Mathematical Modelling in the Early School Years[1]

Lyn D. English and James J. Watters
Queensland University of Technology

In this article we explore young children's development of mathematical knowledge and reasoning processes as they worked two modelling problems (the *Butter Beans Problem* and the *Airplane Problem*). The problems involve authentic situations that need to be interpreted and described in mathematical ways. Both problems include tables of data, together with background information containing specific criteria to be considered in the solution process. Four classes of 3rd-graders (8 years of age) and their teachers participated in the 6-month program, which included preparatory modelling activities along with professional development for the teachers. In discussing our findings we address:

(a) Ways in which the children applied their informal, personal knowledge to the problems;

(b) How the children interpreted the tables of data, including difficulties they experienced;

(c) How the children operated on the data, including aggregating and comparing data, and looking for trends and patterns;

(d) How the children developed important mathematical ideas; and

(e) Ways in which the children represented their mathematical understandings.

> Making modelling, generalization, and justification an explicit focus of instruction can help to make big ideas available to all students at all ages. (Carpenter & Romberg, 2004, p. 5).

We face a world that is shaped by increasingly complex, dynamic, and powerful systems of information, such as sophisticated buying, leasing, and loan plans that appear regularly in the media. Being able to interpret and work with such systems involves important mathematical processes that have been under-emphasized in many mathematics curricula. Processes such as constructing, explaining, justifying, predicting, conjecturing, and representing, as well as quantifying, coordinating, and organising data are becoming all the more important for all citizens. Mathematical modelling, which traditionally has been the domain of the secondary school years, provides rich opportunities for students to develop these important processes.

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A model may be defined as "a system of conceptual frameworks used to construct, interpret, and mathematically describe a situation" (Richardson, 2004, p. viii). By engaging in mathematical modelling students identify the underlying mathematical structure of complex phenomena. Because mathematical models focus on structural characteristics of phenomena (e.g. patterns, interactions, and relationships among elements) rather than surface features (e.g. biological, physical or artistic attributes), they are powerful tools in predicting the behaviour of complex systems (Lesh & Harel, 2003). As such, mathematical modelling is foundational to modern scientific research, such as biotechnology, aeronautical engineering, and informatics (e.g., Gainsburg, 2004).

Many nations are expressing concern over the lack of their students' participation in mathematics and science (e.g., O'Connor, White, Greenwood, & Mousley, 2001; US Department of Education, 2002). However, research has shown that low levels of participation and performance in mathematics are not due primarily to a lack of ability or potential, but rather, to educational practices that deny access to meaningful high-quality learning experiences (e.g., Tate & Rousseau, 2002). Many of these under-achieving students show exceptional abilities to deal with sophisticated mathematical constructs when these understandings are grounded in their personal experiences and are expressed in familiar modes of representation and discourse (Lesh, 1998). It has been shown that a broader range of students emerge as being highly capable, irrespective of their age or classroom mathematics achievement level when they participate in mathematical modelling experiences (Doerr & English, 2003; Lamon, 2003; Lesh & Doerr, 2003). As a consequence, improvements in students' confidence in, and attitudes towards, mathematics and mathematical problem solving become evident.

The primary school is the educational environment where all children should begin a meaningful development of mathematical modelling (Carpenter & Romberg, 2004; Jones, Langrall, Thornton, & Nisbet, 2002; Lehrer & Schauble, 2003; National Council of Teachers of Mathematics (NCTM), 2000). However, as Jones et al. note, even the major periods of reform and enlightenment in primary mathematics do not seem to have given most children access to the deep ideas and key processes that lead to success beyond school.

The study reported here sought to redress this situation by engaging young children and their teachers in a 6-month program, which included preparatory modelling activities culminating in two modelling problems. This paper explores the children's development of mathematical knowledge and reasoning processes as they worked the two modelling problems over several weeks.

Mathematical Modelling for Young Learners

Until recently, mathematical modelling (of the type addressed here) has not been considered within the early school curriculum. Rather, it has been the domain of the secondary year levels (e.g., Stillman, 1998). We argue that the rudiments of mathematical modelling can and should begin in the primary school where young children already have the basic competencies on which modelling can be developed (Carpenter & Romberg, 2004; Diezmann, Watters, & English, 2002; Lehrer & Schauble, 2003; NCTM, 2000; Perry & Dockett, 2002). Indeed, as Carpenter and Romberg documented recently,

> Our research has shown that children can learn to model, generalize, and justify at earlier ages than traditionally believed possible, and that engaging in these practices provides students with early access to scientific and mathematical reasoning. Until recently, however, these practices have not been much in evidence in the school curriculum until high school, if at all. (p. 4).

