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Understanding Linear Measurement: A Comparison of Filipino and New Zealand Children

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An understanding of linear measurement depends on principles that include standard unit size, iteration of units, numbering of a unit at its end, and partial units for measuring continuous length. Children may learn these principles at school, for example through experience with informal measurement, or they may learn them through use of measurement in society. This study compared the application of these principles by children aged 8 and 9 from the Philippines and New Zealand. These countries were selected because they have quite different curricula, societal influences and economies. Ninety-one children were interviewed individually on a common set of unusual tasks that were designed to tap underlying principles. Results showed many similarities and some differences between countries. Most tasks requiring visualisation and informal units were done more accurately by New Zealand children. Some tasks involving the use of a conventional ruler were done more accurately by Filipino children. These differences appear to be related to differences in curricula and possibly to differences in societal use of measurement. We suggest that these results, like those of other writers cited, demonstrate the need for extensive work on the underlying concepts in measurement through work on informal measurement and a careful transition between informal and formal measurement.

Introduction

Children's ability to make measurements depends both on their development and on their cultural experiences. There are concepts that underpin measurement that have to be developed or learned. These include the standard size of units, transitivity, and dealing with a continuous rather than a discrete quantity. These properties appear to be more easily understood in middle-childhood than in early childhood, although comprehending them is influenced by a child's experience.

Development of Measurement Concepts

Much of the research on measurement concepts stems from Piagetian theory which proposes that until children understand conservation of length and a spatial coordinate system they are unable to understand measurement (Piaget & Inhelder, 1967; Vuyk, 1981). Experimental studies have explored different aspects of children's understanding of measurement, with results that vary in degree of agreement with this theoretical view. Davydov (1982) demonstrated the difficulty that young children have in understanding that units have to be of the same size. Bryant and Nunes (1994, cited in Nunes & Bryant, 1996) have drawn attention to the difficulty that children aged 5 to 7 have in using units of different sizes when dealing with centimetres and inches, although older children were better than younger ones on this task. In a further task exploring children's understanding of iterated units (contiguous repeated units), Nunes and colleagues (Nunes, Light & Mason, 1995, cited in Nunes & Bryant, 1996) asked 5- and 6-year-old children to put the numbers on pictures of rulers that had 0.5 cm subdivisions drawn on them. Their placement of the numbers showed that 89% of these young children failed to demonstrate that the first numeral on a ruler showed the end, rather than the beginning, of the first unit. In a different study they explored children's attention to the lines versus the spaces between the lines on a "broken ruler" and on a good ruler and found that although the children were less accurate with the broken ruler than with the good ruler, the difference was not significant. Young children made errors in knowing where to start when using a good ruler as well as when using a broken one. Kamii and Clark (1997) found, when studying children in US grades 1 to 5 that the majority of children did not show understanding of iteration until grade 3 (age 8), near the stage that Piaget predicted. Bladen, Wildish and Cox (2000) found that only 53% of children aged 10 demonstrated understanding of the iterated unit. Hart (1981) reported 48% of 12-year-old children failed to appreciate that a line perpendicular to two parallel lines would be shorter than a line which joined these parallel lines at an acute angle, showing confusion with spatial coordinates. In summary, the research has shown that children can have difficulties with measurement throughout their schooling.

Influence of Schooling and the Wider Culture

The influences of culture and schooling have an important effect on the development of measurement concepts. Children may learn more from school than from the practices of adults in the precise use of measurement. There are overall differences between education in New Zealand and the Philippines, among which are: the money available for education (proportionally greater in New Zealand than in the Philippines); the larger size of classes in the Philippines; and the fact that most children in the Philippines are being educated in English which is not their first language.

Schooling does affect knowledge of measurement topics. Clements, Batista, Sarama, Swaminathan, and McMillan (1997) demonstrated the effect of using a Logo-based unit on children's ability to understand length. Bladen, Wildish and Fox (2000) wrote of 'school contamination' when children learned a measurement procedure without grasping the underlying concepts. There is a marked difference in the curriculum on measurement in New Zealand and in the Philippines, primarily in the emphasis placed on informal measurement. Measurement is one of the five major topics in the

New Zealand mathematics curriculum (New Zealand Ministry of Education, 1992). This curriculum requires children to work with informal units, such as hand spans or paperclips, for about two years (much less than those who draw attention to Piagetian stages would recommend) before moving to standard units. From the list of competencies expected of Filipino elementary children, the use of informal measurement appears only once, and that use is intended for first-grade children. Measurement is covered largely in science and deals almost exclusively with standard units (Philippine Department of Education, 2002).

The development of the use of iteration of units is aided by visualisation. That is, the student needs to be able to imagine the units and transfer them from one setting to another *by eye*. This skill is developed in some school work in which children are asked to judge the length of an object or suggest several things not in their hands that might be of a certain length. Visualisation can be fairly exact rather than just an estimate. The programme described by Clements et al. (1997) requires visualisation.

Students in the Philippines and in New Zealand could differ in their cultural experiences as well as in their school curriculum. An early study of differences in cultural experiences and precision in measurement was that of Gay and Cole (1967) who reported that Kpelle adults estimated the number of cups of rice in a bag more accurately than a group of Peace Corps volunteers, who neither had the experience nor the need for this measurement. In other examples of experience leading to accurate measurement, Millroy (1992) argued that South African carpenters who learn their trade through apprenticeship use a considerable amount of precise mathematics in their work, although they consider themselves unschooled in mathematics. Schliemann and Carraher (1992) reported that relatively unschooled cooks worked out proportional measurement accurately for price and medicine, but were less accurate in cooking, where there was less need for accuracy. We could find no reports that explored the linear measurement concepts of children in different cultures.

