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The Beliefs/Practice Connection in Broadly Defined Contexts

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The findings of a study that examined the connection between the beliefs of secondary mathematics teachers and their classroom practices are reported in this article. Classroom practice was defined in terms of the extent to which classroom environments could be characterised as constructivist. Cluster analysis was used to group teachers according to their responses to a beliefs instrument and to group their classes according to their average responses to a classroom environment survey. Associations between the two sets of clusters were found, suggesting some consistency between broad relatively de-contextualised teacher beliefs and student perceptions considered at the whole class level.

Although beliefs have received considerable research attention in recent decades there is no agreed definition of the concept (McLeod & McLeod, 2002). Researchers in the area thus have a responsibility to clarify the meaning that they attach to the term (Pajares, 1992). In this article *beliefs* is used essentially as it was by Ajzen and Fishbein (1980), to refer to anything that an individual regards as true. Since beliefs must necessarily be inferred from the words or actions of individuals (Pajares, 1992), more certainty can be attached to the existence of a belief than is evident in both. Indeed, the degree to which a subject acts and makes statements in other contexts that are compatible with a given stated belief, the more centrally held (Green, 1971) that belief is likely to be. It is also recognised that individuals may hold beliefs that they do not articulate and, in some cases, are not even aware of (Buzeika, 1996).

In examining beliefs, the larger study, of which what is reported in this article is part, used interviews and observations as well as surveys. The teachers' practices were similarly examined using multiple data sources, namely surveys of teachers' and their students' perceptions of their classroom environments as well as through interviews and observations. This article reports on associations between the teachers' responses to a beliefs survey, and the responses of their students to a classroom environment survey. Details of the larger study are available in Beswick (2003a).

Mathematics teachers' beliefs

Mathematics teachers' beliefs in relation to the nature of mathematics, mathematics teaching and mathematics learning are recognised as relevant to their practice (Ernest, 1989a). In this study Ernest's (1989b) three categories of beliefs regarding the nature of mathematics, Van Zoest, Jones and Thornton's (1994) three categories with respect to mathematics teaching, and an adaptation of Ernest's (1989b) categories regarding beliefs about mathematics learning were used because of their logical interrelationships and grounding in theory (Lerman, 1983; Skemp, 1978) and in empirical research (Anderson & Piazza, 1996; Perry, Howard, & Tracey, 1999; Thompson, 1984; Van Zoest et al., 1994). The connections between the various categories are summarised in Table 1. Beliefs on the same row are regarded as theoretically consistent with one another, and those in the same column have been regarded by some researchers as a continuum (Anderson & Piazza, 1996; Perry et al., 1999; Van Zoest et al., 1994). In this study the various categories proved useful in analysing and discussing the data.

Table 1
Relationships Between Beliefs

Beliefs about the nature of mathematics (Ernest, 1989b)	Beliefs about mathematics teaching (Van Zoest et al., 1994)	Beliefs about mathematics learning (Ernest, 1989b)
Instrumentalist	Content-focused with an emphasis on performance	Skill mastery, passive reception of knowledge
Platonist	Content-focused with an emphasis on understanding	Active construction of understanding
Problem-solving	Learner-focused	Autonomous exploration of own interests

Beliefs and practice

Research interest in teacher beliefs has been premised on the assumption that an individual's behaviour follows from his/her beliefs (Ajzen & Fishbein, 1980; Cooney, 2001; Pajares, 1992). It is unreasonable to attempt to change the practice of teachers without changing their beliefs. Teachers' beliefs have, therefore, long been regarded as critical to the reform of mathematics education (Anderson & Piazza, 1996; Battista, 1994; Cooney & Shealy, 1997). Despite the acknowledged link between teachers' beliefs and their classroom practices its nature remains controversial, with some writers reporting consistency between teachers' beliefs and practice (e.g., Stipek, Givvin, Salmon, & MacGyvers, 2001; Thompson, 1984) and others inconsistency (e.g., Cooney, 1985; Shield, 1999). Even the direction of the beliefs practice connection has been questioned with Guskey (1986, cited in Cobb, Wood, & Yackel, 1990) arguing that rather than beliefs determining behaviour, change in teachers' beliefs is a consequence of change in their behaviours. Cobb et al. (1990) acknowledged the merits of Guskey's (1986) arguments, but concluded that the relationship is not linearly causal in either direction but rather beliefs and practice develop together and are dialectically related. Other writers such as Askew, Brown, Rhodes, Johnson, and Wiliam (1997) acknowledged the subtleties involved by referring to beliefs as *shaping* practice.

Contextual constraints have also been recognised as exerting significant influence on the enactment of beliefs (Sullivan & Mousley, 2001). In their theory of planned behaviour, Ajzen and Fishbein (1980) also emphasised the role of context. They stressed that beliefs are specific with respect to each of four aspects of contexts, namely; place, action or behaviour, time, and subject. They illustrated this with the example of a person's intention to vote in a non-compulsory election. Specifically, intention to vote (an action) may be dependent upon the candidates to choose from (object), the particular polling booth (place), and the date of the election (time). Each possible variation of each of the four variables has the potential to elicit quite different beliefs and so in order to predict behaviour, beliefs must be specified in relation to each of these aspects of context. Green (1971) also asserted the relevance of context, suggesting that the relative strength with which various beliefs are held, that is their relative centrality, is dependent upon the particular context.

Sullivan and Mousley (2001) described beliefs as influencing contextual constraints as well as being influenced by them, and argued that beliefs not only impact on practice, but practice also influences beliefs. In her meta-analysis of case-studies on the relationship between teachers' beliefs and practice, Hoyles (1992) described all beliefs as situated; that is, all of a teacher's beliefs are constructed as a result of experiences that necessarily occur in contexts. Consistent with Ajzen and Fishbein's (1980) view of context, Hoyles (1992) argued that, rather than contextual factors constraining teachers from implementing certain of their beliefs, such factors give rise to different sets of beliefs that are in fact enacted. It is therefore meaningless to distinguish between espoused and enacted beliefs or to examine the transfer of beliefs between contexts, since differing contexts will, by definition, elicit different beliefs (Hoyles, 1992). Pajares (1992) also stressed the contextual nature of beliefs and the implications of their being held as parts of belief systems, as described by Green (1971), rather than as isolated entities.

Many researchers, concerned with change in teachers' beliefs, have drawn upon aspects of Green's (1971) description of belief systems in explaining the relationship between teachers' beliefs and practice (e.g., Arvold & Albright, 1995; Becker & Pence, 1996; Cooney, Shealy, & Arvold, 1998). One aspect of this, of relevance here, is the assertion that beliefs may be held in isolated clusters. According to Green (1971) such clusters of beliefs can develop, and indeed are likely to, when they arise in independent contexts. An important consequence is that inconsistencies between beliefs held in different clusters are likely to go unnoticed and hence unreconciled.

