Mathematics Education Research Journal 2005, Vol. 17, No. 1, 3–23

Primary Preservice Teachers' Understandings of Volume: The Impact of Course and Practicum Experiences

Robyn Zevenbergen
Charles Sturt University

Mathematics teacher education programs are designed with an intention, explicit or implicit, to produce graduates who are competent in both pedagogic knowledge and discipline knowledge. This paper explores students' experiences in coming to learn mathematics and mathematics education. Using an example from a quiz question, supplemented with follow-up interviews with students, the paper raises questions as to the effectiveness of programs that adopt traditional approaches to the teaching of mathematics. It is shown how a functionalist perspective was adopted by many of the preservice teachers, which engendered a reproductive approach to teaching. The data raised concerns about preservice teachers' knowledge of volume, their pedagogic knowledge, and the role of the practicum on their learning.

Within contemporary teacher education, there is an expectation that graduates will be competent teachers of mathematics upon graduation. Arguably, to be considered competent, graduates need both pedagogical content, that is, how to teach mathematics, and discipline knowledge, that is, knowledge of mathematics (Shulman, 1986). The focus on discipline knowledge in primary teacher education has become a focus in contemporary reforms, in part fuelled by a national and international recognition of the importance of numeracy/mathematics. The Board of Teacher Registration, which accredits teacher education courses and provides registration of teachers for the state of Queensland, has recently released its *professional standards* for graduates. The second of these standards states that "Graduates will possess and be able to apply a range of literacies relevant to their professional roles" where this is spelt out to mean "graduating teachers will exhibit *high* levels of personal proficiency in oral and written language and numeracy" (emphasis added) (Board of Teacher Registration, 2002, p. 6)

Part of the task of preservice teacher education is to re-educate students in how they perceive and enact mathematics as a discipline. Contemporary thinking regarding the teaching of mathematics is often antithetical to the lived experiences in many classrooms and/or the memories of preservice teachers. While the research in mathematics education has demonstrated many high quality cases of good teaching in school mathematics, longitudinal data of large cohorts have shown that the teaching of mathematics is very poor. In the Queensland Schools Longitudinal Reform Study (Education Queensland, 2001), the teaching of school mathematics was rated as the most poorly taught of the key learning areas when measured against a productive pedagogies framework. Within this context, it is hardly surprising that studies have shown that preservice teachers hold strong beliefs about the teaching of mathematics which is seen to be an authoritarian discipline governed by rules, formulae and textbooks (Ball, 1990; Schuck, 1996a, Szydlik, Szydlik, & Benson, 2003). This is reinforced through preservice teachers' practical experiences in schools (Frykholm, 1998), so that they believe that learning mathematics is about memorisation rather than autonomous learning. Furthermore, these beliefs have been reported to be resistant to change (Foss & Kleinsasser, 1996) and, as Kagan (1992) contends, students often leave university with the same views that they entered with and in many cases their biases are even stronger.

Within this context, the task of preservice courses is three-fold. First, courses need to ensure high levels of competence/knowledge of mathematics. Often students enter courses with low levels of mathematical knowledge and confidence. Second, courses need to develop the confidence of preservice teachers so that they are not only mathematically competent, but believe in themselves as learners and teachers of mathematics. Third, courses need to challenge preservice teachers so that they embrace new ways of teaching while recognising the strengths (and limitations) of older methods. These purposes are central to most preservice mathematics education programs, the successes of which are often supported or hindered by numerous factors. This article critically explores preservice teachers' reactions and learnings in a course in which pedagogical content knowledge was taught in parallel with discipline knowledge in a way that is congruent with contemporary ideas of teaching mathematics. Two questions were explored as part of the project:

- What are preservice teachers' understandings of volume concepts? and
- What factors impact on student learning in preservice teacher education courses?

Mathematics Discipline Knowledge

Within the context of preservice teacher education, it has been recognized that those entering primary courses often have weak discipline knowledge. Students most frequently enter these courses with minimal understandings or experiences with school mathematics and, in many cases, have completed only minimal compulsory levels of mathematics (Goulding, Rowland, & Barber, 2002). Many primary preservice teachers have tended to specialise in non-mathematical areas after they complete the compulsory years of schooling (i.e., beyond Year 10). This has resulted in many preservice teachers entering their teacher education courses with low levels of mathematics knowledge as well as revealing considerable anxiety towards the subject (Brown, McNamara, Hanley, & Jones, 1999).

Simon (1993) raised concerns about preservice teachers' weak discipline knowledge. Within the Australian context both Taplin (1992) and Schuck (1996a) have reported that preservice teachers had considerable difficulties in coming to learn mathematical concepts, skills, and processes. Studies conducted within some Australian teacher education programs have shown that preservice teachers' knowledge was problematic. For example, Burgess (2000) reported that preservice teachers' mathematical misconceptions were not dissimilar to 11- and 12-year old students'. Similar weaknesses among preservice teachers' mathematics discipline knowledge have been noted by others (e.g., Chick & Hunt, 2001). The carryover of weak discipline knowledge into the teaching profession has been reported by Kanes and Nisbet (1996) who noted the limited mathematical knowledge of practising teachers, both primary and secondary. Within this context, the question of whether discipline knowledge impacts on teaching becomes an important consideration.

Discipline Knowledge and its Impact on Teaching

It has been suggested that mathematics discipline knowledge is not important as teachers can learn it on a need-to-know basis. However, in most studies of teachers' mathematics discipline knowledge, a relationship between discipline knowledge and teaching practice has been reported. Strong discipline knowledge was found to have a positive impact on teacher practice, and the converse was also found to be true.