Cultures differ in the extent to which they value precision. In New Zealand a high degree of precision is required of tradespeople such as carpenters, whose work is regulated by building codes which are usually complied with. There are also building codes in the Philippines, where minimum or maximum measurements are indicated depending on the kind of structure built (e.g., a minimum ceiling height for two-storey houses), but there are likely to be differences between the countries in the ways the codes are enforced. Alleged corruption and inefficiency of Filipino government agencies may contribute to the lack of enforcement, thereby implying or reflecting a lack of valuing of precision by society in general (see, for example, Manila Bulletin, 2004).

Cultures also differ in the measurement systems used in homes and in school. Both countries formerly used imperial units but now primarily use metric measures. New Zealand officially changed all measuring units to metric ones in 1976 and only metric measures are used in all businesses and trades (New Zealand Official Yearbook on the Web, n.d.). This means that most New Zealand children aged 8 and 9 would have parents who were schooled in metric units. The rulers used in New Zealand schools are only in centimetres and millimetres. In the Philippines, a presidential decree imposed the use of the metric system from the year 1974 (Ortiz Munsayac, 1994), but both metric and imperial measurements continue to be used, for example, clothes are sold with imperial measurements for such things as waistlines. Although imperial measures are not used in trade in New Zealand, they continue to be used by a variety of people, for instance in reporting how tall they are. Within schools, Filipino children continue to use rulers that have inches on one side and centimetres on the other. Thus, Filipino children are exposed to imperial and metric linear measurement systems both in school and out of school, and New Zealand children may be exposed to both systems at home but are not likely to be at school. These factors – the importance placed on informal or formal measurement in schools, the degree of precision used in measurement in society, the transfer from imperial to metric units, and the continuation of community use of older units – are different in the Philippines and in New Zealand.

In this study, children's responses in both countries to a set of linear measurement tasks which progressed from relatively easy to relatively difficult were compared. The extent to which different experiences might influence children's accuracy was examined.

Method

The same methodology was used in both countries. This involved individual interviews with children and observing the techniques used to answer questions, as well as interviews with teachers.

Participants

In the Philippines, 48 primary school children attending three different school systems participated in the study. The children's ages ranged from 7 to 10 years, with all but four being aged 8 or 9. For the purpose of analysis, the one 7-year-old child was grouped with the children aged 8 and the three 10-year-old children were grouped with the children aged 9. There were 22 children aged 7 to 8 and 26 children aged 9 to 10. Equal numbers came from three school systems: a traditional public school (6 boys, 10 girls), a traditional private school for boys with an excellent academic reputation (16 boys), and a private Montessori school (9 boys, 7 girls). The children were selected on the basis of availability, except in the traditional public school where the children came from the best class in mathematics. A school administrator or teacher selected which children would participate. The author and interviewer in that country asked for children covering a range of ability, but believes she was given the more capable children.

In New Zealand, 43 primary school children were interviewed. There were 22 children aged 8, and 21 children aged 9. To make the sample as similar as possible to that in the Philippines, these children were selected from two traditional public schools that drew from middle income families (11 boys, 11 girls), one traditional private school for girls that has an excellent record in mathematics (16 girls), and a small private Montessori school (3 boys, 2 girls). There were only 43 New Zealand children in total, largely because the Montessori school had few children. In New Zealand, participation was dependent on parental consent, as well as the cooperation of the school.

Based on their performance on the tasks in the study, there was no significant difference at the 0.05 level in the proportion of items correct by school type in either country, so the results are presented by country and not by school type (Newcombe, 1998).

A sample of the children's teachers was also interviewed on their teaching of measurement at the age level. This sample included three teachers in New Zealand and three teachers in the Philippines who were interviewed in depth.

Interviews

The authors interviewed the children using a common protocol, although there were unintended differences in how it was employed. New Zealand children were interviewed as they worked on tasks. Filipino children were interviewed after they had completed the tasks. Interviews were tape-recorded and transcribed in the Philippines and careful notes were taken during the interviews in New Zealand. The children were asked about their experience of measuring, including what measuring they had seen at home, and how they worked out different tasks—the five tasks are described below. In both countries these interviews lasted 20 to 30 minutes. Some Filipino children took considerably longer to complete the tasks than did the New Zealand children. All questions were presented as given below, but if the children showed any confusion the questions were rephrased. In New Zealand, and in the two private schools in the Philippines, all interviews were held in English, but in the Filipino public school the Filipino author switched to the local language when she thought this would help.

The authors also interviewed the teachers. Some teachers gave only brief responses but three teachers in both countries were interviewed in depth. These interviews provided information on what had been taught to the children, what the children had difficulty with, and the teachers' views on teaching this topic.

Tasks

There were five tasks. The first two tasks explored children's understanding of informal measurement, or measurement without standard units. Both of these tasks explored the children's concepts of iteration of a unit. The next three tasks explored their understanding of rulers in unusual circumstances, and required understanding of all the underlying principles of linear measurement.

The first and second tasks examined children's ability to visualise the iteration of a unit in order to tell how many times one unit would fit into another (see Figures 1 and 2). Although the tasks may appear to relate to area, they require attention to length only. In this sense, the items measured were thick lines.