Context is thus relevant to both the development and the enactment of teachers' beliefs. It influences the specific beliefs that are relevant (and hence likely to be articluated and/or enacted) in a given situation. In fact, Beswick (2003b) argued that failure to account adequately for context explains the apparent contradictions between teachers' beliefs and practices that some (e.g., Cooney, 1985; Frykholm, 1999; Shield, 1999; Sosniak, Ethington, & Varelas, 1991) have reported. She pointed out that in such studies, the contexts in which beliefs were considered or that are evoked by the survey items or questions used to discern them, often differed in terms of one or more of the aspects of context described by Ajzen and Fishbein (1980). Beswick (2003b) concluded that consistency cannot be expected when the contexts in which the teachers' beliefs are considered and their practices observed are not closely matched (Ajzen & Fishbein, 1980; Hoyles, 1992; Pajares, 1992) and that such context matching must include correspondence between the degree of generality of the beliefs and practice being considered. That is, it is unreasonable to expect consistency between broad collections of beliefs that are not closely linked with a specific context, and practice that is not described in equally broad, contextually independent terms. Similarly, if one is interested in specific aspects of teachers' classroom practice then the relevant beliefs must be defined in terms of the same context with equal specificity. The study reported in this article considered both teachers' beliefs and practice in broad terms.

Classroom environment

Classroom environment has been defined as the psychological and social context of the classroom (Fraser, 1991). Shuell (1996) elaborated on this in his description of the milieu in which teachers and students operate:

> Teachers and students work together in the rich psychological soup of a classroom, a soup comprised of cognitive, social, cultural, affective, emotional, motivational and curricular factors. (p. 726)

Other terms such as *learning environment* and *classroom climate* are also commonly used in the literature to describe the same construct and no distinction among them was made in this study.

In this study, teachers' practice was examined in terms of the classroom environments that they created and particularly the extent to which their classroom environments were consistent with the principles of constructivism. This provided a broad view of practice corresponding, in terms of its generality, with the description of the teachers' beliefs provided by the beliefs survey.

The teacher is just one part of, and contributor to, the classroom environment (Shuell, 1996). Amongst contributing factors, Henningsen and Stein (1997) included classroom norms, determined to some extent at least by the teacher, and both teacher and student dispositions. The importance of teachers' beliefs in determining the nature of teachers' impact on the classroom environment has been acknowledged by classroom environment researchers such as Jakubowski and Tobin (1991), who recognised that teachers will endeavour to make significant changes in their classroom environments if and only if they deem such effort worthwhile.

Constructivist classroom environments

In describing the essential tenets of constructivism, von Glasersfeld (1990) asserted that:

1. Knowledge is not passively received either through the senses or by way of communication. Knowledge is actively built up by the cognising subject. (p. 22)

2. The function of cognition is adaptive, in the biological sense of the term, tending towards fit or viability. Cognition serves the subject's organization of the experiential world, not the discovery of an objective ontological reality. (p. 23)

Implicit in the first of these assertions is that learning is dependent upon existing knowledge, which has been constructed in the context of prior experience. This means that individuals are likely to construct differing knowledge in response to a given experience. The second assertion describes constraints on an individual's constructions, in that the activity of construction is purposive in developing constructions that are powerful in the sense that they are useful for making sense of the broadest possible range of experience. Learning thus amounts to the process whereby individuals actively and purposefully modify, or if necessary completely change, their constructions in order to achieve an optimal fit with their experience. It is worth noting that the definition of an individual's beliefs that is used in this study, namely that they are everything that an individual regards as true, includes all of his/her constructions commonly referred to as knowledge. Indeed there is no distinction between the constructivist conception of knowledge as described by von Glasersfeld (1990) and the definition of beliefs used in the current study. Accordingly, the contextual nature of beliefs (Ajzen & Fishbein, 1980; Hoyles, 1992; Pajares, 1992) is consistent with the necessarily contextual nature of knowledge from a constructivist perspective, since an individual's experiences are necessarily contextually bound.

A further dimension of constructivism derives from the work of Vygotsky (Ernest, 1998) who stressed the critical role of language in social contexts in the development of children's thinking. Knowledge is thus socially constructed in a process that leads to the construction of common meanings (Taylor, Fraser, & Fisher, 1993). In essence, other people form part of the context in which individuals construct knowledge and, through interaction, are able to challenge one another's constructions in ways that facilitate the construction of increasingly shared and powerful knowledge.

Constructivism is not prescriptive in relation to teaching (Simon, 2000) and in fact any teaching strategy could be part of a constructivist learning environment (Pirie & Kieren, 1992). Breyfogle and Van Zoest (1998) warned of the danger of equating particular observable classroom characteristics, such as the use of manipulatives or the encouragement of student discussion, with constructivism and losing sight of the essence of the concept in the process. Fundamentally, from a constructivist perspective the role of the teacher is to facilitate, on the part of students, significant cognitive restructuring that goes beyond merely adding to and adjusting existing constructions (Cobb, Wood, & Yackel, 1991). Mathematical learning is thus a problem solving process, in which the social climate of the classroom is a key factor (Cobb et al., 1991). Teaching from a constructivist perspective is therefore a demanding task that requires the teacher to make genuine efforts to understand the mathematical constructions of students in order to structure appropriate learning experiences (Cooney & Shealy, 1997).

A number of writers have attempted to characterise classrooms that reflect constructivist principles in terms of elements that they regard as essential (Cobb, 1988; Confrey, 1990; Pirie & Kieren, 1992). Pirie and Kieren (1992) regarded the teacher's role in the creation of a constructivist classroom environment as crucial and listed four beliefs that they considered that teachers must hold if a constructivist classroom environment is to exist. They are: (a) not all students will progress toward instructional goals as expected; (b) there are different pathways to mathematical understanding; (c) different individuals have different mathematical understandings; and (d) for any given topics there are various levels of understanding, and the process of coming to understand is never complete. Although such a list does not prescribe any particular pedagogy it provides a basis on which to judge the appropriateness or otherwise of various teaching approaches. These beliefs require, respectively, that the teacher has flexible plans that are revised as students' learning occurs; the teacher recognises that students will engage differently with a given task; the teacher regards no particular understanding of a topic as the endpoint for that topic; and that the teacher must establish the expectation that students will justify their answers and explain their thinking (Pirie & Kieren, 1992). Consistent with this, Breyfogle and Van Zoest (1998) contended that in a genuinely reformed, or constructivist, classroom there must be a "fundamental emphasis on students doing the mathematical thinking" (p. 601).

