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Model-Eliciting Activities as a Tool to Develop and Identify Creatively Gifted Mathematicians

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This article addresses the use of Model-Eliciting Activities (MEAs) as a (curricular) tool to develop mathematical creativity and identify students who are creatively gifted in mathematics. The thesis of this article is that by using MEAs, gifted educators can: (a) provide students with opportunities to develop creative and applied mathematical thinking; and (b) analyze students' mathematical thinking when engaged in creative mathematical tasks, aiding in the identification of those students who are especially talented in domain-specific, mathematical creativity. The authors' conclude that MEAs have potential for both developing and identifying creatively gifted mathematicians in the middle grades.

Model-Eliciting Activities (MEAs) were initially created in the mid 1970s by mathematics educators (Chamberlin, 2002; Lesh, Hoover, Hold, Kelly, & Post, 2000; Lesh & Lamon, 1992). These activities have also been called Case Studies for Kids and Thought Revealing Activities, but in this article Model-Eliciting Activities will be used to refer to them, because this name is the one currently being used by most MEA developers and because the name best explicates the mathematical goals of the activities.

MEA developers had two objectives in mind when they created MEAs. First, MEAs would encourage students to create mathematical models to solve complex problems, just as applied mathematicians do in the real world (Lesh & Doerr, 2003). Second, MEAs were designed to enable researchers to investigate students' mathematical thinking—a task endorsed by the National Council of Teachers of Mathematics (NCTM; 2000) and leading math educators (Hiebert et al., 1997; Wood, Merkel, & Uerkwitz, 1996). MEAs have the potential to develop mathematical talent, because they engage students in complex mathematical tasks similar to the tasks that applied mathematicians complete.

It is the thesis of this paper that MEAs may be used to accomplish a third goal, which is to develop and identify students who are creatively gifted in mathematics. The paper begins by defining mathematical creativity. We then describe MEAs in some detail, and interwoven into the description of MEAs is a discussion of the potential of these activities for developing mathematical creativity. Finally, we suggest a role for MEAs in identifying creatively gifted mathematicians, using tools such as the Quality Assurance Guide (Lesh et al., 2000) and the Ways of Thinking Sheets (Chamberlin, 2004).

Creativity and Mathematics

There are many definitions and theories of creativity (Fishkin, Cramond, & Olszewski-Kubilius, 1999; Starko, 1995; Sternberg, 1999). There is also considerable debate about whether creativity consists primarily of domain-general processes, such as divergent thinking, or of domain-specific processes, such as the ability to generate novel representations of the human form using the medium of photography. There are some who see creativity as a phenomenon constructed from the interactions between a domain, a field, and an individual (Csikszentmihalyi, 1999), and others who see creativity as residing primarily in the individual (Gruber, 1989; Gruber & Davis, 1988). Many of the current definitions of creativity are only relevant to adults who are operating on the frontiers of an established domain after years of training and preparation (Fishkin et al.). These definitions are not very useful for identifying or developing creative thinking processes in young people.

In this paper, creativity refers to the domain-specific thinking processes used by mathematicians when engaged in nonroutine problem solving. More specifically, the focus of the article is on developing and identifying students who have an unusual ability to generate novel and useful solutions to simulated or real applied problems using mathematical modeling. Students who have these abilities are defined in this paper as having creative mathematical talent.

The value of creativity in mathematics should not be underestimated. Lamon (2003) suggested that instruction that de-emphasizes different ways of thinking is "incongruous with today's world" (p. 436). Mathematically, creativity may be existent when a nonstandard solution is created to solve a problem that may be solved with a standard algorithm. Divergent thinking (Guilford, 1956) and evaluative thinking (Fasko, 2001) appear to be two prevalent descriptors of mathematical creativity.