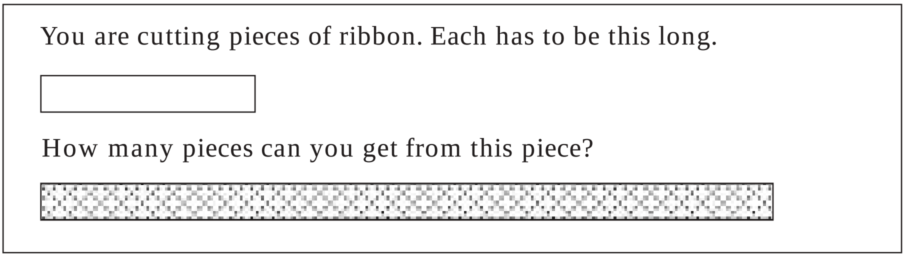

Figure 1. Task 1 – Visualising and informally measuring length with no tool provided.

For the first task (see Figure 1), the children were initially asked to say how many ribbons of the given length could be cut from the longer piece by *using their eyes*, and then they were asked to measure it in some way. They were not given a measuring tool but they had objects around them (e.g., pencil or pen, eraser, or their fingers) that they could use to measure informally. The correct answer is 3 but we also accepted 3 and an indication of a left-over bit.

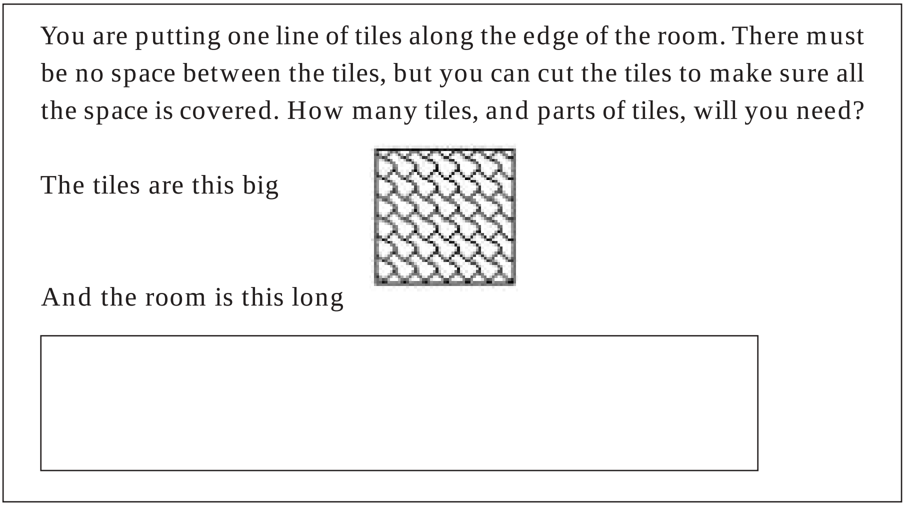

Figure 2. Task 2 – Visualising and informally measuring length with an informal tool.

For the second task (see Figure 2), the children were first asked to give the number of whole tiles and parts of tiles that could fit. After doing this by eye, they were given two card *tiles* to help them measure. A fully-credited answer to this question required a way of expressing parts of units.

For the third task (see Figure 3), the children were given a picture of a ruler with only 4 and 5 marked on it, and lines for what would be 2.5 to 6 as shown in Figure 3. There was space for 10 units. This is an adaptation of a task used by Nunes, Light and Mason (1995, cited in Nunes & Bryant, 1996) with the added requirement of needing to add the missing lines. This task required children to demonstrate that they understood that the first unit ended with 1 rather than starting with it and that all units were uniform in size.

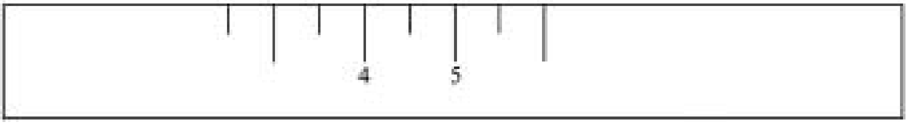

Figure 3. Task 3 – Completing a Ruler

In the fourth task (see Figure 4), children were presented with a picture of a mouse house and asked to measure four dimensions with a *broken ruler*. The broken ruler was a laminated section of a photocopy of a New Zealand ruler with jagged ends that went from 2.8 cm to 7.2 cm on one edge and from 239 mm to 283 mm on the other. This task was based on that of Nunes, Light and Mason (1993, cited in Nunes & Bryant, 1996) but did not compare the use of a real ruler with a broken one. On this task we were interested in whether children counted the length between lines as a unit or whether they counted the lines as the units. The picture to be measured came from the New Zealand Council for Educational Research (NZCER) Assessment Resource Bank item MS2087. However, the questions and measuring instrument differed.

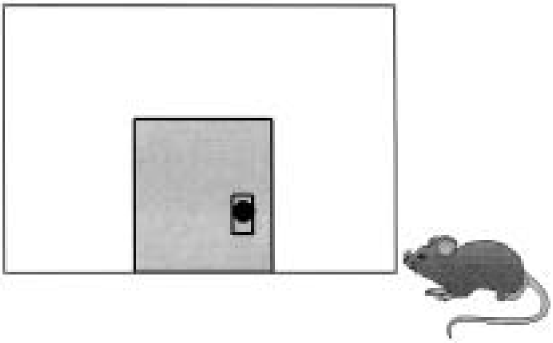

Figure 4. Task 4 – Measuring dimensions with a broken ruler.[1]

[1] Permission for the reproduction of the picture was granted by the New Zealand Council for Educational Research [NZCER].

