*Mathematics Education Research Journal* 2004, Vol. 16, No. 1, 41-70

# Strategies Employed by Upper Secondary Students for Overcoming or Exploiting Conditions Affecting Accessibility of Applications Tasks[1]

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A cognitive/metacognitive framework is presented for analysing applications tasks and responses to these. Conditions facilitating or impeding access to such tasks at the upper secondary level were identified using qualitative data analysis techniques within this framework. Strategies employed in exploiting, or overcoming these conditions were identified. A well-developed repertoire of cognitive and metacognitive strategies together with a rich store of mathematical knowledge, real-world knowledge and experiences, and comprehension skills facilitated access. This was enhanced by metacognitive knowledge encouraging student engagement with the task and by students imagining they were in the task situation. Once moderate skill had been achieved in accessing these applications, coordination and integration of multiple representations, further cues, and mathematical processes and procedures became critical.

The use of applications in secondary classrooms for both teaching and assessment is receiving increasing attention in many countries, for example, Australia (Queensland Board of Senior Secondary School Studies [QBSSSS], 1992, 2000), the Netherlands (Gravemeijer & Doorman, 1999), Uganda (Ekol, 2004), and Canada (Roulet & Suurtamm, 2004). According to Blum and Niss (1991), there are essentially five arguments presented by supporters of their use. These are: (a) as a medium for developing general mathematical competence and attitudes; (b) as a means of developing students' social awareness; (c) to prepare students for using mathematics for problem solving or description in situations in other subject disciplines, occupational contexts, and actual, or future, everyday life-roles; (d) as an essential component of the broader picture of mathematics; and (e) for legitimating mathematical studies by applications providing motivation and relevance.

Ideally, the starting point for a mathematical application is a situation in the real world that can possibly be analysed and/or described using mathematics. This situation is then simplified to allow mathematical analysis and/or description by deliberately selecting particular variables, whilst ignoring others, and making appropriate assumptions. A *real model* that has to be *mathematised* (Blum & Niss, 1991) results: "Mathematization [sic] is the process from the real model into mathematics" (p. 39). Conditions and

[1] The research reported in this paper was supported in part by an Internal Research Award from the School of Education, James Cook University, QLD, Australia, and was conducted as part of my doctoral studies at the University of Queensland.

assumptions in the real model are translated into a *mathematical model*. *Model formulation* is the entire process from the real situation through the idealised real model to this mathematical model. This is the first phase of *mathematical modelling*. The remaining phases involve identification of relevant mathematical field(s); use of mathematical methods and insights to obtain results; translation of results back into the situation; and a “reality check” (Pollak, 1997, p. 102) to verify the results are practical, reasonable, and acceptable within the original situation. If verified, the results are communicated to potential users; otherwise, the model must be analysed for flaws and the process repeated. “What is different about real-world mathematical problem solving [mathematical modelling] is that the standards and mental processes of two masters – the real-world situation being studied as well as mathematics – are involved” (Pollak, 1997, p. 102).

Classroom application tasks are often much more modest in their “modelling” of the real world. Model formulation is usually curtailed to the last step or the mathematical model is given. The identification of the field(s) of mathematics of relevance to the model can also be considerably reduced as the appropriate mathematics to use can be fairly obvious if only applications related to recently studied work are set. Although the results are usually translated back to the original situation, the reality check is rarely more than cursory.

Embedding mathematical tasks in meaningful task contexts for both teaching and assessment purposes can be enriching according to Van den Heuvel-Panhuizen (1999). She claims task contexts do this by enhancing accessibility, revealing more about students’ abilities, giving students more latitude in their approach, and by providing students with solution strategies inspired by their imagining themselves in the situations portrayed. She acknowledges, though, that locating tasks in familiar contexts is not always supportive of students’ solution attempts and may create difficulties, particularly in assessment. Such difficulties include students challenging the intended mathematical interpretation of the problem by deliberately taking an alternate reading of the situation which is consistent with a plausible real-world scenario, or “ignoring the context” entirely and therefore excluding their “real-world knowledge and realistic considerations” (Van den Heuvel-Panhuizen, 1999, p. 137) from the solution process. Many researchers (e.g., Greer & Verschaffel, 1997; Reusser & Stebler, 1997; Verschaffel, De Corte, & Vierstraete, 1999; Verschaffel, Greer, & De Corte, 2000; Yoshida, Verschaffel, & De Corte, 1997), working in a variety of countries, have shown that across a range of contextualised problems students have predominantly answered without engaging realistic constraints when it has been prudent to do so. Other researchers raise further concerns. Cooper (1998) and Busse and Kaiser (2003), for example, point out that contexts are open to many interpretations. Furthermore, Lubienski (2000, p. 457) questions “the use of open, contextualised mathematics problems” as a means of “promoting equity for lower SES students”. In her study, lower socio-economic status students did not solve contextualised problems in ways that helped them learn more "powerful generalizable, methods" (p. 477) nor were they able to see that the same mathematical idea was being encountered in a variety of task contexts. These concerns are critical if they indicate that many students are facing insurmountable difficulties in accessing such tasks because of the contextual demand of these tasks.

Stillman and Galbraith (1998) infer from the first author's previous study of applications tasks, that applications teaching should focus on reducing the time students spend on orientation activities. This can be achieved by "developing cognitive skills that facilitate more effective problem representation and analysis, and by promoting the development of metacognitive strategy knowledge" (p. 185) to facilitate appropriate decision making during orientation. Kadijevich (1999) highlights the lack of focus in research studies on this important phase of an application-centered approach. He believes that the demanding interplay between students' cognitive, metacognitive, and affective domains that such an approach requires, has been neglected in both research and teaching. It is imperative, then, that the conditions affecting students' access to applications tasks and the cognitive and metacognitive strategies they employ to take advantage of conditions facilitating access or successfully overcoming conditions which impede access, be identified. In accordance with Flavell (1987), a *cognitive strategy* is taken to mean "one designed simply to get the individual to some cognitive goal or subgoal" (p. 23), whilst a *metacognitive strategy* is one used to monitor or regulate what is being done in a cognitive strategy.

## Cognitive / Metacognitive Framework for Analysing Applications Tasks

The cognitive / metacognitive framework (Figure 1) has been developed by the present author and used as the basis for generating an analysis system for examining the complexity of applications tasks (Stillman, 2002). The research reported in this paper comprises a small part of this larger study. Here the framework is used to identify conditions facilitating or impeding students' access to applications tasks through an analysis of responses to such tasks. As well as including students' cognitive and metacognitive resources, the framework incorporates cognitive technologies (Pea, 1987) such as graphing calculators, as these tools present opportunities for freeing up limited cognitive resources for other activities during task solution.

The framework consists of a set of information processing structures and associated resources; an operating system which perceives the stimulus cues in the task presentation, retrieves information from the external sources, working memory and long term memory, and operates on primary and secondary productions (Fong, 1994); and a monitoring and evaluation system. There are five main components:

(a) the external source (Fong, 1994) which is the problem statement and any visual cues;
(b) the working memory (WM) (Baddeley, 1997);
(c) long term memory (LTM) (Bruer, 1993; Fong, 1994);
(d) external working memory which includes processing units of calculating, graphing, and modelling devices; and
(e) external memory (Mayer, 1983) or secondary external source which can include pen and paper, calculator displays, or calculator memory.

As components (a), (b), and (c) are relevant to the study reported in this paper they will be elaborated further.