Mathematical modelling activities differ from the usual problems that young children meet in class. Problem solving in the early years has usually been limited to examples in which children apply a known procedure or follow a clearly defined pathway. The “givens,” the goal, and the “legal” solution steps are usually specified unambiguously—that is, they can be interpreted in one and only one way. This means that the interpretation process for the child has been minimalised or eliminated. The difficulty for the child is basically working out how to get from the given state to the goal state. Although not denying the importance of these existing problem experiences, it is questionable whether they address adequately the mathematical knowledge, processes, representational fluency, and social skills that our children need for the 21st century (Carpenter & Romberg, 2004; English, 2002; Steen, 2001).

In contrast to the typical “word problems” presented to young children, mathematical modelling problems involve authentic situations that need to be interpreted and described in mathematical ways (Lesh & Harel, 2003). The information given, including the goal itself, can be incomplete, ambiguous, or undefined (as often happens in real life). Furthermore, information contained in these modelling problems is often presented in representational form, such as tables of data or visual representations, which must be interpreted by the child.

In recent years there has been a strong emphasis on providing young children with equal access to powerful mathematical ideas (Carpenter & Romberg, 2004; Diezmann & Watters, 2003; English, 2002; Perry & Dockett, 2002). Mathematical modelling problems provide one avenue for meeting this challenge. Key mathematical constructs are embedded within the problem context and are elicited by the children as they work the problem. The generative nature of these problems means that children can access mathematical ideas at varying levels of sophistication. For example, as we indicate later, young children can access informal ideas of rate by considering how time and distance could determine the winner of a paper plane contest.

The importance of argumentation in young children's mathematical development has also been highlighted in recent years (e.g., Perry & Dockett, 2002; Yackel & Cobb, 1996). Although Piaget (e.g., Inhelder & Piaget, 1955/1958) claimed that the ability to argue logically is beyond the realms of young children, recent work has demonstrated otherwise (e.g., Dockett & Perry, 2001). As Perry and Dockett (2002) noted, it is important for us to be aware of and nurture the early genesis of argumentation, especially since it will form the basis of mathematical proof in later years. Mathematical modelling activities provide a solid basis for young children's development of argumentation because they are inherently social experiences (Zawojewski, Lesh, & English, 2003) and foster effective communication, teamwork, and reflection. The modelling activities are specifically designed for small-group work, where children are required to develop sharable products that involve descriptions, explanations, justifications, and mathematical representations. Numerous questions, conjectures, conflicts, resolutions, and revisions normally arise as children develop, assess, and prepare to communicate their products. Because the products are to be shared with and used by others, they must hold up under the scrutiny of the team members.

Description of the Study

Setting and Participants

All four 3rd-grade classes (children approximately 8 years old) and their teachers from a state school situated in a middle-class suburb of Brisbane, participated in the study. The principal and assistant principal provided strong support for the project and attended some of the workshops and debriefing meetings that we conducted with the teachers.

Tasks

In collaboration with the teachers, we developed four preparatory activities, which were followed by two modelling problems.

The preparatory activities. These were designed to develop children's skills in: (a) interpreting mathematical and scientific information presented in text and diagrammatic form; (b) reading simple tables of data; (c) collecting, analysing, and representing data; (d) preparing written reports from data analysis; (e) working collaboratively in group situations; and (f) sharing end products with class peers by means of verbal and written reports. For example, one preparatory activity involving the study of animals required the students to read written text on "The Lifestyle of our Bilby," which included tables of data displaying the size, tail length, and weight of two types of bilbies. The children answered questions about the text and the tables.

The modelling problems. The contexts of the modelling and preparatory activities were chosen to fit in with the teachers' classroom themes, which included a study of food, animals, and flight. The first modelling activity, "Farmer Sprout," comprised a story about the various types of beans a farmer grew, along with data about various conditions for their growth. After responding to questions about the text, the children were presented with the "Butter Beans" problem comprising two parts. The children had to examine two tables of data displaying the weight of butter beans after 6, 8, and 10 weeks of growth under two conditions (sunlight and shade; see Table 1).