Task 5 was a released item from the Third International Mathematics and Science Study – Repeat (1999). It was item P12 as is shown in Figure 5. The question associated with this was: *If the string in the diagram is pulled straight, how long will it be?* This task required visualising, or some measurement technique, and understanding that units rather than numbers were to be counted. Additional complexity is added by the need to measure the length of a curve against a centimetre ruler.

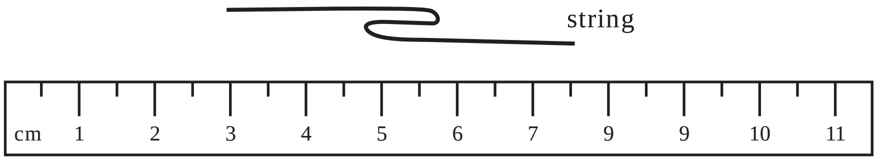

Figure 5. Task 5 – (from TIMSS-R)

Results

In all tasks, we analysed the methods that children used as well as their accuracy. Additional information on the methods used with the New Zealand sample and the Philippine sample are available in Irwin and Ell (2002) and Vistro-Yu (2003). To provide an overview of the findings, the percentages of correct results from each country on each task are presented in Table 1 before a more detailed analysis of responses on individual tasks. The percentages given combine both age groups and take the number of children given the task into account.

Table 1
Percentage of all Children Accurate on each Task

	Philippines n=48	New Zealand n=43
Task 1 (measuring)	94%	91%
Task 2 (measuring)	19% *	74%
Task 3 (unit before 1)	56%	53%
Task 3 (even units)	42%	44%
Task 4 (all 4 items)	23%	40%
Task 5	38%	26%

* based on n=32

As can be seen from the data in Table 1, the tasks were of increasing difficulty for New Zealand children, but there was irregularity in the order of difficulty for Filipino children. The nature of answers on each task is given below.

Task 1 required children to visualise and measure one length, given a picture of an informal unit. While children were not given a movable unit to measure with, some chose to measure with their eraser or the clip on the mechanical pencil provided. Others used their fingers. Children in New Zealand were more likely to use visualisation as their main way of measuring on this task while more Filipino children chose to find a measuring tool. Children who visualised did not use any measuring instrument except measuring by eye. As explained above, the correct answer was either 3 or 3 and some left over. Table 2 gives both the number of children giving a particular answer and the method used, by age groups.

Table 2
Number of Children who were Accurate when using Visualising (a) and Informal Measurement (b) on Task 1

	Philippines		New Zealand	
	7-8 yr* n=22	9-10 yr* n=26	8 yr n=22	9 yr n=21
a) Visualising				
Whole Number	11	7	8	17
Number with a bit left over	0	0	4	3
Total number (and percentage) of accurate children	11 (69%)	7 (44%)	12 (55%)	20 (95%)
b) Informal Measure				
Whole Number	19	15	2	5
Number and bit left over	2	9	17	15
Total number (and percentage) of children giving accurate measurement	21 (95%)	24 (92%)	19 (86%)	20 (95%)

Note. *Due to a misunderstanding between the authors, only 32 of the Filipino children were asked to visualise. All of the children were asked to measure.

For Task 2, the children first visualised and then were given two tiles to use for measuring. The accurate answer for measuring the number of tiles needed was 5.33. When visualising, answers of either 5, or 5 and a fractional portion were accepted. For the measuring portion, 5 and any clear expression of a fractional portion was required, the most common being 5 and a bit of a tile or fractions like a half. However, if children gave only the whole number 5 when measuring with tiles this was seen as a systematic error. Common measuring errors included: leaving a space between the tiles when iterating, sometimes as wide as their finger; overlapping the tiles; and spreading the tiles out to fill the space. The results are shown in Table 3.

As can be seen on Table 3, for Task 2 the Filipino children were much more likely than the New Zealand children to give only a whole number rather than a number and a fractional part when measuring. Children from the two countries differed more on this task than on any other tasks.

Table 3
Number of Children who were Accurate when using Visualizing (a) and Informal Measurement (b) on Task 2

	Philippines		New Zealand	
	7-8 yr * n=16	9-10 yr * n=16	8 yr n=22	9 yr n=21
a) Visualising				
Accurate number and fractional part	0	0	1	0
Whole number only	6	9	7	9
Number (and percentage) of accurate children when either answer was accepted	6 (38%)	9 (56%)	8 (36%)	9 (43%)
b) Informal Measure				
Accurate number and fractional part	1	5	18	13
Systematic error of giving whole number only	15	9	0	2
Number (and percentage) of children giving accurate answers which included fractional part	1 (6%)	5 (31%)	18 (82%)	13 (62%)

Note. *Due to a misunderstanding between the authors, only 32 Filipino children were asked this question.

Task 3 required the children to complete a drawing of a ruler. The challenges on this task were starting the ruler with 0 or at least the space of a unit before 1 and producing relatively even units for the rest of the ruler. Responses to this task are summarised in Tables 4 and 5. There was little difference between the children in the two countries on starting the ruler correctly, with roughly half of the children being scored as accurate.