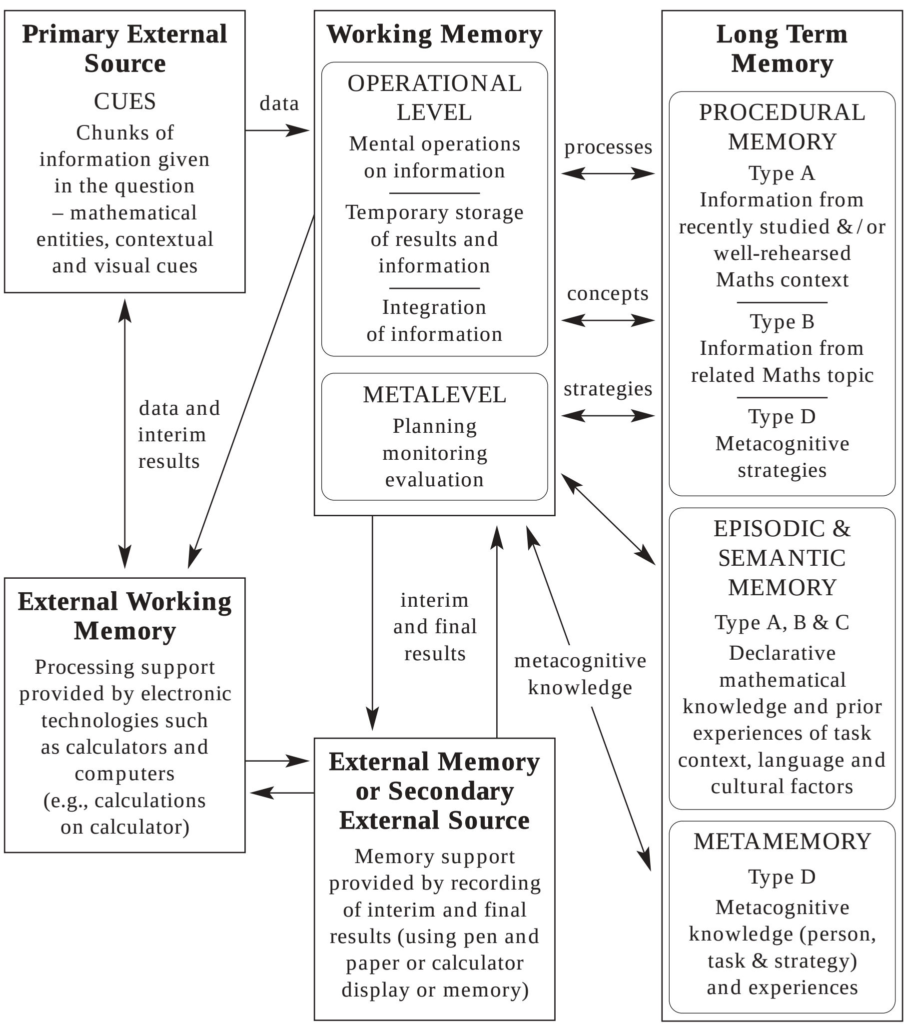


*Figure 1.* Cognitive/metacognitive framework for analysing applications tasks.

## *External Source*

The primary external source refers to the data or information that can be extracted directly from the problem statement or any accompanying visual representations such as tables, graphs, photographs, and diagrams. It is the primary source of information external to the problem solver and takes the form of mathematical entities, contextual cues, and visual cues. Not all data in the problem presentation have the same strength in cuing facts, concepts, processes, prior experiences, semantic knowledge, or metacognitive knowledge and strategies from LTM (Stillman & Galbraith, 1998). Wagenaar (1986) uses the term *cuing efficiency* to refer to the likelihood of retrieval of a specific memory as a function of the particular prompt that is used. This concept is of especial relevance here to the likelihood of retrieval of appropriate knowledge from LTM as a function of the particular piece of data presented in the task presentation. According to Kaplan and Simon (1990), the salience of specific cues for a particular task solver contributes to performance on a task. Cue salience interacts with the task solver's domain specific prior knowledge and knowledge of global strategies. Prior knowledge can have a beneficial effect constraining the search space for a task representation by ignoring irrelevant details and focussing on that section of the data that is critical for a solution. However, with tasks involving insight, prior knowledge can have the reverse effect leading to search in the wrong part of the problem space (Kaplan & Simon, 1990). Similarly, aberrant readings of a verbal task resulting from irrelevant prior knowledge interfering with the task solver's initial reading can result in a search in the wrong problem space. Although "it is not possible for a teacher to predict the many readings students may produce from a single text" (Peirce & Stein, 1995, p. 63), multiple readings are less likely in the highly ritualised social context of written test conditions in which these applications tasks are usually assessed because of the influence of students knowing "the customs of the classroom microculture" (Voigt, 1998, p. 192).

In applications tasks, cue salience and its interaction with prior knowledge are of critical importance. It has been suggested (Masingila, Davidenko, & Prus-Wisniowska, 1996) that secondary students may be unable to consider all the constraints in a realistic task situation, or keep track of them, due to not being exposed to many classroom tasks with realistic levels of constraints; however, a lack of anything in their experience that really could raise the salience of limiting constraints to a critical threshold is an alternative explanation. Prior experience in the real situation, rather than the vicarious experience of reading a verbal description of it, can raise the salience of a particular cue. This alternative explanation has been explored in the larger study and reported elsewhere (Stillman, 1998; 2000).

Not all task solvers are able to extract the same amount or the same type of information from a task presentation. In many experimental studies of the problem solving abilities of school children in the Soviet Union, Krutetskii (1962/1969, 1968/1976) found that "under identical conditions of perception of mathematical material, pupils with different abilities in mathematics obtain information of a different nature” (Krutetskii, 1962 / 1969, p. 81). Furthermore, the more capable problem solvers always acquired more information and had “an ability to actively extract from the given terms of a problem the information maximally useful for solution” (Krutetskii, 1968 / 1976, p. 233).

Successful problem solvers, therefore, not only need an ability to be able to gather relevant information from the task, but also to selectively focus during this information gathering process, so as to select only a minimal subset of that information in an efficient and mathematically perceptive manner. Sometimes this selective focussing on the available information needs to be done iteratively rather than on a single pass through the data. If only “maximally useful” information is extracted and attended to, students can then effectively reduce the demand on their working memory as resources related to attention are not wasted in focussing on extraneous data.

This ability of successful problem solvers to extract, by whatever means, “maximally useful” information may be related to how they perceive the data in the task presentation. According to Broadbent (1987), “incoming material is ‘chunked’ by recognising a sequence of several items as forming a familiar unit” (p. 182). In the classroom, students’ ability to chunk information relates to the conceptual and procedural knowledge base they have developed through their prior classroom experiences. For example, senior secondary students studying calculus could reasonably be expected to perceive $A'(t) = 25t - c$ as one composite mathematical entity which they can chunk, whilst a student at the beginning of secondary school would be more likely to perceive and process the individual symbols separately. Krutetskii’s (1962 / 1969) observations support this. Language and cultural factors also contribute to this differential ability to extract relevant information from the task presentation.

