Pajares (1992) agreed with this connectedness notion, arguing that beliefs are prioritised according to their connections to other cognitive and affective structures.

Mathematics teachers' beliefs have an impact on their classroom practice, on the ways they perceive teaching, learning, and assessment, and on the ways they perceive students' potential, abilities, dispositions, and capabilities. Van Zoest, Jones, and Thornton (1994) compared the beliefs about mathematics and mathematics teaching of pre-service primary mathematics teachers involved in a mentorship program to the beliefs of peer pre-service teachers not involved in such a mentorship program. The mentorship intervention was grounded on a socio-constructivist approach to mathematics instruction, and the treatment group teachers participated in small-group teaching activities. Van Zoest et al. (1994) used a variation of Kuhs and Ball's (1986) model for their study. They found that the intervention group acted as if, initially, its views had been influenced by the socio-constructivist approach to mathematics instruction promoted by the mentorship program. Later on in the program however, their actions indicated that they favoured a more traditional set of beliefs about mathematics teaching.

Perry and his colleagues (Howard et al., 1997; Perry et al., 1996; Perry et al., 1999) contributed to research on teachers' beliefs by providing a model that aimed to describe the espoused beliefs of both primary and secondary mathematics teachers, as a result of a series of studies they conducted in Australia. They identified two factors, the *transmission view* and the *child-centredness view*, which they considered sufficient for a full description of the espoused beliefs of mathematics teachers. Perry et al. (1999) stressed that they considered the two factors making up their model as being distinct factors and not as the two ends of a continuum of one belief factor.

Similarly, Fennema, Carpenter, Franke, Jacobs, & Empson (1996) conducted a study of mathematics teachers' beliefs and instructional practices as they learned about students' thinking and decided how to utilise that knowledge to enable them to finalise instructional decisions. They categorised teachers' beliefs into four levels:

- Level A: Teachers in this category believed that students learn best by being told how to do mathematics.
- Level B: Teachers in this category were beginning to question the idea that children needed to be shown how to do mathematics, but they have conflicting beliefs.

- Level C: Teachers thought that children would learn mathematics as they solved many problems and discussed their solutions.
- Level D: The beliefs of teachers in this category were characterised by the acceptance of the idea that children can solve problems without direct instruction and that the mathematics curriculum should be based on children's abilities.

The researchers reached the conclusion that with regard to the relationship between levels of instruction and beliefs; "there was no consistency in whether a change in beliefs preceded a change in instruction or vice versa" (Fennema et al., 1996, p. 423). This scheme has been used in the analysis of the case study in this project.

Based on their research, Cooney and his collaborators (Cooney, Shealy, & Arvold, 1998) developed a characterisation of secondary mathematics teachers' belief structures that has also been used in the analysis of the case study. These authors acknowledged the contributions earlier researchers have made on shaping the notion of reflection, and proposed the following partial scheme for the characterisation of teachers' knowledge belief structures:

- *Isolationist:* Teachers in this category tend to have belief structures in such a way that beliefs remain separated or clustered away from others. Accommodation is not a theme that characterises an isolationist.
- *Naïve idealist:* Teachers in this category tend to be received knowers in that, unlike the isolationists, they absorb what others believe to be the case but often without analysis of what he/she believes.
- *Naïve connectionist:* This position emphasises reflection and attention to the beliefs of others as compared to one's own. The naïve connectionist fails, however, to resolve conflict or differences in beliefs.
- *Reflective connectionist:* This position emphasises reflection and attention to the beliefs of others as compared to one's own. The reflective connectionists, however, resolve conflict through reflective thinking.

Cooney (1999) stated that the "inculcation of doubt and the posing of perplexing situations" (p. 173) would seem to be central to the shift from being a naïve idealist (or even an isolationist) to being a reflective connectionist. Inciting doubt and making the previously unproblematic problematic can have significant impact on a person's world and could lead to varied and perhaps unsettling responses.

Relationship between teachers' beliefs about mathematics and mathematics teaching and learning, and their instructional practice

Research findings (Cooney, 1999; Ernest, 1989, 1989b; Nespor, 1987; Pajares, 1992; Thompson, 1992) suggest that teachers' beliefs about the nature of mathematics and mathematics teaching and learning, have an impact on teaching practice. Some researchers have stressed the importance of teachers' beliefs on the success or otherwise of contemporary mathematics curricular reforms (Askew, Brown, Rhodes, Johnson, & Wiliam, 1997; Pajares, 1992; Thompson, 1992; Van Zoest et al., 1994). Pre-service high school mathematics teachers experienced difficulties in the implementation of their espoused beliefs in practice, according to Parmelee (1992, cited in Van Zoest et al., 1994).