Table 4
Number and Percentage of Children Indicating that there was the Length of a Unit to the Left of 1 on a Ruler

	Philippines		New Zealand	
	7-8 yr n=22	9-10 yr n=26	8 yr n=22	9 yr n=21
Number (and percentage) of accurate children when either answer was accepted	11 (50%)	16 (62%)	14 (64%)	9 (43%)

Table 5
Evenness of Units on the Ruler Drawn for Task 3

	Philippines		New Zealand	
	7-8 yr n=22	9-10 yr n=26	8 yr n=22	9 yr n=21
Even throughout	5	15	12	7
Partially even	10	6	4	0
Number (and percentage) of children even throughout	5 (23%)	15 (58%)	12 (55%)	7 (32%)

The authors judged evenness of units, with an inter-rater reliability of 94%, on a sample of 16 responses, eight from each country. The results are shown in Table 5. Some children were careful with the size of their units at one end of the ruler and not at the other end. These children were scored as being partially even.

When the placement of children's lines were considered as partially even, they were usually inaccurate at the right-hand end of the ruler, the portion usually drawn last, and where more units fit. Several Filipino children and one New Zealand child wrote numbers to 12, as might have been expected on a 12-inch ruler.

Task 4 required the children to measure four lengths with a broken ruler. We were interested in whether they counted the lines or the space between the lines on the ruler – this is a way of assessing their understanding of a unit – and determining what other methods they used to account for the fact that the segment of the ruler that they had was not long enough to measure two of the lengths. The segment of ruler used was from a New Zealand ruler. Accurate answers were 5.4 cm, 7.7 cm, 3.1 cm and 2.7 cm. An answer was judged accurate if it was within 0.5 cm of these measurements. If children gave the measurement in other units, like inches, they were not penalised. A measurement that was approximately 1 cm greater than the accurate number was scored as the systematic error of counting lines rather than spaces, a feature we were interested in. The children's accuracy for each of the four lengths is summarised in Table 6.

The results in Table 6 show that at these ages many children cannot accurately measure lengths when the ruler given does not begin at 0. The fact that the ruler was not long enough to cover the first two dimensions did not affect the accuracy or inaccuracy of the children's measurements. The inaccuracies in the measurements mainly stemmed from the fact that the ruler did not begin at 0. Reasons for inaccuracy, apart from counting lines instead of the unit lengths, included starting with the large numbers (mm) for longer lengths and referring to the last number on the ruler as the measure of the dimension. For example, the ruler went from 3 cm to 7 cm, and some children declared the length to be 7 cm, referring to last number on the ruler as the measurement. Alternatively, the measurement from 3 to 7 cm

Table 6
Number of Children Giving Accurate and Inaccurate Measurements for Four Lengths on the Mouse House

	Philippines		New Zealand	
	7-8 yr n=22	9-10 yr n=26	8 yr n=22	9 yr n=21
a) Height of house				
Accurate	3	6	10	8
Systematic error of counting lines	0	1	1	1
b) Width of house				
Accurate	3	5	9	7
Systematic error of counting lines	0	1	1	3
c) Height of door				
Accurate	3	6	7	7
Systematic error of counting lines	0	2	6	6
d) Width of door				
Accurate	9	10	11	8
Systematic error of counting lines	0	1	3	3
Mean percentage accurate on all measures	20%	26%	42%	36%

was seen to indicate a length of 3 + 7 = 10 cm, the sum of the first and last number. Faced with these confusions some children decided to estimate and not obtain an exact measurement.

In coping with the task of measuring a length that was longer than the ruler more accurately, the most common strategy was making a mark at the end of the ruler and then sliding the ruler over. Other strategies were creative, for example imagining the first 3 centimetres and adjusting the placement of the broken ruler accordingly, and imagining the beginning as 0 and counting accordingly.

It was also observed that several of the children preferred to ignore fractional or small units and used whole numbers for all measurements. Some did not know what to call the small units (mm). While explaining her work, one girl used the Filipino term for unit – kabuuan, indicating that she understood the importance and function of a unit. While her numerical values for all four items were accurate, she kept on calling them inches instead of centimetres.

Overall, more New Zealand children were successful in their measurements but more also made the systematic error of counting the lines rather than the units. Filipino children were more successful in measuring the shorter lengths that did not require iteration of the ruler segment.

Filipino children expressed more surprise when shown the broken ruler than did the New Zealand children, probably because it represented part of a New Zealand school ruler (an unexpected complication). Several Filipino and some New Zealand children called the units 'inches'. If Filipino children were correct on the first measurement they tended to be correct on all.

A number of New Zealand children did not attempt to iterate the ruler on the longer items but visualised the number of units before and after the end of the ruler segment. Two children from one school measured the missing portion using the width of a finger as one centimetre. Teachers confirmed that they had had children measure different parts of their body, but had not specifically taught children to make this transfer to using the body part as a measuring tool.

Task 5 required a combination of the skills assessed separately in the previous tasks. In particular, it required accurate visualisation and understanding of a unit that did not start at the beginning of a ruler. This was the task with a picture of a curved string above a ruler for which children were asked to say how long the string would be if pulled straight. The need for visualisation did not stop some children from each country from trying to measure by using their fingers or pencils. Table 7 shows the number of children who were correct ± 0.5 cm or incorrect using either visualising or measuring strategies. Some children used both strategies, and for others it was not possible to tell what strategies were used.