Theories of problem-solving processes and of language comprehension processes unite in analyses of written mathematical task representations. For example, situation-based reasoning plays a central role in Nathan, Kintsch, and Young’s theory (1992) of algebra word problem comprehension incorporating Kintsch’s (1988) model of text comprehension which is specifically designed to cope with the effects of task context. These authors view the comprehension and solving of word problems (and by extension, applications tasks) as entailing the construction of three mental representations of the problem: (a) a *textbase* representing textual input, (b) a *situational model* of the events described in the problem statement and inferred or elaborated from it using the task solver’s general knowledge base, and (c) a model of its mathematical structure (the *problem model*). Accordingly, “a complete theory of problem solving must include the language comprehension process, the resulting mental representations, the role of inferences and real-world knowledge, and the necessary formal calculations for deriving a solution” (Kintsch, p. 331).

To understand a task completely, students must possess sufficient relevant knowledge in LTM to adequately understand the situation described, and appropriate strategies to generate necessary inferences and elaborations to understand the situation fully. Such tasks are highly reading-oriented. Lack of relevant prior knowledge and poor text comprehension lead to serious errors (Nathan et al., 1992). According to the Nathan et al. (1992) model, omissions from written mathematical models constructed by students “match information that is unstated in the text but necessary for complete understanding of the situation” (p. 334), whereas misspecifications in the students’ models are often due to miscomprehension of the text or misapplication of mathematical procedures.

## *Working Memory*

The working memory system is vital for understanding how students perform on complex tasks such as solving mathematical applications. This is because task solvers must combine temporarily remembering information such as problem states, goals, interim results, and appropriate strategies, with processing and coordination of this information (Monsell, 1984). Working memory contains the information that can be accessed currently. This can include input from the sensory system, information retrieved from long term declarative memory (i.e., the storage for facts, concepts, and beliefs), information produced from the primary external source, or secondary productions of information from processed information or secondary external sources (Fong, 1994). Differences among theoretical working memory models provide different predictions for the effects of practice of procedures or skills, irrelevant and redundant information, and use of differing modalities (e.g., visual or auditory) on the load on working memory.

Carlson, Khoo, Yaure, and Schneider (1990) have found that “use of working memory in problem solving changes substantially with practice, but the nature of that change cannot be characterized as a simple reduction in storage or processing demands” (p. 213). At first, all information for task performance must be loaded and maintained in working memory by control processing and temporary context storage. At modest levels of practice, permanent knowledge about the task should reduce the need for control processes to load working memory. Only after extended practice are automatic processes sufficient for loading and maintaining information, but attention is still needed to access and manipulate information. The results of Carlson et al. (1990) “support the view that coordinating or integrating representations is a separable component of complex skills” (p. 214). This is in keeping with Schneider and Detweiler’s (1988) claim that, once a modest degree of skill has been achieved through practice, the central constraint on performance is coordination of multiple items of information.

Carlson, Sullivan, and Schneider (1989) review predictions for the impact of irrelevant information on the load on working memory. In single workspace models (e.g., Atkinson & Shiffrin, 1968), if this information is stored in working memory these models predict interference even if the knowledge is not accessed because activation has to be distributed over all elements. In a distributed model (e.g., Baddeley, 1997; Schneider & Detweiler, 1987), no interference is predicted if the information is merely stored and not accessed, as separate subsystems are available for storage and for processing. If the irrelevant information is held in working memory and accessed, cost of access is minimal for single workspace models as storage and processing take place in the same cognitive workspace, and only coordination of representations is required if the data are in different forms such as a graph and text. With distributed systems, access costs are high as coordinating representations requires establishing communication between separate subsystems or regions.

Similar effects are predicted for redundant information. Sweller (1992) claims that "when redundancy occurs integration has negative consequences" (p. 59). He bases this on a non-mathematical research study (Chandler & Sweller, 1991) in which it was found that the use of a diagram and a verbal description, when both sources of information could stand alone, had negative rather than positive or neutral effects. At the very least, this work indicates that redundancy cannot be considered to be neutral. The redundant information acts as a distractor using up cognitive resources needlessly as task solvers mentally integrate information.

It appears that coordination and integration of information and representations, and allocation of attention resources are of vital importance to the efficient functioning of WM, especially when solving tasks such as mathematical applications which potentially require skills of different degrees of practice, contain irrelevant and redundant information, as well as a variety of representations in the task presentation. The role of a central executive in controlling and regulating these cognitive activities appears compelling. At the metalevel then, it is argued that the framework must include a monitoring and evaluation system, and that the metacognitive activity that oversees these cognitive processes occurs in the WM. Executive control decisions directed at controlling or monitoring cognitive activities (such as planning how a task is going to be approached) may initiate metacognitive strategies to monitor cognitive progress (Kluwe, 1987) or be informed by metacognitive knowledge recalled from LTM. *Executive regulation decisions* deal with the regulation of processing capacity, what is being processed, processing intensity, and the speed of processing (Kluwe, 1987). Regulation decisions do not necessarily result in changes in cognitive activity, as it may be decided to continue with the current activity.

In terms of the cognitive/metacognitive framework (Figure 1) then, working memory is characterised as having an *operational level* and a *metalevel*. At the operational level, mental operations are performed on information, temporary storage is provided for the results of such operations and other information, and the integration of information and representations is performed. At the metalevel, coordination of the operational level actions occurs through planning, monitoring, and evaluation decisions.

## *Long Term Memory (LTM)*

Long Term Memory is no longer considered to be a single storage system but is believed to be characterised by a number of systems which serve different purposes and operate differently but not necessarily separately (Tulving, 1985). Tulving (1985) identifies three systems (procedural, semantic, and episodic) but concedes there are probably many more. *Procedural memory* is the system for storing knowledge of what can be done with facts, concepts, and episodes as opposed to knowledge of what these entities are. It consists of knowledge that has become automatic through practice (Bourne, Dominowski, Loftus, & Healy, 1986). The contents of procedural memory are not always open to conscious recall, nor can they always be expressed or described (Taylor, 1991). *Semantic memory* stores general, encyclopaedic knowledge of the world, and word meanings. The distinguishing feature of this information is that it is not linked to an individual or a particular time or place. *Episodic memory* is a memory system based on knowledge gained by an individual through their own experience. It is, therefore, personal and linked to events that happened at a particular time and place. Both episodic and semantic memory are available for conscious recall during task solving and can be readily expressed and described in interviews.

Two types of memory are involved in problem solving and higher-order cognition—procedural and semantic memory (Bruer, 1993). However, as task context usually plays an important role in applications tasks, episodic memory may also play a significant role, as prior knowledge influences both what students notice in a task and how it is interpreted. Information stored in both semantic and episodic memory is the relevant knowledge in LTM that Nathan et al. (1992), in their task comprehension model, refer to as necessary for understanding a task.

Two types of mathematical knowledge are stored in LTM according to Fong (1994): (a) information related to the recent content area of study or information which is well rehearsed, and (b) knowledge of related mathematical topics. Fong calls these *Type A* and *Type B knowledge*, respectively, claiming that the former is more readily retrieved than the latter. In keeping with this classification, two further categories are proposed. *Type C knowledge* includes memories of prior knowledge of the task contexts of applications of an encyclopaedic or experiential nature, word meanings, and cultural knowledge including classroom culture. In most circumstances, it is suggested that *Type A* knowledge is easier to retrieve than *Type C*. *Type D knowledge* includes metacognitive knowledge about the person, task, or strategies, as well as metacognitive experiences and knowledge of strategies (Flavell, 1979). In the framework being proposed (Figure 1), LTM is shown as consisting of: (a) procedural memory which stores both *Type A* and *B* mathematical knowledge as well as *Type D* knowledge in the form of metacognitive strategies, (b) *episodic and semantic memory* which stores declarative mathematical knowledge of both these types as well as *Type C* knowledge, and (c) *metamemory* which is designated as storing *Type D* knowledge consisting of metacognitive knowledge and experiences.