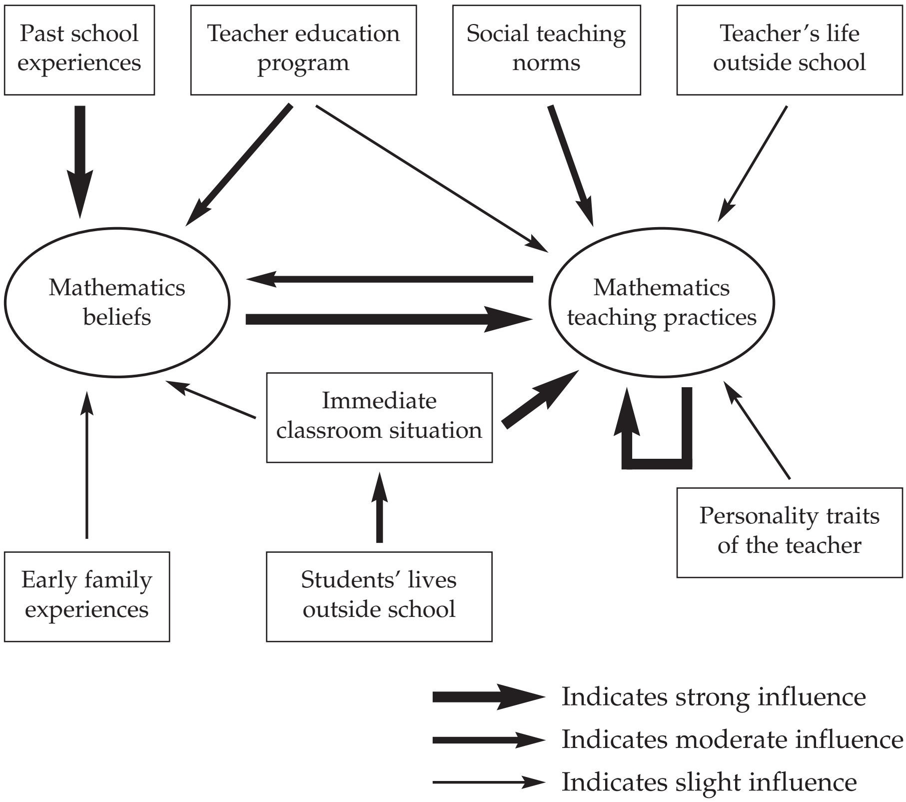
Nisbet and Warren's (2000) review of the relevant research investigating potential interrelationships between mathematics teachers' espoused beliefs and their classroom practice offers some insights into the current state of affairs. For example:

- the relationship between teachers' beliefs and classroom practice is dynamic with each influencing the other;
- teachers' practices are shaped by their beliefs about the nature of mathematics teaching and learning;
- changes in teachers' beliefs about teaching and learning are derived largely from classroom practice; and
- changes in teachers' beliefs about teaching and learning are influenced by the production of valued outcomes (i.e., student learning) resulting from classroom experimentation. (Adapted from Nisbet & Warren, 2000)

Regarding the causes of disparities between teachers' espoused (stated) and enacted beliefs (classroom actions), three possible explanations have been offered by Ernest (1989), namely:

- the depth of the teacher's espoused beliefs and the extent to which they are integrated with his/her other knowledge and beliefs, especially pedagogical knowledge;
- the level of the teacher's consciousness of his/her beliefs and the extent to which the teacher reflects on his or her teaching practice; and
- the influence of the social context and its effect on teachers' actions and behaviours.

Raymond (1997) proposed a model of relationships between teachers' mathematics beliefs and their teaching practice which was used in this study. The model is presented in Figure 1 and provides a useful starting point and a way of structuring discussion.



Mathematics beliefs: About the nature of mathematics and mathematics pedagogy

Mathematics teaching practices: Mathematical tasks, discourse, environment, and evaluation

Immediate classroom situation: Students' (abilities, attitudes, and behaviour) time constraints, the mathematics topic at hand

Social teaching norms: School philosophy, administrators, standardised tests, curriculum, textbook, other teachers, resources

Teacher's life: Day-to-day occurrences, other sources of stress

Students' lives: Home environment, parents' beliefs (about children, school, and mathematics)

Teacher education program: Mathematics content courses, field experiences, student teaching

Past school experiences: Successes in mathematics as a student, past teachers

Early family experiences: Parents' view of mathematics, parents' educational background, interaction with parents (particularly regarding mathematics)

Personality traits: Confidence, creativity, humour, openness to change

Figure 1. Raymond's (1997) model of the relationships between teachers' mathematics beliefs and their teaching practice.

The model suggests complex relationships between mathematics teachers' beliefs and their practice, and it contributes to a better understanding of some factors that are considered to be important when teachers attempted to explain the existence of inconsistencies between them. Raymond (1997) suggested that the main causes of the inconsistencies reported in her study were not so much teachers' beliefs, but school and classroom practices. The cumulative effect of these other influences (i.e., past school experiences, teacher education program, personality traits of the teacher, social teaching norms, etc.) was the main cause of inconsistencies between beliefs and practice. The manifestation of teachers' beliefs in mathematics classrooms would be likely to be influenced by: (a) the teacher's prior experiences of learning mathematics, classroom teaching, peer interactions, awareness of research results in mathematics education, using mathematics in other life experiences, teacher education programs; and (b) by various socio-historical, socio-economic, socio-political, and socio-cultural factors.

Purpose of the study and research questions

The central concern of this study was to contribute to an understanding of secondary mathematics teachers' beliefs about mathematics, mathematics teaching, mathematics learning and their teaching practice. To explore these complex affective factors, this study utilised both qualitative and quantitative research methods. The research questions were as follows:

(1) What are the beliefs of Greek secondary mathematics teachers with regard to mathematics as a discipline, and to the learning and teaching of mathematics? Specifically, does there exist a typology of mathematics teachers' beliefs that correspond to the categorisations postulated in the research literature?

(2) Are there inconsistencies between the teaching practice of Greek secondary mathematics teachers and their beliefs about mathematics and mathematics learning and teaching?

An extensive search of the literature revealed that no studies investigating the relationship between teachers' beliefs about mathematics and mathematics teaching and learning, or their instructional practices, had been conducted in Greece.

Methodology

This section describes the methods used to investigate the research questions. The study was conducted in two parts. The first part was a qualitative survey and the second part a case study of an experienced teacher.