In a large study of 9000 Year 7 students, Mandeville and Lui (1997) concluded that the level of teacher discipline knowledge impacted significantly on the learning of the students. They reported that teachers with high levels of mathematical understandings provided higher quality learning opportunities for their students than did their colleagues with limited understandings of mathematics. They reported that teachers with strong discipline knowledge were able to provide quality mathematics learning as they could mathematise examples and mathematically extend students and their thinking. Similarly, Irwin and Brit (1999) noted that the discipline knowledge of teachers impacted on their subsequent willingness to reflect on, and change, their teaching practice. They reported that one of the teachers with "limited mathematical background…appeared not to be able to solve problems herself …[gave] confused and tentative responses … [and] did not stimulate class discussion and investigation that might have lead students to adjust their understanding" (p. 97). The limited background knowledge of primary preservice students must be considered in their preparation for teaching in the light of studies such as that of Madeville and Lui (1997), in which teachers' discipline knowledge was found to correlate positively with the quality of learning environments and with the subsequent learning of their students.

Links between discipline knowledge and planning for learning have also been observed. In a large study in the UK, Goulding et al. (2002) found that there was a significant link between "poor subject knowledge [being] associated with weaknesses in planning in teaching primary mathematics" (p. 699). Recognising that correlation does not imply causation, the authors elaborated further that the positive links were potentially due to the connections that preservice teachers make between subject knowledge and pedagogical content knowledge. They contended that the links were due to both cognitive and affective dimensions of the teachers. Being strong in content knowledge, they claimed, offered a sense of confidence which, in turn, can be realised through teacher actions. Offering a strengthened program in subject knowledge would give preservice teachers resources upon which they could draw as they planned their teaching. The authors concluded that when preservice teachers have secure mathematical foundations, they have greater confidence in their own knowledge as teachers.

Bibby (2002) powerfully illustrated the impact that fear or shame have on how teachers, as adults, viewed their learning of mathematics. Recognising that weak content knowledge impacts on a teacher's sense of identify, Bibby demonstrated the flow-on effect this had on teaching. In particular, she cited the notion of mathematics being about "correct answers" creating pressure on teachers to believe that they needed to know the answers. She showed how teachers who had come to construct this particular view of mathematics developed a sense of fear and shame at not being able to undertake the work needed and so relied on rigid approaches to teaching mathematics.

Goulding, Rowland, and Barker (2002) identified the importance of preservice teachers making connections between discipline and pedagogical content knowledge. In attempting to break the distinction between content knowledge and how it was taught, Ball (1990) argued that preservice teachers need to develop connections between discipline knowledge and pedagogical content knowledge. She contended that strength in discipline knowledge can be transferred to pedagogical content knowledge. This view was supported in the Mandeville and Lui (1997) study where teachers with a strong mathematical knowledge were able to provide "greater depth in dealing with concepts, [and were] better equipped to lead students to use their knowledge and use more higher-order content than teachers less knowledgeable about the content" (p. 406). These studies have been powerful in demonstrating the impact of discipline knowledge on teaching. In all cases, it was found that there was a positive effect of strong discipline knowledge on quality teaching with the converse also holding true. No studies could be found that contradicted this research.

While discipline knowledge impacts on student learning, other studies have focused on teachers' beliefs about learning and the nature of mathematics. In such studies the link between the beliefs that teachers hold about mathematics and the ways in which teaching was subsequently organised have been documented. It has been noted that there is a strong relationship between discipline knowledge and how the teacher implements the lesson (Foss & Kleinsasser, 1996). For example, if a teacher were to hold the view of mathematics as procedural knowledge, then a pedagogical approach would be adopted that embraced this view of knowledge. The converse can also be noted in the work of Irwin and Brit (1999) who drew on the work of Ernest (1991) and argued that teachers who see mathematics as "fallible, changing and like any body of knowledge, the product of human invention" (p. 95) are more likely to adopt problem solving approaches in their teaching. While Irwin and Brit were positive about linking beliefs and practice, the evidence they cite from their cohort of teachers lends support to the notion that preservice (and practising) teachers are more likely to see teaching mathematics as rule-governed and teach in such a manner. These studies illustrate the impact of discipline knowledge on teaching (and planning) in mathematics. As such, they demonstrate the importance of discipline knowledge in preservice programs, as this knowledge impacts significantly on subsequent teaching and student learning, once graduates enter their classrooms as teachers. The recognition that the responsibility for preservice teachers' knowledge base was that of teacher educators has been known for a number of decades (Tirosh & Graeber, 1989). Concerned teacher educators have argued that it is the responsibility of the field to ensure that graduates exit programs equipped with appropriate knowledge, skills, and dispositions, and that they do not have the misconceptions with which they entered preservice teacher education so that they do not reproduce these misconceptions when they assume responsibility for their own classrooms.

In summary, it has been argued that through their exposure to school practices, preservice teachers enter courses with conceptions of how to teach mathematics, and that these views were often reinforced through the practicum experiences. Bibby (2002), Bischoff, Hatch and Watford (1999), and Gellert (1999) identified that the participants in their studies regarded mathematics as a discipline that was learnt through rote-and-drill procedures, that there were right and wrong answers, and that the teacher always had the answer. Within the Australian context, this observation was also documented by Schuck (1996b). To counter these issues, Ball (1990) argued strongly that time needs to be allowed for preservice teachers to unlearn what they have learnt as school students, and to learn what needs to be done as a teacher – in particular, the unlearning of rote-and-drill procedures.

Attempts to challenge widely-held views about the procedural teaching of mathematics are often thwarted by preservice teachers' experiences on practicum, where their views are reinforced by teachers in schools (Frykholm, 1998). Studies have shown that teachers reproduce the practices of the past and encourage practicum students to do likewise (e.g., Wilcox, Schram, Lappan, & Lanier, 1991). When preservice students valued the practicum component of their courses more highly than the theoretical components, there was considerable potential for the theoretical learning to be devalued or rejected when the practicum experience was in conflict. Teacher education courses thus need to consider the impact of the practicum on what is learnt and how this complements the students' theoretical learnings.