# The Study

This article reports part of a much larger study which investigated the effects of context on students' approaches to, and performance on, applications tasks in the main pre-tertiary mathematics subject in Queensland secondary schools, Mathematics B (QBSSSS, 1992). Specifically, this part of the study addressed the following research questions:

1. What conditions facilitate or impede students' access to applications tasks? These conditions may be: (a) inherent in the nature of the task, or (b) conditioned by the local situational context (e.g., classroom practices or teacher idiosyncrasies).
2. What strategies are employed by students to: (a) overcome accessing difficulties, or (b) take advantage of ease of access? Of the strategies, which are: (a) cognitive, or (b) metacognitive in nature?

## *Participants*

The study was conducted in two public high schools in a large provincial city. Data collection occurred in four phases over a six month period during which contact with both schools was continuous. Forty-one students in the last two years of high school (Years 11 and 12) participated. All students were studying Mathematics B which is a two year integrated course in which students study: applied geometry; trigonometric, periodic, exponential, and logarithmic functions and their applications; introductory calculus and its application; networks or linear programming; financial mathematics; and applied statistical analysis. "The intent of Mathematics B is to encourage students to develop positive attitudes towards mathematics by an approach involving problem solving and applications" (QBSSSS, 1992, p. 2). Through the study of mathematical applications in life-related situations, students are expected to "develop a set of procedures to be used in approaching modelling problems" (QBSSSS, 1992, p. 7).

# Method

## *Administration of the Tasks*

All students were videotaped as they attempted to solve applications tasks individually, with each student completing up to four tasks. Students were able to ask for clarification of words if needed. Examples of the tasks are included in the appendix to this article. These tasks differed in terms of familiarity, complexity, and degree of contextualisation. The requirement for *unfamiliarity* is that either "the context is unfamiliar while the techniques are familiar" or "familiar techniques are combined in an unfamiliar way". Unfamiliarity implies that a student has to interpret the context and to decide which techniques are to be applied to solve the problem. The choice of techniques may not be immediately apparent to a student" (QBSSSS, 1994, p. 1). The *complexity* of the task "depends on how obvious the choice of techniques is, the number of techniques required, and the amount of guidance given to students" (QBSSSS, 1994, p. 1). The term *degree of contextualisation* is meant to convey the range of embeddedness that exists between the mathematics that can be used to model a situation and the description of the situation. This can range from the simple case where there is virtually no integration of the model within the context, the context merely acting as a border surrounding the mathematics which can be readily removed without loss of meaning (e.g., Microwave Ovens Task, see Appendix), to the situation where the two are totally integrated and separation becomes difficult as the mathematics derives its meaning from the context (e.g., Tide Task, see Appendix). Although the majority of the tasks used in the study were closed rather than open-ended, the nature of some tasks provided scope for students to move in the direction appropriate to their perception of the problem. This occurred when tasks were more like true modelling tasks than mere applications, or a fair degree of leeway was allowed in students' interpretation of the task.

## *Student Interviews*

In total, 64 semi-structured, stimulated recall interviews were conducted and recorded immediately following task completion. Some students did several tasks. The videotapes of their task solving sessions were reviewed by the students, in conjunction with the script of the task, during the interview. Students were also asked to draw diagrams, when appropriate, to illustrate their understanding of the task context and the task goal. Use of the videotaped task solving sessions as a visual stimulus throughout the interviews allowed both the interviewer and the interviewees to track the students' developing understanding of the task context by discussing their changing perceptions of the task, as observed in their changing use of diagrams throughout the review of the task solving session.

## *Analysis*

All interviews and task solving sessions in which there were interactions between the student and the researcher were transcribed. The interviews were analysed using the qualitative data analysis software QSR NUD*IST (Qualitative Solutions and Research, 1997). NUD*IST facilitates 'grounded theory' construction (Strauss & Corbin, 1990) which attempts to capture and interrogate the meanings emerging from data. This is achieved by constructing and exploring new categories and themes as they arise from the data, then refining these through a "process of progressive category clarification and definition" (Tesch, 1990, p. 86). This was done using a variety of matrix displays.

Thematic conceptual matrices (Miles & Huberman, 1994) were used initially to show *conditions* that facilitated or impeded task accessibility by year level at School A, and the *strategies* that were used by students to overcome, or avail themselves, of these conditions. An illustrative row of such a table is shown in Table 1. The *conditions* entries in these matrices were summary descriptive phrases, together with representative illustrative quotes from the interviews to clarify their meaning, and indicators of the extent and nature of multi-case activity for each cell in the table. The last of these was shown by case numbers and references to illustrative NUD*IST text-units (e.g., A06[Task 3] 15-21) from the interviews, or data from the task tapes. All conditions included were confirmed by at least one case at the site and were not contradicted by other cases. The *strategy* entries in these matrices were interpretations of the behaviour observed during the task sessions (indicated by the symbol, R), or statements by the students about their thinking and actions during the interviews.

Table 1
*Conditions and Strategies Facilitating Task Accessibility at School A (Year12)*

| Conditions Facilitating Task Accessibility | Facilitating Strategies | |
|---|---|---|
| | Cognitive Strategies | Metacognitive Strategies |
| Recognition of mathematical cues – trigger words and visual features<br>"Like when I saw the k sort of thing, the constant there, I knew that I had to sort of find that to work out the rest of the question."<br>A06(Task3)15-21;<br>A09(Task5)14-16;<br>A13(Task5)12,67-70;<br>A17(Task5)12-14;<br>A18(Task7)76-80,112-112;<br>A48(Task5)13-17 | Uses perception (Procedural skill). | Regulates what is being processed by deciding to focus attention on particular stimuli and not others. (R)<br>Confirms matching of selected stimuli with key elements in mental schema. (R) |

*Note.* The numbers following the task numbers are the text units of the NUD*IST documents. (R) means inferred by researcher.

In order to focus on the content of the matrices and to view the data at a more conceptual level to elicit the main trends across cases, a content-analytic summary table (Miles & Huberman, 1994) was produced, combining the *conditions* data from the matrices for both phases of the data collection at School A (i.e., the Year 12 data and the Year 11 data). The new tabulation deliberately dropped the case identification of data and the entries were reduced to the summary phrases. To confirm the applicability of these conditions to similar settings, the data from School B were categorised and displayed in thematic conceptual matrices as before. A cross-site content-analytic table was then produced where the entries consisted only of conditions that were confirmed at both sites together with their frequency of occurrence.

## Results

### *Conditions Affecting Task Accessibility*

As seen in Table 2, there were eight conditions facilitating task accessibility which occurred at both sites. These have been grouped into three clusters in order of frequency of occurrence: (a) Memory-related facilitating conditions (39), (b) perceptual conditions (27), and (c) engagement conditions (14). Thirteen conditions were confirmed as impeding task accessibility at both sites. These have been clustered in Table 2 into five groups in descending frequency of occurrence: (a) Language-related conditions (63), (b) representational conditions (40), (c) memory-related impeding conditions (32), (d) organisational conditions (25), and (e) task-specific conditions (6).