Part 1: Survey

A survey was administered to explore the areas of interest and to produce data to examine the research questions. The survey was sent to a random selection of six hundred grade 7–12 mathematics teachers. The return rate was 78%, (n = 465) and the resulting sample comprised 465 (276 males, 145 females, 44 no gender specified) participants, which included 431 secondary mathematics teachers (244 males, 143 females), 24 secondary mathematics teachers holding a principal's position (22 male, 2 female), and 10 regional mathematics consultants (10 males), in 45 State High Schools throughout Greece. The returned surveys reflected a reasonably well-balanced distribution of grade-level experience.

The 34-item questionnaire, designed by the first author, was administered during 1999–2000 in State High Schools in Greece, and covered the following participant demographics: gender, age, professional development background, length of teaching experience, position held, postgraduate studies background, and beliefs about mathematics, mathematics learning, and mathematics teaching. A Likert-type scoring format was used, and teachers were asked to indicate the extent to which they agreed (or disagreed) with each statement presented. A five point scoring system was used – strongly disagree (SD) to strongly agree (SA). A score of 1 was assigned to the SA response and a score of 5 to the SD response. The survey contained three subsets of items: Beliefs about Mathematics (BM: 6 items), Beliefs about Mathematics Learning (BML: 7 items) and Beliefs about Mathematics Teaching (BMT: 21 items). A space was also provided for teachers to comment on any aspect (of each part) of the instrument and its items.

Part 2: Case Study

In the second part of the study, a case study focused on the work of a veteran secondary mathematics teacher in Greece named Ann (pseudonym). Data were collected through lesson observations and pre- and post- lesson interviews. Excerpts from the case study are reported in a later section. The analyses drew on her own words as well as on detailed analyses of her lessons and the pre- and post- lesson interviews. Transcripts of audiotapes and videotapes were used to ensure that the analysis was based as closely as possible on the reality of the classrooms observed. Professionals transcribed the dialogue as completely and accurately as the equipment would permit, and the final transcript was double-checked by the first author. The transcription process was completed when a professional translator, an Englishman who has lived and worked in Athens as a translator for thirty years, translated all Greek transcripts into English. These were in turn double-checked by the first author using the back-translation technique and by paying special attention to the accuracy of mathematical nomenclature.

Verification procedures

When the analysis of the videotaped sessions was completed, Ann was given a copy and she was asked to comment on its veracity and accuracy. She expressed no disagreement or concern with the analysis. Two further checks of the transcripts of the videotaped sessions were administered. Randomly selected sections were analysed by another experienced coder (a Senior Lecturer in mathematics education at the University of Athens) for inter-coder reliability. The entire analysis was then re-examined "blind" by the first author to provide an intra-coder reliability check.

Data analysis

Questionnaire responses regarding beliefs about mathematics, mathematics teaching and mathematics learning were analysed using SPSSwin. Principal Components Analysis, Cluster Analysis, Multiple Discriminant Analysis, and Trends Analysis were used in the study. In what follows, only the Principal Components Analysis results are presented. Detailed analyses of the extensive statistical procedures used in the study can be found in Barkatsas (2003).

Principal Components Analysis

A confirmatory Principal Components Analysis (PCA) was used in order to interrogate the 34 questionnaire items for a typology of teachers' espoused beliefs. Given the exploratory nature of the study and guided by the Analysis of Variance (ANOVA), the scree plot and the interpretability of the factors, a five components orthogonal solution (Table 1) was accepted after the extraction of principal components and a Varimax rotation. The solution accounted for 39% of the variance, and 22 of the 34 items were used to delineate the components. For this study, the Kaiser-Meyer-Olkin Measure of Sampling Adequacy was 0.679, and Bartlett's Test of Sphericity was significant ($p < .01$), so factorability of the correlation matrix was assumed.

The analysis yielded 13 components with eigenvalues greater than 1. Given the exploratory nature of the study and guided by the interpretability of the components, as well as the scree plot, a five-component orthogonal solution was accepted after the extraction of principal components and a Varimax rotation.

The PCA depicted in Table 1 followed the elimination of psychometrically "poor" items. Variables loading on more than one component were eliminated, Cronbach's alpha coefficient of internal consistency was used to ensure that the items comprising the resulting components produced a reliable scale, and a normality test was carried out to assess normality among single variables by examining their skewness and kurtosis. If a variable has substantial skewness or kurtosis, it needs to be transformed (Tabachnick & Fidell, 1996). In this study, no variable had any substantial skewness or kurtosis, so no transformation of variables was necessary. The rotation method was a Varimax with Kaiser Normalisation. The rotation converged in seven iterations.

Table 1
Components Related to Views about Mathematics, Mathematics Teaching and Mathematics Learning.

Item	Item description	Loading
Factor 1: A socio-constructivist orientation to mathematics, mathematics teaching and mathematics learning		
24	It is important for students to be provided with opportunities to reflect on and evaluate their own mathematical understanding	.594
14	Ignoring the mathematical ideas generated by the students can seriously limit their learning	.594
32	The education system should be preparing critically thinking citizens who are able to utilise their mathematical skills	.583
17	Mathematics teachers should be fascinated with how students think and intrigued by alternative ideas	.482
16	Teachers should encourage their students to strive for elegant solutions when they solve problems	.385
1	Justifying the mathematical statements that a person makes is an important part of mathematics	306
Factor 2: A dynamic problem-driven orientation to mathematics, mathematics teaching and mathematics learning		
13	Mathematics learning is enhanced by challenging activities within a supportive environment	.650
30	The comprehension of mathematical concepts by students should correspond to their cognitive development and it should be a decisive factor in the content sequence to be taught	.608
34	Teachers should respect the mathematical knowledge of their students, which is consisting of a nexus of experiences, beliefs, attitudes, representations, concepts, strategies, connections, values, judgements and emotions	.558
6	Mathematics is a beautiful, creative and useful human endeavour that is both a way of knowing and a way of thinking	.463
19	Teachers always need to hear students' mathematical explanations before correcting their errors	.447
4	Mathematics is the dynamic searching for models and problems and their results are open to review	.358
Factor 3: A static-transmission orientation to mathematics, mathematics teaching and mathematics learning		
17	The most effective way to learn mathematics is by listening carefully to the teacher explaining a mathematics lesson	.718