In the current education context where there is recognition of the need for strong mathematical content knowledge, courses need to be developed that foster the development of mathematical knowledge in preservice primary teachers, and that create strong links with pedagogical content knowledge. In the remainder of this article, students' responses to one question on an examination, and subsequent interviews with the students, are considered. It is suggested that some students are able to make the transition to thinking mathematically, while others remain entrenched in their strongly-held views of mathematics.

Context of the Study

A third year course was offered in which students were expected to develop a strong understanding of mathematics discipline knowledge. The course was offered in ways that sought to challenge the beliefs students held about mathematics, and was aligned with the approaches advocated by Civil (1993) and Szydlik, Szydlik, and Benson (2003). The approach adopted within the course supported the practice whereby students are encouraged to unlearn their beliefs that mathematics is about algorithms, authoritarianism, rote learning, and application of formulae. Study groups were encouraged so that students could learn from each other and offer support to peers (Zevenbergen, 2000). The course focused on developing mathematical meaning rather than procedures (Wilcox, Schram, Lappan, & Lanier, 1991). Students were not given formulae, and were encouraged to make sense of the questions being posed. Alternative methods were encouraged, and students were encouraged to provide examples and counter-examples of how they solved problems. Students were also encouraged to model to their peers (rather than the teacher) how they solved tasks. Problems posed were ones that had practical or realistic orientations. Throughout the course, students were asked: "What does this question mean?" and "What do I need to do?" They were also encouraged to "estimate" before doing any calculations. As part of the course, students needed to pass a quiz in which they answered questions which covered esoteric tasks and tasks using everyday contexts. Responses to one of these questions is examined in this article. After the students completed the work and the work had been corrected, several students were asked to participate in interviews about the quiz, based on their responses to one particular question. The interviews were aimed at accessing and clarifying how the students had worked through the question, that is, the interview aimed to explore their mathematical thinking, how they felt about the quiz in general, and how confident they were with their responses to the particular question.

Students were posed the following question: *What amount of concrete would be needed to fill a barbeque area 8.5m long, 3.2m wide and 30cm deep? Express your answer in the way you would if you were to phone the concrete company to place the order*. The figures had been selected so that it enabled easy estimation when the figures were rounded to workable numbers (8 x 3 x $\frac{1}{3}$).

In the supporting tutorial activities provided in the mathematics education course, students constructed a benchmark of a cubic metre. In this activity, a model of the cubic metre was constructed and students estimated how many people will fit into it. They then filled the cubic metre with a range of objects – including people (for informal units) and 1000s cube of the base ten blocks. In the same module, students also undertook work with conversions so that they played with models of litres (to see how many cubic centimetres were in a litre), the weight of a litre of water, and so on. Throughout these activities, the discussions in the tutorials drew out the students' informal understandings of conversions and, in so doing, provided mental models of the conversions. In this case, students were encouraged to create a visual model of the cubic metre and the process of placing the 1000s cubes along the dimensions of the cubic metre. This gave them a greater sense of how many units (litres or cm^3) were in the cubic metre. As a result they constructed an image that did not rely on rote learned conversions. The focus was on developing understandings of models so that students moved away from reliance on rote formulae and tables for conversions. Throughout the module there was an emphasis on questioning so that students came to construct their own understandings of conversions and units of measure. Problems were posed (such as how many cubic centimetres in a base ten 1000s block?) and students used equipment to explore, while the course team posed questions to the small groups that formed in the tutorials. This was undertaken to facilitate debate within and between these groups. The approach is one commonly used in mathematics education courses and one that Dole and Beswick (2002) claim supports change in students' beliefs about the teaching of mathematics.

Variation in Responses

Depending on the year of offer, response rates to the volume question have varied but there was a consistency in the types of responses offered. In most years, up to 50% of the students offered the correct response of 8.16 cubic metres (or $8.16m^3$) or students rounded numbers so that responses such as 8.2 cubic metres, $8.2m^3$ or even $8m^3$ were accepted. Almost full credit was awarded to responses that were expressed in cubic centimetres. The flexibility in "acceptable" responses was considered in concert with the ways that the students demonstrated their understandings in the presentation of their work. Many students' working sections reflected very clear understanding, and many offered explanations for their responses such as "The answer is $8.2m^3$ but the concrete company will only sell in full or half metres so I would order $8m^3$ and then adjust when laying the concrete."

To give some sense to the breadth of responses offered, Table 1 shows the types and numbers of student responses. Those responses that were deemed to be appropriate are in bold.

Table 1
Responses offered by students to the Volume question (categories and frequencies)

Answer in cubic metres	No of responses	Answer in cubic cms	No of responses	Answer in litres	No of responses
***8.16m^3**	**32**	8 160cm^3	1	8.16	2
Incorrect single digit	6	70 000cm^3	2	11L	1
81.6m^3	3	***8 160 000 cm^3**	5	81.6L	2
Incorrect two-digit numbers	4			272L	1
816m^3	9			***8 160L**	3
Incorrect three-digit numbers	1			*Incorrect four-digit numbers*	2
8 160m^3	1			81 600L	1
Incorrect four-digit numbers	3			*Incorrect five-digit numbers*	1
81 600m^3	2				

* indicates those responses that were identified as acceptable
NB. 5 students also provided answers in cm^2; 1 student provided an answer in kilograms and another in dm^3.

Table 1 shows the diversity in the responses offered by the students. These data suggest that only 32 out of 98 students were able to calculate a result and transfer the result into an appropriately communicable form (i.e. approximately 8 cubic metres). However, other answers (in litres and cubic centimetres were seen as acceptable in that they were mathematically correct, but contextually inappropriate in terms of communication). As indicated, a range of responses was seen as appropriate depending on how students interpreted the demands of the task. Others converted the measurements to centimetres but failed to convert 8 million cubic centimetres to cubic metres. Since the question required them to undertake this final part in order to communicate the result to a cement company it was seen as incomplete. Similarly, others converted their responses to 8000 litres. These responses have been bolded in the table and, while technically correct, they are not usually the style in which to express a purchase of concrete. This suggests that the students have the esoteric knowledge of school mathematics but have not transferred it to the practical context, and that there has been a prioritising of school mathematical knowledge over practical mathematical knowledge (or numeracy).