Table 1
Data presented for the Beans Problem

Sunlight				**Shade**			
Butter Bean Plants	**Week 6**	**Week 8**	**Week 10**	**Butter Bean Plants**	**Week 6**	**Week 8**	**Week 10**
Row 1	9 kg	12 kg	13 kg	Row 1	5 kg	9 kg	15 kg
Row 2	8 kg	11 kg	14 kg	Row 2	5 kg	8 kg	14 kg
Row 3	9 kg	14 kg	18 kg	Row 3	6 kg	9 kg	12 kg
Row 4	10 kg	11 kg	17 kg	Row 4	6 kg	10 kg	13 kg

Using the data of Table 1, the children had to (a) determine which of the conditions was better for growing butter beans to produce the greatest crop. As a culminating task the children were required to write a group letter to Farmer Sprout in which they outlined their recommendation and explained how they arrived at their decision; and then (b) predict the weight of butter beans produced on week 12 for each type of condition. The children were to explain how they made their prediction so that the farmer could use their method for other similar situations. On completion of the activity, each group reported back to the class. Following the reporting back, the group's peers asked questions and provided constructive feedback.

The second modelling activity, "The Annual Paper Airplane Contest," (see Appendix) presented children with a newspaper article that described an annual airplane contest involving the flight performance of paper airplanes. The children were given information regarding the construction of the planes and the rules for the flight contest. After completing a number of comprehension questions, the children were given the problem information and associated investigation shown in the Appendix.

Procedures

Teacher meetings. We implemented a number of workshops and debriefing sessions for the teachers throughout the year. We conducted two half-day workshops with the teachers in term 1 to introduce them to the activities and to plan the year's program more thoroughly. In these workshops, the teachers worked on the activities they were to implement and identified various approaches to solution. Two more workshops were conducted during the middle and at the end of the year for planning and reflective analysis of the children's and teachers' progress. Several shorter meetings were also conducted throughout the year, including those before and after the teachers had implemented each activity. During these debriefing sessions the teachers discussed with the researchers issues related to student learning, the activities, and implementation strategies.

Task implementation. The preparatory activities were implemented weekly by the teachers towards the end of first term and part of second term. During the remainder of second term and for all of third term, the teachers implemented, on a weekly basis, the two modelling problems. There was approximately one month's lapse between the children's completion of the Butter Beans Problem and the Airplane Problem.

Each modelling activity was explored over 4–5 sessions of 40 minutes duration each and conducted as part of the normal teaching program. After an initial whole class introduction to the modelling activity, the children worked independently in groups of 3 to 4 on the activity. The teachers monitored each group and provided scaffolding where necessary. Such scaffolding included questioning children for explanation and justification, challenging the children, querying an inappropriate action, and providing overall encouragement and motivation. The teachers also focussed on supporting children's writing and the development of group skills. In the final session the students provided a group report to the class and their conclusions were discussed.

Each of the teachers had previously established procedures for group work and for class reporting. For example, each group of children had a group-appointed manager who was responsible for organising materials and keeping the group on task. The importance of sharing ideas as well as explaining answers was also emphasised in class group work.

Data Collection and Analysis

In each of the four classes, we videotaped the teacher's interactions and exchanges with the children in each of the sessions. The teacher was fitted with a radio microphone so that her dialogue with children was the focus of data collection. We also audiotaped each of the teacher meetings.

Given the naturalistic setting and the desire to be as least intrusive as possible, videotaping of children was limited to a focus group in each of two classes. Another focus group in each class was audiotaped. These focus groups were selected after discussions with the teachers and were of mixed achievement levels and gender. One of the main criteria for selecting the focus groups was children's willingness to verbalise while working on tasks.

Other data were collected in response to critical events. That is, the camera would focus on a group who were engaged in resolving some specific aspect of a problem. Other data sources included classroom field notes, children's artefacts (including their written and oral reports), and the children's responses to their peers' feedback in the oral reports.

In our data analysis, we employed ethnomethodological interpretative practices to describe, analyse, and interpret events (Erickson, 1998; Holstein & Gubrium, 1994). This methodological approach allowed us to describe the social world of the classroom by focussing on what the participants said and did, rather than by applying predetermined expectations on the part of researchers. In our analyses, we were especially interested in (a) the nature and development of the mathematical ideas and relationships that the children constructed, represented, and applied; (b) the nature and development of the children's thinking, reasoning, and communication processes; and (c) the development of socio-mathematical interactions taking place within groups (children) and whole-class settings (teacher and children), with particular interest in those interactions involving mathematical argument and justification (Cobb, 2000).

We thus constructed detailed descriptions of the classes to capture the socio-cultural interactions that afford opportunities for children to engage in mathematical learning and reasoning. At a more specific level we used iterative refinement cycles for our videotape analyses of conceptual change in the children (Lesh & Lehrer, 2000). Through repeated and refined analyses of the transcripts and videotapes we were able to identify themes and perspectives that enabled us to make generalisations or assertions about the teachers' and children's behaviours (Cresswell, 1997).