2	Mathematics is a static and immutable knowledge with objective truth	.675
28	The role of the mathematics teacher is to transmit mathematical knowledge and to verify that learners have received that knowledge	.637
Factor 4: A mechanistic-transmission orientation to mathematics, mathematics teaching and mathematics learning		
12	The memorisation of mathematical facts is important in mathematics learning	.623
27	Teachers or the textbook – not the student – are the authorities for what is right or wrong	.602
10	Mathematics knowledge is the result of the learner interpreting and organising the information gained from experiences	.532
Factor 5: A collaborative orientation to mathematics, mathematics teaching and mathematics learning		
20	An effective way to teach mathematics is to provide students with interesting problems to investigate in small groups	.551
11	Students are rational decision makers capable of determining for themselves what is right and what is wrong	.536
33	All students are able to be creative and do original work in mathematics	.469
8	Students can learn more mathematics together than by themselves	.447

Six items loaded on component 1, six items on component 2, three items on component 3, three items on component 4, and four items loaded on component 5 (Table 1). The naming of components was guided by the nature of the items. The factors were as follows:

- Teachers whose beliefs are expressed by component 1 (15.3% of total sample) can be assumed to espouse a *socio-constructivist orientation* to mathematics, mathematics learning and mathematics teaching.
- Teachers whose beliefs are expressed by Factor 2 (23.2% of total sample) can be assumed to espouse a *dynamic problem-driven orientation* to mathematics, mathematics learning and mathematics teaching.
- Teachers whose beliefs are expressed by Factor 3 (30.8% of total sample) can be assumed to espouse a *static – transmission orientation* to mathematics, mathematics learning and mathematics teaching.
- Teachers whose beliefs are expressed by Factor 4 (26.2% of total sample) can be assumed to espouse a *mechanistic – transmission orientation* to mathematics, mathematics learning and mathematics teaching.

- Teachers whose beliefs are expressed by Factor 5 (4.5% of total sample) can be assumed to espouse a *cooperative orientation* to mathematics learning and mathematics teaching.

In summary, the results of the PCA seemed to reveal two orientations that are characteristic of secondary mathematics teachers' beliefs with regard to the nature of mathematics and the learning and teaching of mathematics:

- A *contemporary – constructivist orientation*, consisting of the following (not mutually exclusive but complementary) views:
  (i) the socio-constructivist view (component 1);
  (ii) the dynamic problem driven view (component 2); and
  (iii) the cooperative view (component 5).
- A *traditional – transmission – information processing orientation*, consisting of the following (not mutually exclusive but complementary) views:
  (i) The static view (component 3); and
  (ii) The mechanistic view (component 4).

The analysis revealed that mathematics teachers' beliefs about mathematics could not be separated from their beliefs about teaching and learning mathematics.

Van Zoest et al.'s (1994) factor analysis of 175 pre-service primary teachers' beliefs revealed only one significant factor, which the authors labelled a *socio-constructivist orientation* to mathematics, mathematics teaching and mathematics learning. This particular orientation shares many similarities with the socio-constructivist orientation revealed by the present study, given the fact that a number of the items used in Van Zoest et al. (1994) study had been used in this study.

Ernest (1989a) stated that "teachers' conceptions of the nature of mathematics by no means have to be considered to be consciously held views; rather they may be implicitly held philosophies" (p. 20); and that "three philosophies of mathematics are distinguished because of their observed occurrence in the teaching of mathematics" (p. 21); and also that "beliefs about mathematics are reflected in teachers' models of the teaching and learning of mathematics, and hence in their practices" (p. 22).

Our interpretation of Ernest's (1989) statements – in conjunction with the views regarding teachers' beliefs expressed by Askew, et al. (1997), Cooney (1999), Fennema, et al. (1996), Pajares (1992), Tanner and Jones (1998), and Van Zoest et al. (1994) – is that it is not possible to separate mathematics teachers' views about mathematics from their views about mathematics teaching and learning. In this study, a dynamic problem-driven component was obtained in the survey's factor analysis, by treating secondary mathematics teachers' views regarding mathematics, mathematics teaching and mathematics learning as an intertwined, interconnected, and inseparable belief system deeply embedded into a personal philosophy or worldview.

Case study: The work of Ann

This section focuses on the work of a veteran Greek mathematics teacher Ann. It provides insights into both research questions of the study by exploring the interrelationships between the espoused beliefs held by Ann and her beliefs in practice and through a description of factors that influence and shape those beliefs. In what follows, a selection of interview questions and responses are provided to illuminate these interrelationships.

Introducing Ann

Ann is in her late forties and has been teaching in the Greek State School system for twenty–two years. She is a veteran teacher having taught at schools with students from low socio-economic backgrounds as well as in middle-class suburbs. Ann holds a four-year Bachelor of Science degree, with a major in pure mathematics, and a Master of Mathematics Education degree. Ann has taught Years 10–12 (students aged 16–18) for many years and in the year the study took place she was teaching Year 10 and Year 11 classes.