The major conditions (incidence rate ≥ 10) facilitating access to the applications tasks used in the study were the recall of a similar or parallel task, and prior knowledge of the task context (memory-related conditions); the recognition of mathematical cues in the form of trigger words or visual features, and being able to visualise the situation (perceptual conditions); and engagement with the task context (an engagement condition). Minor facilitating conditions included the mathematics involved in the solution of a task being well rehearsed, metacognitive knowledge that encouraged engagement with the task (memory-related conditions), and intuitive instantaneous recognition of mathematical cues (a perceptual condition).

By far the most frequently reported conditions impeding task access were: language problems related to technical language used in the context, comprehension difficulties, and the unusual wording of tasks. A factor contributing to the high incidence of this condition was the deliberate use of longer verbal tasks in the study than the students were used to in the classroom. Other major conditions (incidence rate ≥ 10) impeding task accessibility were cuing words (of either a mathematical or contextual nature) not being salient (a language-related condition); all the representational conditions, namely, inability to mathematise the context, difficulties with the integration of given or derived contextual information, and difficulties extracting mathematical information from the task context; difficulties recalling a concept, formula, procedure or relevant prior knowledge of the task context (a memory-related condition); and the organisational condition involving difficulties formulating a plan of attack. Minor impeding conditions included difficulties establishing a global goal for the task, lengthy task statements, metacognitive person or task knowledge discouraging access, the mathematics involved in a task not

Table 2
*Cross-Site Content-Analytic Summary Table: Conditions Affecting Task Accessibility*

| Facilitating Conditions | Impeding Conditions |
|---|---|
| *Memory-related Conditions* | *Language-related Conditions* |
| Similar or parallel task recalled (19) | Language problems (37) |
| Prior knowledge recalled (18) | Cuing words (mathematical or contextual) not salient (19) |
| Maths involved is well rehearsed (6) | A lot of words in task statement (7) |
| Metacognitive knowledge encouraging engagement with task (2) | *Representational Conditions* |
| *Perceptual Conditions* | Can't mathematise task context (18) |
| Trigger words and visual features recognised (14) | Difficulties extracting mathematical information from task context (10) |
| Being able to visualise the situation (10) | *Memory-related Impeding Conditions* |
| Instantaneous recognition of mathematical cues (3) | Difficulties recalling a concept, formula, procedure or relevant prior knowledge (15) |
| *Engagement Condition* | Metacognitive knowledge discouraging access (8) |
| Successful engagement with task context (14) | Lack of recency of maths involved (6) |
| | Interference from prior knowledge (3) |
| | *Organisational Conditions* |
| | Difficulties formulating plan (16) |
| | Difficulties establishing task goal (9) |
| | *Task-specific Conditions* |
| | Maths not obvious (6) |

*Note.* The numbers in brackets are the number of cases across sites that met this condition. Only conditions that have been confirmed at both School A and B have been included in this table.

being recently studied (i.e., more than one month since last studied), the fact that the mathematics involved in particular tasks was not obvious, and interference from prior knowledge. A fuller description of these conditions is provided in Stillman (1999, 2002).

## *Strategies Used to Exploit Conditions Facilitating Task Access*

Several strategies were identified that students in the study were able to use unassisted to take advantage of the conditions that were identified in the cross-site analysis as facilitating task access.

*Strategies for benefiting from facilitating memory-related conditions.* Students used retrieval strategies to recall all four types of information identified in the Cognitive / Metacognitive Framework (see Figure 1) in order to benefit from memory-related facilitating conditions. This is illustrated in the following examples.

I: (Petri Dish Task 1) You persisted a lot more than you did with the other one [involving probability].

Jan: Yeah, maybe, it's just because we did probability ages ago and the unit *that we have just been doing* is on that. [Recall of *Type A* information (information from a recently studied mathematical topic).]

Ben: (Road Accident Task) I dredged into my memory *and I remembered a bisector of chords goes through the centre point*. [Recall of *Type B* information (information from a related mathematical topic but not usually seen in the context of the question).]

I: (Road Construction Task) And what was at the top of the bank?

Kit: Umm, some existing properties.

I: And what did you interpret that to be?

Kit: *People's blocks like fence lines* coming right to the top of the slope. [Recall of *Type C* information (word meanings and general, encyclopaedic knowledge of the world).]

Jan: (Petri Dish Task 2) Well, *if I've got a question, a Maths question, that has got a simple first part to it … that's enough to get me into the context* of the question. [Recall of Type D information (metacognitive task and person knowledge from metamemory).]

The only metacognitive activity reported in relation to this cognitive activity was Ben's local assessment of his state of knowing in the Road Accident Task when he had to reconcile the prior knowledge of the task situation he had recalled with the description of the task context.

Ben: (Road Accident Task) *That was what I was worried about*. How can he brake when his brakes had failed? But they obviously failed here [points to end of skid mark].

The researcher inferred that the choosing of a mathematical strategy on the basis of having seen a similar or parallel task would have required some comparative judgements to be involved and these constitute meta-cognitive activity.

*Strategies for benefiting from facilitating perceptual conditions*. Cognitive strategies that were used to take advantage of facilitating perceptual conditions included recognition strategies, mental imaging strategies, and perception.

I: (Shaft Task) Uhmm, now *you saw that straight off* as a derivative, why is that?

Rob: Oh, when I saw it was D, ah, …

I: D prime or D dash?

Rob: Yeah, D dash t, I knew that was the first derivative of the function.

Ann: (Ecosystem Task) I couldn't *visualise* that one [referring to Road Construction Task] but I could [this one] … *I could actually know what was going on*.

Jo: (Road Construction Task) Umm, I knew it had something to do with triangles straight away when I looked at it.

I: Why was that?

Jo: Because it was talking about angles and just when I read it *the picture I got in my head* … just talking about slopes and everything it formed a triangle sort of thing.

There was one reported example of a student monitoring the use of her perceptual skills by making a local assessment of the accuracy of her instantaneous recognition of mathematical cues.

Tui: (Fertilizer Task) I thought from the first, 'Ah, it's Simpson's Rule' but I was just *trying to convince myself* that it was. Intuition.

On other occasions, however, this phenomenon appeared to be automatic with students not being conscious of having thought about their thinking as Taylor (1991) would have predicted.

The researcher inferred that the cognitive activity described above had the potential to initiate metacognitive activity to regulate processing capacity, as executive decisions would need to be made about the amount of information processing capacity allocated to various forms of processing (Kluwe, 1987). For example, when mental imaging strategies were being used the student would have decided that precedence be given to visual processing. Recognition strategies would also involve executive decisions about the regulation of what was being processed, as the student would have to decide to focus on particular words or phrases or visual features at the expense of others and confirm the matching of these with key elements of schema recalled from memory.

*Strategies for benefiting from facilitating engagement conditions.* Engagement with task context was facilitated by retrieval strategies to assist in the recall of prior knowledge of the task context (*Type C* information in Figure 1) and strategies such as elaboration to facilitate the integration of contextual and mathematical information. The use of elaboration and the recall of prior knowledge are evident in the following excerpt from the task-solving session with Kay, when the researcher has been asked for assistance with the Road Construction Task. The student is asked where the new lane might be placed in the drawing she has made of the situation. She suggests two possibilities and by listening to "the voice of 'practical reasoning'" (Wistedt & Martinsson, 1996, p. 178), obviously informed by her previous experiences, correctly convinces herself that the second interpretation is the better fit for the situation described in the problem statement (see Appendix).

I: (Road Construction Task) Now, where … If you are going to put this new lane in, where will the new lane be?

Kay: It would have to go like *on the slope* would it not? Or *would they have to drill out?* Yeah, they would do that. So they would knock that out and make that.