Findings

In reporting our findings, we first address the children's progress on the Butter Beans Problem and then examine their developments on the Airplane Problem. We also consider how children applied their informal, personal knowledge in working the problems.

Butter Beans Problem: Part (a)

Across the four classes, we noted an initial tendency for the children to want to record an answer from the outset, without carefully examining and discussing the problem and its data. The children had to be reminded to think about the given information and share ideas on the problem prior to recording a response. We also observed the children oscillating between analysing the data and discussing at length the conditions required for growing beans. The children drew on their informal knowledge acquired through past experiences in trying to account for the variations in the data. At times, they became bogged down discussing irrelevant issues because their informal knowledge was taking precedence over their task knowledge (i.e., the children's recognition of the specific information presented in the problem). We illustrate this point in later excerpts of the children's work.

We noted at least three approaches that the four classes of children adopted in analysing the data in Table 1. The first approach was to focus solely on the results for week 10 and systematically compare rows 1 to 4 for each condition (i.e., compare 13 kg with 15 kg, 14 kg with 14 kg and so on). A variation of this approach was to make the comparisons for each of weeks 6 and 8 as well. A second approach was to add up the data for week 10 in each condition and compare the results. A third but inappropriate variation of the last approach was to sum all of the weights in each table and compare the results. As one child explained, "Sunlight has 146 to 118 (shade). So plants are in sunlight." A further approach (again, inappropriate) was to add the quantities in each row for each condition and compare the end results (i.e., 9 kg + 12 kg + 13 kg for sunlight and 5 kg + 9 kg + 15 kg for shade, and so on).

As the children explored the data initially, they were looking for trends or patterns that would help them make a decision on the more suitable condition. They were puzzled by the anomalies they found and used their informal knowledge to account for this, as can be seen in the following group discussion (hereafter referred to as Amy's group):

Students collectively: 10 against 6, 11 against 10, and 17 against 13.

Amy: So this is obviously better than that, but working out why is the problem.

Oscar: Yes, because the more sunlight the better the beans are. For some reason…

Amy: In some cases, it's less; but in most cases, it's more the same.

Tim: It would depend on what type of dirt it has been planted in.

Oscar: I've got an idea. Perhaps there were more beans in the sunlight.

Tim: We're forgetting one thing. Rain. How much rain!

Amy's group spent quite some time applying their informal knowledge to identify reasons for the trends in data. In doing so, the children engaged in considerable hypothetical reasoning and problem posing, which eventually led them back to a consideration of the task information:

Amy: We're stuck. I can't work this out.

Oscar: I've got an idea. If we didn't have any rain, the sunlight wouldn't …it wouldn't add up to 17 (kg). And, if we didn't have any sunlight, it wouldn't be up to 17 either. But if we had sunlight and rain…

Tim: Do you want me to jot that down?

Oscar: Don't jot that down because that's wrong. OK, 15 kilograms.

For the remainder of this lesson, Amy's group cycled through applying their informal knowledge to find reasons for why they thought sunlight was better, reviewing the task information by re-examining the sets of data, and attempting to record their findings. In the following excerpt, the group explains to the researchers the dilemma they were facing and the explanations they were considering.

Oscar:	But our problem is, we thought it would be because of the rain. It can't get in as well with the shade cloth on. But then we found these results. And we've got a problem. We can't work out why this has popped up. So we're stuck here.
Amy:	We thought that it was probably that they accidentally put-when they planted the plants, they probably accidentally put slightly bigger plants in this row 1; or the row could have been accidentally longer so it would weigh more. But otherwise, we're sure that sunlight's the best.
Tim:	I think sunlight's best.
Researcher:	Why do you think sunlight is better?
Tim:	Because of the results, like here or here (pointing to week 10 in each condition)
Amy:	Like, look at 17 to 13 or 18 to 12.
Researcher:	Or 14 to 14, or 13 to 15.
Amy:	Yeah, these two are just a bit of a problem, and we've worked out it was probably the row size.