The responses to this question over time have been relatively consistent in terms of types of responses and percentages of responses in different categories. In most years, approximately 40% to 50% of students were able to offer an appropriate answer. It was alarming that so many students offer responses that have errors either in calculations or interpretations. While some error can be attributed to examination pressure or anxiety, what was alarming was that the arithmetic demands of the task were not complex and were ones that an upper primary school teacher would be expected to teach. Despite the approach advocated in the course, so many students consistently responded inappropriately to such questions. The error patterns noted were of particular concern. They suggest that students have not developed a strong sense of number, measurement, or space. When responses of 8000 cubic metres are considered, there is a sense that some preservice teachers may be working at levels lower than would be needed for effective teaching.

In order to make sense of the responses, in 2003 30 students were interviewed after the quiz had been corrected to gain insights into their thinking and to gain an appreciation of the diversity in responses. A range of responses was identified and students were selected on the basis of the responses they offered. Students were given their responses and asked to talk through their thinking as they had gone about working through the question. They were also asked how confident they were with the answers they had given. The following sections of the article reveal focused types of responses offered by students and excerpts from the interview data to illustrate their thinking.

Strategies Used to Answer the Volume Problem

Throughout the course, students were encouraged to develop conceptual knowledge so that they would have deeper understandings of mathematical ideas and processes. The consistent patterns in responses offered to the volume question over a 4-year period suggested that there were common patterns in students' ways of thinking and working mathematically. While the coursework sought to empower students to *understand* rather than *do* mathematics, the data suggest that such an approach may not have been as successful as envisaged. The interview data suggested that there was heavy reliance on procedural knowledge, that is, algorithmic methods in which lock-step strategies were used to solve the task. These strategies suggest that the students relied on particular ways of knowing in mathematics.

From the interview data, several strategies were articulated by the preservice teachers. Some indicated that they had developed insights into working through problems, while others still relied on lock-step approaches and had incorporated their coursework into their existing rote schema of how to work mathematically.

Using Estimation

One of the frequently discussed strategies in contemporary classrooms has been the move away from focusing on the use of rote, rule-governed methods to those in which students are encouraged to make sense of the problems and to estimate. Seeing estimation as a legitimate process in mathematics often requires a significant shift in preservice teachers' thinking, as it challenges their conceptions of mathematics as being an exact science with only one correct answer. Throughout their coursework, students are encouraged to use estimation in their work and in their experiences in schools. For some students, this strategy has been most useful, and as the responses would suggest, many of the students used the strategy in working through the problem.

> Amanda[1]: When I got to this question, I remembered how we always talked about estimation so I did that first. I thought, well, it is 8 and 3 so that is 24. I rounded 30cms to one third of a metre and then found $\frac{1}{3}$ of 24 which is 8. So I had a rough idea of what the answer would be. Then I worked it out properly so I then multiplied the 27.2 by 3 which gave me 81.6. When I did this at school, I would have usually just jumped my decimal point along. I was never really sure which way it went, but now I know that the answer is about 8, so I knew it was 8.16 and not 81.6. I felt pretty confident that I had the right answer.

Amanda (and many others) reported the estimation strategy as one which they were more confident that they had the right place value in their responses. As Amanda noted, she was never sure when she "jumped my decimal point along" as to whether the decimal point was in the correct place or not. The use of estimation had made her more confident about her response. Other students used both estimation and moving the decimal point along the numbers to ensure that they had the correct response. Michael reinforced this:

> Michael: I'm pretty confident in moving the decimal point along when I do multiplications with decimal numbers. So, I used both methods to check that I was right. First I estimated and knew I had to have an answer about 8 so when I multiplied the numbers together, and then moved the decimal point along the number of points I had, I was very confident that the answer was going to be 8 something and not 0.8 or 80. It was comforting to see them line up.

These types of comments represented students' strategies to estimate the reasonableness of their answers. They were indicative of the ways in which the strategies can empower students to use mathematics in ways that are meaningful rather than being rote procedures. Ideally, these were the responses that aligned with the goals of the program in which it was planned that students would use mathematical thinking, problem solving, and estimation to work through their problems.

---

1 Pseudonyms have been used in this article.

Drawing a Diagram and Estimation

While the comments in the previous section suggest that the strategies being espoused can be empowering for students, a similar ethos towards strategies can be adopted that reinforces previous algorithmic approaches. With most problem solving approaches, students are encouraged to use a range of strategies (Clement & Konold, 1989; Hiebert et al., 1996). So throughout the course students were encouraged to use a diverse range of strategies, including drawing diagrams, to make sense of what the questions or tasks were asking of them. As with any approach, there is a danger that students use it without engaging at a deep level, seeing the strategies as tools by which to obtain answers. While some students began to employ alternative methods for working through tasks, there was some evidence to suggest that the moves were not always for appropriate mathematical reasons. In the responses offered by Justin and Marguerite (see below), it appeared that they adopted the diagram strategy without seeing its potential – just as something that was expected of them in this course. Marguerite made this quite explicit.

Justin: The first thing I did was to draw it out and then to label the different dimensions. I then calculated it out. So, I did the surface first and got 27.2. I then multiplied by 30 cms which is 0.3 m and that gave me 8.16. I rounded it up to 8.2 because you wouldn't order 8.16m^3. I was pretty confident that was right as it made sense to me because 82 m^3 was too much – that would have been 3m deep. I wanted to make sure though so then I did an estimation – 8 x 3 x $\frac{1}{3}$. I could see once I wrote it down like that, that the 3 and the $\frac{1}{3}$ was 1, so it was 8 x 1 which was just 8. I then was really happy with my 8.2 m^3. I wondered whether or not I should write 8.16 or 8 or 8.2 because I don't think that a cement company delivers 8.16 m^3 of concrete. I nearly wrote that but then figured that it was a maths quiz so you probably should be giving the 8.16 m^3 answer.