Ann's beliefs about mathematics

Interview questions were divided into three sections, in concert with the three sections of the questionnaire used in the quantitative part of the study. This part of the interview discussion focused on Ann's espoused beliefs about mathematics.

When asked what she thought about mathematics, Ann replied that she held a non-traditional view about mathematics, considering it not as a static collection of unrelated facts but a dynamic science that evolved as a result of the enculturation of the human race and it can be used to improve understanding of natural phenomena. When probed further about school mathematics, Ann said that she was disappointed by the way it was being taught, and by the strong emphasis being placed on the preparation of students for final examinations. She held a strong belief against mathematics being used primarily as a means to gain entry to University studies.

Ann's beliefs about mathematics learning

This part of the interview discussion focused on Ann's espoused beliefs about mathematics learning:

> I: What can a good mathematics student do?
>
> A: I would like the students to be able to work on their own and in teams, to be able to do problem solving and modelling and to be able to use mathematical proofs in their work. A good mathematics student (in Years 10–12) should be able to use abstractions and to generalise. In my classes I would like my students to be meticulous and conscientious, to work hard and persist on set tasks irrespectively of the fact of getting the answers or not. In addition, I expect a good student to perform well on tests as well as in class.

When discussing good students Ann stated that students should be motivated, they should have a desire to learn and to be autonomous learners.

Ann held a strong view about cooperative learning. Students in her classes were expected to be involved in cooperative problem solving and modelling activities, since according to her, learning is about achieving shared knowledge.

I: What accounts for the differences between a good and a poor mathematics student?

A: A good mathematics student should be organised and know how to study. In mathematics, the process is more important than the product. The student needs to be motivated, to have a desire to learn and to be an autonomous learner.

According to Ann, motivating students to realise that mathematics means more than just preparing for final examinations empowers them, and may result in better performance in mathematics. Ann stated that a fundamental condition for improved performance in mathematics on the part of the students is a desire to learn. Participation in meaningful activities was the key to success in mathematics according to Ann.

Students' learning of mathematics depends, according to Ann, primarily on the learner. Ann reiterated her strong contention that students learn mathematics through active participation in meaningful activities.

I: How do you define comprehension of a mathematical concept of topic?

A: There is no simple answer to that question. Being able to recall the required facts and algorithms when necessary represents evidence that understanding of a concept has taken place in mathematics. When students can apply their knowledge in contexts other than the ones presented in their textbooks, it is another piece of evidence that understanding has taken place. To be able to use knowledge and skills at any time, to model any given context, and to have developed an inquiring mind, is additional evidence that mathematical understanding has taken place.

Ann espoused a socio-constructivist view of mathematics learning, one grounded on her firm belief that all students can learn mathematics and that mathematics learning is the result of students making connections and interpreting and organising the information gained from experiences.

Ann's beliefs about mathematics teaching

This part of the interview discussion focused on the interviewee's espoused beliefs about mathematics teaching:

I: Where did you learn to teach mathematics that way? Have you ever had graduate and/or in-service courses on how to teach mathematics?

A: Life taught me (she laughs). By that, I mean that I had to learn many things on my own during my teaching career. I am currently participating at a regional mathematics teachers' cluster and I think that there are some positive outcomes because we discuss our daily problems and we formulate common strategies for more effective teaching. It is very important for my professional life. As far as graduate courses are concerned, I feel that I have learned a few new things in my Master's course.

Cooperation among teachers in the region offered a platform for shared understanding and the emergence of a framework of common values, according to Ann. The impact of professional development did not seem to have had a great influence on shaping Ann's beliefs about teaching. Her daily experiences appeared to have strongly affected her beliefs about mathematics teaching.

I: Have you ever tried something different? Why?

A: Firstly, I believe that a number of traditional teaching techniques are outdated and need to be replaced by new teaching approaches. Secondly, I am constantly trying to discover new approaches and educational environments that will enable the students to fully develop as mathematics apprentices. Thirdly, I believe that contemporary teaching ideas based on constructivism have given me a new purpose in teaching mathematics. I have learned a lot from my students too. If I had something in mind and tried it in class, the interaction with my students created a dialectic relationship from which a number of unexpected new ideas kept emerging.

Ann's firm socio-constructivist stance about mathematics teaching and learning was further reinforced by her statements, the fact that she espoused a mathematics microcosm culture in her classes, and she envisioned students' interaction as a dialectic relationship necessary for new ideas to emerge. Ann valued communication and encouraged continuous dialogue between all members of her classes.

I: What happened while attempting to introduce a new approach?

A: I have encountered a number of problems by practising contemporary teaching methods because students are very apprehensive about their potential to enhance learning. Some students do not perceive them to be as effective as the traditional chalk and talk, and paper and pencil techniques with which they are accustomed. Senior students who are accustomed to working individually find it very difficult to work in groups. Some students worry that their teacher will not appreciate their efforts, partly because they like to be protagonists in class and partly because they fear that the outcome of the group's work won't be their work alone. They also worry about how they are going to be assessed in groups and the fact that the work of their group may not be awarded top marks. Some of the students take a long time to get used to contemporary teaching techniques. The fact of the matter is, however, that contemporary techniques help even the weakest students to participate in classroom work, and to learn more effectively.

Ann experienced resistance to cooperative learning from her own students who felt that they would not be rewarded for their individual efforts and that they will not gain full credit for their work by participating in group work.

This is an indication of the adverse impact the system may have on students' attitudes to learning mathematics when it is founded on traditional chalk-and-talk approaches, resulting in individual efforts and competitiveness being valued and rewarded as the approved approach to learning mathematics.