Several individuals have stressed the significance of creativity in mathematics. Sternberg (1999) has underscored the importance of creativity in the application of classroom material. He stated that in order for one to be an effective mathematician in the real world, one must be able to creatively apply material that has been learned as opposed to merely regurgitating it for a test. He further stated that possessing analytical (mathematical) reasoning skills is not always consistent with being able to apply knowledge to real-world problems. The creative, applied mathematician needs a strong combination of creative, practical, and analytical abilities. Going even further, Piirto (1998) stated that without creativity, gifted mathematicians compromise the ability to think abstractly. In addition, others have suggested that thinking abstractly and engaging in higher order thinking are both gateways to success in upper-level mathematics (Hjalmarson, 2001; Schoenfeld, 1992). Higher order thinking, in turn, involves both analytic and synthetic thinking. In other words, logical, mathematical thinking is necessary but not sufficient for success in upper-level mathematics. Creative thinking abilities are also needed.

A requisite component of problem solving is that the problem is novel and therefore the solution is not immediately known (Carpenter & Moser, 1983; NCTM, 2000). Problem solving should not be a process whereby one simply remembers and regurgitates algorithmic processes (Schoenfeld, 1992). Instead, problem solving should be a process whereby one has the opportunity to use various processes to solve the task. By focusing on nonroutine problem solving as a critical component of mathematics, creativity will become more significant and creative talent can be developed.

Several themes exist in math creativity research. A recurring theme in the literature deals with viewing creativity as conceptual and imaginal thinking abilities: flexibility, fluency, and originality of thinking (Bejat, 1972; Tuli, 1982). Second, creativity in mathematics is not consistent with creativity in other disciplines as a result of domain specificity (Feist, 2005; Kaufman, 2004; Milgram & Livne, 2005). For example, individuals creative in language arts may not be creative in mathematics, and vice versa. Third, and likely a result of the poor correlation between creativity in other domains and mathematics, teachers can not accurately predict creativity in mathematics unless they have the opportunity to observe students working on problems that elicit mathematical creativity (Houtz, Lewis, Shaning, & Denmark, 1983).

Sadly, little has been accomplished in recent years to develop creative mathematical talent and identify students who are creatively gifted in mathematics. Instead, most of the focus of mathematical talent development programs has been on accelerating students through the K–12 mathematical curriculum as quickly as possible (Chamberlin, 2005). MEAs provide gifted educators with an alterative to acceleration. MEAs engage students in interdisciplinary, nonroutine problem solving and provide opportunities for students to develop the type of creative mathematical talent needed in fields such as business and engineering, as well as the mathematical modeling abilities needed for higher level mathematics.

Model Eliciting Activity Description

The purpose of this section of the paper is to describe MEAs and explore their usefulness for developing creative mathematical talent. A typical MEA is described in some detail. The design principles are described and their curricular and learning characteristics are reviewed, with special emphasis on the aspects of MEAs that foster the development of mathematical creativity.

MEAs are mathematically based activities designed for use with students in grades 4–12, with special emphasis on grades 5–8. The value of MEAs is that they help educators accomplish goals determined to be best practices by mathematics educators. For example, MEAs foster communication and problem solving, two principles outlined in the NCTM principles and standards document (NCTM, 2000). Examples of MEAs may be found at online (see http://www.edci.purdue.edu/casestudiesforkids).

Description of a Typical MEA

Each MEA is comprised of four sections. The first two sections set up the problem context and parameters, and the final two sections present the problem. The first section of an MEA is a reading passage. Passages are one-page simulated newspaper articles written to generate student interest and discussion about the context of the problem. As an example, an MEA entitled *Sue the Dinosaur* (Carmona, 2001) has a reading passage about students visiting a museum where they see a dinosaur recreation. Students are subsequently asked to create a model that will enable them to reassemble Sue. The second section of an MEA is the readiness question section. These are questions a student answers about the preceding article. Some questions are simple comprehension questions, such as "Who was the director of the sports camp?" Other questions are inference questions, such as "Why might arriving on time be more important to some travelers than other travelers?" A third type of question asks for an interpretation of data, such as "Review the charts on the next page. In what year was the quickest average time recorded for the 400-meter run?" The purpose of this section is to ensure that students have the foundational knowledge they need to solve the problem. In these first two sections, there is little emphasis on creativity development; rather, the emphasis is on helping students understand the context of the problem situation.

