Mathematics Education Research Journal 2005, Vol. 17, No. 2, 9–38

Using a Schematic Model to Represent Influences on, and Relationships Between, Teachers' Problem-Solving Beliefs and Practices

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Schematic models have been used extensively in educational research to represent relationships between variables diagrammatically, including the interrelationships between factors associated with teachers' beliefs and practices. A review of such models informed the development of a new model that was used to plan an investigation into primary school teachers' problem-solving beliefs and practices. On the basis of the findings from the research, the model was revised to include the important variable of prior mathematics learning, as well as a repositioning of the influence of teaching experiences in classrooms.

Considerable advice has been given to teachers about the importance of developing students' problem-solving skills in mathematics classrooms (e.g., Lester, 1994). Much research has focused on developing students' problem-solving abilities and on programs to enhance teachers' understanding of students' thinking (Schoenfeld, 1992). However, there is evidence that teachers have not responded to this advice (Hollingsworth, Lokan, & McCrae, 2003) with the suggestion that the culture of schooling and teachers' own beliefs about what is important to teach hinder the implementation of problem-solving approaches in classrooms (McLeod & McLeod, 2002; Stigler & Hiebert, 1999).

There has also been research into relationships between teachers' beliefs and practices in mathematics with the identification of several associated key variables (e.g., Thompson, 1992). Some investigators have used flow chart style schematic models to represent these variables and relationships (e.g., Ernest, 1991; Fennema, Carpenter, & Peterson, 1989; Raymond, 1997). Schematic models can provide structures that aid discussion and investigation by providing frameworks, which allow for explanation and prediction (Keeves, 1997). These various models include common elements or variables but also differ in some components, affirming the complexity of the relationship between practices and beliefs. Relatively few studies have examined relationships between teachers' problem-solving beliefs and practices in detail, and we claim that some synthesis of the different types of models and the development of a new model is a valuable addition to research in this important area.

This paper presents some influential schematic models, considers similarities and differences between the models, and proposes a new model that encapsulates the factors impacting on teachers' beliefs and practices, particularly in relation to the teaching of mathematical problem solving in primary classrooms. The new schematic model is then evaluated using a study that aimed to identify teachers' problem-solving beliefs and practices as well as to investigate the factors that may be impacting on teachers' lack of implementation of problem-solving practices. Data included surveys, interviews and classroom observations of primary school teachers in New South Wales [NSW] classrooms (for other aspects of this study, see Anderson, 1997, 1998, 2003; Anderson, Sullivan, & White, 2004). A revised model is presented that provides for greater dynamic action and interaction between variables, encompasses a wider range of variables, and better represents the interrelationships between beliefs and practices, particularly in the case of problem solving.

Schematic Models Linking Beliefs and Practices

Schoenfeld (2000) defines "model" as "a representation of a particular phenomenon, in which objects and relationships characterised in the model correspond to selected objects and relations in the phenomenon being represented" (p. 248). Schematic models have been used variously in educational research to provide a pictorial, or flow chart style, diagrammatic representation of relationships between variables (e.g., Schoenfeld, 1992). Keeves (1997) argued, "the use of a symbolic or diagrammatic form can often serve to make explicit and definite the structure of the model that would otherwise remain hidden in an excess of words" (p. 387). Romberg (1992) suggested that models are often used as a "heuristic device to help clarify a complex phenomenon" (p. 51).

While the use of diagrams might more readily communicate the relationships between factors than the use of lengthy descriptions, it is difficult to show the complexities of interrelationships. By their very nature, schematic models are static rather than dynamic and tend to suggest an oversimplification of real situations. However, as stated earlier, models can be useful for explanation or prediction and have been used to describe the connections between teachers' beliefs and practices in several studies. Models have the potential to aid discussion about change in practice by allowing teachers to focus on critical factors that might be inhibiting change.

Examples of Schematic Models linking Beliefs and Practices

Six models are presented to provide a comprehensive yet diverse summary of research into the relationship between beliefs and practices. One schematic model was used by Romberg (1984) to explore the differences between classrooms where the same mathematics content was taught. The model aided the investigation of teachers' intentions, as evidenced in their plans, teachers' classroom actions, and students' performance. The model proposed that teachers' beliefs, and the particular mathematical content to be taught, impacted on teachers' plans and actions and, ultimately on students' performance (Figure 1).

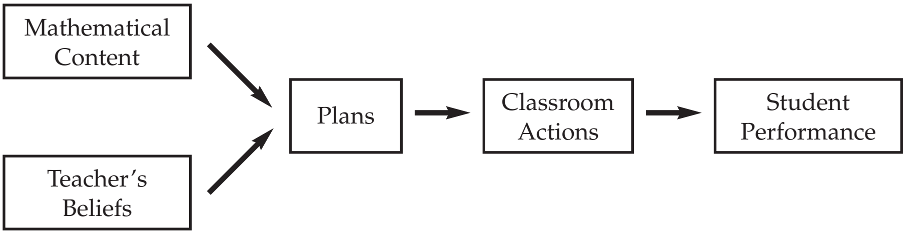

Figure 1. The elements of a model of mathematics pedagogy (Romberg, 1984, p. 125).

Romberg's model is a linear representation of the relationship between teachers' beliefs and practices and does not allow the possibility that actions and student performance could in turn impact on teachers' beliefs and future planning of mathematics lessons. However, it does recognise that teachers may teach different mathematics content in different ways.

A different linear model is Guskey's (1986, 2002) model of teacher change that proposes professional development precedes the implementation of new ideas in classrooms, which when implemented could lead to a positive change in student learning outcomes, and subsequently a change in teachers' beliefs and attitudes (Figure 2). This model places teachers' beliefs at the opposite end to the Romberg model, but it is a model of teacher change, and Guskey (2002) argued that "significant change in teachers' attitudes and beliefs occurs primarily after they gain evidence of improvements in student learning" (p. 384). Interestingly, Leder, Pehkonen and Törner (2002), in a review of relevant literature, suggested that "no consistent pattern has yet been identified for facilitating teacher change" (p. 2), since, as reported in many intervention studies, some teachers change their practice while others do not.

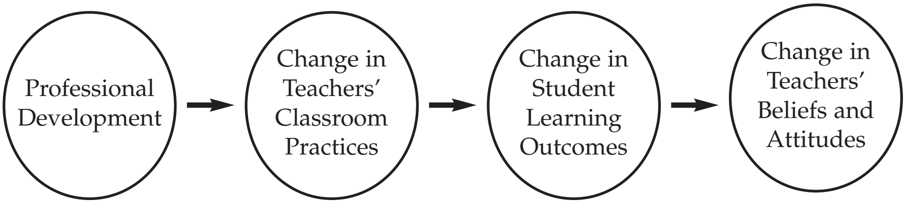

Figure 2. A model of teacher change (Guskey, 2002, p. 383)

Guskey (2002) noted that change in student learning outcomes could include changes in attendance, involvement in class activities, behaviour and motivation, as well as improvements in knowledge and understanding. He did acknowledge that the model "overly simplifies a highly complex process", that exceptions did exist and that "the process of teacher change is probably more cyclical than linear" (Guskey, 2002, p. 385).

A further model that includes a feedback loop or circular relationship is the curriculum development model of Fennema et al. (1989) (Figure 3) which emphasises the influence of teachers' knowledge and beliefs on student learning. It proposes that classroom instruction is determined by teachers' decisions, which in turn are influenced by the interaction between knowledge and beliefs. This model includes three aspects relating to students including their cognitions, learning, and behaviours. There is recognition that students' behaviours will influence teachers' decisions but the model does not accommodate how student behaviour and response might directly influence teachers' beliefs and knowledge. In this schematic model, the interaction between variables is represented with two-way arrows, thus suggesting a more dynamic relationship.