Marguerite: Well I drew out the dimensions but I did not really need to do it. It might help some people to work out what they need to do but I know that I just need to multiply the numbers all together. I thought, though, I should draw the diagram because you had talked about it helping us make sense of things.

Much like the rote-and-drill procedures that have been challenged by problem solving methods, the comments offered by Justin and Marguerite suggest that students may tend to see them merely as means to an end, without engaging with the intended purposes. Just as they have learned that step-by-step algorithms were effective means for getting correct answers in mathematics, they have come to see other strategies in the same light. In this case, the students saw the drawing of the shape as part of the repertoire of the responses, as opposed to a tool for enabling thought. The conditions of assessment can create situations where students put down anything to gain marks – a 'just in case' strategy.

Using Liquid Measures

Conversion between measures for liquid (ml) and solid (cm^3) was undertaken by students in tutorials so that they gained a sense of the metric system as being integrated. For some students, they recalled the tutorial work related to the relationship between mls and cm^3 and applied this learning to their responses.

> Paul: What I did was to work out the volume. I firstly converted everything to cms and then just multiplied the numbers together and got that answer [points to the response 816 000]. Because it was all in centimetres, I know that it was cm^3 for the right answer so that gave me 816 000cm^3 [again points to the response]. When we did the tutorials, I remember the activities that we did when we calculated volumes of the different boxes and then filled them with water and then found out how many millilitres there were. I was really amazed as I did not know that and it really stuck in my head. So, I knew that concrete was a liquid so figured that what we had to do was to convert the cm^3 to millilitres. I knew from the work in the tutes that what you have as centimetres cubed is the same in millilitres so I just changed my answer from cm^3 to millilitres. I was pretty confident it was right because what we'd done in tutes really stuck in my head.

Paul drew on what had been undertaken in tutorials to respond in the way described. One of the long standing difficulties with examinations is that students read more (or less) into the questions or feel that there is some hidden trick to a question. Students who responded using litres may have seen that there was more to the question than met the eye and hence sought out more for the response. In this category of question, students drew on their in-class experiences to provide further information that might help them gain extra marks.

> Sam: This was a good question because it really made me think about what I had to do. I remember you emphasising estimation and drawing diagrams so that we really understood what the question was asking. So, I did the diagram but don't think I really needed it. But, hey, you might give us extra marks if we did it, right? Anyway, I then estimated so I could see it was really about 10 x 3 x $\frac{1}{3}$ so I thought it has to be about an answer of 10. I then changed everything to metres because I don't feel confident when I have to calculate with big numbers. So, I multiplied 8 by 3, got 24. Then I multiplied that by 0.3 and got 7.2. I know that I had to move the decimal place. My estimate was 10, so that meant that the answer had to be about 10, so that meant it was 8.16. So, I had 8.16 cubic metres. I thought that there had to be more to the question because you asked us to represent the answer in a form whereby you would

> order the concrete so I figured I had to do something else to the answer. I remember that we did that activity with seeing how many litres there were in a cubic metre and thought we must need to change the cubic metres to litres so then had to remember what the conversion was. I remembered that we got 10 of the 1000s blocks (base ten blocks) and put them along the sides and built them up so I knew that there were 10 x 10 x 10 and that is 1000 so I remembered that there 1000 litres in a cubic metre so changed it to 8 160 litres of concrete. I felt confident that the answer was right but not sure if it was what you meant. I was not sure how you buy concrete but knew it would not be in millilitres. I think it is right.

What is evident in Sam's response is that he used an estimation strategy (10 x 3 x $\frac{1}{3}$) which was then modified so that the estimation produced a more accurate response of 7.2. Sam then recalled the tutorial activity in which the students used the 1000s cube from the Base 10 blocks to calculate the number of blocks needed for a cubic metre (as opposed to trying to recall the conversion factor). The in-tutorial activity also had the students converting the 1000s cube to volume which Sam recalled as being the equivalent of 1 litre. These activities are brought into the considerations for developing a response.

Sam's comment indicated how students respond by drawing on experiences in courses without analysing the question and lacking the confidence to answer what they think will be an appropriate response. Sam's response was enlightening to teacher educators as it highlights the functionalist thinking of students in their learning when the assessment discourse overpowers the numeracy discourse.

Finding Kilograms

Three students offered responses but had no logical explanation for them. This was evident in responses such as that provided by a student who gave an answer in kilograms. Throughout the interview, it was clear that the student had little of the conceptual knowledge needed for the task and relied on piecemeal approaches to solving the task.

> Marcia: I'm not sure how I got this answer and I don't think it is right but I don't know how to get it to the right answer. I did the multiplying out and that gave me how many grams I had. I then divided my answer by 1000 because there are that many grams in a kilogram and so I got that answer [points to answer]. I then know how many kilograms of concrete I need but I don't think it is the right answer but I don't know how to make it right.

Marcia's comment highlighted the ad hoc process that some students adopt. It was not possible to deduce from the interview whether the examination process was causing anxiety and hindering Marcia's approach or whether it was her understanding of the task and the numeracy demands that were the problem.

Algorithmic Approaches

One of the key strategies used in mathematics is to teach algorithmic methods for problems. This approach dominates most school mathematics practices so that students coming into teacher education have been indoctrinated into this way of working. When they experienced success, it was through the adoption of steps and procedures. Such approaches were very obvious in many of the incorrect responses for which students had little idea of how to solve the task mathematically. It was also evident in some of the approaches that successful students used. However, in the case of the successful students, the conversion between units of measure was accurate whereas in other cases, such as the one below, conversion was a difficulty. In Jessica's response, a lock-step approach was evident. Unfortunately, this approach was commonly used by students, despite the ethos adopted in the course.