The third section of an MEA is the data section. This section can take many forms, including a diagram, chart, map, table of times or performances, table of sales, and so forth. The third section is often referenced in the readiness question session and it is always used in the final question. The fourth section of an MEA is the problem-solving task. This question or statement is generally no longer than a paragraph and it asks students to solve a mathematically complex problem for a hypothetical client. One unique characteristic of MEAs is that students solve the problem given to them and then generalize their model to subsequent situations. These last two sections comprise the bulk of the mathematics of the MEA and create an ill-structured problem for students to solve, eliciting mathematical creativity, as well as mathematical modeling. The fourth section is the one in which creativity may become most apparent. Interestingly, anecdotal data suggests that some students gifted in mathematics strive to finish the problem as quickly as possible and their solutions are often minimally creative. Students who complete the problem in a longer period of time may seek particularly creative solutions. Time spent on task is not indicative of creativity per se and the extent to which students are creative in this section is most indicative of students' mathematical creativity in MEAs.

Six Principles for Designing MEAs

To create MEAs, researchers in mathematics education at universities throughout the United States and New Zealand follow specific guidelines. These guidelines are referred to as the six principles for designing MEAs (Chamberlin, 2004; Lesh et al., 2000). The principles ensure that each MEA will have the intended curricular and learning characteristics. It is possible for anyone to create MEAs, but it is difficult to succeed in this task without at least minimal training from individuals who are intimately acquainted with how they are constructed. For this reason, most teachers and curriculum coordinators may choose to use the preexisting database of problems found on the aforementioned Web site. Closely following the principles enhances the likelihood that the MEA meets all standards and will stimulate model-eliciting behaviors. Field testing must occur and interviews typically are performed to ensure that these principles have been adequately met. This careful attention to the design of MEAs is one reason why they are such powerful tools for the development and identification of mathematical creativity.

Model Construction Principle. The model construction principle states that a successful response to the problem demands the creation of a model. A model is a system that consists of elements, relationships among those elements, operations that describe how those elements interact, and patterns or rules that apply to the relationships and operations. A model is evident when one system describes another system. This first and most important characteristic of MEAs suggests that these activities are inherently designed to elicit creative behaviors and higher level thinking, especially at the level of synthesis.

The Reality Principle. The reality principle has also been referred to as the meaningfulness principle. This principle states that the scenario presented should be one that realistically could occur in the life of a student. Paying close attention to the reality principle is intended to increase student interest and simulate the kinds of activities that real, applied mathematicians engage in when solving problems for clients. It is possible that the more realistic the problem, the more potential that exists for creative solutions due to students' familiarity with the problem.

The Self-Assessment Principle. The self-assessment principle states that students must be able to measure the appropriateness and usefulness of solutions without input from the teacher. In turn, students may use this information to refine responses in subsequent iterations. Again, this principle is consistent with the development of creativity because individuals engaged in creative work must be skilled in self-evaluation.

The Construct Documentation Principle. This principle is the reason these activities have sometimes been called "thought revealing activities" (Lesh et al., 2000). The construct documentation principle indicates that students must be able to reveal their own thinking while working on the MEA and that their thinking processes must be documented in their response. This principle is related to the self-assessment principle, which requires students to evaluate how closely their solution is reflected in their final documentation. The demands of documenting the solution involve technical writing. As such, MEAs are instrumental for the gifted community because writing technically may facilitate higher level thinking and metacognition. The construct documentation principle also helps to ensure that teachers who implement MEAs focus on the thinking processes of their students during problem solving, as well as on their final answer or model. A focus on process helps to nurture creative thinking.

The Construct Shareability and Reusability Principle. The construct shareability and reusability principle states that the product should be able to be used in a parallel situation. If the model developed can be generalized to other situations requiring a similar model, then the response is a successful one. This principle is closely related to the next one.