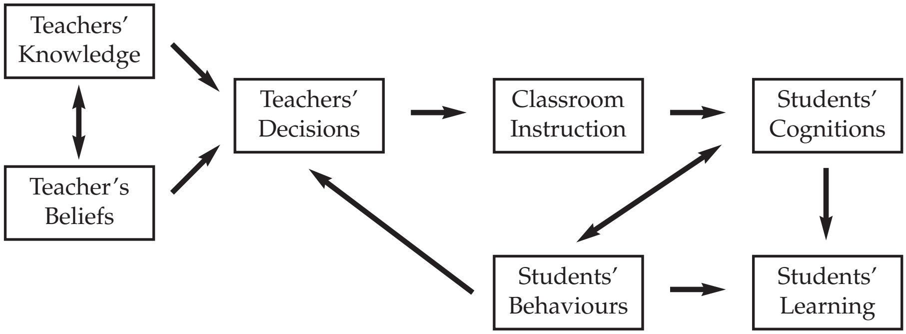

Figure 3. A model for curriculum development (Fennema et al., 1989, p. 180).

Another model that includes multiple interrelationships was proposed by Flexer, Cumbo, Borko, Mayfield and Marion (1994) (Figure 4). This model posits that three key sets of beliefs impact on classroom practice including: beliefs about how children learn; beliefs about what mathematics is important to teach children; and beliefs about instruction and assessment. Flexer et al. examined the impact of an "intervention" in the form of a professional development program that focused on alternative assessment procedures. The authors recognised the importance of beliefs on practices but also acknowledged that changes in practices may alter beliefs. In their study, teachers appeared to change their beliefs as a consequence of successfully using new assessment approaches.

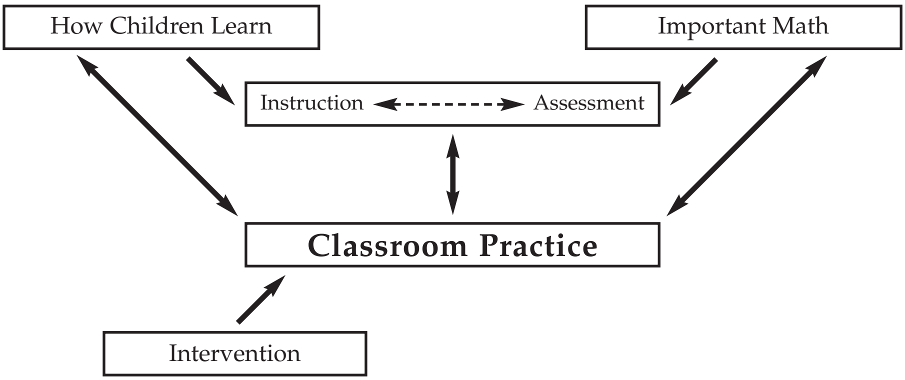

Figure 4. Belief system of teachers (Flexer et al., 1994, p. 3).

Several studies conducted in the mid-eighties reported a mismatch between teachers' espoused beliefs and enacted practices (e.g., Cooney, 1985). In response to these findings, new factors were introduced into Ernest's (1991) schematic model that attempted to account for the disparity (Figure 5). This schematic model distinguishes between teachers' models (a term used by Ernest differently from this article) of teaching and learning mathematics as well as between espoused models and enacted models. Ernest (1991) proposed that the espoused models, which represent teachers' beliefs, are "mediated by the constraints and opportunities provided by the social context of teaching" (p. 290). The schematic representation reveals that a primary component of any teacher's perspective is his or her personal philosophy of mathematics.

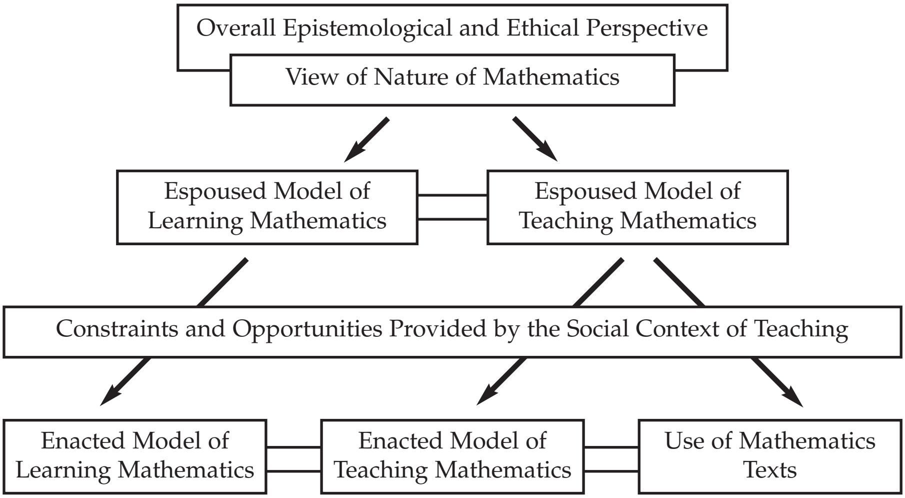

Figure 5. The relationship between espoused and enacted beliefs of the mathematics teacher (Ernest, 1991, p. 290).

The mismatch between beliefs and practices may be the result of many influences, including social context and teachers' level of thought. Social context includes expectations of other people such as students, parents, peers, and superiors. It also includes the "institutionalised curriculum" which includes the mandated curriculum, adopted texts, and assessment procedures. Ernest (1991) noted that a teacher's espoused model of teaching also determines the use of textbooks since they "embody an epistemology, and the extent to which the presentation and sequencing of school mathematics is followed is a crucial determinant of the nature of the implemented curriculum" (p. 290).

The findings from these studies led to other investigations of the causes for the mismatch between beliefs and practices (e.g., Van Zoest, Jones, & Thornton, 1994). In Raymond's (1997) study of the inconsistencies between beliefs and practices, she sought to investigate the level of inconsistencies as well as the reasons why these occur. Other factors were identified as contributing to the mismatch, including prior school experiences, which help to frame a teacher's view of mathematics. The model presented in Figure 6 provided the conceptual framework for an investigation with social teaching norms including the school setting, programs, other teachers, and parents of students. The immediate classroom situation included the current mathematical topic of study, the students, and time constraints.

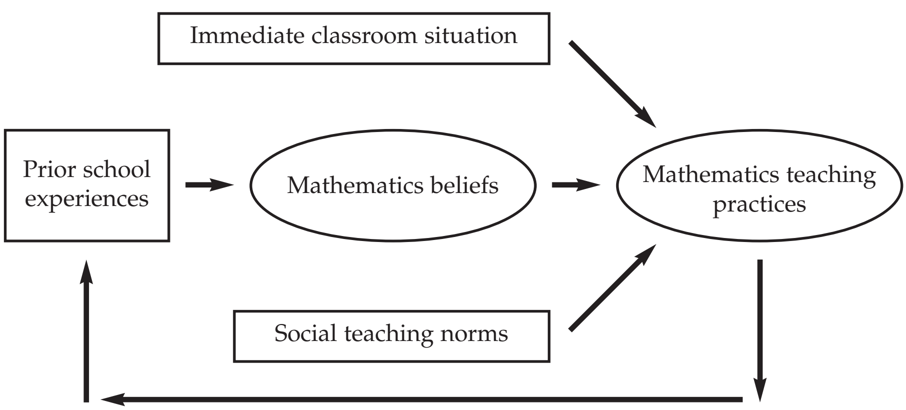

Figure 6. A model of relationships between mathematics beliefs and teaching practices (Raymond, 1997, p. 551).