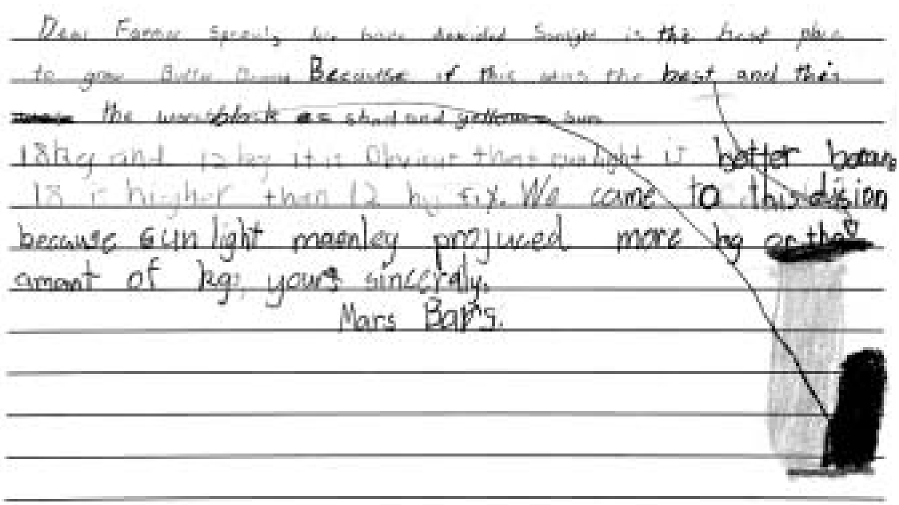

Figure 1. Amy's group's representation of the beans' growth in sunlight and shade.

Amy's group made further progress in the next session where they were more focused on the task information with Amy creating a diagram to show the difference in mass between the two conditions (see Figure 1). Amy directed her peers' attention to row 3, week 10, where the difference was the greatest ("Here's the best and here's the worst"). Amy attempted to show the other group members this difference by drawing a simple bar graph ("picture graph", as she described it) with the first bar coloured yellow to represent the 18kg (sunlight) and the second bar coloured black to represent the 12kg (shade).

In the excerpt below, Amy is explaining the diagram to the group. At the same time, she is trying to get her peers' attention back onto the problem.

Amy: Ok, guys, if you said this was shade and here's the worst and here's the best (pointing to row 3, week 10)-shade's about there (pointing to her diagram). Here's the best and here's the worst…and that represents the sunlight beans, that would be about the sunlight there (pointing to the yellow shading on her diagram). …what I'm trying to say is the shade is about half as good as sunlight.

Amy's peers, however, were not listening to her so she decided to pose this question to bring them back on task: "This here is sunlight and this here is shade. Which one's better?" Still not happy with her peers' lack of enthusiasm, Amy posed a more advanced question for her peers:

Amy: Oscar, if this long piece was shade, and the short piece was sunlight, and they represented the weight of the beans, which one would be better?

Oscar: This.

Amy: No, shade would be because it's bigger. A bigger mass of kilograms.

The difficulty for many of the children across the four classes was completing the letter for Farmer Sprout. As Amy explained to her teacher, "You see, I've drawn a picture graph and we've worked out the answer, but we can't put it into words… I know! We can draw this (her representation) on our letter and explain what it means in words. And that'll get us out of it." The group finally produced the letter shown in Figure 1, choosing to focus solely on the largest difference between the conditions.

When asked where they obtained their information for this conclusion, Amy explained, "Well, we basically added all of this up (week 10 data for each condition) and we found that shade produced about half as much as sunlight altogether."

Other children produced reports that were embellished with their personal knowledge but limited in reference to task knowledge. For example, a group of boys reported to their class as follows:

Dear Farmer Sprout. We have measured the conditions that you should grow the butter beans in summer because they will grow better. Butter beans will grow more in the sun than in shade which will make it taste better. They will make you

> *strong. Farmer sprout the beans you are growing are good beans. We think you should pick the beans on Sunday. You should have lots of good beans. Get some spray to kill the bug. Sunlight has 146 kg to 118 kg. So plants, it is in sunlight.*

Butter Beans Problem: Part (b)

In responding to the second component of the Butter Beans Problem, the children generally relied on patterns in the data to predict the mass of the beans after 12 weeks. For example, another group in Amy's class reported their predictions for the sunlight condition as follows: "Our findings show that in row 1, week 12, you will get 15 to 17 kilograms, and in week 12, row 2, you'll get 17 kilograms, and in row 3, week 12, you will get 19 to 21 kilograms, and in week 12, row 4, you shall get 18 to 20 kilograms. That's what we think for sunlight." When asked how they got these findings, the children explained, "The data, because we went to week 10 and we counted 2 on…because they've sort of gone up like, in twos and it was another two."

When the teacher asked the class if the pattern in each row of the table "was exactly the same, that is, increasing by one or increasing by two," the children agreed that it wasn't. When asked for some reasons why, Amy responded, "Because they're (plants) not made to be a counting pattern." The teacher then discussed with the children various external factors that could be responsible for the different rates of growth.