The Effective Prototype Principle. The effective prototype principle suggests that the model created should be easily interpretable by others. This principle differs from the construct shareability and reusability principle in that students may use this prototype in a similar, but not parallel situation. These final two principles help young mathematicians to learn that creative solutions to applied mathematical problems are useful and generalizable. The best solutions to nonroutine mathematical problems are robust enough to work in different situations and easy for others to understand. Very similar prototype design principles are used in creative fields like engineering, computer technology, and even applied chemistry.

Summary. Taken together, the six design principles highlight the most important characteristics of a good MEA and clearly demonstrate why these activities are helpful in nurturing mathematical creativity. MEAs require the creation of something new—a mathematical model—and engage students in a creative process that is very similar to the process used in applied design fields. The design principles are used to guide the creative process of developing new MEA problems. MEAs that are designed according to the six principles have specific curricular and learning characteristics, which are discussed below.

Curricular Characteristics of MEAs

MEAs have five curricular characteristics: interdisciplinarity, well-structured problems, realistic problems, metacognitive coaching, and explication of student thinking. These are not characteristics that developers use to create MEAs. Instead, they are characteristics that flow from the design principles and are inherent in well-designed MEAs.

The interdisciplinary nature of MEAs enables educators to integrate other disciplines. The primary content emphasis of MEAs is mathematics. MEAs, however, cannot be completed without reading, communicating with peers, and writing an explanation of the solution. In addition to math and literacy, MEAs have a context related to social studies, science, art, and/or physical education. For example, the MEA entitled *Historic Hotels* (Carmona, 2002) has a historical context, *Aluminum Bats* (Chamberlin & Hjalmarson, 2002) has a materials engineering context, the *Quilt Problem* (Carmona & Hjalmarson, 2001) has a creative arts context, and *Summer Sports Camp* (Chamberlin, 2000) has a physical education context. Enabling students to use knowledge from various subjects may increase the likelihood that highly creative solutions will be produced. In routine mathematics problems, students simply mimic the process previously illustrated by the teacher (Hiebert et al., 1997). However, in nonroutine problems and interdisciplinary problems, students may use a wider range of knowledge, and potentially increase the likelihood of using creativity.

MEAs are well-structured problems. An ill-structured problem is one in which persons solving the problem need to do research to find data or information to solve it (Boyce, VanTassel-Baska, Burruss, Sher, & Johnson, 1997; Hmelo & Ferrari, 1997; Stepien & Gallagher, 1993; Stepien, Gallagher, & Workman, 1993). On the other hand, a well-structured problem is one in which information necessary to solve the problem is contained within the problem so that no research is required to solve the problem. In each MEA, the third page contains adequate information to solve the problem.

MEAs are realistic problems that are relevant to the lives of students (Lesh et al., 2000). The use of realistic problems in mathematics is likely to better promote learning for understanding than the use of problems without a context (Cooper & Harries, 2003). For instance, elementary and secondary students are likely to have engaged in a summer book reading contest, played softball, attended a track and field camp, mowed a lawn, attended an amusement park, and so forth. Many problem-solving tasks advertise the reality of the problem, but they are not real at all. Authors of MEAs have invested extensive time ensuring that contexts in the problems are in fact realistic by field-testing the problems with students and gathering informal qualitative data, in the form of interviews, after the problems have been administered. Realistic problems may increase the likelihood for creative solutions because students are acquainted with the context.

For MEAs to be successfully administered, the teacher serves as a metacognitive coach. As a metacognitive coach, a teacher interacts with students by posing rather than answering questions. For instance, students might ask a teacher if a solution is correct. Because one of the six principles of MEAs is self-assessment, a teacher may respond by asking students if their reasoning makes sense. As a metacognitive coach, a teacher indicates that wrong answers do exist, along with varying degrees of correct answers. However, steering students to a single, correct way of thinking is not an element of metacognitive coaching (Gallagher & Stepien, 1995). Hence, metacognitive coaching encourages creative thinking.