> I: Grouping: on what basis? Why?
>
> A: I started using group work in 1983. I recall that I wanted every single student to be involved in class work instead of having them passively listening to me. I (then) thought that if the class was divided in small groups then I could have a small community in each group with its potential to differentiate among its members and with its leaders unknowingly acting as my assistants – without the rest of the class knowing about it. I will not be satisfied unless all students participate in classroom activities.

The promotion of a community environment with its potential to encourage active participation, brainstorming, ideas exchange, and to maximise each student's potential in mathematics learning, had been the driving force behind Ann's collaborative teaching approach. Ann claimed that her ability to teach mathematics has been influenced by her beliefs about two teaching prerequisites (and her desire to improve on both): that the teacher should have a strong background in mathematics (mathematical content knowledge), and be knowledgeable of contemporary didactical, psychological, and pedagogical approaches.

> I: Which factors do you consider have had a significant influence on your beliefs about mathematics teaching?
>
> A: I believe that the [Greek] education system limits the teacher's options to innovate and experiment in her classes. I sometimes feel that I have to apologise to my colleagues and to my superiors for attempting to introduce innovations in my teaching. The system – as it is today organised – forbids the teacher to use any approaches that are not "tried and true". Sometimes I feel like an outsider having to overcome – on a daily basis – time constraints, peer pressure, overt and covert opposition, or disapproval from the administration. On top of that there may be opposition and disapproval from some parents and students who believe that schooling means teaching to the test, and that a collaborative approach is not the best way for individual students to achieve top marks.

Ann's beliefs about mathematics teaching seem to have grown out of a desire to oppose a stagnant and inflexible system which limits the teachers' options to innovate, to try new approaches, and to strive to develop an active learning and collaborative teaching environment in her classes.

> I: Which factors may explain the existence of inconsistencies between espoused beliefs and beliefs in practice?
>
> A: The first factor concerns the expectations of the education system that could sometimes act in ways that inhibit innovations. The second

> factor concerns the things you are used to do that turn out to be a lot easier than introducing innovations in your daily routine. The third factor is the fact that it is very difficult to overcome every obstacle on a daily basis in order to do what you believe is the best practice. The fourth factor is the fact that I do not have my own mathematics classroom. The fifth factor is the fact that regional mathematics consultants are more decorative than anything useful. We never get to meet them and discuss our concerns. The sixth factor concerns my family obligations, which in practice drastically limits my available time. The seventh factor concerns the lack of communication with some of my students, since they have been bred to think that schooling means teaching to the test. The eighth factor is the fact that I do not have all the required recourses at school. The ninth factor concerns the fact that the system does not provide any incentives for innovative teaching practice and the tenth factor is the fact that there is no teaching evaluation or appraisal which could indirectly force me to be more prepared on a daily basis and to perform even better in class.

The absence of a systemic teacher evaluation/appraisal system, and the lack of support for innovative teaching approaches suggested to Ann an impaired vision of education by Regional and State educational policy makers.

Conclusions and Implications

The investigation of the existence of a typology of beliefs categories that could be used to characterise secondary mathematics teachers' beliefs relating to mathematics, and mathematics teaching and learning, resulted in two orientations being identified: a *contemporary – constructivist orientation, and a traditional – transmission – information processing orientation*.

The analysis showed that, overall, neither the static nor the mechanistic view of mathematics, mathematics teaching, and mathematics learning were rated highly among mathematics teachers. The static view, however, was stronger than the mechanistic view. It was also noticed that all dimensions of the contemporary view, that is the socio-constructivist, the problem solving, and the collaborative dimensions, rated more highly among mathematics teachers than both dimensions of the traditional view, that is, the static and the mechanistic dimensions. Replication of particular findings would substantiate some of the conclusions drawn.

The case study of the beliefs and practices of Ann has provided some insights into the complex interactions at work. Ann demonstrated that her beliefs were not always consistent with her instructional practices, as has been documented in previous studies of teachers (Cooney, 1999; Ernest, 1989a; Pajares, 1992; Raymond, 1997; Thompson, 1992). Ann's beliefs about mathematics were more traditional than her beliefs about mathematics teaching and learning.

In a number of recent studies, teachers' beliefs have been found to be flexible regarding what was central to the task of teaching (Malone, 1996), to be influenced by effective practice within the boundaries of their classrooms (Buzeika, 1996), and to be influenced by a perceived pressure of known expectations of other teachers when adopting a classroom approach (Shield, 1999). Ann's case study highlighted the need to address the contribution of cultural influences in the formation of teachers' beliefs, and the transformation into practical approaches. It seems that prior experiences and social norms are decisive factors in the beliefs of mathematics teachers as is the belief that if they adhere to the "tried and true" approaches in mathematics teaching and learning they will not go wrong.

The major causes of inconsistencies suggested by Ann were the classroom situation, prior experiences, and social norms. Teachers may believe, for instance, that group work is the best environment for exploring mathematical ideas and learning mathematics, but that preparing their students for university entrance examinations and the pressure to achieve the highest scores possible for each student may keep them from implementing this belief into practice. This illustrates how a single element in the classroom situation, or the influence of societal and parental expectations, and teaching social norms can affect teaching practice to a greater extent than the teacher's espoused beliefs.

Another finding of this study is that prior school experiences and personal world-views and ideologies were the main influence on the teacher's beliefs about mathematics, but her own school experiences and her teaching experiences were the main influences on beliefs about teaching, learning, and assessing mathematics.