Whilst reviewing her task-solving videotape after the session she elaborates further and it is obvious she sees her engagement with the task context at this point as being crucial to her drawing a mathematical diagram and solving the task.

Kay: And then you had to figure out like the highway would cut off because you would have to *blow up some of the ridge*. So, I knew the situation now with the bank and everything. I just wanted to label everything so that I understood what it is.
I: Uhmm.
Kay: And I knew that you had to build the lane.
I: Yes.
Kay: And blow it up. So then that's when I figured out, 'Ok, so then that's what we have to do!' And then I drew the diagram.

## *Strategies Used to Overcome Conditions Impeding Task Access*

Strategies that students in the study were able to use unassisted to overcome the conditions that were identified in the cross-site analysis as impeding task access were also identified.

*Strategies for overcoming language-related problems.* Language related conditions elicited the recall and application of a variety of comprehension strategies such as re-reading, drawing a diagram, visualising, underlining key words, selecting key features and points, prioritising information, and processing "bit by bit". Re-reading was by far the most commonly used of these strategies.

Kay: (Road Construction Task) I'd *read through that about a hundred times.*
I: Uhmm.
Kay: Just to try and figure out what it said and I knew that this was the situation and I hadn't messed it up finally.

Re-reading was not always successful by itself, so students used diagram drawing, visualising, and information organising strategies to increase its effectiveness.

Amy: (Road Construction Task) I had to read it like ten times and still I am thinking, 'All right, so how do I do this?' But *after I drew all the pictures* and stuff I thought, 'Okay, this has got something to do with Trigonometry.'

Greg: (Road Construction Task) [After interview, commenting on why it took him so long to access the task.] *You had to get a picture in your head!*

Bill: (Shed Task) At the start I thought it was going to be difficult because I think I looked too hard and I couldn't understand it.

> And was hard until I drew a diagram and then it was easier. Umm, I think *after I just drew the diagram then I was just … I understood it*.

Strategies designed to organise information into a form that could be more readily processed concentrated on reducing the amount of information that had to be processed. This was done by selectively focussing attention on what was considered important and ignoring the other data in the task statement.

> Amy: (Road Accident Task) Most of the *information at the start* is kinda basically telling you what's happened, like an *introductory* sort of thing and then the information as you get *towards the end is the more important stuff*. I didn't really read the start too much, probably twice and then I read through the vital information.

An alternative strategy designed to assist with the organisation of information for processing did not reduce the total amount of information processed but focussed on the speed at which the processing was being done by concentrating on a bit at a time.

> I: (Road Construction Task) When you first saw the question what was your initial reaction to it?
>
> Ann: I'm thinking … umm … I thought, 'Well, I'd better *go through this bit by bit to understand it*.'

It was inferred by the researcher that cognitive activities such as re-reading, drawing a diagram, and visualising could initiate metacognitive activity of a regulatory nature. In particular the regulation of processing capacity involves executive decisions about the amount of information processing capacity that needs to be allocated to the various mental operations involved, that is, reading, creating a visual image, and those associated with the transformation of the mental image into a drawing. There was the potential for executive control decisions to be involved as students monitored the state of their understanding of the problem during re-reading of the task statement. Information organising strategies, as mentioned above, also had the potential to elicit metacognitive strategies related to the regulation of what was being processed in which order, whilst deciding to process "bit by bit" was expected to involve metacognitive strategies designed to regulate the speed of information processing.

*Strategies for overcoming representational problems.* Cognitive strategies that were used to overcome representational problems arising from students not being able to mathematise the task context or to extract mathematical information from the task context consisted of selectively focussing attention on relevant aspects of the problem presentation by re-reading, visualising, or re-drawing a pictorial representation of the task situation.

> Sue: (Petri Dish Task 2) … until I read this last sentence, when I started reading, 'Oh, that was the rate', umm, not the rate, 'the area'. You like got given the rules, yeah. Then when I like got to this sentence and I re-read it I realised, I thought of that first. It was just like, it just seemed logical …

Bea: (Petri Dish Task 1) So, I thought the growth rate must have been like a derivative or something.
I: Yes.
Bea: And then I thought *I was imagining the function of it.*
I: Yes.
Bea: And then I was thinking, 'Well, what is the gradient? The gradient is the rate of change and that is actually the growth rate.' So.
I: Yes, yes.
Bea: So, that would have been the derivative of it.

I: (Road Construction Task) Now up in the top corner what you were saying was, 'Now, I'm not certain I've got this,' so you were going back writing and *drawing the picture of what you thought was happening?*
Alice: Uhmm [agreeing].

In order to overcome representational problems resulting from difficulties integrating given or derived contextual and mathematical information, students used cognitive strategies such as constructing a mental representation or creating a visual representation such as a diagram or graph where they were forced to integrate the various pieces of data.

Bill: (Shed Task) Umm, there I am just reading the thing. Just reading the question and just drawing the diagram *trying to match the question up* just so it is easier to visualise.

Use of these cognitive strategies instigated a number of the reported incidences of metacognitive strategies being used to monitor cognitive activity. These included self questioning to provide the student with information about the status of their cognitive activity at a particular point in the task solving process, using visualising to check the progress of the results of cognitive activity and evaluating the adequacy of actions when trying to mathematise a context.

Sue: (Petri Dish Task 2) What exactly the question was asking and like, '*Where am I going? Why was I …*'
I: *Heading in the particular direction?*
Sue: Yeah.

Elle: (Hockey Task) Well, *I was pretending I was a hockey player and, umm, you know the angle you had to view the goal.*
I: Yes, and that was useful in checking what you were doing, was it?
Elle: Uh-huh, yeah.

Kym: (Shed Task) *I thought, 'That can't be right'* because, you know, it is not joining the side. You know, I thought, 'Well, it has to be a triangle.'

The researcher inferred, however, that the cognitive activity mentioned also had the potential to initiate metacognitive activity that was regulatory. This may have involved the regulation of processing capacity by making decisions about how much information capacity was to be allocated to reading, visual imagery and the transformation of a mental image to a written representation such as a diagram or a graph. The integration of information would also be expected to involve some sort of coordination of different representations in working memory (Carlson et al., 1990).

*Strategies for overcoming memory-related problems.* Retrieval strategies were mainly used to overcome memory related impeding conditions, although in a lot of cases these problems were not overcome. Tom, for example, was having difficulty remembering his trigonometrical ratios. He decided to recall all of the trigonometry he could, then, by a process of elimination, figured out which concept applied.

Tom: (Rugby Goal Task) I was trying to, umm, I was *trying to remember all the trigonometry that I had done before.*
I: Yes.
Tom: Because I haven't done it for … I haven't done it for a month or something like that.
I: Yes. And that's what you were … you were looking at all the trig ratios and *trying to work out which one was relevant?*
Tom: Yeah.

Mary's belief that tides repeated every 12 hours, recalled from her encyclopaedic knowledge of the world, interfered with her use of the data given in the Tide Task until she was able to overcome this by recalling the problem conditions and correcting her error.

Mary: (Tide Task) It was 12 hours 30 minutes and *I would have assumed it was 12.*
I: Now what did you have to rub out for?
Mary: I just plotted the point in the wrong spot.
I: What told you it was out of place? How did you realise it was out of place?
Mary: Umm, *I just reminded myself it was 12 and a half hours and not 12.*

The recall of metacognitive task and person knowledge (Flavell, 1979) that discouraged task access was combated by strategies that were specific to the particular case involved. Jenny, for example, was reluctant to begin the Ecosystem Task because of the format of the task (one A4 page of text and diagrams) which was not her preferred style, but, after recalling and applying comprehension strategies, she chose to ignore this and began to solve the task.