> Jessica: I remembered that we always have to use the same units when doing volume or any measurement so changed them all to centimetres. I then multiplied them all together because that's what you do when you do volume. So that was 8 500 by 3 200 by 30. It was a big number and I didn't really like it. I wasn't really sure about it because it was so big. I checked all my zeroes and I had the right number. I then had to convert back to metres. I know that there are 100 cms in a metre so moved the decimal point in by two places. That gave me an answer of 8 160m^3. I think that it was pretty right because I remembered all the steps that we learned when I was at school. I had a really great teacher who helped me with all of this because I could never understand it. She taught us how to do things like this as a series of steps. I remember the steps so knew that it was right. I checked all the workings out and had not made any mistakes so I was pretty confident I had it right.

The disempowerment of this process was clearly evident in Jessica's comments. Not only did she calculate an incorrect response, she had no idea that it was incorrect. Relying on this lock-step approach, she had followed basic rules she had learned at school and had applied them (incorrectly) to this problem. Within the comfort of this approach, she was blissfully unaware of the mistake she had made but also had little idea of the inappropriateness of her answer. This type of response was evident among many of the students and is worthy of further examination since it points to key considerations for teacher education. Many of the challenges the students made to the teaching staff were often on the grounds that they were sure that they had correct results (as did Jessica) and had considerable difficulty in understanding where they had made errors. This misunderstanding is more fully discussed in the next section of the paper.

The follow up interviews were useful in highlighting the methods students used to solve problems; they also revealed the impact of coursework on their learning. In some cases, the ideas encountered in the course were enacted by students who came to see that there were other, more effective ways to work mathematically and to understand what the tasks required of them. However, in other cases, the coursework had little impact on student thinking. For example, strategies explored in the coursework (such as using diagrams) were incorporated as just another step that could be taken. Students did not see these as strategies that could be more or less useful depending on the task but rather as just another thing that they had to incorporate into their workings. In other cases, ideas encountered in the coursework were rejected outright. This rejection was evident in a number of cases, one of which will be discussed in more detail below. These examples provide insights into students' thinking and the forces that work against the implementation of contemporary ideas in teacher education courses. While only six of the 30 students interviewed offered similar comments to Jessica, they do highlight the difficulties for contemporary teaching in teacher education programs.

The Case of Jessica

The case of Jessica warrants further scrutiny since it points to a number of common issues. Two key concerns will be discussed: the algorithmic approach used by students, and their apparent lack of number sense. Each will be discussed in detail. As there was a considerable number of responses of the algorithmic format, interviews were conducted with six students who had offered this type of response. Furthermore, throughout the interview, there was evidence that other forces had impacted on Jessica's thinking – her experiences as a school student, and as a preservice teacher in the practicum situation. These forces were powerful in shaping her views on how school mathematics is taught and thus explaining her resistance to the on-campus coursework.

Based on her responses, what was of concern was that Jessica had not shifted from her own practices as a school student, and that there was a resistance to the ideas encountered in her preservice coursework. She remained with the rigid step-by-step approach she had been shown at school (and had had success with). Furthermore, she brought misconceptions to the situation (that because there were 100 cm in a metre, therefore there were 100 cm^3 in a cubic metre) and used such misconceptions to frame her solution. Her reliance on the lock-step process hindered her thinking about the meaning of the question and her response. Her final response of 8 000 cubic metres suggested that she had little sense of what this response meant mathematically. The interview demonstrated the resilience of school-learnt methods and the impact these have on the potential (or lack of) to understand the meaning of responses offered.

Initially Jessica was asked how she had solved the task. In her response, her reliance on strict compliance with the lock-step approach was evident. Furthermore, her subtle resistance to other approaches – such as those encountered in her teacher education course – became apparent:

R[2]: Can you explain why you use this method?

S: I was not very good when I was at school with maths. I had this teacher who helped me pass and the way he did it was to make me go step-by-step. That helped me a lot. I know it works so why would I change?

R: What about what you have been learning at uni. Do you think that is useful?

S: Well no. I can get the questions right. I think estimation is a waste of time. I mean why would you estimate and then work out the answer. That seems like it is making more work than you need to. I like to know the exact answer so I don't think estimation is good. You don't get the right answer anyway so why not just work it out.

The success Jessica had with school-learnt procedures had become embodied in how she construed mathematics. The success of lock-step thinking had conditioned her to see that this was the way that mathematics could be successfully undertaken. Making sense or meaning of the question or the answer was not part of her thinking about mathematics and indeed, as she indicated, only created more work.

When questioned further as to the implications this had for her own teaching practice, the work of Frykholm (1998) (cited earlier) was reinforced. Frykholm indicated that the practicum often hindered students' development of new approaches by reinforcing practices from the past. Jessica was adamant that what happened in university did not resonate with her practicum experiences. This enabled her to dismiss what she had experienced in her on-campus learning, and she was thus resistant to change.

R: Have you tried this [algorithmic teaching] with your students when on prac?

S: Yes, and it works. The children get the right answer – provided they know their times tables [multiplication facts].

R: How do your teachers rate you when on prac?

S: They see me as being very good with teaching maths. I have got good reports.

R: So how do your teachers teach maths?

S: Every class I have been in, I have seen them teach this way. In fact, I am sorry to say but I have not seen a teacher yet who gets the children to estimate or that sort of stuff. They do it the same way I was taught.

R: So how useful has what you have done at Uni been?

S: Well, for me it has not been much good. I have not learnt anything I did not already know.

---

2 R is for 'researcher'; S is for 'student'.

This section of Jessica's transcript reinforces the claims made by Frykholm (1998) that the practicum experience can override what was being taught in the university context, and that the experiences in the field can have greater sway and power than on-campus learning. As appears in this case, the student had successfully rejected her on-campus learning since it did not conform with her own learning experiences nor with her practicum experiences.