Taylor et al. (1993) incorporated the principles of constructivism espoused by von Glasersfeld (1990), and the social context in which the constructive process occurs, into their Constructivist Learning Environment Survey (CLES) used in the current study. They considered a constructivist classroom environment to be one in which: students were able to act autonomously with respect to their own learning; the linking of new knowledge with existing knowledge was encouraged and facilitated; knowledge was negotiated by participants in the learning environment; and the classroom was student-centred in that students had opportunities to devise and explore problems that were of relevance to them personally (Taylor et al., 1993). Such elements are consistent with the literature discussed above and also align well with the principles and standards promoted by the National Council of Teachers of Mathematics (NCTM) (2000).

Research questions

The study reported here addressed the following research questions:

1. What beliefs do secondary mathematics teachers hold regarding the nature of mathematics, mathematics teaching, and mathematics learning?
2. To what extent do secondary students perceive their mathematics classrooms to be constructivist?
3. What connections exist between the beliefs held by secondary mathematics teachers and their classroom environments as perceived by their students?

Method

Participants

The study involved 25 teachers (17 male, 8 female) comprising all who identified as mathematics teachers in six secondary schools catering for students in grades 7 to 10 and situated in a rural region of the Australian state of Tasmania. Geographically and demographically similar schools were selected in order to minimise variability of the school level contexts in which the teachers were operating. This was because the focus of the study was on individual and classroom level variables in relation to the ways in which teachers' beliefs influenced their practice. The teachers varied in experience from 1 to 38 years, had varied mathematical and educational qualifications, and occupied a range of positions within their schools.

Each of the 25 teachers was asked to administer the CLES survey to two of their mathematics classes. Fourteen teachers did this whereas 11 gave the survey to just one class, with most of the latter group citing time constraints as the reason. A total of 39 classes thus completed surveys about their mathematics classroom environment.

Instruments

The beliefs survey consisted of 26 items with which teachers were asked to indicate the extent of their agreement on a five-point Likert scale ranging from Strongly Agree to Strongly Disagree. An initial survey consisting of 40 items was constructed comprising all 35 items from a survey of teachers' beliefs about mathematics teaching designed by Van Zoest et al. (1994), and an additional 5 items relating to the nature of mathematics taken from a similar survey by Howard, Perry, & Lindsay (1997). The 40-item survey was trialled with a group of 35 mathematics teachers who would not be involved in the actual study. Factor analysis of the pilot study results revealed two factors essentially corresponding with the respective views of mathematics teaching and learning that were identified as theoretically consistent with Instrumentalist and Problem-solving views of mathematics (Ernest, 1989b). The factors identified were similar to those found by Howard et al. (1997)

and described as *traditional* and *constructivist*, whereas Van Zoest et al. (1994) identified just one factor, which they referred to as *socio-constructivist orientation*. This close match between the factors that emerged from the new survey and those that were identified using the surveys from which its items were derived is evidence of the validity of the survey used in the current study.

Fourteen of the items included in the initial survey were omitted on the basis of feedback from participants in the pilot study concerning an appropriate length for the survey, and factor analysis of the pilot study results. The omitted items were either not significantly associated with either factor (< 0.3) or correlated approximately equally with both factors. Of the final 26 items, 24 were from the Van Zoest et al. (1994) survey and just 2, relating to the nature of mathematics, were from the Howard et al. (1997) survey. Alpha reliability coefficients for the groups of retained items associated with each of the two factors (consistent with Ernest's (1989b) Instrumentalist and Problem-solving views of mathematics) were 0.78 and 0.77 respectively.

The beliefs survey was scored such that a higher score indicated greater consistency with a Problem-solving view of mathematics and corresponding views of mathematics teaching and learning (see Table 1). The survey also asked for personal data including gender, years of teaching experience, mathematics qualifications, education qualifications and the grade level and course codes of the mathematics classes that the teachers were currently teaching.

The CLES required respondents to indicate the frequency of occurrence of various events in their classroom on a five-point Likert scale ranging from Very Often to Never. The survey was described by Taylor et al. (1993). It comprised four subscales of seven items each which measured the extent to which the respondent perceived each of four aspects of classroom environments consistent with constructivism. Taylor et al. (1993, p. 6) described the four aspects as follows:

> Autonomy:
> Extent to which students control their learning and think independently.
>
> Prior Knowledge:
> Extent to which students' knowledge and experiences are meaningfully integrated into their learning activities.
>
> Negotiation:
> Extent to which students socially interact for the purpose of negotiating meaning and building consensus.
>
> Student-Centredness:
> Extent to which students experience learning as a personally problematic experience.

Taylor et al. (1993) provided data establishing both the internal consistency and discriminant validity of the CLES scales. The surveys were scored by assigning the numbers 1 to 5 to the various response options, with 5 indicating that the event described occurred *Very Often*. The higher the CLES score the greater the conformity of the classroom environment with constructivist principles, as perceived by the respondent. Items on the teacher and student versions of the survey differed minimally with, for example, the words "the students" on the teacher version substituted for "I" on the student version.

Procedure

Teachers completed and returned the beliefs survey during the first few weeks of the school year. After a delay of several weeks, they administered the student version of the CLES to the students in, in most cases, two mathematics classes. In order to facilitate the survey completed by each teacher being linked to those completed by that teacher's students, the teachers were asked to use a code name on the survey they completed and to instruct their students to write their teacher's code name on the CLES that they completed.

Data analysis

In analysing and discussing the data in this study it was useful to characterise the beliefs of teachers with respect to the nature of mathematics, mathematics teaching and mathematics learning, using the categories described in Table 1. The views described in the last line of that table are most consistent with a constructivist view of mathematics learning. It was therefore expected that teachers whose classroom environments could be described as constructivist (Taylor et al., 1993) would be likely to hold beliefs in these categories.

The beliefs survey data of the 25 teachers and the average responses to the student version of the CLES of the 39 classes were each subject to cluster analysis as described by Hair, Anderson, Tatham, and Black (1998) using SPSS (SPSS Inc., 2002). Cluster analysis is a technique aimed at discerning structure in a data set by grouping respondents according to the similarity of their responses. In this study the technique was used to identify groups of teachers who were similar in terms of their espoused beliefs and also to identify classes with broadly similar classroom environments as perceived by the students. This enabled associations between various groups and between the membership of the various groups and relevant personal and class data to be examined.

The particular cluster analysis procedure used in this study for both the beliefs and CLES data was hierarchical and agglomerative. That is, the respondents were initially considered to each comprise a separate cluster. Subsequent steps in the process combined the two most similar clusters from the previous step into a single cluster. In this study Ward's method (Hair et al., 1998) was the clustering algorithm and the squared Euclidean distance was the similarity measure.