As can be seen from Table 7, a larger percentage of Filipino children than New Zealand children were accurate on Task 5. They were more accurate

Table 7
Number of Children Giving Accurate Answers and Method used by Children from both Countries on Task 5, a Diagram of a Curved String

	Philippines				New Zealand			
	7-8 yr n=22		9-10 yr n=26		8 yr n=22		9 yr n=21	
	correct	wrong	correct	wrong	correct	wrong	correct	wrong
Visualised	6	8	4	12	4	7	1	7
Measured	3	0	1	4	2	8	3	9
Used both strategies	0	1	1	1	0	1	1	0
Strategy unclear	1	3	2	1	0	0	0	0
Number (and percentage) accurate	10 (45%)		8 (31%)		6 (27%)		5 (24%)	

at both ages. Overall, more children did the task by visualising than by trying to measure. In all groups, except 7-8 year old Filipinos who measured rather than visualised, more children using either technique were incorrect than were correct.

Results of Interviews with Children and Teachers

The interviews with children showed us that very few of them recalled having much experience with precise measurement outside school. Only one child in each country reported having seen precise measurement at home. In New Zealand this was related to renovating the family home, and in the Philippines it related to dressmaking. All other uses of measurement were relatively inexact, such as is commonly done in cooking. New Zealand children mentioned measuring the field with *the wheel thing* and using measuring tools such as the ruler on the paper cutter, or seeing an older sibling use a protractor. Filipino children reported seeing parents measuring before buying furniture or adding décor to the walls of the house, and rulers being used as a straight edge not necessarily for measuring. Some children reported being given rulers early on and allowed to measure *stuff* around the house, but were not necessarily guided in this measurement. In school, more children reported using metre sticks than short rulers. At play, children reported using informal measurements to determine relative heights. Informal and formal measurement experiences were limited both at home and in school.

The Filipino teachers reported limited teaching of informal measurement, dealing almost entirely with formal measurement. In contrast, two of the New Zealand teachers said that they did something on measurement every term. The New Zealand teachers expressed the opinion that more time should be spent on informal measurement before moving to formal tools, despite children being anxious to use rulers. They taught informal measurement using such units as hand spans, strides, finger widths, and the length of outstretched arms. They also spent time on estimation such as estimating the length of a metre. They thought that they probably spent too short a time in the transition from informal to formal measurement.

Discussion

When embarking on this research project, we assumed that the tasks would be of increasing difficulty in both the Philippines and in New Zealand, and that older children would do better than younger ones. We expected to find some differences between the countries that might be attributed to practices in schools and in society. The tasks did prove to be of increasing difficulty for New Zealand children, but the picture was not smooth for the sample from the Philippines. Results showed that for all tasks except Task 1 (94% and 91% correct) children in this age range from both countries found these tasks challenging. This may reflect the point made by Bragg and Outhred (2000) that while children are taught the techniques for measuring length, they are not given enough time or instruction on the underlying principles. The fact that The Philippines scored below the international mean, and New Zealand scored near the international mean, on assessment of measurement in the TIMSS-R in 1999 (Chamberlain & Walker, 2001; Vistro-Yu, 2002), suggested that children may be not be strong in the underlying concepts essential to measurement, and that their schooling is not helping them with these underlying concepts as much as it might.

Giving possible reasons for the differences found requires speculation, as does any error analysis. We acknowledge that such speculation is always open to counter suggestion.

Issues Relating to Age and Development

Despite some differences between the accuracy of children in the two countries, there were some tasks on which the proportion of successful children was very similar. This was true for Task 1 (94% and 91% correct), which required an understanding of iteration for a small number of items. It was true for both aspects of Task 3, both starting a ruler with a unit before 1 (56% and 53% correct) and regularity of units (42% and 44% correct). These tasks required understanding of the size and identity of units, as explored by Davydov (1982) and Bryant and Nunes (1994, cited in Nunes & Bryant, 1996). There were differences in the percentage of tasks done correctly on Task 4 (23% and 40%) and 5 (38% and 26%), and we speculate on the reasons for these differences below. Overall, the results suggest that, compared to development, differences in curriculum and culture play a relatively minor role in the two countries where these studies were carried out. This would conform to the Piagetian view of necessary precedents for accurate measurement (Bladen, Wildish, & Cox, 2000; Piaget & Inhelder, 1967).

Although there were differences between the 8- and 9-year-old children in this study, they are not consistent across tasks or countries. On most tasks a higher percentage of older Filipino children than younger ones were successful, but a higher percentage of young New Zealand children than older New Zealanders were successful on the last three measuring tasks. It could also be that as New Zealand children had done informal measuring more recently, they transferred concepts from that context to these unusual measuring tasks. Alternatively, it might be that formal measuring was relatively novel to New Zealand children, and they might therefore have been more careful than the older children. One insight into the reasons for older children doing less well might lie in the systematic errors that were made on Task 4 when measuring with a broken ruler. The error of counting lines rather than spaces was slightly more common among older children, possibly because they forgot or ignored the principles learned in informal measurement. This difference would be worth investigating further.

The children in this study performed with greater success on indicating that the first unit on a ruler ended in 1 on Task 3 than the younger children in a task reported by Nunes, Light, and Mason (Nunes, Light, & Mason, 1995, cited in Nunes & Bryant, 1996). This is consistent with the developmental differences reported by Davydov (1983) and Piaget and Inhelder (1967). The relatively poor performance of children from both countries on Tasks 3, 4, and 5 supports the suggestion, made by both Bragg and Outhred (2000) and Bladen, Wildish, and Cox (2000), that many students do not have the necessary underlying understanding when they are taught formal measurement.