In the case of Ann, it was clear that her beliefs about mathematics, mathematics teaching, learning and assessment were of a contemporary/ socio-constructivist nature. The fact that Ann named her political ideology as a primary influence on her non-traditional teaching practice, especially on her firmly held beliefs about the power and utility of collaborative teaching and learning approaches, shed light on, and enabled the interpretation of, her classroom practices which were in concert with her beliefs about mathematics and mathematics learning and teaching. It was clear, however, that beliefs about mathematics pedagogy and about the role of societal and cultural factors were the dominant influences in Ann's teaching practice.

The culmination of a number of factors constituted Ann's beliefs system regarding the nature of the learning-teaching process. Most of them had to do with the difficulty of introducing and sustaining innovative teaching practices in a system which had solid traditional foundations and where teaching was seen primarily as a public servant's job. The absence of a systemic teacher evaluation/appraisal system and the lack of support for innovative teaching approaches point to a myopic vision by Regional and State educational policy makers.

Ann's beliefs about mathematics teaching and learning and her consequent instructional decisions were in line with Fennema et al. (1996) level D. Her beliefs were characterised by the acceptance of the idea that children can solve problems without direct instruction, and that the mathematics curriculum should be based on children's abilities. Ann believed that her role was to find out what children knew and to use that knowledge to structure the learning environment. Ann also believed that what the children knew should be a major influence on all her instructional decisions.

Overall, Ann could be classified as a reflective, connectionist mathematics teacher. Ann reflected on and attended to the beliefs of others as compared to her own. She attempted to resolve conflict through reflective thinking, but although she did not make connections, she was able to weave them into a set of beliefs held from a relativistic and committed perspective while listening to and valuing others. Ann, as a true reflective connectionist, analysed and integrated the merits of various positions, and came to terms with what she believed in a committed way.

Final words

Teachers' beliefs influence their classroom practices, their beliefs are formed early, and beliefs about teaching are well established by the time a prospective teacher starts attending university classes. Therefore, it is important for mathematics educators, researchers, and curriculum designers to understand the impact teachers' beliefs have on their cognitions and classroom practices.

The awareness of one's beliefs system could stimulate the examination and development of beliefs about mathematics, and mathematics teaching and learning because teacher preparation and in-service programs are likely to exert more influence on beliefs than on instructional practice. Initial teacher education has the power to influence teaching practices significantly, but in-service programs could prove more effective in this endeavour. It could be conjectured that both pre-service and in-service teacher education programs may indirectly affect teachers' instructional practice if their objective concerned influencing the belief systems of teachers. Hiebert, Gallimore, and Stigler (2002) argued that effective professional development programs yield the best results when they are:

> Long-term, school based, collaborative, focused on students' learning and linked to curricula. In such programs, teachers examine student work, develop performance assessments and standards-based report cards, and jointly plan, teach, and revise lessons. Teachers, who traditionally have worked in isolation, report favourably on programs that bring them in close contact with colleagues in active work on improving practice. (p. 3)

A growing number of professional development programs draw from the available research evidence in their effort to change teachers' beliefs about their instructional practices. Yet Hiebert et al. (2002) reminded us that "teachers rarely draw from a shared knowledge base to improve their practice" (p. 3) and that "there is a persistent concern that educational research has too little influence on improving classroom teaching and learning" (p. 3). It could be claimed that professional development programs, such as those espoused by Hiebert et al. (2002), might lead to a "professional community mind-set" among mathematics teachers by providing them with the opportunity to rely on the dynamics and the collective wisdom of the team. In that way, teachers may develop an increased predilection for reflection and critical analysis of their own teaching and of their own beliefs about teaching and learning. The results of this study support the idea that mathematics teachers need to be provided with systematic guidance in developing the skills for critical reflection and self-appraisal.

Data such as those gathered from the lessons of a veteran mathematics teacher could be used in pre-service and in-service professional development courses in several ways. Excerpts could be used as the basis of discussion on mathematics teaching and learning matters, of those factors that influence teachers in their daily practices, of the pressures they are faced with, how they respond to those pressures, and of the inconsistencies identified between espoused beliefs and beliefs in practice. Such excerpts could be used in an open-ended way for participants to consider what they might have done in the same situation and why. Teachers could acquire valuable knowledge by exploring the daily routines they use and by examining the inconsistencies identified. Observed inconsistencies may also point to deficiencies in the planning and the implementation of professional development courses.

Consideration might be given to the development of a scale which measures the extent to which mathematics teachers consider mathematics as a static-mechanistic or a dynamic body of knowledge, and the extent to which mathematics teaching and learning are viewed as constructivist-oriented or as transmission-oriented endeavours.

The findings of this study suggest that the broad social and cultural climate of the classroom may impact on teachers' espoused and enacted beliefs about mathematics, and mathematics learning, teaching and assessment. This possibility invites further investigation.

References

Askew, M., Brown, M., Rhodes, V., Johnson, D., & Wiliam, D. (1997). *Effective teachers of numeracy, Final report*. Report of a study carried out for the Teacher Training Agency 1995–96 by the School of Education, King's College London.

Barkatsas, A. (2003). *Secondary mathematics teachers' beliefs about teaching, learning and assessment and factors that influence these beliefs*. Unpublished Ph.D. Thesis. Curtin University of Technology.

Buzeika, A. (1996). Teachers' beliefs and practice: The chicken or the egg? In P. C. Clarkson (Ed.), *Technology in mathematics education* (Proceedings of the 19th annual conference of the Mathematics Education Research Group of Australasia, pp. 93–100). Melbourne: MERGA.

Cooney, T. J., Shealy, B. E., & Arvold, B. (1998). Conceptualising belief structures of preservice secondary mathematics teachers. *Journal for Research in Mathematics Education, 29*(3), 306–333.

Cooney, T. J. (1999). Conceptualizing teachers' ways of knowing. *Educational Studies in Mathematics, 38*, 163–187.