Given that consistency between teachers' beliefs and practice can be expected if and only if both the relevant beliefs and practice are defined in equally specific or broad terms (Beswick, 2003b), cluster solutions containing small numbers of clusters, and hence encompassing broadly similar respondents, were sought. In this instance both beliefs and practice were broadly defined - the survey data provided a necessarily incomplete picture of teacher's beliefs and classroom practice, and the beliefs survey in particular was not clearly linked to a specific classroom context. For each cluster analysis, inspection of the agglomeration schedule revealed a stage at which large changes in the clustering coefficient occurred compared with the previous stage. In each case this corresponded with a small number of clusters (three or five) having been formed. The identification of stages between which large changes in the agglomeration coefficient occur is acknowledged as a valid means of determining at which stage of a hierarchical cluster analysis the agglomeration of clusters should be stopped (Hair et al., 1998).

The results of any cluster analysis are dependent upon a number of essentially subjective decisions, such as the measure of similarity between clusters and the stopping rule used (Hair et al., 1998). Furthermore, it is essentially a descriptive technique that will produce clusters whether or not there is in fact any inherent structure in the data (Hair et al., 1998). Hair et al. (1998) stressed the importance of examining a variety of possible cluster solutions with a view to identifying a final solution that is optimal in the sense that the cluster profiles correspond with theoretical categories or categories expected on the basis of other reliable evidence, and that correlate in expected ways with variables that were not used in the formation of the clusters. In this study, several variations of the clustering procedure were considered and the final cluster solutions were characterised by significant correspondence with the belief categories identified in the literature (Ernest, 1989b; Van Zoest et al., 1994) and important associations with either the teachers' personal data or data relating to the students in various classes. Such associations indicate that the clusters identified may thus have predictive validity (Hair et al., 1998).

Results and Discussion

The results relating to each of the research questions are presented and discussed in turn.

Research Question 1: Teacher beliefs

The complete teacher beliefs data set

The possible range of scores for the beliefs survey was 26 to 130, with the actual scores ranging from 88 to 113. The numbers of teachers who indicated that they agreed or strongly agreed, were undecided, and disagreed or strongly disagreed with each statement in the beliefs survey are shown in Table 2. The items are arranged in descending order according to the numbers of teachers who agreed or strongly agreed.

Table 2
Teacher Belief Survey Responses

	Beliefs About Mathematics, Its Teaching and Its Learning: Items	No. SA or A	No. undecided	No. D or SD
1	A vital task for the teacher is motivating children to solve their own mathematical problems.	25	0	0
2	Ignoring the mathematical ideas that children generate themselves can seriously limit their learning.	24	1	0
3	It is important for children to be given opportunities to reflect on and evaluate their own mathematical understanding.	23	2	0
4	It is important for teachers to understand the structured way in which mathematics concepts and skills relate to each other.	23	1	1
5	Effective mathematics teachers enjoy learning and 'doing' mathematics themselves.	22	2	1
6	Knowing how to solve a mathematics problem is as important as getting the correct solution.	22	1	2
7	Teachers of mathematics should be fascinated with how children think and intrigued by alternative ideas.	22	1	2
8	Providing children with interesting problems to investigate in small groups is an effective way to teach mathematics.	22	0	3
9	Mathematics is a beautiful, creative and useful human endeavour that is both a way of knowing and a way of thinking.	19	3	3
10	Allowing a child to struggle with a mathematical problem, even a little tension, can be necessary for learning to occur.	18	6	1
11	Children always benefit by discussing their solutions to mathematical problems with each other.	18	5	2

12	Persistent questioning has a significant effect on children's mathematical learning.	17	7	1
13	Justifying the mathematical statements that a person makes is an extremely important part of mathematics.	16	9	0
14	As a result of my experience in mathematics classes, I have developed an attitude of inquiry.	16	6	3
15	Teachers can create, for all children, a non-threatening environment for learning mathematics.	16	5	4
16	It is the teacher's responsibility to provide children with clear and concise solution methods for mathematical problems.	15	3	7
17	There is an established amount of mathematical content that should be covered at each grade level.	14	7	4
18	It is important that mathematics content be presented to children in the correct sequence.	12	8	5
19	Mathematical material is best presented in an expository style: demonstrating, explaining and describing concepts and skills.	8	10	7
20	Mathematics is computation.	5	5	15
21	Telling the children the answer is an efficient way of facilitating their mathematics learning.	4	3	18
22	I would feel uncomfortable if a child suggested a solution to a mathematical problem that I hadn't thought of previously.	3	1	21
23	It is not necessary for teachers to understand the source of children's errors; follow-up instruction will correct their difficulties.	2	2	21
24	Listening carefully to the teacher explain a mathematics lesson is the most effective way to learn mathematics.	1	9	15
25	It is important to cover all the topics in the mathematics curriculum in the textbook sequence.	1	1	23
26	If a child's explanation of a mathematical solution doesn't make sense to the teacher it is best to ignore it.	0	0	25

The overwhelming majority of the teachers agreed with the first eight items listed in Table 2. All of these items, with the exception of number 4 that suggests Platonist leanings, are consistent with a Problem-solving view of mathematics (Ernest, 1989b) and Student-Centred views of mathematics teaching and learning (Van Zoest et al., 1994).

Items 9 to 15 are similarly consistent with these views, and although at least 64% of the teachers indicated their agreement with them and only small numbers dissented, approximately one fifth to one third of the teachers were undecided about these items. Interestingly, all of these items except numbers 9 and 14 relate to specific pedagogic strategies in contrast to the first eight items. It thus appears that although the vast majority of teachers held Problem-solving and Student-Centred beliefs, significant numbers of them were uncertain as to the appropriateness or otherwise of particular teaching strategies in their implementation. Sosniak et al. (1991) reported a similar phenomenon in the results of a much larger survey of grade 8 mathematics teachers, in that teachers appeared to have increasingly traditional views as items became closer to the classroom context. Their interpretation of their results led them to conclude that the teachers had incoherent views. This is not the inference drawn in the current study. Instead, it is likely that the extent to which the teachers' beliefs, as described by items 1 to 8, are manifested in their classrooms is a function of a range of contextual variables, principal amongst which are other, perhaps more centrally held beliefs (Green, 1971). It should also be noted that it cannot be assumed that any specific teaching practices are necessarily associated with the beliefs that the teachers professed to hold in agreeing with items 1 to 8.

Items 16 to 21 in Table 2 are consistent with either Platonist or Instrumentalist views of mathematics and corresponding views of mathematics teaching and learning. Interestingly, although 60% of the teachers agreed that teachers should provide "clear, concise solution methods" (item 16), only 16% approved of telling students the answers (item 21). It is thus clear that the teachers distinguished between these two strategies. It could be that many of the teachers who were undecided about item 10, and even some of those who agreed with it, believed that although allowing students to struggle is worthwhile, at some point they have a responsibility to provide resolution. This belief may even be driven, to some extent at least, by their perceptions of curriculum pressures given that more than half of the teachers believed that there were prescribed amounts of content to cover each year (item 20). Many of the items in this group also attracted relatively high numbers of undecided responses, suggesting that the teachers felt some tensions around these issues. Item 19 divided the teachers more evenly than any other and perhaps best encapsulates the tension that is evident between traditional and alternative pedagogies.