Children's Responses to the Airplane Problem

As indicated in the Appendix, the Airplane Problem required the children to determine the winner with respect to: (a) The plane that stays in the air for the longest time; (b) The plane that travels the greatest distance in a straight-line path; and (c) The overall winner for the contest. This problem may be considered more challenging than the Butter Beans Problem in that relationships between variables are involved. The Airplane Problem also engages children in a consideration of rules and conditions that anticipate some decision being made.

Across the four classes we observed a variety of approaches to working the problem, with these approaches displaying important mathematical developments. We also noted a few difficulties in the children's interpretation of the table of data and their ways of operating on the data.

On commencing, many children were absorbed in applying their personal knowledge to dealing with the problem. For example, they discussed the nature of the wings, the cabin, the luggage area, and possible flight paths. Some groups physically acted out a plane's flight path, while others made a simple paper plane. We consider this initial discussion and physical representation to be of benefit to the children in familiarising themselves with the problem. Children's application of personal knowledge to this problem was less intrusive than in the Butter Beans Problem, with the exception of the notion of "scratch". Many children associated the term "scratch" with physical marks on a plane, rather than its meaning of elimination. The teacher's intervention was needed here to elicit this alternative meaning from the children.

On continuing with the problem, most groups across the four classes focused on one variable only, be it the number of scratches, the distance travelled, or the time taken by each team. For example, Team C was considered "The winners of the time in the air" and Team E, "The winners of the distance travelled" (the children arrived at these results by adding the respective time and distance data for the three attempts).

There were a few groups who initially operated inappropriately on the data by adding metres to seconds. When probed by their class teacher, one group indicated that they did not fully understand what the data represented, as can be seen in the excerpt below. Notice, however, that Matt had doubted the appropriateness of his group members' actions from the outset.

Teacher: How do you know they would be the winners all the time?

Susie: Because we added up. They are overall winners.

Teacher: Why are they overall winners?

Susie: Because we added up … we added up 32 onto 5.

Teacher: What are these numbers all about? What are you adding up?

Matt: That's what I tried to ask them.

Teacher: Well, why don't you look at your labels? The labels are so important.

One group member acknowledged that they had been looking at the labels (units of measure) but responded that "They are the points", indicating that she had difficulty in interpreting the data. Children who added data inappropriately in the Butter Beans Problem (i.e., summing all the weights in each table) also had problems with data interpretation.

Several groups across the four classes initially used the notion of scratch as the sole criterion for deciding on possible winners. That is, winners were teams who were not scratched on any trial. The teachers' input here was necessary to challenge this claim. Alex's group, for example, had decided that Team C was the overall winner on the basis that it was not scratched. When the teacher drew attention to the fact that both Team B and Team E had not been scratched, the group quickly reconsidered their answer and stated: "We thought it was either Team C or Team E." On the other hand, children who used the number of scratches as one of the criteria for determining the winner revealed elementary probability ideas when they stated that a team had less chance of winning if it were scratched. This understanding is illustrated in the letter of Tom's group, cited later.

We were especially pleased to see children across all four classes develop at least an informal understanding of rate (speed) as they tackled the issue of an overall winner. We provide examples of this development in the following excerpts and begin by returning to Amy's group. In solving the Airplane Problem, Amy explained, "Actually, me and Douglas have worked it out.

The people who have the least amount of seconds to the most amount of metres with the least amount of scratches." The teacher asked the group to clarify this statement:

Amy: The least number of seconds with the most metres. So like they spend barely any time flying like 12 metres. They spend one second in 12 metres.

Teacher: So that's one way of looking at it. So you're thinking that it's going to be travelling very fast but a long distance. So would that be to decide the distance travelled?

Oscar: No, the overall.

In later discussion, when Amy's class was presenting their reports, we (the researchers) asked one group of students how their approach to problem solution differed from that of the group who had presented before them. Notice in the discussion below, how a stronger understanding of speed was emerging.

Researcher: An interesting letter. Who can tell me, was the letter that this group wrote…did it have the same information as the first group's letter, or was it different information?

Chris: Different.

Researcher: In what way was it different?

Chris: Different strategies…they took notice of the scratches.

Researcher: Anything else different from the first group?

(Inaudible student response)

Researcher: Yes, they looked at the least number of seconds, whereas Chris, your group looked at the most number of seconds.

Amy: We thought the least, because it would obviously be a better plane if it could have (inaudible). 13 metres in just 2 seconds means it'd fly really fast rather than say, 13 metres in 20 seconds…it would be just gliding along. We thought about the speed as well.