Dewey, J. (1933). *How we think: A restatement of the relation of reflective thinking to the educative process*. Boston: D. C. Heath.

Ernest, P. (1989). The knowledge, beliefs and attitudes of the mathematics teacher: A model. *Journal of Education for Teaching, 15*, 13–34.

Fennema, E., Carpenter, T. P., Franke, M. L., Jacobs, V. R., & Empson, S. B. (1996). A longitudinal study of learning to use children's thinking in mathematics education. *Journal for Research in Mathematics Education, 27*(4), 403–434.

Green, T. (1971). *The activities of teaching*. New York: McGraw-Hill.

Hiebert, J., Gallimore, R., & Stigler, W. (2002). A knowledge base for the teaching profession: What would it look like and how can we get one? *Educational Researcher, 31*(5), 3–15.

Hollingworth, S. (1989). Prior beliefs and cognitive change in learning to teach. *American Educational Research Journal, 26*, 160–189.

Howard, P., Perry, B., & Lindsay, M. (1997). Secondary mathematics teachers' beliefs about the learning and teaching of mathematics. In F. Biddulph & K. Carr (Eds.), *People in mathematics education* (Proceedings of the 20th annual conference of the Mathematics Education Group of Australasia, pp. 231–238). Rotorua, NZ: MERGA.

Kuhs, T., & Ball, D. (1986). *Approaches to teaching mathematics: Mapping the domains of knowledge, skills and dispositions* (Research Memo). Lansing, MI: Centre on Teacher Education, Michigan State University.

Leder, G. C. (1993). Reconciling affective and cognitive aspects of mathematical learning: Reality or a pious hope? In I. Hirabayashi, N. Nohda, K. Shigemathu & F-L. Lin. (Eds.), *Proceedings of the 17th conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 46–65). Tsukuba, Ibaraki: University of Tsukuba, Japan.

Leder, G. C., Pehkonen, E., & Törner, G. (Eds.) (2002). *Beliefs: A hidden variable in mathematics education?* Dordrecht, The Netherlands: Kluwer.

Malone, J. A. (1995). Determining preservice teachers' beliefs about their role in teaching mathematics. In A. Richards (Ed.), *FLAIR: Forging links and integrating resources* (Proceedings of the 15th biennial conference of the Australian Association of Mathematics Teachers, pp. 264–269). Darwin: AAMT.

Malone, J. A. (1996). Preservice secondary teachers' beliefs: Two case studies of emerging and evolving perceptions. In I. Puig & A. Guteriez (Eds.), *Proceedings of the 20th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 313–320). Valencia, Spain: PME.

Malone, J. A., Thornton, C. A., Langrall, C. W., & Jones, G. A. (1997). Modyfying preservice teachers' beliefs about their role in teaching mathematics. In D. Fisher & T. Rickards (Eds.), *Science, technology and mathematics education development* (pp. 355–363). Perth: Curtin University Press.

McLeod, D. B. (1989). The role of affect in mathematical problem solving. In D. B. McLeod & V. M. Adams (Eds.), *Affect and mathematical problem solving: A new perspective* (pp. 20–36). New York: Springer-Verlag.

McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualisation. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 575–596). New York: MacMillan.

Nespor, J. (1987). The role of beliefs in the practice of teaching. *Journal of Curriculum Studies, 19*, 317–328.

Nisbet, S., & Warren, E. (2000). Primary school teachers' beliefs relating to mathematics, teaching and assessing mathematics and factors that influence these beliefs. *Mathematics Teacher Education and Development, 2*, 34–47.

Pajares, M. F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research, 62*(3), 307–322.

Perry, B., Howard, P., & Conroy, J. (1996). K-6 teachers' beliefs about the learning and teaching of mathematics. In P. C. Clarkson (Ed.), *Technology in mathematics education*. (Proceedings of the 19th annual conference of the Mathematics Education Group of Australasia, Vol. 1, pp. 453–460). Melbourne: MERGA.

Perry, B., Howard, P., & Tracey, D. (1999). Head mathematics teachers' beliefs about the learning and teaching of mathematics. *Mathematics Education Research Journal, 11*(1), 39–57.

Raymond, A. M. (1997). Inconsistency between a beginning elementary school teacher's mathematics beliefs and teaching practice. *Journal for Research in Mathematics Education, 28*(5), 550–576.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334–370). New York: Macmillan.

Schon, D. A. (1983). *Educating the reflective practitioner: How professionals think in action*. New York: Basic Books.

Shield, M. (1999). The conflict between teachers' beliefs and classroom practices: Ethnicity and the stereotyping of mathematics. In J. M. Truran & K. M. Truran (Eds.), *Making the difference* (Proceedings of the 22nd annual conference of the Mathematics Education Group of Australasia, pp. 439–445). Adelaide: MERGA.

Tabachnick, B. G. & Fidell, L.S. (1996). *Using Multivariate statistics* (3rd ed.). New York: Harper Collins.

Tanner, H., & Jones, S. (1998). Dynamic scaffolding and reflective discourse: Successful teaching styles observed within a project to teach mathematical thinking skills. In C. Kanes, M. Goos, & E. Warren (Eds.), *Teaching mathematics in new times*. (Proceedings of the 21st annual conference of the Mathematics Education Group of Australasia, Vol. 2, pp. 596–604). Brisbane: MERGA.

Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127–146). New York: Macmillan.

Van Zoest, L. R., Jones, G. A., & Thornton, C. A. (1994). Beliefs about mathematics teaching held by pre-service teachers. *Mathematics Education Research Journal, 6*(1), 37–55.

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