Raymond's (1997) research led to a revised model of relationships between beliefs and practices (Figure 7). She stated that "although the model cannot be applied universally without amendment, it suggests complex relationships between beliefs and practice and builds towards an understanding of factors that contribute to the inconsistency between them" (Raymond, 1997, p. 570).

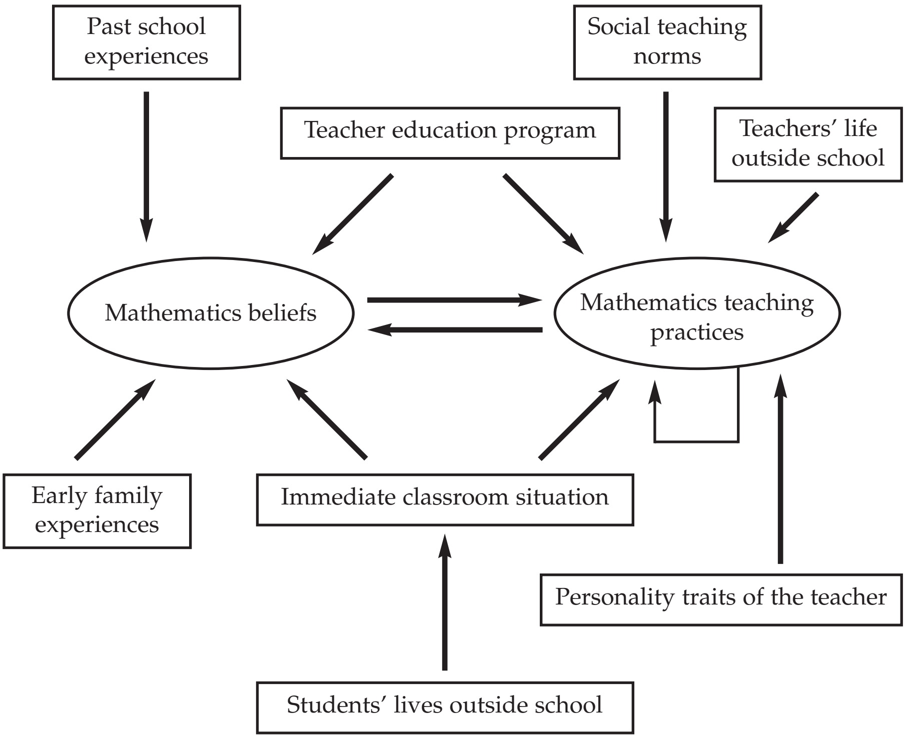

Figure 7. A revised model of relationships between mathematics beliefs and practice (Raymond, 1997, p. 571).

Raymond (1997) concluded that social teaching norms and the immediate classroom situation play key roles in influencing practice. It is these elements that are most likely to create inconsistencies between beliefs and practice, as they will change depending on the school the teacher is working in and the class that she is teaching. Further observations suggested that "... in many instances, teaching style is governed more by the sum of these other factors despite the teachers' perception that beliefs should play a major role in determining practice" (Raymond, 1997, p. 570). Even though this model is clearly more complex, it provides a better representation of what was the actual situation in classrooms. It is an example of a model informing research, and being revised by subsequent findings.

The models described in this section indicate the complexity of the relationship between teachers' beliefs and practices and suggest a variety of contributing factors. Table 1 summarises the features of each of these models.

Table 1
A summary of models used to investigate teachers' beliefs and practices

Researchers	Summary of models and key factors
Romberg (1984)	A model of mathematics pedagogy that includes teachers' beliefs and mathematics content as determining factors in teachers' plans, classroom actions and student performance.
Guskey (1986, 2002)	A model of teacher change that proposes a linear sequence of events from professional development to new practices in classrooms with a change in teachers' beliefs and attitudes if there is a change in student learning outcomes.
Fennema, Carpenter, & Peterson (1989)	A model for curriculum development that connects teaching and learning and includes teachers' knowledge, beliefs and decisions as influencing factors on instruction and students' learning.
Flexer, Cumbo, Borko, Mayfield, & Marion (1994)	A model of the belief system of teachers that includes beliefs about children's learning and appropriate mathematics content with beliefs about instruction and assessment as factors influencing practice.
Ernest (1991)	A model of espoused and enacted beliefs recognising the influence of teachers' conceptions of knowledge and mathematics, their views about mathematics teaching and learning, and acknowledges the constraints and opportunities of the classroom and school setting.
Raymond (1997)	A model of the relationships between teachers' beliefs and practices that recognises the influence of a range of new factors including teacher education programs, experiences, teachers' and students' lives outside of school, and teachers' personality traits. Key factors that account for inconsistencies are social teaching norms and the immediate classroom situation.

These models are valuable as they provide lenses for a wide range of important factors governing teachers' problem solving practices. Comparison allows for key specific factors to be identified. In particular, one common factor emerging is the relationship between teachers' beliefs and practices and teachers' knowledge. This knowledge could incorporate mathematical content knowledge, and advice from teacher education programs. Other factors noted in these studies include assessment, constraints and opportunities, the mathematical content to be taught, resources, and the teacher's personality as well as outside influences. The constraints and opportunities could incorporate assessment practices, the immediate classroom situation including students' behaviours, and social teaching norms. This recognises the impact of the school context through programs, parents and other teachers. These factors can be summarised as the social context of teaching.

A source of difference in the models may be attributed to differences in definitions of beliefs and the role of cognitive and affective factors. Furinghetti and Pehkonen (2002) examined differences in definitions of beliefs in a range of studies in relation to the emphasis placed on cognitive and affective elements. They suggested:

> The place of beliefs on the dimension affective-cognitive may be seen in different ways. If we were to stress the connections between beliefs and knowledge, we would see beliefs mainly as representatives of the cognitive structure of individuals. However, to see beliefs as a form of reactions toward a certain situation, means that we consider beliefs to be linked to the affective part of individuals. (p. 40)

They noted that some researchers focus on cognitive aspects, others focus on affective aspects, and some have a mixed orientation. This would appear to be the case for the models presented here with a cognitive focus (e.g., Romberg, 1984), a more affective focus (e.g., Raymond, 1997), and a mixed focus (e.g., Guskey, 2002) all represented. However, an overriding commonality of definitions of teachers' beliefs is the recognition of an affective component that is reflected in teachers' reactions to certain situations (Furinghetti & Pehkonen, 2002). In the study described below, both cognitive and affective elements are considered in the representation of teachers' beliefs about mathematics, about the teaching and learning of mathematics, and about the role of problem solving in learning mathematics.

A New Schematic Model

The previous section identified the general areas of knowledge and social context as general components in the relationship between teachers' beliefs and practices. The knowledge component includes:

- mathematical content and plans (Romberg, 1984);
- professional development (Guskey, 2002);
- teachers' knowledge and decisions (Fennema et al., 1989);
- important mathematics and assessment procedures (Flexer et al., 1994);
- use of mathematics texts (Ernest, 1991); and
- teacher education program (Raymond, 1997).

The social context of teaching includes:

- student performance (Romberg, 1984);
- student learning outcomes (Guskey, 2002);
- student cognitions, learning, and behaviours (Fennema et al., 1989);

- constraints and opportunities provided by the social context (Ernest, 1991); and
- social teaching norms and immediate classroom situation (Raymond, 1997).