Only small numbers of teachers agreed with items 22 to 26 indicating that the vast majority of the teachers believed that they have a responsibility to assist students actively in their learning (items 23 and 26), are open to alternative approaches from students (item 22), and are not reliant on textbooks in determining the sequence of their teaching (item 25). Only one teacher believed that, "Listening carefully to the teacher explain a mathematics lesson is the most effective way to learn mathematics", but more than one third was undecided.

In summary, whereas many of the teachers appeared to hold beliefs consistent with Ernest's (1989b) Problem-solving view, a considerable number also held more traditional beliefs, although it would appear that very few had beliefs that could readily be classified as Instrumentalist.

Clusters in the teacher beliefs data

Cluster analysis (Hair et al., 1998) led to the identification of three clusters - sufficient to accommodate all of the teachers and likely to represent appropriately broad categories of teachers in terms of their beliefs. The average response to every item of the beliefs survey of teachers in each of the three clusters was calculated in order to profile the clusters. The items that varied most significantly across the clusters are shown in Table 3.

Table 3
Beliefs Survey Items Distinguishing the Clusters

		Cluster average item totals		
Item No. in Table 2	Beliefs Survey Item	Content and clarity	Relaxed problem solvers	Content and understanding
20	Mathematics is computation.	2.8	1.6	3.5
22	I would feel uncomfortable if a child suggested a solution to a mathematical problem that I hadn't thought of previously.	1.9	1.1	4.5
21	Telling children the answer is an efficient way of facilitating their mathematics learning.	2.6	1.7	1.1
16	It is the teacher's responsibility to provide children with clear and concise solution methods for mathematical problems.	3.9	2.7	4.5
17	There is an established amount of mathematics content that should be covered at each grade level.	3.9	2.7	4.5

18	It is important that mathematics content be presented to children in the correct sequence.	3.5	3.0	4.5
2	Ignoring the mathematical ideas that children generate themselves can seriously limit their learning	4.2	4.5	5.0
5	Effective mathematics teachers enjoy learning and 'doing' mathematics themselves.	4.5	3.8	3.5
25	It is important to cover all the topics of the mathematics syllabus in the textbook sequence.	1.9	1.4	1.0

Scores close to one indicated strong disagreement while scores close to five indicated strong agreement with the statements. Of the 25 teachers, 13 were in the *Content and clarity* cluster, 10 in the *Relaxed problem solvers* cluster and 2 in the *Content and understanding* cluster.

The characteristics of teachers in each of the clusters are summarised in Table 4. In addition, the Pearson product moment correlation coefficient between each factor and the average total beliefs score for each of the three clusters was calculated.

Table 4
Summary of the Three Teacher Beliefs Clusters

Cluster characteristics	Content and clarity	Relaxed problem solvers	Content & understanding	Correlation coefficient
Average total beliefs score	97.6	103.4	89.0	
Average teaching experience (years)	13.9	18.2	23.5	–0.64
% of teachers with 3rd year Uni maths	23.1	40.0	0.0	1.00
% of teachers with B. Sc or B. App. Sc.	61.5	60.0	50.0	0.86
% of teachers with Masters degrees	7.7	30.0	0.0	0.93
% of female teachers	38.5	30.0	0.0	0.81

Teachers in the *Content and clarity* cluster can be described as believing that the teacher has a responsibility to explain mathematical content clearly and may even need to tell students the answers. These teachers believed that they must cover the prescribed content and that the content must be correctly sequenced. They believed that computation is a major part of mathematics and that effective mathematics teachers should themselves enjoy the discipline. Although the cluster included teachers with 1 to 38 years experience, four of the five least experienced teachers were included. This cluster also included all of the teachers who identified as having no tertiary mathematics, as well as three who had studied mathematics to 3rd year university level.

The *Relaxed problem solvers* cluster of teachers is defined primarily in terms of what they were not concerned about rather than what they believed, and hence can be characterised as *relaxed*. Teachers in this cluster can be described as viewing mathematics as more than computation and were less inclined to believe that it was their role to provide answers or even clear solution methods. They were also less concerned about either content coverage or sequencing. Although the majority of the teachers indicated that they held some beliefs consistent with Problem-solving and Student-Centred views, the teachers in this cluster were those least likely to also hold beliefs consistent with other orientations, and hence these teachers could be described relatively unequivocally as *Problem solvers*.

Teachers in the *Relaxed problem solver* cluster were, on average, more experienced than those in the *Content and clarity* cluster with between 8 and 31 years of teaching experience. None of the teachers in this cluster clearly indicated that they had studied no tertiary level mathematics. In addition, this cluster included three of the four teachers who indicated that they had Masters degrees as well as all of the teachers who were in charge of mathematics. The inclusion of teachers in charge of mathematics in this cluster is consistent with the finding of Perry et al. (1999) that head mathematics teachers held more progressive beliefs than other mathematics teachers.

The two teachers in the third cluster, *Content and understanding*, can be described as the most concerned about the coverage and sequencing of the content, but the least likely to seek guidance regarding sequencing from a textbook. They were very focussed on students' understanding of the content, and their discomfort with students suggesting alternate solutions may have stemmed either from a concern that multiple methods may create confusion among students, or from a tendency to want to regard themselves as experts who have thought of everything - after all, they know the correct sequence of topics without referring to a textbook. Although the small number of teachers in this cluster make it difficult to make general statements, their average teaching experience was greater than that for teachers in either of the other clusters and they had neither mathematics majors nor Masters degrees. In addition, their overall beliefs scores indicated that this was the cluster least aligned with a Problem-solving view of mathematics and its corresponding views of mathematics teaching and learning.

In summary, the three clusters identified represent distinct types of secondary mathematics teachers, so aligning them to some extent with the categories identified in the literature described in Table 1. The major difference is the fact that there was no cluster that clearly represented teachers with an Instrumentalist view of mathematics and associated views of mathematics teaching and learning. Although the *Content and understanding* cluster teachers were the most likely to regard mathematics as computation, it was not clear that they believed that this was all that constitutes mathematics. In addition, given the concern of these teachers for understanding, they appeared to have a view of mathematics teaching described by Van Zoest et al. (1994) as Content-focussed with an emphasis on understanding, and this is consistent with a Platonist view of mathematics. The *Content and clarity* cluster teachers clearly had a Content-focussed view of mathematics teaching, but their view of mathematics was not Instrumentalist. It would thus seem that these two clusters both fall within the Content-focussed with an emphasis on an understanding view of mathematics teaching, but to differing degrees, with the *Content and clarity* cluster including some elements of a Content-focussed with an emphasis on performance view. The *Relaxed problem solvers* cluster represents teachers whose beliefs were markedly consistent with a Problem-solving view of mathematics (Ernest, 1989b).