Accuracy of Each Task in Each Country

As Task 1 was done well by most children in both countries, this indicated that children from these countries had adequate understanding of the basic principles of unit size and iteration when the number of units was small. Many children could visualise one unit being repeated in another context, and others invented ways of iterating this unit despite not being given a unit to use. Their high rate of success on this task that required a small number of units (91% and 94%) is in contrast to the success in iteration reported by Kamii and Clark (1997), in which 55% of third grade (aged 8) children were successful on a unit iteration task. This may be due to the difference in the tasks. Our children’s high rate of success on this task makes the difference in their rate of success on Task 2 of more interest.

While an average of 74% of New Zealand children succeeded on Task 2, an informal measurement task requiring an awareness of partial units, only 19% of Filipino children succeeded on the task. This was the lowest performance for Filipino children on any task. These percentages refer to children who gave 5 and a fractional part as their answer. In contrast, 75% of Filipino children gave only a whole number as the answer, while only 5% of New Zealand children gave this response. We cannot say that Filipino children did not understand the principle of partial units in continuous measurement. It is more likely that they thought them unimportant. Filipino children tended to relate informal measures to estimates and thus whole number answers were deemed to be acceptable. This ignoring of fractional parts is consistent with other situations, such as Filipinos' use of money. For example, when a person owes P243.55, that person will pay the rounded whole number figure of P243, dismissing the P0.55, possibly because of the low value of the Peso. A common practice among Filipinos is to give a bill of a large denomination when paying for a purchase, essentially transferring the burden of counting the exact change to the seller. The approximation inherent in such transactions is also found in other estimation tasks, and is learned by young children from their parents. Thus in Task 2, we suspect that Filipino children saw whole number as adequate for this measurement.

On Task 3 (drawing a ruler), while similar proportions of children from each country succeeded in giving a unit before 1, 44% and 46% of children from these countries failed to show that they knew that the number 1 stood for the end of the first unit. In numeration, counting starts with one and it appears that those who did not start the ruler with a space before 1 did not appreciate the difference between numeration and measurement units. The results that we gained do indicate that the children in this study did better than children aged 5 and 6 on a similar task reported by Nunes, Light, and Mason (1995, cited in Nunes & Bryant, 1996) on which 89% of children failed to show this knowledge. Similar proportions of Filipino and New Zealand children drew units of similar size on their rulers. For some children the perceived need to fit in 12 units was stronger than the need to make units of the same size. This is likely to relate to the need to complete a 12 inch ruler, but as this error occurred in both countries it cannot be attributed solely to school rulers which differ in the two countries. It may be that the home experiences of children in both countries included 12 inch rulers.

Differences between the countries on Tasks 4 (measuring with a broken ruler) and 5 (giving the length of a curved string in a diagram using the pictured ruler below) were larger than the differences on Task 1 or 3, but not as large as the difference on Task 2. Tasks 4 and 5 required understanding of what a unit was on a ruler, its standard size, how it was iterated, and the relation between measurement units of different sizes and the number of units used to measure a length. In both countries, the majority of children attempted to use visualisation to measure on this task rather than measuring with their fingers or other tools, but this visualising was often inaccurate. A possible reason for the Filipinos' less competent performance on Task 4 could be because teachers spend little time on informal measurement without attention to these underlying principles. The Filipino curriculum does not require work on informal units, but they are included in the New Zealand curriculum largely so that children will attend to the nature of a unit. At the same time, New Zealand teachers admitted to spending less time on the transition from informal to formal units than they thought would be advisable.

We are puzzled by the different results on Task 5, with Filipino children more accurate than New Zealand children. This task explored children's strategies in obtaining the length of a curve against a centimetre ruler, a difficult task. The Filipino children who were accurate on this task did it by visualising, yet only one of these children had been accurate in visualising on Tasks 1 and 2. We speculate that because the ruler was also in the picture, they visualised the units on the ruler in comparison to the string as a way of measuring, but we cannot tell if this was so from our interviews. It may be that they took more care on this task, which looked more like the measurement that they were used to than did Tasks 1 and 2.

Effect of Differences in Curricula on Methods used by the Children

The methods that the children used for these unusual measuring tasks appeared to reflect different emphases in the curricula of each country. The New Zealand Curriculum has informal measurement as the main focus of Level 1, which covers about the first two years of school depending on children's competence (Ministry of Education, 1992). New Zealand also emphasises visualisation or imaging as a major aspect of its Numeracy Project (New Zealand Ministry of Education Numeracy Projects, 2003). The visualisation required is not an estimate; children need to have an exact picture in their minds. For example, they might visualise a number ending in 9 on a number line, see that it is close to a multiple of 10 which is easier to calculate with, do the calculation and then carry out the appropriate compensation. Although we can never be sure what children are visualising, this exercise is given considerable time in class, in the hope that they will develop this skill. The curriculum in the Philippines does not emphasise visualisation and gives little emphasis to informal measurement (Philippine Department of Education, 2002). It treats measurement primarily as the use of tools. Tasks such as those presented here would be very unusual in the Philippines. This suggests that, without curricular emphasis on visualising, children rely more on external frameworks such as rulers for ways of judging and remembering length in the same way that older people do (Bryant, 1974, summarised in Vuyk, 1981). Accuracy is valued highly in Filipino schools, if not always in society. When the Filipino children were presented with Tasks 1 and 2 and asked to visualise, several drew units of the same size as the one presented in order to see how many would fit on the length to be measured. When asked to give the length of a curved string, they marked off units with their pencil and counted these.