Consistent with the different definitions of beliefs is the potential difference between what teachers profess to believe and what they actually believe (Schoenfeld, 2000; Thompson, 1992), and the difference between what teachers profess to do in their classrooms and what they actually do (Schoenfeld, 2000; Stigler & Hiebert, 1999). Hence, this proposed new model includes both "professed beliefs" and "beliefs" as well as "reported practices" and "practices". The only schematic model described earlier which represents these potential differences was that of Ernest (1991). Several studies have found that in relation to mathematical problem solving, teachers frequently profess a commitment to this teaching approach and yet there is little evidence of this in their classrooms (e.g., Cooney, 1985).

A new model is now proposed incorporating the components mentioned above: knowledge, beliefs, practices and the social context of teaching (Figure 8). This model acknowledges the influence of knowledge about mathematics, problem solving and how children learn, advice from preservice and inservice education, as well as curriculum and resource materials, on beliefs (Fennema & Franke, 1992). The model is a simple linear model consistent with Ernest's (1991) model where beliefs have been placed between knowledge and practices since they seem to filter what knowledge teachers embrace. According to Pajares (1992), beliefs influence perceptions in that they filter situations to make them more comprehensible. He also indicated that beliefs are held with different intensities. Practices are also influenced by the social context of schooling as suggested by Ernest (1991) and Raymond (1997).

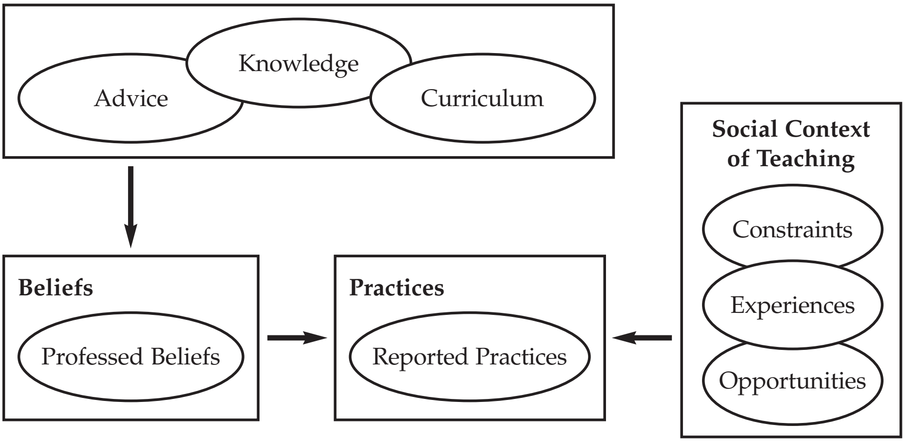

Figure 8. A new model of the factors that impact on teachers' problem-solving beliefs and practices.

This new schematic model was used to guide instrument design and data analysis in a study of teachers' problem-solving beliefs and practices. Data from the study were used to test the selection of variables and the relationships represented in the schematic model. The next section of the paper describes the study and presents data which support changes to the model.

Teachers' Problem Solving Beliefs and Practices

There have been substantial opportunities for teachers to build knowledge about teaching problem-solving skills and using problems as a focus of learning in mathematics (Wilson & Cooney, 2002). It has been argued that the use of non-routine problems and problem-centred activities form the basis of classroom activity in a reform or inquiry-based classroom (Clarke, 1997). Advice to teachers has been provided in papers in research and professional journals (e.g., Sullivan & Mousley, 1994), in national curriculum statements (e.g., Australian Education Council, 1991) as well as in curriculum documents (e.g., Board of Studies New South Wales [BOSNSW], 2002). Such advice has been accompanied by considerable efforts through preservice and inservice programs to change teaching practices from more traditional approaches to contemporary or reform methods (Artzt, 1999; Schifter, 1998; Van Zoest et al., 1994).

As there is evidence to suggest that teachers of Year 8 mathematics classes in Australia may not have responded to this advice (e.g., Hollingsworth et al., 2003), it is of interest to determine whether primary school teachers support problem-solving approaches. It is also of interest to explore the impact of, and the relationships between, each of the four key components proposed in the new schematic model, and to seek to identify other components that might be impacting on the implementation of problem solving in mathematics classrooms.

Methodology

Data collection methods for the study included a survey, interviews, and observations. A survey was designed to gather information about teachers' problem-solving knowledge, beliefs, practices, and the impact of the social context of teaching, and administered to 162 primary school teachers in New South Wales.

The Survey. Finding ways to infer beliefs from behaviours can be challenging but data are typically gathered using surveys, interviews, or observations (Leder & Forgasz, 2002). Hollingsworth (2003) criticised the use of surveys to gather information about teaching practices as teachers can forget classroom events and may not even be aware of what they have done. Recognising these limitations, a survey was used, which combined Likert scales and open-ended questions, and was informed by instruments used in similar studies into teachers' beliefs and practices (e.g., Howard, Perry & Lindsay, 1997; Raymond, 1997; Van Zoest, et al., 1994). To overcome the possibility of misinterpretation, the survey referred to the use of particular types of problems in mathematics lessons and examples were provided to illustrate the meaning of the terms "exercise", "application problem", "unfamiliar problem", and "open-ended problem" (Anderson, 2003).

To develop the items, an artificial continuum of teaching and learning was used. The concept of a continuum of teaching and learning with descriptions of particular perspectives was informed by earlier studies (e.g., Ernest, 1991) in which a range of approaches to teaching problem solving in mathematics classrooms had been described. At one end of this continuum was the belief that mathematics is a fixed body of facts to be delivered by teachers and internalised by students, referred to as a *traditional* teaching approach. This perspective is associated with individual student work, rehearsal of routine questions, and reliance on textbooks or worksheets. This view may be accompanied by a belief that problem solving is an *end* and that problems should be presented to students after they have mastered basic facts and skills. At the other end of the continuum, termed a contemporary teaching approach, mathematics is seen as a dynamic subject to be explored and investigated. Classroom practices associated with this perspective usually involve group work and the use of non-routine questions that promote mathematical thinking, and the development of problem-solving skills, an approach that has been described as representing a reform classroom (Clarke, 1997). This teaching approach may be accompanied by a belief that problem solving is a *means* to learning mathematics, an approach that has been referred to as teaching "*through* problem solving" (e.g., Lampert, 2001). Each of these perspectives was used to develop a set of items on the survey.

To set the items in a meaningful context, as recommended by Ambrose, Philipp, Chauvot and Clement (2003), the first two sets of survey items sought information about teachers' problem-solving beliefs through statements made by two imaginary teachers. Naomi's perspective suggested that she used problem solving as an end in learning mathematics. For example, one of her supposed statements was that "students should learn algorithms before they do application and unfamiliar problems". Gwendolin's perspective suggested that she used problem solving as a means for learning mathematics. For example, she "stated", "it is essential for students to explore their own ways of doing mathematics questions before being shown the teacher's methods". Respondents indicated whether they "strongly agreed", "agreed", "disagreed" or "strongly disagreed" with the statements. It was anticipated that respondents' levels of agreement or disagreement with the beliefs of each imaginary teacher would provide an indication of their professed beliefs.

Another item sought data on teachers' reported practices and listed 20 statements related to teaching approaches. The items were chosen on the basis of strategies mentioned in the literature (e.g., Clarke, 1997), for example, "you explain in detail what the students have to do to solve problems", and "you present unfamiliar and open-ended problems to the class with very little indication of how to solve them". Teachers rated the frequency of their use of each of the strategies as "hardly ever", "sometimes", "often", and "almost always". It was anticipated that the frequency with which teachers reported that they used these strategies would provide an indication of the perceived importance of the practices.