Correlations between small numbers of data points need to be viewed cautiously. However, as shown in Table 4, it appears that a higher average total beliefs score is associated strongly with teachers who have studied mathematics and/or have a Masters degree. There were also significant correlations between the average total beliefs scores and both the percentage of the teachers who have science degrees, and the percentage of teachers who are female. The latter is consistent with the findings of Li (1999, cited in Nisbet & Warren, 2000) that female teachers are more likely than male teachers to use student-centred teaching approaches such as collaborative group work and whole class discussions. There is a moderate negative correlation between average total beliefs score and the teachers' average experience.

The connection between beliefs that are consistent with constructivist ideas, as Problem-solving beliefs are, and the extent to which teachers have studied either mathematics or education may be related to Lerman's (1997) notion of a second voice. Lerman (1997) claimed that this second voice could come from literature or from colleagues. Clearly teachers who have studied are more likely to have been exposed to a broader range of literature and to have had greater opportunities to reflect on their understandings in collegial contexts. These teachers were therefore most likely to have had available an alternative paradigm (Nespor, 1987) when deficiencies in their beliefs and practice became evident to them. Accordingly, teachers in the Content and clarity cluster, being the least experienced, have had the least opportunity in this regard. Indeed their emphasis on clarity resonates with Cooney's (1999) contention that beginning teachers are likely to have beliefs relating to the importance of providing clear explanations for students. The fact that this cluster included inexperienced teachers as well as others with more than 30 years experience suggests that findings reported in the literature in relation to beginning teachers may also be relevant to some experienced teachers.

The negative correlation with teaching experience demonstrates that time alone is not sufficient for teachers to acquire beliefs consistent with a constructivist perspective. Rather, this is likely to occur only if experienced teachers avail themselves of opportunities to engage with such ideas, as would be likely in the course of Masters level study.

Research Question 2: Student perceptions of their classroom environments

The complete student CLES data set

The total average CLES scores range from 81.0 to 87.7. The narrow range is to be expected due to the averaging of a large number of scores and the consequent reduction in the influence of extreme values. The average totals for the four CLES scales, from all 39 classes were: Autonomy –23.2; Prior Knowledge –22.1; Negotiation –23.7; and Student-Centredness –15.0. The overall impression of the average classroom environment as perceived by the students was thus one in which they believed, to a significant extent, that they engaged in the negotiation of meaning with their peers, thought for themselves and had opportunities to integrate their learning with their past experiences. However, they considered the teachers to be in charge of assigning tasks and time frames, and regarded them as both the sources and arbiters of correct methods and solutions.

The numbers, and percentages, of classes for which the average responses were Very Often or Often, Sometimes, and Seldom or Never are shown, for each CLES (student version) item, in Table 5. The items are grouped according to the CLES scale to which they contributed.

Table 5
CLES (student version) Responses

CLES Item Number	Constructivist Learning Environment Survey (Student Version): Items	No. Often or Very Often (%)	No. Sometimes (%)	No. Seldom or Never (%)
Negotiation:				
1	In this class I ask other students about their ideas.	2 (5)	36 (92)	1 (3)
5	In this class I don't ask other students about their ideas.	0 (0)	20 (51)	19 (49)
9	In this class I am not aware of other students' ideas.	0 (0)	18 (46)	21 (54)
13	In this class I talk to other students about the most sensible way of solving a problem.	18 (46)	21 (54)	0 (0)
17	In this class I try to make sense of other students' ideas.	12 (31)	27 (69)	0 (0)
21	In this class I pay close attention to other students' ideas.	5 (13)	34 (87)	0 (0)
25	In this class I don't pay attention to other students' ideas.	1 (3)	14 (36)	24 (62)
Prior Knowledge:				
2	In this class the teacher helps me to think about what I learned in past lessons.	6 (15)	32 (82)	1 (3)
6	In this class I get to see if what I learned in the past still makes sense to me.	8 (21)	31 (79)	0 (0)
10	In this class there's not really enough time to think.	1 (3)	31 (79)	7 (18)
14	In this class I get to think about interesting, real-life problems.	3 (8)	35 (90)	1 (3)
18	In this class I learn about things that interest me.	12 (31)	27 (69)	0 (0)
22	In this class what I learn has nothing to do with real-life.	0 (0)	31 (79)	8 (21)
26	In this class the things I learn about are not really interesting.	4 (10)	35 (90)	0 (0)

Student-Centredness:				
4	In this class the teacher gives me problems to investigate.	16 (41)	23 (59)	0 (0)
8	In this class the teacher expects me to remember important ideas I learned in the past.	33 (85)	6 (15)	0 (0)
12	In this class the activities I do are set by the teacher.	39 (100)	0 (0)	0 (0)
16	In this class the teacher expects me to remember things I learned in past lessons.	34 (87)	5 (13)	0 (0)
20	In this class I learn the teacher′s method for doing investigations.	25 (64)	14 (36)	0 (0)
24	In this class the teachers insists that my activities be completed on time.	36 (92)	3 (8)	0 (0)
28	In this class the teacher shows the correct method for solving problems.	35 (90)	4 (10)	0 (0)
Autonomy:				
3	In this class I think hard about my own ideas.	35 (90)	4 (10)	0 (0)
7	In this class I do investigations in my own way.	16 (41)	23 (59)	0 (0)
11	In this class I try to find my own way of doing investigations.	11 (28)	28 (72)	0 (0)
15	In this class I decide how much time to spend on an activity.	0 (0)	29 (74)	10 (26)
19	In this class I decide if my solutions make sense.	18 (46)	21 (54)	0 (0)
23	In this class I decide if my ideas are sensible.	17 (44)	22 (56)	0 (0)
27	In this class I decide how much time I spend on an activity.	0 (0)	33 (85)	6 (15)

Consideration of items contributing to the Negotiation scale indicates that in at least 97% of classrooms, the students, on average, believed that they asked one another about their ideas and ways of solving problems, and that they paid attention to other students' ideas at least sometimes. A similar majority (at least 97%) of class average responses to items contributing to the Prior Knowledge scale indicated that their classrooms were such that they were at least sometimes able to reflect on and evaluate their prior learning, and all of the class averages indicated that the content was interesting and life-related at least some of the time.

With regard to Student-Centredness, there were no classrooms in which the students' average perception was that the teachers rarely set tasks and time limits, required students to remember past learning, or provided correct solutions. In fact, 100% of the class averages indicated that the teacher Often or Very Often set the activities, 92% indicated that the teacher insisted on deadlines with similar frequency, and 90% indicated that the teacher Often or Very Often showed correct solution methods.