Filipino children did equally well or better when tasks required understanding of a standard measurement tool (Task 3, starting with the length of a unit, and Task 5). Some Filipino children took much longer to complete the tasks than did New Zealand children. We assume that this was related to a desire to be accurate. This is likely to reflect the emphasis of the Filipino curriculum at the ages of these children.

One possible cause of differences in achievement between children in the two countries may relate to the achieved curriculum, or what actually is learned in the classroom, rather than the intended or mandated curriculum in the two countries. In the Philippines many schools have large classes and relatively formal teaching. This means that teachers are less able to spot weak or false understanding of underlying principles in measurement among their students than is the case in New Zealand classrooms, where there are usually fewer than 30 students. This difference in class size, and the resultant difficulty in teachers' ability to spot misconceptions, was the only possible difference between the two countries that we could identify that might be related to their differences in economic status.

Effect of Differences in Measurement Systems in Each Country

The extent to which imperial measures are still used in each country contributed to some differences in outcomes. Filipino children were baffled at the sight of the broken ruler that had big numbers on one side (mm) and small numbers on the other side (cm), as they were used to seeing inches on one side and centimetres on the other side of rulers. Some of them used the side with large numbers for long measurements and the side with small numbers for small lengths. When children from both countries gave names for units, those from both countries sometimes mixed centimetres and inches. For example, they might say that a length was 7 cm and 3 inches when they meant 7 cm and 3 mm. When drawing rulers in Task 3, four Filipino children and one New Zealand child drew a ruler that went up to 12, making their units small enough for this to fit. The authors and interviewers in New Zealand were particularly surprised to hear how often inches were mentioned, as these units have not been in the school curriculum for nearly 30 years. This is an example of culture holding on to a measurement despite government decree and changes in the curriculum.

Wider Cultural Influences

Wider cultural influences were harder to pinpoint, although some evidence was gained from the interviews. In the Philippines there is a statement, Pueda na yan, which indicates that something is good enough—that further accuracy is not needed. This phrase may reflect an attitude that could have influenced children's lack of care in informal measuring, but the influence was counterbalanced by the accuracy required in school tasks in formal measurement. The greater emphasis on informal measurement in New Zealand may delay the children's experience with formal measurement accuracy. In both countries the children's young age meant they were not given responsibility for accurate measuring, and thus were removed from the measuring requirements of trade or industry.

Implications for Educators

The research findings reported here have important messages for educators in both countries. The results indicate that the principles of measurement are difficult for children and require more attention than they are usually given in school. This result is similar to the findings of Bladen, Wildish and Cox (2000), Bragg and Outhred (2000), Hart, Johnson, Brown, Dickson, and Clarkson (1989), Kamii and Clark (1997), Nunes and Bryant (1996), and the results of the Third International Study of Mathematics and Science Study-Repeat (1999). Our findings support the view of the New Zealand teachers interviewed. They said that not enough time was spent on informal measurement, where use of measurement principles is evident. In the Philippines, informal measurement needs more emphasis. In both countries the transition from informal to formal measurement needs much more time and care.

We believe that it is important for educators to differentiate between visualisation that can be fairly exact, and visualisation used as an estimate. The visualisation that we assessed here, and see as an important skill, is a fairly accurate transposition, by eye, from one position to another. This was done well by nearly all children in Task 1 for which the number of units was small. It was well used on Task 3, drawing the units on a ruler, by some children who examined their units to be sure that they were equal in length. Some children used it in Task 4, when visualising the units that would come before the start of the broken ruler. It was also the main way to carry out Task 5, for which no tools were provided. This was a much more difficult task because the children had to visualise a curled string on a ruler, or the units of a ruler on this string. Unlike Task 1, it was not easy to visually line up the start of the object to be measured with the unit.

Certainly, there are situations where estimated measures in whole numbers are useful, as well as situations where accurate measures are important. Emphasis on one skill does not negate the other. Understanding the difference between these situations could be considered another underlying principle for measurement.

The findings from this study indicate that the transition to formal measurement needs to be built on concepts developed in informal measurement. Essential concepts to cover in this transition are an understanding of the fact that units are counted at their end. Thus, on a ruler the first unit ends with 1. Units for a measurement need to be of the same size. When units of different sizes are used, as with centimetres and millimetres, there is an exact relationship between these units. This relationship can be used to express fractional parts of a unit. Children need enough practice with units to be able to visualise how many of one would fit in a particular length with some exactness. The transition from informal to formal measurement can be aided by having children discover a body part of a known size which can be used to check formal measurement. We were particularly interested in the children who invented this transitional unit for themselves, and believe that educators could learn from their example. The exercise of drawing a ruler would be useful in this transition, as it requires creating equal units and writing 1, not at the left-hand end, but one unit in from the left. Having students carry out this activity would also enable teachers to see what underlying concepts still needed to be emphasised. In both countries, children would benefit from more emphasis on accuracy in measurement, especially the use of parts of a unit, an essential concept for continuous quantity.

Our experience in this study suggests that schools cannot depend on measurement in the wider community to teach children to use linear measurement, as they do not get enough experience for this to be so. This suggests that teachers should also set up some practical experiences for which exact measurement is essential, in order for children to value this exactness. Children should use these practical activities to help them build the concept of accuracy in formal measurement upon their knowledge of informal measurement.

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