Jenny: (Ecosystem Task) When I got down here I realised what it was on about and so I picked it up.

This would also have involved an evaluation of her confidence in her understanding of the task. Similarly, Rob, who was approaching the Shaft Task with caution because it involved derivatives which he felt he was not very adept at using, chose to ignore this when he found he was proceeding with ease after tentatively beginning the problem.

Rob: (Shaft Task) After I got started on this one I found it to be fairly easy to complete.

*Strategies for overcoming organisational problems.* Both cognitive and metacognitive strategies were in evidence when students attempted to overcome difficulties experienced in formulating a plan of attack. Bob recalled and applied the strategy of drawing a diagram in an effort to formulate a plan.

I: (Road Accident Task) What were you thinking about possible plans of attack before you started doing that?

Bob: I had no idea. I thought *if I drew this something might come into my head.*

Greg was more successful formulating a plan for the task he was attempting, however. He used an information organising strategy which enabled him to focus on particular aspects of the task and thereby produce a viable plan.

I: (Ecosystem Task) Now, at this stage, had your thoughts crystallised as to what you were going to do with this?

Greg: Sort of … I thought that maybe *if I wrote it down that maybe it would be clearer to me.*

I: So, you write it all out and then … ah … Now, I think you are sitting there and having a good think before you start doing anything. So, at this stage what are you thinking of doing?

Greg: Awh, I was trying to actually see if I could put what I had written down into like a formula type thing.

Metacognitive strategies were brought into play by another student trying to select possible techniques to use and a global strategy to begin. In the Ecosystem Task, Ann made a local assessment of her confidence with the use of a particular strategy before rejecting it.

Ann: (Ecosystem Task) I thought I might have a look at the proportion of how much because it said, 'Will it still stay the same?' Umm, I was having a look at the proportion of each. How much they've dropped down to and how much they have gained there and then I thought, *'I am not too confident working in this area so I'll try something else.'*

She also rejected an alternative plan after evaluating its time effectiveness.

Ann: … from that stage I was going to work out each division that he had lost on the way down but then *I figured that that would take too long …*

In the Hockey Task, she used verbal mediation to decide on using a divide and conquer strategy to attack the problem.

Ann: (Hockey Task) I'm thinking, *'I know how to do this but I have to work out which part I have to do first and then work out the other parts.'*

None of the strategies that students tried to employ to overcome difficulties establishing a global goal for the task were successful.

*Strategies for overcoming task-specific problems.* The one task specific condition that occurred at both sites was the emergence of problems that developed from the fact that the mathematics to be used was not obvious in tasks such as the Road Accident Task, the Births Task, and the Fertilizer Task. Only one student, Tui, was successful in overcoming this using a mixture of perception and visualising.

> Tui: (Fertilizer Task) I had to think a bit. I thought. I just read it and *tried imagining it in my mind* what it was saying. And then I thought, *'It looks like an area problem. Find the area underneath the graph.'*

Later she drew a graph of the table of values to confirm her intuition that she should use Simpson's Rule to find this area because, although she had recognised it as an area problem almost immediately, she still had to work out what mathematical techniques she could use to solve it and she did not think this was obvious.

> Tui: … you have to work out what you are supposed to be doing and this one doesn't exactly tell you what you are supposed to be doing … actually I realise *why I was drawing the graph*. I was seeing whether it was linear or not. If it was linear I was going to do … to do trapezoidal. If it was like this I was going to do, umm, extended Simpson's and it was obviously like that so I was going to do extended Simpson's. Actually, actually I thought … *I thought from the first, 'Ah, it's Simpson's rule!' but I was just trying to convince myself that it was.*

## Discussion

The cognitive/metacognitive framework in Figure 1 proved useful in identifying and examining the conditions that facilitated or impeded task access for the students in the study through an analysis of students' responses to the tasks. When the conditions facilitating task access are examined some conditions appeared to reduce the difficulty level of a task for particular students whilst others contributed to reducing the complexity of the task. The conditions that reduced difficulty were either personal in nature (e.g., being able to visualise, possessing metacognitive knowledge that encourages task access, prior knowledge of the task context) or attributes of the task that were susceptible to individual variation when a particular student interacted with the task (e.g., how recent a particular piece of mathematics required in the task has been studied, or how well rehearsed the required mathematics is). On the other hand, the complexity of a task appeared to be reduced by particular attributes of the task (e.g., the presence of salient cues (Kaplan & Simon, 1990) in the form of trigger words or visual features). Similarly, impeding conditions that increased the difficulty of a task for particular individuals often resulted from the interaction of a student's personal attributes with the attributes of the task (e.g., reluctance to make assumptions, cuing words not being salient, recall difficulties, interference from prior knowledge, metacognitive task knowledge which discouraged access) but sometimes were purely personal (e.g., possessing metacognitive personal knowledge that discouraged access). Impeding conditions that increased the complexity of the task were task attributes such as the mathematics or the goal of the task not being obvious, the need to integrate given and derived contextual information in order to construct a mental representation of the situation described in the task, or the need to make assumptions in order to formulate a mathematical model for the task.

It is suggested that task difficulty may vary from student to student whilst task complexity appears to be fixed as it is determined by the attributes of the task. This is in agreement with Williams' (2002) distinction between these two terms. It is foreshadowed, however, that these attributes may be related to particular solution methods rather than the task per se (e.g., one solution approach may require a deeper level of integration of information than another). Personal attributes of the student also appear to act as intervening conditions between task complexity and task difficulty. These would explain the different consequences that occur (e.g., different difficulty levels or whether or not impeding conditions were overcome) when different students attempt tasks of the same complexity. These points will be investigated in the future.

For students to benefit from facilitating conditions in applications tasks, they need: a well-developed repertoire of cognitive and metacognitive strategies as well as a rich store of mathematical concepts, facts, procedures, and experiences; vicarious general encyclopaedic knowledge of the world and word meanings; and truly experiential knowledge from personal experiences outside school or in more practical school subjects. In particular, a variety of retrieval, recognition, mental imaging, perceptual, and integration strategies, together with metacognitive strategies for monitoring, regulating, and coordinating the use of these cognitive strategies are necessary. As suggested by Nathan et al. (1992), the tasks in this study are highly reading-oriented and thus rely on: (a) accessing a good store of relevant prior knowledge for generating the inferences and elaborations necessary for understanding the situation fully, and (b) good comprehension skills to enable the student to specify a valid problem model for the task through the application of mathematical procedures. In some instances use of both cognitive and metacognitive strategies was enhanced by students (e.g., Elle in the Hockey Task) imagining or pretending to be in the situation described, confirming Van den Heuvel-Panhuizen's (1999) assertion cited at the beginning of this article. This facilitation of access was also enhanced by the development of metacognitive knowledge which encouraged students to engage with the task (e.g., Jan in Petri Dish Task 2). However, once a modest degree of skill has been achieved in accessing complex tasks such as applications, the work of Schneider and Detweiler (1988) and Carlson et al. (1990) point to coordination and integration of multiple representations, further cues, and mathematical processes and procedures as becoming critical as the solution attempt progresses.