In another class, Tom's group explained how they arrived at the overall winner by considering three variables, namely, time, distance, and number of scratches. However, this group considered the greatest time in the air, rather than the least, to be an important variable:

Dear Judges
We have found a way to see who is the winner.
You have to time the team to see who is in the air for the longest.
You have to measure to see who goes the furtherest.
You look closely to see who goes straight and whoever gets the longest gets a prize and whoever stays in the air longest gets a prize.
If a team gets scratched, it has less chances.
Whoever gets the longest in the air and the distance is the overall winner.

It could be that the structure of the problem questions influenced Tom's group (and others) to choose the greatest distance / greatest time relationship when completing their report. We challenged Tom's class by asking the question, "If two paper planes were thrown and one went 12 metres in 6 seconds and a second plane went 12 metres in 3 seconds, who would be the winner?" The children immediately identified the first plane as the winner, explaining, "Because they (the first plane) stayed in the air longest and both went the same distance. The six made the difference." There was agreement with this response across the class.

In yet another class, Menassa's group provided a detailed report that included the order in which the teams should win and also referred to an inverse relationship between time in the air and points that should be awarded. The group members took turns in explaining the system they had developed:

First member: Longer seconds they take in the air, the less points they get. The less time in the air and the longer they go in the air, the more points.

Second member: Team E was the group you should choose (the child made reference to the use of trundle wheels and stop watches to measure distance and time respectively).

Third member: We think that Team E should win the contest. They should win because nobody else managed to fly 13 metres in two seconds. Team A would come second; They went 12 metres in 2 seconds. Team B would come third; They got 3 seconds in 12 metres, and they had no scratches. Team D would come fourth; They got to go 12 metres in three seconds but they had one scratch. Team C would come fifth because they got 11 metres in two seconds and Team F would come last. Team F's best score is 11 metres in 2 seconds with one scratch.

One of the researchers queried the group:

Researcher: Would you like to tell us more about those teams? You said that a team went 12 metres in three seconds. Is that better than a team that goes 12 metres in six seconds?

Children: Yes, yes.

Researcher: Why did you say that?

Children: Because they took less time in the air.

Researcher: What else were you thinking about?

Children: How far they go.

Discussion and Concluding Points

The modelling problems used in our study encourage young children to develop important mathematical ideas and processes that they normally would not meet in the early school curriculum. The mathematical ideas are embedded within meaningful real-world contexts and are elicited by the children as they work the problem. Furthermore, children can access these mathematical ideas at varying levels of sophistication.

In both modelling problems we observed the interplay between children's use of informal, personal knowledge and their knowledge of the key information in the problem. At times children became absorbed in applying their personal knowledge to explain the data, which resulted in slowed progress, especially on the Butter Beans Problem. At other times, children's informal knowledge helped them relate to and identify the important problem information (e.g., understanding the conditions for the airplane contest). Some groups embellished their written reports with their informal knowledge, such as referring to additional conditions required for growing beans. We also observed children recognising when their informal knowledge was not leading them anywhere and thus reverting their attention to the specific task information. We hypothesise that, in doing so, the children were showing recognition of and respect for the presentation and organisation of the data in the problems.

We consider it important that children develop the metacognitive and critical thinking skills that enable them to distinguish between personal and task knowledge, and to know when and how to apply each during problem solution. The role of the teacher in developing these skills has been highlighted by Lehrer, Giles, and Schauble (2002). Teachers need to walk a tight rope in capitalising on the familiar in data modelling and in "deliberately stepping away from it" to assist students in considering the data themselves as objects of reflection (p.23).

The need to expose young children to mathematical information presented in various formats, including tables of data, is evident from this study. While the children developed facility in interpreting and working with the tables of data, some groups experienced initial difficulties. For example, the cumulative nature of the data in Table 1 was not apparent to some children, who added all of the data for sunlight and compared this with the aggregate of the data for shade. Activities in which children collect and record their own data can assist here. In the present study, this was achieved through the preparatory activities leading up to the modelling problems.

In both modelling problems we saw the emergence of important mathematical ideas that the children had not experienced during class instruction. Children's elementary understanding of change and rate of change was evident on both problems, while notions of aggregating and averaging were seen on the Butter Beans Problem. Of particular interest though, is children's informal understanding of speed observed in the Airplane Problem. Some groups focused on the relationship, "shortest time, longest distance" to determine the winning teams, and in so doing, referred to the "speed" of a plane or how "quick/quickly" a plane flew. Other groups considered the relationship, "longest time, longest distance" to be the determinant of the winning plane. The latter could be due in part to the way in which the problem questions were worded. Nevertheless, we see this Airplane Problem as providing opportunities for children to explore quantitative relationships, analyse change, and identify, describe, and compare varying rates of change, as recommended in the Grades 3-5 algebra strand of the Principles and Standards for School Mathematics (NCTM, 2000). In addition, we saw elementary probability ideas emerging when children linked the number of scratches with a plane's chances of winning.