In relation to items contributing to the Autonomy scale, in all of the classrooms, the students, on average, believed that they thought hard about their own ideas at least some of the time, with the average perception in 90% of the classrooms being that this happened Often or Very Often. In addition, the average student perception in all classes was that they at least sometimes did investigations in their own ways. In contrast to the results for item 24 in the Student-Centredness scale, in approximately three quarters of the classrooms the average student perception was that they at least sometimes decided how much time to spend on activities.

Clusters in the student CLES data

Cluster analysis (Hair et al., 1998) of the average CLES responses of the students in each class revealed five clusters. In order to profile the clusters, the average total for each of the four CLES scales was calculated for the classes in each cluster. The results of these calculations are shown in Table 6 along with the numbers of classes in each cluster.

Table 6
CLES (student version) Average Scale Totals for each Cluster

		Cluster 1	Cluster 2	Cluster 3	Cluster 4	Cluster 5
	Number of classes	11	7	6	7	8
CLES scales	Negotiation	24.3	23.7	25.7	22.6	22.4
	Prior Knowledge	22.2	23.6	24.3	21.1	21.0
	Student-Centredness	13.8	15.1	11.7	17.3	16.5
	Autonomy	23.7	23.2	24.3	21.7	23.4
	Totals	83.9	85.5	85.9	82.8	83.3

The five profiles are shown graphically in Figure 1.

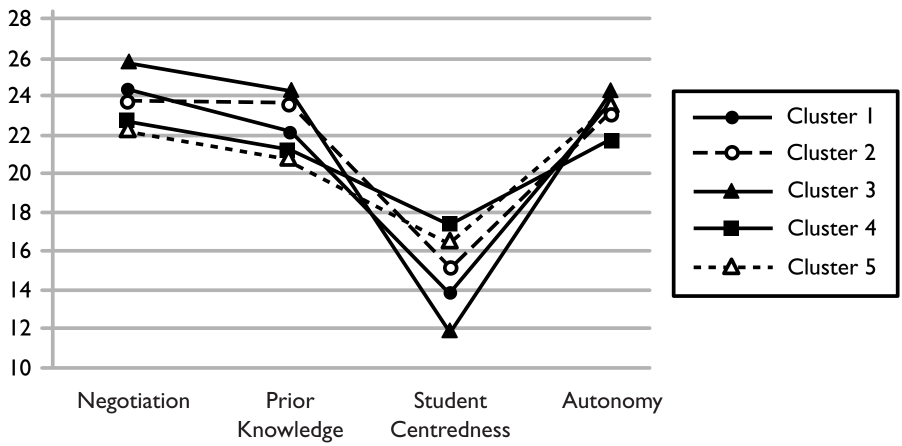

Figure 1. Cluster profiles for CLES (student version) data

While the overall shapes of the profiles for the five clusters are similar, there are important differences and the clusters are also distinguishable in terms of factors relating to the nature of the classes and average characteristics of their teachers. Interestingly, the order of the scores for each of Negotiation, Prior Knowledge and Autonomy for each of the clusters is almost exactly the reverse of the order for the Student-Centredness scale. In other words, according to the students' perceptions, the more that they were able to interact socially to construct understanding, integrate their learning with their existing knowledge and exercise control over their own learning, the less likely they were to be personally responsible for the choice of content, amount of time spent on various tasks and the evaluation of their own solutions. Factors that may contribute to this phenomenon are discussed below in relation to Cluster 3, for which the disparity between Student-Centredness and other scales is particularly pronounced.

The characteristics of each of the five CLES clusters are summarised in Table 7. Because of the inverse relationship between it and the other three dimensions of classroom environment measured by the CLES, Student-Centredness is excluded from average totals. The grade level ranges represent the range of grades outside of which no more than one class in the cluster fell, while the ability level of typical classes in each cluster reflects the courses undertaken by streamed classes. Higher ability indicates that the non-heterogeneous classes were studying the most demanding syllabus of a compulsory mathematics subject or an optional subject designed for students intending to study pure mathematics in the senior secondary years, while lower ability indicates that homogeneous classes were studying less demanding syllabi of compulsory mathematics. Mixed ability indicates heterogeneous classes.

Table 7
Summary of the Five CLES (student version) Clusters

	Cluster 1	Cluster 2	Cluster 3	Cluster 4	Cluster 5
Average total CLES without Student-Centredness	70.2	70.5	74.3	65.4	66.8
Average teacher beliefs score	97.3	103.1	107.4	99.4	98.9
Average teacher experience (years)	19.9	22.4	14.8	11.4	17.5
% of teachers with 3rd year Uni maths	45.5	28.6	50.0	28.6	25.0
% of teachers with Masters degrees	18.2	42.9	16.7	0.0	0.0
% of female teachers	45.5	0.0	66.7	57.1	25.0
Ability level of typical class	higher	mixed or lower	higher	mixed or lower	mixed or lower
Grade range of classes	8–10	7–9	9–10	7–10	9–10

One teacher had a class in each of Clusters 1 and 2, providing evidence of the influence on the classroom environment of variables relating to the nature of the class. Since beliefs are highly contextual (Ajzen & Fishbein, 1980; Hoyles, 1992; Pajares, 1992) it appears that teachers may hold different beliefs regarding mathematics and its teaching and learning depending upon the nature of the particular class that they are teaching. The case of the particular teacher mentioned here was described in detail by Beswick (2004).

It appears that a number of variables relating to both the classes and the teachers are relevant to the differences between the classroom environments of the classes. Both the average teachers' beliefs score, and the percentage of teachers with a Masters degree, varies across the clusters similarly to the average total CLES (student version) score, suggesting that there is a relationship between each of these variables and the extent to which students perceived their overall classroom environments to be constructivist. There were no clear relationships between any other variables and the students' overall perceptions of their classroom environments. However, it seems that higher ability classes, predominant in Clusters 1 and 3, tended to be characterised by lower levels of Student-Centredness, and were more likely to have teachers with 3 years of tertiary mathematics and perceived levels of the dimensions of a constructivist classroom environment other than Student-Centredness that were related to the teacher's beliefs. There was evidence that for classes of mixed or lower ability, typical of Clusters 2, 4 and 5, the extent to which the classroom environment was characterised by Autonomy, Negotiation and Prior Knowledge was also related to the average total teacher beliefs score. Cluster 4 classes had the least constructivist classroom environments overall, with the difference greater when Student-Centredness was excluded. This cluster suggests that classes of mixed or lower ability taught by relatively inexperienced and less mathematically qualified teachers whose beliefs are typically less aligned with a Problem-solving orientation are the least likely to experience classroom environments that could be described as constructivist.