Open-ended questions requested that teachers describe a recently used problem, explain why they prefer to use particular types of mathematics questions or problems, and describe the professional development needs of the staff at their school in relation to the implementation of problem-solving approaches. The analysis and results of these survey questions are described in the next section.

The Interviews. Semi-structured interviews were conducted with a sample of nine respondents who represented the spread of problem-solving beliefs and practices. These teachers also taught in a range of school contexts including different socio-economic areas with diverse populations. The purpose of the interviews was to gather more in-depth information about particular teachers' beliefs and practices, and to explore further the social context of teaching. The protocol used for the interviews was to use each respondent's survey responses as a starting point for ongoing discussions about:

- their knowledge and understanding of problem-solving approaches;
- their planning and programming for problem solving; and
- the impact of their class and the school context on their plans.

The interviews were audio taped and transcribed to identify links between knowledge, beliefs, practices, and the social context of teaching for each participant.

The Observations. From the nine teachers who were interviewed, two were selected for classroom observations because they had a deeper knowledge and understanding of problem-solving approaches, and they were able to articulate clearly their beliefs about the role of problem solving in learning mathematics. In addition, they each reported using particular practices for quite challenging classes, even though issues in each of their school contexts might have militated against such approaches. Each teacher was observed on three occasions teaching "problem-solving lessons", and interviewed after the lessons to discuss their actions, lesson outcomes, and student responses. The lessons were videotaped and the interviews were audio-taped for subsequent transcription. These data were then used to identify additional variables for the revised schematic model.

Results and Discussion

A summary of some of the key findings from the surveys, interviews, and observations is presented, with a discussion of the results in relation to the model. These findings either provide evidence to support the new model, or challenge the structure and assumptions of the model so that it can be revised to better represent the relationships between beliefs and practices for problem solving in mathematics.

Survey Data

In order to identify broad groups of responses from the beliefs statements, surveys were initially sorted into five categories referred to here as very traditional, traditional, very contemporary, contemporary and mixed. This categorisation was based on each respondent's level of agreement with either the *traditional* perspective or the *contemporary* perspective (Anderson, 1997). Responses that were either "strongly agree" or "agree" were grouped together, as were those for "disagree" and "strongly disagree".

As an example of how this categorisation was achieved, a teacher was placed in the very traditional category if there was agreement with six or more of the seven traditional statements combined with disagreement with six or more of the seven contemporary statements. A similar arrangement, but with the reverse requirements, was used for the *very contemporary* category. From this, 6 respondents (4%) were placed in the very traditional category, 17 (11%) were traditional, 8 (5%) were contemporary and 12 (7%) were very contemporary. The remainder were placed in the mixed group (73%). To explicate trends in the data, the following discussion combines the traditional and very traditional responses as well as the contemporary and very contemporary responses (Anderson et al., 2004). Responses to three of the traditional belief statements and three of the contemporary belief statements by each of these groups of teachers are presented in Table 2.

Table 2
Levels of Agreement (%) of the Traditional Teachers (n=23) and the Contemporary Teachers (n=20) to Selected Belief Statements

Belief statements	Traditional Agreement	Contemporary Agreement
students should learn *algorithms* before they do application and *unfamiliar* problems	100	10
application and *unfamiliar* problems are best left to the end of the topic in mathematics	61	5
mathematics lessons should focus on practising skills	78	15
mathematics lessons should focus on problems rather than on practice of algorithms	0	95
students can learn most mathematical concepts by working out for themselves how to solve *unfamiliar* and *open-ended* problems	0	95
it is essential for students to explore their own ways before being shown the teacher's methods	0	95

These responses indicate that there are teachers from the total population who appear to have polarised views, but within the groups there is strong agreement, matching the hypothesised endpoints of the continuum. To determine whether each group of teachers reported using practices that were consistent with their professed beliefs, Table 3 presents the proportion of teachers in each group who reported using a particular strategy either "often" or "almost always".

Table 3
Proportions (%) of each of the traditional teachers (n=23) and the contemporary teachers (n=20) who reported using selected teaching approaches "often" or "almost always"

Selected teaching approaches	Traditional	Contemporary
you ensure that students work alone	43	0
you explain in detail what the students have to do to solve problems	61	15
you set exercises to allow the students to practise their skills	87	45
you encourage the students to work in small, cooperative groups	43	80
you present *unfamiliar* and *open-ended* problems to the class with very little indication of how to solve them	0	35
you encourage students to record their own procedures and methods of solving problems	35	80
you pose *open-ended* problems to allow students to explore mathematical situations for themselves	9	55

Generally it seems that the professed beliefs and the reported practices are linked. The *traditional* teachers reported using strategies that are compatible with a transmissive style of teaching in that they frequently have students working alone, they prefer to provide detailed explanations, and most of this group frequently set exercises for skills practice. The *contemporary* teachers reported using practices that give responsibility to the students by encouraging group work, providing less initial explanation, encouraging individual recording, and allowing students to explore mathematical ideas.

One aspect of the survey was to determine which types of mathematics questions or problems are chosen by teachers and for what purposes. Others have found that providing an innovative curriculum that advocates a focus on problem solving and investigative approaches does not necessarily change teachers' practice (Norton, McRobbie, & Cooper, 2002). Table 4 presents the responses for frequency of use of each of the question types.

Table 4
Proportions (%) of each of the traditional teachers (n=23) and contemporary teachers (n=20) who report using each of the question types often or sometimes/rarely

Types of Questions	Often		Sometimes/Rarely	
	Traditional	*Contemporary*	*Traditional*	*Contemporary*
Exercises	100	30	0	70
Application Problems	82	65	18	35
Open-ended Problems	14	25	86	75
Unfamiliar Problems	5	10	95	90

From Table 4, there is a clear difference between the *traditional* and *contemporary* teachers in the frequency of use of exercises in mathematics lessons. There is less difference between the frequencies of use of other question types. However, more of the contemporary teachers use open-ended and unfamiliar problems. The literature about reform classrooms suggests that teachers should more frequently use inquiry approaches that could include the use of open-ended and unfamiliar problems (Clarke, 1997). At least some of these teachers appear to have responded to this advice but certainly not all of those who profess to support *contemporary* belief statements. It may be that "the teacher holds the belief about problem solving subservient (or peripheral …) to the belief that the teaching of mathematics is about certainty and procedural knowledge" (Wilson & Cooney, p. 131).

To gain further insights into teachers' knowledge, beliefs, practices, and the context of teaching, an open-ended question was included to ask teachers to explain why they use particular question types. The comments were categorised into themes. These themes related to:

- considerations of the students in relation to learning, knowledge and understanding, age and affective factors;
- characteristics of the question types including language and the promotion of student thinking;
- confidence of the teacher; and,
- the availability of suitable resource materials.

The students' levels of understanding of mathematics were perceived to be a critical factor in choice of question type. Many teachers suggested that open-ended and unfamiliar questions were more appropriate for more able students. One example of this follows:

> Only the top 10 or so in my class can handle unfamiliar problems. The whole class need strategies to help them solve problems therefore

> application problems are more appropriate to my class as they can use known strategies. Only the top 10 would explore unfamiliar problems.

While many respondents agreed that problem solving was an important part of learning mathematics, the students in the classroom clearly determined the practices they chose as well as the mathematical question types they believed to be most appropriate.