Our study has also highlighted the contributions of these modelling activities to young children's development of mathematical description, explanation, justification, and argumentation. Because the problems are inherently social activities, children engage in numerous questions, conjectures, arguments, conflicts, and resolutions as they work towards their final products. Furthermore, when they present their reports to the class they need to respond to questions and critical feedback from their peers. We see this as another area where the teacher's role is important, specifically, in scaffolding the quality of discursive practices.

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Authors

Lyn D. English, Department of Mathematics, Science and Technology, Queensland University of Technology, Victoria Park Road, Kelvin Grove Queensland 4059.
Email: <l.english@qut.edu.au>

James J. Watters, Department of Mathematics, Science and Technology, Queensland University of Technology, Victoria Park Road, Kelvin Grove Queensland 4059.
Email: <j.watters@qut.edu.au>

Appendix

THE STONEY CREEK TIMES

Students fly away in the Annual Paper Airplane Contest at local school

If the Wright Brothers, pilots, and aircraft engineers can do it, surely the students in 'Stoney Creek' State School's year three classes can do it.

What will you be doing, that a couple of inventors, some of the best pilots in the world, and the brightest minds in the world do everyday? Fly!

You will attempt to be like the Wright Brothers and design an airplane that will meet today's airplane standards.

However, you won't be using aluminium, various metal parts or jet engines for these planes. All you will need are pieces of paper – or any other craft materials – and a whole lot of imagination.

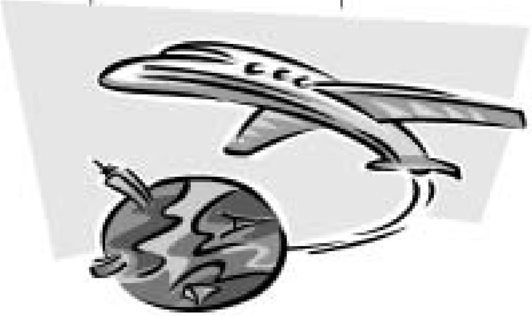
You have the opportunity to design planes that will be able to fly long distances. In the contest, you will need to design a plane that will travel in a straight path.

However, with every contest there is a set of rules that you must follow to try to win the contest's grand prize. Some of these rules are:
1. No cuts can be made in the plane's wings,
2. Parts may be cut off from the plane entirely, and
3. You must build your own planes.

You will be working in groups to design and test your planes before contest day. Each group gets *three attempts*.

Scratches may occur in this contest. A scratch means that the plane did not travel in a straight path for any of the flight.

I have heard that some of you are getting way into this – someone said that you are bringing in the in-flight refreshments! This will be an interesting contest.



Reflection Questions:

1. What is the Annual Paper Airplane Contest about?

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2. What needs to be done to design an airplane that will be successful for the contest?

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3. What does it mean if your plane is scratched in one of your attempts?

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4. What units of measurements are used in contests in which distance and time are measured?

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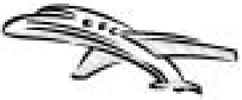
The Annual Paper Airplane Contest



This year, the Stoney Creek State School will hold their annual paper airplane flying contest on 15th September. Students in year three will be working in groups and will design one plane.

All planes will be designed to fly for as long as possible in the air *(time)* and over a long *distance* from a target. The plane will need to travel in a *straight-line path*.

Three awards will be given at this contest. One will be given to the group whose plane stays in the air the longest – another to the group whose plane travels the longest straight-line path – and the final award is an overall award given to the group who wins the contest.

Results from the Annual Paper Airplane Contest 2002

Team	*Attempts*	*Time in the Air (seconds)*	*Distance travelled in a straight path (metres)*
Team A	1	2	11
	2	2	12
	3	scratch	scratch
Team B	1	3	12
	2	1	7
	3	1	8
Team C	1	1	9
	2	3	11
	3	2	11
Team D	1	3	12
	2	scratch	scratch
	3	1	8
Team E	1	2	9
	2	1	10
	3	2	13
Team F	1	1	9
	2	2	11
	3	scratch	scratch

Investigation

In the past years, the judges have had problems with deciding how to select a winner and how to judge the contest. Using the given data from the previous years, find a way to help the judges decide on the overall winner of the contest.

Write a letter to the judges of the contest explaining to them how to determine who wins each of the categories (time in the air and distance travelled in a straight-line path) and how to decide the winner of the overall award for the contest.