Although no quantitative analysis of the impact of the ability or grade level of the class was conducted, it is evident from Table 7 that there is no clear connection between these characteristics of classes and the overall extent to which their classroom environments were perceived by students to be constructivist. However, since the groups of classes in each cluster were clearly distinguishable in terms of their ability and grade levels, it can be inferred that these factors, in conjunction with other variables, are related to particular patterns in students' perceptions of their classroom environments. It therefore seems that while classes with constructivist classroom environments are more likely than other classes to be taught by a teacher with a Problem-solving view of mathematics, corresponding views of mathematics teaching and learning, and with Masters level qualifications, other variables including the grade and ability level and the teacher's mathematics background interact to influence the classroom environment.

Research Question 3: Relationships between the teacher beliefs clusters and the student CLES clusters

The percentages of teachers of classes in each of the five clusters identified in the CLES data that fell in each of the three teacher beliefs clusters are shown in Table 8. The average total CLES score for each cluster is also shown. The data show that although there is no direct correspondence between the two sets of clusters there are definite and important links.

The CLES clusters with the highest total CLES scores, Clusters 2 and 3, had the highest percentages of *Relaxed problem solver* teachers. That is, classroom environments that can be described as constructivist are more likely to have teachers: (a) whose beliefs are to a large extent aligned with a Problem-solving orientation to mathematics and with associated views of mathematics teaching and learning; (b) are more likely than other teachers to have Masters degrees and to be in charge of mathematics in their schools; (c) are reasonably experienced; and (d) are highly likely to have studied tertiary mathematics.

Table 8
Connections Between Teacher Beliefs Clusters and CLES (student version) Clusters

	Cluster 1	Cluster 2	Cluster 3	Cluster 4	Cluster 5
Average total CLES score	83.9	85.5	85.9	82.8	83.3
% of Content & clarity teachers	60.0	28.6	20.0	57.1	50.0
% of Relaxed problem solver teachers	20.0	71.4	80.0	42.9	37.5
% of Content & understanding teachers	20.0	0.00	0.00	0.00	12.5

Classes in Clusters 2 and 3 had respectively low and very low average scores for Student-Centredness whereas their most likely teachers, the *Relaxed problem solvers*, had beliefs that indicate that they were less concerned than other teachers with providing answers and correct solution methods or with covering prescribed content. This apparent contradiction is unsurprising in view of the context-specific nature of beliefs, and in particular the claim that the contexts in which beliefs are inferred and practice observed must be matched if correspondence is to be expected (Beswick, 2003b). In this case the teachers' beliefs were articulated in one context (in response to a beliefs survey completed in relation to no specified context) and enacted in another (in particular classrooms with particular students). It is likely that specific contextual factors including the fact that many of the CLES Cluster 3 classes were studying the most demanding high school mathematics subjects with inherent pressures, are relevant. Indeed, in relation to Cluster 3, the inverse relationship between Student-Centredness and the other CLES scales that characterised the set of CLES clusters may be a result of a combination of differing curriculum and time pressures perceived by teachers in relation to classes studying different courses, and the teachers' perceptions of the students' capabilities (Beswick, 2004).

Content and clarity teachers were most prevalent in CLES Clusters 1, 4 and 5 which had slightly lower overall average total CLES scores, and Clusters 4 and 5 had considerably lower average totals when Student-Centredness was removed. From Table 7 it can be seen that the predominantly *Content and clarity* teachers of Cluster 1 classes and the predominantly *Relaxed problem solver* teachers of Cluster 3 classes were quite similar in terms of their qualifications and that the typical grade and ability levels of their classes were also similar. However, in addition to their differences in beliefs, which were reflected in the proportions of teachers from the two teacher beliefs clusters, they differed in terms of experience and likelihood of being female, with Cluster 1 teachers being, on average, more experienced and more likely to be male.

Both of the *Content and understanding* teachers taught classes in Clusters 1 and 5. These teachers were more experienced and their average teacher beliefs scores were lower than typical teachers in other beliefs clusters. Thus they fit the pattern of experience and beliefs scores observed across CLES clusters 1 and 5. They are also typical of teachers of classes in CLES Clusters 1 and 5 in terms of their qualifications.

Conclusion

Implications

Given that very few teachers appear to hold beliefs about the nature of mathematics that are exclusively consistent with a Problem-solving view (Ernest, 1989b) it is likely that very few students are acquiring an appreciation of mathematics that is consistent with the paradigm shift that Cooney and Shealy (1997) maintained has occurred in the way in which mathematicians view their discipline. The results also indicate that the extent to which classroom environments can be characterised as constructivist is related to, among other variables, the extent to which the teacher has a Problem-solving view of mathematics. Since many teachers do not have this view it follows that many students are likely to be learning mathematics in classroom environments that are not consistent with the principles of constructivism.

The fact that many of the teachers in the current study expressed beliefs about pedagogies that were consistent with constructivist ideas and yet many also held beliefs that were counter to constructivist principles (Table 2) suggests that some teachers may have inadequate understandings of constructivism - perhaps as Pirie and Kieren (1992) suggest, equating it with particular teaching strategies. This has implications for those involved in teacher education and in the provision of professional learning for mathematics teachers who may inadvertently contribute to impoverished understandings of the implications of constructivism for teaching by contrasting teaching that is consistent with constructivist theories with practices, such as telling, commonly associated with traditional or transmission views of teaching (Richardson, 2003).

The results of this study further underscore the complexity of the relationship between teachers' beliefs and practices and highlight a number of variables that appear to interact with teacher beliefs and with one another to influence classroom environment. The findings suggest that context is indeed an important consideration in the relationship between beliefs and practice. In this study teachers' beliefs were considered in terms of broad relatively de-contextualised views of mathematics and mathematics teaching and learning and it appears that these have some predictive capacity in relation to teachers' practice in contexts defined at the level of class characteristics. Specifically, the ability level (as perceived by the teacher) and grade level of the class, and possibly the relative curriculum pressures perceived by teachers in some courses, impacted the way in which teachers enacted their beliefs.

Recommendations for future research

The use of cluster analyses of survey data was effective in this study in distinguishing patterns of responses among participants and in identifying relationships between broad categories of beliefs and classroom environments. The technique could be usefully employed in subsequent studies of teachers' beliefs, including the beliefs of pre-service and primary teachers.

While this study found that teachers who have Masters degrees are more likely than other teachers to have constructivist classroom environments, the current study did not examine the content or focus of these qualifications. A study examining the elements of Masters level study that are most effective in facilitating changes in beliefs that increase the likelihood of teachers creating constructivist classroom environments is therefore recommended. Its findings would be of relevance to those involved in the facilitation of such study and to mathematics educators generally.

There remains much to be done in terms of examining the ways in which particular teacher beliefs influence their classroom environments. What specific teacher practices for example, lead to classroom environments that students perceive as more constructivist? And, what specific teacher beliefs, or systems of beliefs, underlie these practices?

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