A key component of teachers' thinking involved their comments about affective issues, with 18% of all respondents commenting about this aspect of problem solving in mathematics. Some comments related to negative feelings such as frustration, anxiety and the tendency of students to give up and not persevere when solving problems. Other comments mentioned the positive aspects of problem solving, and related the need to develop students' confidence and to provide experiences that are less threatening. Several teachers wanted to challenge students, but commented on the need to ensure success and enjoyment while problem solving. For example:

> With Year 6 students this year, many of whom feel anxiety over maths performance, these problems [exercises and application problems] were a way to establish confidence and success when worked through …

Teachers seemed to have identified different purposes for each of the question types based on curriculum requirements and question characteristics (Anderson, 2003). Teachers reported that exercises were necessary as they provided valuable practice in basic skills and procedures. Application problems enabled students to apply these basic skills and procedures in real-life contexts. Open-ended problems provided challenge, developed higher-level thinking, and allowed students to respond at their own level. Unfamiliar problems challenged and motivated more able students, and provided opportunities for students to apply problem-solving strategies. A traditional, experienced teacher of upper grades indicated:

> I don't use problem solving as much as I would like to, as I just don't have the time to get organised. I feel strongly that each child still needs to have a grasp of concepts of +, –, x, ÷, before they can begin to understand fully and explore the problem-solving concept, especially unfamiliar problems. Open-ended problems, I really haven't used that much (maybe I need to reassess). Application problems are essential for purpose and reinforcement of WHY we learn mathematics.

The general view was that all problems were valuable but choice was made on the basis of accessibility for students in the class, although in some schools there were other considerations. A traditional, experienced Year 4 teacher stated:

> This school has a very formal curriculum and examinations twice each year. Each type must be taught and revised frequently. We enter all competitions and do very well. The open type are common in competitions.

This confirmed the strong influence of the social context of teaching on teachers' practices.

Several teachers mentioned confidence and competence in using problem-solving approaches, suggesting a need for further support, particularly in the form of additional resources. Another open-ended question on the survey asked teachers to indicate "the professional development needs of teachers at your school in relation to problem solving". Suggestions in common between the *contemporary* teachers and the *traditional* teachers were:

- practical strategies in the use of the full range of problem types;
- better resources of appropriate problems;
- adapting problem solving to the varying needs of students including lower ability and students with poor literacy skills; and
- peer mentoring and modelling of approaches.

In addition, the contemporary teachers suggested:

- integrating problem solving with curriculum outcomes;
- grouping children for exploration and reflection;
- using other approaches than focusing on teaching algorithms;
- using other resources than textbooks; and
- teaching mathematics *through* problem solving.

These last five suggestions indicate that the *contemporary* teachers support reform methods and would like additional professional development support so that their colleagues appreciate the approach.

It appears from these responses that the knowledge, advice and curriculum efforts to date have not provided enough support for teachers. It is possible that the support has not challenged teachers' beliefs enough for them to reconsider their current practice in order to fully embrace reform recommendations. It is also evident that the impact of the social context of schooling is quite strong. Teachers were particularly concerned about meeting the needs of the students, while acknowledging the impact of the use of textbooks and particular assessment practices.

In seeking to identify other variables that influenced reported practices, teachers made considerable reference to the teaching grade level of the students. Table 5 indicates the proportion of traditional and contemporary teachers who were teaching in each of the categories Years K–2, Years 3–4, Years 5–6, and those with a specialist role such as supporting students with learning difficulties.

Table 5
Proportions (%) of each of the traditional teachers (n=23) and contemporary teachers (n=20) in each of the teaching grade levels

	Years K–2	Years 3–4	Years 5–6	Specialist
Traditional	4	39	35	22
Contemporary	70	20	10	0

There seems to be some link between teaching grade level and association with *traditional* and *contemporary* views since 74% of the traditional teachers were teaching in Years 3 to 6 compared to 30% of the contemporary teachers. A possible explanation for support of the more traditional statements by teachers who were teaching in Years 3 to 6 is that algorithms are introduced and developed in these years for most students (BOSNSW, 2002). As the curriculum mentions recall of facts, mental strategies, and written algorithms at this level, this might encourage teachers to focus on these skills and, as a consequence, leave learning *through* problem solving until these have been established. Indeed one teacher mentioned this:

> The syllabus identifies Year 3 outcomes as being based on the basic algorithm as in the example for Exercises. I also like to use context-based problems involving language as in Application problems.

This is an example of the way that advice from curriculum documents influences practice. In the schematic model, the link between *curriculum* and *reported practices* is represented as being filtered through *beliefs*. It is possible that there should be a more direct link between curriculum and practices in the model.

Data from the surveys confirm a link between professed beliefs and reported practices, and highlight the impact of the social context of schooling, including the evidence for perceived constraints and opportunities existing in particular school contexts (Anderson et al., 2004). Some teachers have embraced the advice about reform classrooms while others have not. The interviews enabled an investigation of the particular knowledge, beliefs, and practices of a sample of survey respondents who held a range of views about problem solving in mathematics.

Interview Data

From the survey analysis, nine teachers representative of the spread of problem-solving beliefs were selected for the interviews. One was *very traditional*, three were *traditional*, three appeared to hold *mixed* beliefs, and two were *very contemporary*. Forty-five minute interviews focused on each teacher's knowledge about problem-solving approaches, their professed beliefs and reported practices from the surveys, and the impact of the particular context of their school and classroom. These discussions helped to ascertain the reliability of the survey and to determine the validity of the classification process. In addition, the interviews sought to test the new schematic model as representing relationships between teachers' problem-solving beliefs and practices.

A comparison of views as reported by teachers on the survey and views described by teachers in the interviews indicated that these were the same for six of the nine teachers. The interview discussions confirmed that all of the *very traditional* teachers and the *traditional* teacher were correctly classified since they held traditional views and consistently used traditional classroom practices. For these four teachers, exercises and application problems were the main student question types used in mathematics lessons since there was a strong belief in the need to develop skills and procedures before problem solving. All mentioned a structured approach with a teacher-centred presentation of ideas, more individual student work, and a reliance on textbooks. Problems were usually used as extension questions for the more able members of the class.

For two of the *mixed* teachers, additional comments on the open-ended questions suggested that they may hold more contemporary beliefs and used contemporary teaching approaches. This was confirmed in the interviews as both were supportive of problem-solving approaches and believed that problem solving was an important and necessary life skill for all students. Discussions with the third teacher who had been placed in the mixed category confirmed a mixed set of beliefs, particularly in relation to her lower ability mathematics group. For the two teachers who were classified as *very contemporary*, interviews confirmed this for one, while the other tempered her beliefs and was considered to be contemporary.

Knowledge and understanding of the advice about problem solving was determined by each teacher's comments about their problem-solving practices as either an end, a means, or both. Those teachers with more traditional beliefs appeared to view problem solving as an end, or as the solution of a variety of types of problems. This view was usually accompanied by a belief that problems were difficult mathematics questions that were appropriate for more able students. The teachers with more contemporary or mixed beliefs acknowledged that problem solving could be a means, or an important process that supports mathematics learning for all students.

The interviews sought to identify other factors that impacted on teachers' problem-solving efforts in their current schools. The factors identified as influencing this set of teachers' problem-solving practices were diverse. For the teachers who held more *traditional* views about mathematics teaching and learning, factors included the level of ability of the students in streamed classes, frequent use of textbooks, formal assessment procedures, approaches used in local high schools and after-school tuition classes, students' success in mathematics competitions and in gaining entry to selective high schools, and parents' expectations. One traditional teacher discussed the pressure from parents to get students into selective high schools:

> So we've got that sort of pressure and that sort of pressure necessitates that we do have to do a lot of drill … we do have to do tables and do the old fashioned method, and it works. I mean I'm not saying anything against old fashioned methods.