When Jessica realised that her response was incorrect, she was at a loss to understand how the procedure had failed her. She went over her work to check the multiplications and workings but could not identify her mistake. When probed about the relationship between centimetres and cubic metres, she insisted that the conversion was a factor of one hundred. The only way to convince her was to work with the Base 10 blocks where she could visualize the unit block, the tens block, the 100s block and then the 1000s block. When asked how many cubic centimetres in the cubic metre, she was unable to work this out. After some time working on modelling with the blocks she gained a sense of "how many" blocks were needed for a cubic metre. Counter to the work of Dole and Beswick (2001), in Jessica's case (and some of the other students), practical, hands-on activities in the classes had been rejected. There had been no shift in thinking and understanding, based on practical experiences, that challenged preconceived notions (such as conversions).

Further questioning with Jessica focused on the reasonableness of her response to draw out her understandings of what the answer meant. Up to this point in the interview, Jessica had been convinced that her response was correct and had no sense of what her response meant nor had she any inclination to make sense of that response. She appeared to cling to her belief that if the calculations were correct, then the answer was correct.

R: You have an answer here of 8 200 cubic metres. Do you know what they would look like?

S: No.

R: Well you have an idea now of a cubic metre. Here you are saying that there are 8000 of them. How big do you think that is?

S: A lot.

R: Let's look at the question. Can you guess at how big this area is – 8m x 3m?

S: I guess it is a bit smaller than this room.

R: OK and 30cms is about the length of a ruler. So if you put the concrete on here, say to make a stage, do you think you would need 8000 cubic metres to make that platform?

S: I guess so

R: Why?

S: Well that's how much I calculated [but not convincingly]. I have checked it out and the workings are all right … is it because of the mistake I made with the centimetres?

At this point, Jessica began to show some hesitancy in her responses. It appeared that she was retaining her original workings since the calculations all appeared to be correct, but had yet to make the link between her misconception and the impact it had on the overall answer. Jessica's response was one that was common among the students who made this error, in that they had little conception of what their answer actually meant. They had no tangible sense of the magnitude of 8000 cubic metres (or other large quantities). Further probing about the concepts of metres and kilometres forced Jessica to visualize 8000 metres as being equivalent to 8 kms and allowed her to create a sense of the magnitude she was proposing in her response.

The case of Jessica was not uncommon among the cohort of students. The figures cited at the start of this article give some indication of students' responses to the question. While some recognition must be given to the "shame" (Bibby, 2002) that students may feel as they enter testing situations, and the power of such emotions on their performance, it is important to recognise that other factors – such as those identified through the interview with Jessica – were at play when students work with mathematical ideas and processes. A disconcerting number of students, despite their on-campus experiences, rely on old habits and views of mathematics. There was considerable encouragement from the responses offered by students that they were able to embrace their on-campus learning; this should not be ignored. However, the concern was how to move students such as Jessica forward in their pedagogical and content knowledge so that they will not reproduce the practices that they had embraced.

Implications for Preservice Teacher Education

The espoused intentions of the course were to encourage students to think mathematically and to provoke preservice teachers to move away from their school-learnt methods to understand mathematics rather than to do mathematics. The success to which this had been achieved was mixed. As documented elsewhere, study groups have been powerful tools for encouraging change (Zevenbergen, 2000), but whether such approaches transfer to learning and outcomes has not been documented. In using one of the assessments of the course, it was found that preservice teachers may not develop deep understandings of mathematics in their teacher education program. Some of the responses documented in this article may be due to the pressure of an examination context. Similarly, the outcomes documented here may be constrained by the tight timeframes in which radical changes were expected, when the ways of working have been developed by the students over many years of school mathematics. Such change may be too optimistic within a short period.

The data presented in this article highlight a number of key issues. First, preservice teachers' understandings of volume may be problematic. This reinforces findings from other studies such as those on probability in which it was found that preservice teachers' understandings were similar to primary school students' understandings (Burgess, 2000). The diversity of responses offered by preservice teachers in this study could be comparable to what would be expected from students in an upper primary classroom. These data raise concerns about primary preservice teachers' discipline knowledge in this area of mathematics. Second, the ways of working mathematically were documented through the interviews. While the course had an emphasis on working mathematically as opposed to algorithmic methods, the data presented here suggest that some preservice teachers have engaged with this change, but others were quite resistant to it. The case of Jessica highlighted that she did not see the value in such change. It raises concerns as to how to move students like Jessica into new ways of thinking in order to evoke change at the classroom level. It would be difficult to imagine a teacher like Jessica engaging with reforms once she graduates. The data also suggest that some fundamental thinking was not present in some students' responses – namely there was a lack of number sense, measurement sense, and spatial sense. These attributes or qualities would be most useful in the classroom. In the responses offered by Jessica and others, it would appear that their capacity to identify errors in their teaching or in the responses offered by their students will be hindered. This is a critical disposition that needs to be developed among preservice teachers in order for them to be competent in the mathematics classroom. Despite the multifarious methods used in the preservice education course to develop these dispositions, there would appear to be a worrying number of students who have not achieved them. In some cases, as for Jessica and some others, this may have been due to their rejection of the ideas encountered in coursework (and schools). This is highly problematic for teacher education.

In considering the students' responses reported in this article, a final point has to be made. Their responses highlight the power of the practicum. As documented here, Jessica and others were able to reject their on-campus work due to their field work experiences in schools. As has been documented by the Queensland School Longitudinal Reform Study (Education Queensland, 2001), the teaching of mathematics in a significant number of schools is poor, yet preservice teacher education students use this as a justification for rejection of on-campus learning. As Frykholm's (1998) and this study have shown, preservice teachers' experiences in schools can override their on-campus experiences enabling them to reject the transformative approaches being advocated in preservice courses. In that it engenders a reproductive model of teaching, this is highly problematic for teacher education and for those schools where innovation and best practice are embedded. Ideally, it would be useful to expose students to schools and classrooms that demonstrate the values embedded within teacher education courses if such courses are to effectively change teaching practice.