Amit (1994) has warned that "as assessments become more complex and more connected to real-world tasks there is a greater [chance] that the underlying assumptions and points of view may not apply equally to all students" (p. 15) and this has been borne out in this study. Failure of a student to possess a well-developed strategic repertoire or rich store of mathematical, encyclopaedic, semantic, or experiential knowledge can lead to the situation where conditions that facilitate one student's access to the task become impeding conditions for another student. For example, a task such as the Road Construction Task, which has apparently obvious mathematical and contextual cues for one student (e.g., Jo) may prove to be inaccessible for another who does not have the appropriate knowledge base or fails to activate an appropriate knowledge base because of a poorly developed strategic repertoire. If the student does have the appropriate knowledge and strategic bases but fails to activate either initially, the student (e.g., Greg) may experience an initial period of difficulty but then overcome the impeding condition. At other times, particular attributes of an applications task, such as unusual wording, a lengthy problem statement, or the required mathematical model or method not being obvious, can impede access. These difficulties can be overcome, however, by students possessing and activating a well-developed strategic store together with an appropriate knowledge base. A wide variety of cognitive strategies that include retrieval, comprehension, information organising, attention focussing, information representing, and visualising are necessary for overcoming the potential array of impeding conditions that a student may encounter in attempting to access an applications task. The effective use of these strategies is enhanced by an equally rich and varied store of metacognitive strategies.

# Conclusion

The current emphasis on real-world applications, for whichever combinations of the reasons identified by Blum and Niss (1991), has made the task of assessing student performance in mathematics even more problematic as students grapple with the confounding effects of task context knowledge and the need for higher text comprehension skills. Teachers are being asked to teach in new ways and assess in new areas which are different from those in which they learnt mathematics themselves and were educated as teachers. The purpose of this article has been to identify the strategies that students employ in taking advantage of conditions that facilitate access to applications tasks or successfully overcoming conditions that impede access initially. By documenting these strategies and conditions affecting task access, teachers are provided with the means to develop learning experiences that focus on reducing the time students spend on these orientation activities (Stillman & Galbraith, 1998) during task solution, and this should have a positive effect on students' ability to access the tasks during assessment. Such learning experiences should promote the development of: (a) cognitive skills that ensure more effective problem representation and analysis, and (b) metacognitive strategy knowledge which facilitates appropriate decision making during orientation, and coordination and monitoring as the solution progresses.

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# Appendix: Examples of Application Tasks

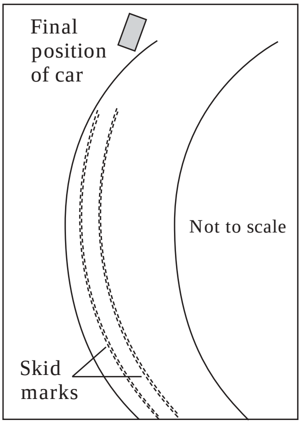
*Tide Task:* Greenough-on-Sea has an ideal deep water harbour for transportation of goods by sea. The depth of water at any point in the harbour varies with the rise and fall of the tide. The major limitation of this harbour is that during certain parts of the year the depth of water at the entrance to the harbour is shallower at low tide than necessary for some ships to enter the harbour. Water depth is no problem at the wharf all year round.

On a particular day the depth of water at the entrance at high tide is 8 metres and the time between successive high tides is 12 hours 30 minutes. A low tide at the harbour entrance occurs at 12 noon and the depth of water is only 4 metres. A ship drawing 5 metres below the water-line arrived just outside the entrance at 11 am. The ship entered the harbour as soon as it could. If it took four and a half hours to enter, unload, load and sail back to the harbour entrance, what was the depth of the water at the harbour entrance when it left? Fully justify your answer stating any assumptions made. (Teacher constructed task.)

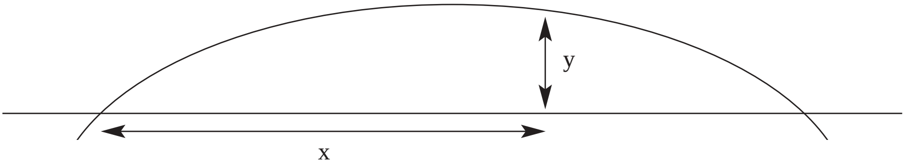
*Road Construction Task:* A new traffic lane (of minimum width 6 metres) is to be added to a section of highway which passes through a cutting. To construct the new lane, engineers need to excavate an existing earth-bank at the side of the roadway which is inclined at 25˚ to the horizontal. This will make the inclination steeper. Local council regulations will not allow slopes greater than 40˚ due to the potential for erosion. Determine if the new traffic lane can be excavated without expensive resumption of the properties at the top of the bank which is 7 metres above the road surface. [List any simplifying assumptions you have had to make.]

*Microwave Ovens Task:* The number of radioactive emissions from a certain faulty microwave oven is given by $N_1 = 64(0.5)^t$ at $t$ years from the first time of use, and the number from another faulty microwave oven is given by $N_2 = 4^{15}(0.0625)^t$ at $t$ years from the first time of use. Find out when both microwave ovens will emit the same number of radioactive units. (Teacher constructed task.)

*Road Accident Task* (Adapted from Smith & Hurst, 1990, pp. 67-80.): A car screeched round a bend and ended up in the ditch by the side of the road. The police were called and when they arrived they made detailed measurements of the skid marks left on the road by the car. These measurements were used to draw a plan of the scene of the accident as shown below.




When interviewed the driver said that the car's brakes failed as he came into the bend and so he could not slow the car down. He also said that he was driving at about 45km/hr (the speed limit for that road) when he entered the bend. An examination of the car confirmed that the brakes were not working at the time of the accident.




A reference line was used to measure the skid marks. The distance *x* was measured along the reference line and the distance y perpendicular to it. For the outer skid mark the values obtained were:

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *x* | 0 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 | 27 | 30 | 33 |
| *y* | 0 | 1.19 | 2.15 | 2.82 | 3.28 | 3.53 | 3.54 | 3.31 | 2.89 | 2.22 | 1.29 | 0 |

(All distances were measured in metres.) The police also measured the incline of the road and found that this particular stretch was flat.

When a car moves round a curved path with its wheels rotating but slipping sideways as in the accident above, a simple model to obtain its speed is to use the equation: $s^2 = dr$ where $s$ is the speed, $d$ is the drag factor for the road surface and $r$ is the radius of the curve. Test skids conducted at the scene found the drag factor to be 6.64 m/s/s.

Using the data above the police determined the driver's speed allowing a 10% error in the final calculation in the driver's favour (to compensate for any errors in measurement). Did the police calculations show that the driver was telling the truth about the speed of the car?

*Petri Dish Task 1.* A bacterial culture is being produced in a petri dish containing a culture medium. If the area covered by the culture is given by $A = 1.2te^{-0.01t}$ where $t$ is the time in hours since it began to grow, find the number of hours until the bacteria stop spreading.

*Petri Dish Task 2.* A bacterial culture is being produced in a petri dish containing a culture medium. The area covered by the culture is given by $A = 1.2te^{kt}$ where $k$ is a constant and $t$ is the time in hours since it began to grow. If the area was 1 square unit when $t$ was 0.913 hours, find the number of hours until the bacteria stop spreading.

*Shaft Task.* The diameter of a cylindrical shaft is gradually reducing through wear. The rate at which the diameter is changing is given by $D'(t) = -0.144t^2$ mm per month for $t > 0$. After 1 month of continuous use the diameter is 500.12 mm. How many months will the shaft have been in use before the wear is 1 mm?