For the teachers who held more *contemporary* views, a common influencing factor was poor school experiences in mathematics that often seemed to result in teachers not wanting to teach the way they had been taught. One example of this was from a teacher who had been disappointed with her mathematical experiences at school and so returned to evening classes at a technical college to repeat a senior secondary mathematics course.

> … [we] went over all of the things that I hadn't really understood. I was doing it but didn't know why and I had this most brilliant teacher who just opened the whole door for me and from then on it went just from being a bogey to one of my favourites and you know if you don't get someone who inspires you with that confidence and then teaches you how, I think it's often going to be a bogey for a lot of children.

Other factors from this group of teachers included enthusiasm for mathematics, a desire to promote positive attitudes, knowledge of more contemporary approaches, and the influence of other teachers' views.

Lack of time was a recurring theme for both groups of teachers. Even when they supported doing more problem solving, time was commonly given as an issue. For example:

> … well I really think it develops critical thinking skills and I think that's very important for children to develop higher level thinking skills and that goes through all their work … I would like to do more … critical thinking skills, the six hats and all of that sort of stuff, yeah, I would love to do that but we just haven't got the time …

A rich data set was obtained from the nine teachers who were interviewed. All teachers were aware of at least some of the advice about problem-solving approaches, particularly through the curriculum documents and textbooks, which were used in most schools. However, a factor that seemed to impact on beliefs for several participants was their early experiences as learners of mathematics. This appeared to have an impact on both beliefs and practices. The impact of the social context of teaching was critical in determining what happened in many classrooms. The students' knowledge and understanding of mathematics, their parents' expectations, the influence of other staff members, and the general lack of time to prepare innovative approaches were all cited by these teachers, as well by many teachers on the surveys. There was also evidence that classroom experiences added to teachers' knowledge in important ways. This was further explored in the observations.

Observations

The Teachers. The final stage of data collection involved classroom observations and further discussions, or reflective interviews, with two of the nine teachers who had participated in the interviews. Rose and Gaye had a deeper knowledge and understanding of problem-solving approaches, and held strong beliefs about the importance of problem solving. Each described a range of school cultural issues that were potential constraints and yet each reported implementing problem-solving approaches. One consideration to support the findings was to explore the level of consistency and coherence of beliefs (Wilson & Cooney, 2002) and practices across a variety of data collection methods.

Rose was an experienced, dedicated teacher of a Year 2 class who believed that children needed to experience challenge and success in mathematics learning (Anderson, 1998). On her survey responses, Rose had been classified as *very contemporary* and the interviews confirmed this. Rose consistently and coherently reported the same beliefs and practices on the survey and during interviews. Rose believed that her position as a member of the school executive as well as her increased knowledge through further study had empowered her to resist the constraints operating in her school. Rose's own learning of mathematics played an important role in her views about teaching mathematics. She described herself as "a person who failed maths" and that because of this she is a different kind of teacher of mathematics from those she had experienced. She stated:

> Maths is a bogey in my life, but I maintain that actually because of that I'm probably a more sensitive teacher in the area because I remember we had a teacher who couldn't understand why we had problems when I went to school and I hated maths and she used to just dismiss our problems as being stupid therefore you know I had a problem and I do think that maths is taught fairly poorly …

Gaye began teaching as a second career and currently had a Year 6 class, but she taught mathematics to a streamed group of lower achieving Year 6 students. She had been classified as having *mixed* beliefs from the survey but during the interviews there appeared to be some inconsistencies in her views. She believed that problems provided a purpose for learning about mathematical ideas since having learned basic facts and algorithms, students needed to be able to apply this knowledge to solve problems in the "everyday world". She also believed that mathematics was best learnt in "streamed" classes to better cater for students' needs and to enable the presentation of problems appropriate for that particular group of students. Like Rose, Gaye had undertaken postgraduate study and described negative experiences as a learner of mathematics.

It was apparent that Gaye had a sound knowledge and understanding of *contemporary* approaches and she believed that they were worthwhile. However, her experiences with lower ability students, particularly in upper primary classrooms, had led her to reconsider this and to adapt her practice to better meet the needs of this group. In this way, her belief in more contemporary approaches was tempered by the need to support these students and to provide substantial guidance. This suggested that the inconsistencies in Gaye's responses were related to the conflict between answering questions in relation to notions of "good practice", or responding in relation to the needs of the students she was teaching. Alternatively, it could highlight the possibility that for some teachers there may by different sets of beliefs that depend on aspects of the current class or school context, a notion that has been referred to as situated beliefs (Hoyles, 1992).

The Lessons. Each teacher was asked to teach three problem-solving lessons. These lessons were videotaped and "reflective" interviews were held after the lessons, which enabled each teacher to reflect on the outcomes of the lesson and the successful problem-solving efforts of the students. Rose's class was involved in a unit of work on "money" which was integrated across other key learning areas in the curriculum including English. She used a variety of question types including open-ended problems, encouraged students to work together and share their thinking, provided concrete materials to support student learning, and used whole-class discussion to demonstrate, consolidate, and share ideas.

Classroom observations also suggested that there were factors that both supported and constrained Rose's efforts. Supporting factors included the classroom layout as desks were positioned in groups to enable discussion between students, the availability of concrete materials and calculators, and the general attitude of the students towards the activities as they responded well to the tasks and willingly attempted each activity. In addition, Rose had spent considerable time and effort planning and creating the ideas and problems for the students that included several real-life applications and teaching ideas that were highly motivational. The use of toy catalogues to plan how the students might spend $100 was clearly a source of enjoyment and interest, and the discussions about possible purchases from the school canteen for lunch were both relevant and pertinent to students' everyday world.

However, there appeared to be several constraining factors that were impacting on Rose's efforts. The diverse needs of the students in her class were apparent. There were several students who had considerable difficulty remaining on task, other students needed additional explanation and assistance in order to understand and attempt tasks, and a few students worked quickly to achieve solutions and then requested extra work. Rose needed to encourage these students constantly to find additional solutions to the open-ended questions. There was evidence from the observations that Rose attempted to put her beliefs into practice, but it was also clear that her class was as challenging as she had described. In spite of the difficulties encountered in catering for a variety of students' needs, the students were motivated by Rose's attempts to engage them in meaningful activities.

Gaye's class was doing a unit of work on "scale" from the Measurement strand of the syllabus (BOSNSW, 2002). The students had been given a set of six cards, each containing different activities related to measurement. They were to work on these in small groups with opportunities to report back to the whole class on what they had discovered. The main discrepancies between Gaye's reported strategy use and observed use included less frequent use of whole class discussion and application problems. There appeared to be more frequent use of student choice of problems, presenting problems that related to students' experiences, and open-ended problems.

During the classroom observation period there seemed to be no real constraints operating in Gaye's classroom that prevented her from implementing the problem-solving approaches that she wanted to use. There appeared to be a reasonable correlation between her reported strategy use and her actual practice, which in turn was consistent with her mixed beliefs. However, there were also identifiable differences in that she reported hardly ever allowing students to spend several lessons on the same problem, and only sometimes using problems that arose from the school context or open-ended problems. During the observations, it was apparent that the approach taken did incorporate each of these three strategies. She made a real effort to incorporate many of the strategies that she believed were appropriate and desirable, with drill-and-practice number facts exercises at the beginning of each lesson.