References

Ball, D. L. (1990). The mathematical knowledge that prospective teachers bring to teacher education. *Elementary School Journal, 90*, 449–466.

Bibby, T. (2002). Shame: An emotional response to doing mathematics as an adult and teacher. *British Educational Research Journal, 28*(5), 705–722.

Bischoff, P. J., Hatch, D. D., & Watford, L. J. (1999). The state of readiness of initial level preservice middle grades science and mathematics teachers and its implications on teacher education programs. *School Science and Mathematics, 99*(7), 394–399.

Board of Teacher Registration. (2002). *Professional standards for graduates and guidelines for teacher education programs*. Brisbane: Board of Teacher Registration.

Brown, T., McNamara, O., Hanley, U., & Jones, L. (1999). Primary student teachers' understanding of mathematics and its teaching. *British Educational Research Journal, 25*(3), 299–325.

Burgess, T. (2000). Are teachers' probabilistic concepts more sophisticated than those of their students? In J. Bana & A. Chapman (Eds.), *Mathematics Education Beyond 2000* (Proceedings of the 23rd annual conference of the Mathematics Education Research Group of Australasia, Vol. 1, pp. 126–133). Perth: MERGA.

Chick, H., & Hunt, D. (2001). Preservice primary teachers' judgements about the probability of everyday events. In J. Bobis, B. Perry, & M. Mitchelmore (Eds.), *Numeracy and Beyond* (Proceedings of the 24th annual conference of the Mathematics Education Research Group of Australasia, Vol. 1, pp. 147–154). Sydney: MERGA.

Civil, M. (1993). Prospective elementary teachers' thinking about teaching mathematics. *Journal of Mathematical Behaviour, 12*, 79–109.

Clement, J., & Konold, C. (1989). Fostering problem-solving skills in mathematics. *For the Learning of Mathematics, 9*(3), 26–30.

Dole, S., & Beswick, K. (2002). Maths anxiety self-assessment as a quality assurance measure. In B. Barton, K. C. Irwin, M. Pfannkuch, & M. O. J. Thomas (Eds.), *Mathematics Education in the South Pacific* (Proceedings of the 25th annual conference of the Mathematics Education Research Group of Australasia, Vol. 1, pp. 236–243). Sydney: MERGA.

Education Queensland. (2001). *The Queensland school longitudinal reform study*. Brisbane: GoPrint.

Ernest, P. (1991). *The philosophy of mathematics education: Studies in mathematics education*. London: Falmer Press.

Foss, D. H., & Kleinsasser, R. C. (1996). Preservice elementary teachers' views of pedagogical and mathematical content knowledge. *Teaching and Teacher Education, 12*(4), 429–442.

Frykholm, J. A. (1998). Beyond supervision: Learning to teach mathematics in a community. *Teaching and Teacher Education, 14*(3), 305–322.

Gellert, U. (1999). Prospective elementary teachers' comprehension of mathematics instruction. *Educational Studies in Mathematics, 37*, 23–43.

Goulding, M., Rowland, T., & Barber, P. (2002). Does it matter? Primary teacher trainees' subject knowledge in mathematics. *British Educational Research Journal, 28*(5), 689–704.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K., Human, P., & Murray, H. (1996). Problem solving as a basis for reform in curriculum and instruction: The case of mathematics. *Educational Researcher, 25*(4), 12–21.

Irwin, K., & Brit, M. (1999). Teachers' knowledge of mathematics and reflective professional development. In B. Jaworski, T. Wood, & S. Dawson (Eds.), *Mathematics teacher education: Critical international perspectives* (pp. 91–101). London: The Falmer Press.

Kagan, D. M. (1992). Implications of research on teacher beliefs. *Psychologist, 27*(1), 65–90.

Kanes, C., & Nisbet, S. (1996). Mathematics-teachers' knowledge bases: Implications for teacher education. *Asia-Pacific Journal of Teacher Education, 24*(2), 159–171.

Mandeville, G. K., & Lui, Q. (1997). The effect of teacher certification and task level on mathematics achievement. *Teaching and Teacher Education, 13*(4), 397–407.

Schuck, S. (1996a). Chains in primary teacher mathematics education courses: An analysis of powerful constraints. *Mathematics Education Research Journal, 8*(2), 119–136.

Schuck, S. (1996b). Reflections on the dilemmas and tensions in mathematics education courses for student teachers. *Asia-Pacific Journal of Teacher Education, 24*(1), 75–82.

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4–14.

Simon, M. A. (1993). Prospective elementary teachers' knowledge of division. *Journal for Research in Mathematics Education, 24*(3), 233–254.

Szydlik, J. E., Szydlik, S. D., & Benson, S. R. (2003). Exploring changes in pre-service elementary teachers' mathematical beliefs. *Journal of Mathematics Teacher Education, 6*(3), 253–279.

Taplin, M. (1992). Difficulties with basic mathematics amongst pre-service teachers. *New Horizons in Education, 6*(5) 3–19.

Tirosh, D., & Graeber, A. O. (1989). Preservice elementary teachers' explicit beliefs about multiplication and division. *Educational Studies in Mathematics, 20*(1), 79–96.

Wilcox, S. K., Schram, P., Lappan, G., & Lanier, P. (1991). The role of a learning community in changing preservice teachers' knowledge and beliefs about mathematics education . *For the Learning of Mathematics, 11*(3), 31–39.

Zevenbergen, R. (2000). Using study groups to support mathematics learning in preservice teacher education. In J. Bana & A. Chapman (Eds.), *Mathematics Education Beyond 2000* (Proceedings of the 23rd annual conference of the Mathematics Education Research Group of Australasia, Vol. 2, pp. 680–687). Perth: MERGA.

Author

Robyn Zevengergen, Murray Education Unit, Charles Sturt University, PO Box 789, Albury, NSW, 2460. Email: <rzevenbergen@csu.edu.au>