Reflective Interviews. Discussions with Rose confirmed that her students were challenging, but she was satisfied with her lessons. As the conversation continued, a new set of issues arose that could have acted as constraints. Staff disagreement over streaming of classes, an unhappy school culture, and a coercive Principal were raised in this last opportunity to discuss Rose's experiences in her current school. She did not really feel constrained by his views and practices, but her passion for other teachers to adopt similar practices would not be met until there was a change in school leadership.

Rose's beliefs about learning in general and learning mathematics in particular have been influenced by several factors. Her experiences of learning mathematics were fraught with difficulties and poor teaching. Her life-long learning revealed alternative approaches to teaching and learning, and has made her question the way she was taught. Returning to the classroom after a period of extended leave provided her with the experience of feeling like a beginning teacher and further changed her views about teaching and learning as well as the role of professional development. She has responded to advice given in curriculum documents, and in undergraduate and postgraduate courses. Her experience as a teacher of mathematics had added to her knowledge and reinforced her belief that this is the best way for children to learn. She employs teaching strategies that support these approaches even though there are constraints operating in her school that support more traditional teaching approaches.

Gaye was somewhat critical of her lessons, as she had wanted to demonstrate her knowledge and understanding of reform approaches. Discussions during the reflective interviews were informative and revealed several critical issues in her school context. For Gaye, potential constraints included particular aspects of the school culture: a prescribed textbook, formal assessment procedures, and rigid programming. She seemed to be able to resist these constraints by adapting her practice to accommodate the requirements of the school executive. For example, the prescribed textbook was used as a source of homework exercises to support the activities planned for class time. As Gaye had trained as a mature-age student and had a strong character, she seemed to be able to resist the imposition of these potential constraints.

Evaluating the Model

The data showed that knowledge was a major factor impacting on teachers' practices. However, this knowledge was more diverse than represented in the proposed model. Not only was knowledge about the curriculum important but knowledge relating to teachers' understanding of their students and their individual needs was influential. Hence a teacher may profess a strong belief in problem-solving practices, but when confronted with a particular group of students decides that the approach is not appropriate for them. This was consistent with Rokeach's (1968) concept of centrality, with the notion of central and peripheral beliefs, as well as stability of beliefs versus beliefs that may be subject to change. He proposed a "wide spectrum of beliefs", with deep beliefs at one end and surface beliefs at the other, and suggested that when researching beliefs, two types of knowledge (objective and subjective) need to be considered, with beliefs belonging to subjective knowledge.

Many teachers were aware of the emphasis on problem solving in curriculum documentation. They had received advice in either preservice or inservice education, and they had at least some knowledge of problem types and problem-solving teaching approaches. However, it appeared that for at least some teachers, their early experiences as learners of mathematics provided additional knowledge that influenced their beliefs and practices. For example, both Rose and Gaye discussed their early learning of mathematics and a desire to teach differently.

The impact of the opportunities and constraints in particular contexts was strong although this was related to teachers' beliefs. Student ability or grade level, for example, was seen as a constraint for problem solving for the more traditional teacher but an opportunity for problem solving for the more contemporary teacher. Students' stages of schooling as well as knowledge and understanding of mathematics were other elements of the social context of teaching which impacted on teachers' decisions. It was unfortunate that there appeared to have been few opportunities to enhance teachers' problem-solving efforts in each of the contexts for the nine teachers who participated in the interviews. There seemed to be many more constraints that interfered with the implementation of problem-solving approaches than opportunities that supported such endeavours.

The elements in the model were useful in analysing the interaction between beliefs and practices for problem solving. However, the analysis also showed areas of early mathematics learning, subjective knowledge, and aspects of the social context of teaching, which would benefit from greater emphasis. To better represent the links between all of the factors, a revised cyclic model that acknowledges the important influence of the social context of teaching on knowledge (both objective and subjective) as well as on practices is now presented (Figure 9). This is supported by Guskey's (2002) claim that successful student learning outcomes influence teachers' beliefs and attitudes.

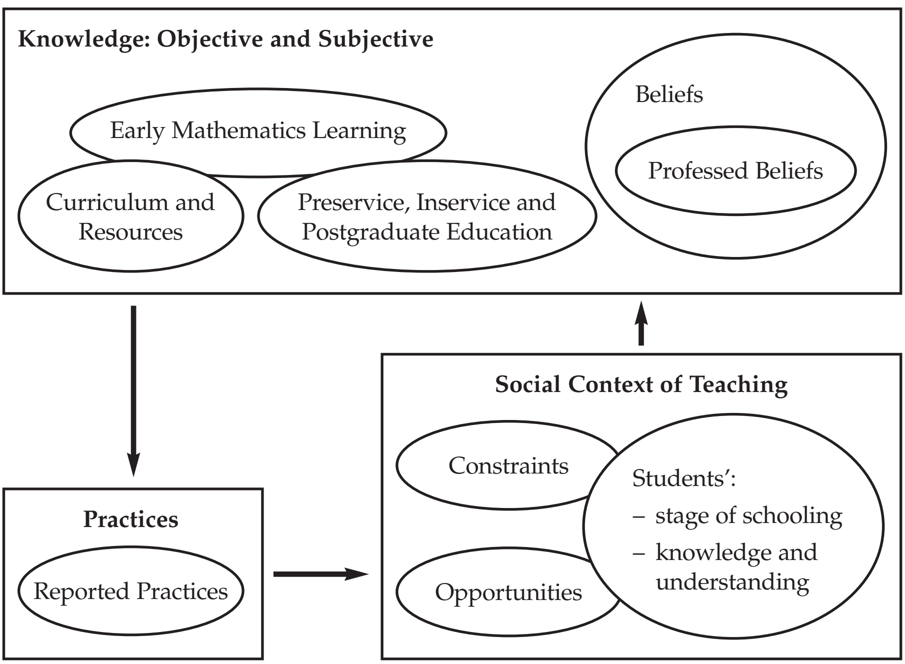

Figure 9. A revised model of the factors that impact on teachers' reported beliefs and practices.

The original model did not directly link beliefs and the social context of teaching. The new model takes into account the influence of contextual factors from classroom and school experiences as potentially influencing beliefs, or subjective knowledge, in important ways (Furinghetti & Pehkonen, 2002). This model recognises that teachers are constantly changing their plans and reacting to students' responses, and hence they are more adaptive in their practice (Wilson & Cooney, 2002). This adaptation is particularly apparent when teachers become more reflective (Artzt, 1999; Wilson & Cooney, 2002), a process that is usually promoted when teachers undertake postgraduate study. The courses that Rose and Gaye studied seemed to have changed their perspectives about the importance of problem solving in learning mathematics, particularly for Rose. It should be noted that for any particular teacher, some of the factors in the model might have a stronger influence than others, depending on previous experiences and current school context (Raymond, 1997). Also, this model is a generalisation of a complex situation and is used to provide an illustrative guide to aid research into teachers' beliefs and practices.

Conclusion

Even though this study confirmed that school experiences have an impact on subsequent behaviour in classrooms, in the case of Rose there was a clear resistance to the approach she had endured at school. Perhaps her desire to teach differently from the way she had been taught stems from her not wanting to perpetuate failure. "Maths as a bogey" is a common view among preservice primary teachers, and mathematics educators can certainly provide opportunities for students to confront such beliefs. We argue, though, for more — that consideration of a model such as the one presented here provides a reflective tool which can assist teachers in implementing problem-solving approaches in their classrooms.

The analysis of the investigation presented here supports schematic models that describe relationships between the factors influencing teachers' beliefs and practices, being useful for guiding individual teacher reflection and group discussions, especially as part of planned and sustained teacher professional development programs. The models can also be used to guide further research into important areas of the curriculum that teachers struggle to implement.

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