MEAs provide teachers with the opportunity to investigate students' thinking. This investigation often leads to insight regarding how to refine curricula. MEAs provide richer information on students' thought processes than a worksheet, a closed-answer test, or a simple word problem. After gaining authentic information about students' reasoning, a teacher can decide whether to reteach a concept, to stay with the initial curriculum plan, or to accelerate to an advanced topic. MEAs thus have great assessment potential and, in particular, they may help teachers identify creative solutions.

The curricular characteristics of MEAs are consistent with other curricular approaches designed to enhance student creativity such as creative problem solving (Schack, 1993), problem-based learning (Gallagher, 1997), and independent study (Chamberlin, 2005). The most creativity friendly curricular characteristics of MEAs are interdisciplinarity, metacognitive coaching, and problem solving. These same characteristics are seen in other problem-solving programs, such as the Future Problem Solving Program, designed to enhance student creativity (Hoomes, 1986). The difference is that MEAs are specifically focused on developing mathematical creativity.

Learning Characteristics of MEAs

MEAs have five learning characteristics when viewed in relation to student learning: collaboration, multiple processes, self-directed learning and self-assessment, fostering of ownership, and model development. As with the curricular characteristics, these characteristics are not used as a guide to create MEAs. Instead, they can be viewed as tasks in which a student might ordinarily engage to successfully solve an MEA. Student collaboration is fostered when doing MEAs because students work in groups of three or four. When engaging in MEAs, students depend on the expertise of peers in mathematics, literacy, and the context for the specific MEA. If, for example, a student has a relative who is a doctor and the context for the MEA is epidemiology, then student background knowledge may be an asset. Pulling on the expertise of peers mimics what happens in real life and workers operate in teams like they do in the real workforce (Dark, 2003).

Students focus on multiple processes when doing MEAs. Unlike simplistic word problems, one or two simple computations will not solve any MEA. For instance, a group cannot merely add up a series of numbers and divide by the number of entries in that column to solve an MEA. The answer would be inadequate. Processes required and models created to solve MEAs are in much greater depth than simple algorithmic solutions and thus promote creative thinking because they are nonroutine. Hence, higher order thinking (Wieczerkowski, Cropley, & Prado, 2000) is required to complete each MEA. Students must also engage in metacognition to solve MEAs (Lesh, Lester, & Hjalmarson, 2003). As an example, students may create a plan to solve the problem and monitor their progress as they execute the plan.

Self-directed learning and self-assessment are also trademarks of MEAs. Once the problem statement has been discussed in class, students work in groups. The process used to solve the problem is a decision made by students. Identifying efficient mathematical processes requires some creativity while it simultaneously forces students to be consumers of mathematics. Self-assessment is promoted when doing MEAs because students are regularly reminded by the teacher that they are producing a product for a client/customer (Lesh et al., 2000). Thus, the students must decide whether or not the product meets the demand(s) of the client. For instance, in the MEA entitled *Summer Sports Camp* (Chamberlin, 2000), students are asked to form rules to equitably design a track team. Scott Memmer, the client, expresses concern that their current system of friends selecting friends for teams has not worked in the past. Each year one team defeats the other team by a significant margin. To solve this problem, students can create a model to form teams and then score the meet to see if their scores are close. Not only does this level of autonomy foster independent learning, but it also alters responsibilities for the teacher from lecturer to metacognitive coach.

Moreover, MEAs foster ownership because students create their own models to solve realistic mathematical problems. Students are no longer seeking the single appropriate answer known only by the teacher. Prior to introducing an MEA, teachers do not illustrate an algorithmic process to solve the problem, as is done in a didactic approach (Steiner & Stoecklin, 1997). Instead, students are encouraged to work heuristically, inventing methods and models that will solve the problem. As a result of increased ownership, students often take pride in describing their solution to peers. Discussing and describing solutions, also referred to as debriefing, in which students engage during the presentation, show that although similarities may exist in reasoning, each group solved the problem using their own methods. This solution debriefing is nothing new to many teachers and classrooms because it is a tool frequently used to create meaning and infuse communication into mathematics. When students create specific solutions, a context for talking about mathematics can be developed (Wood, Merkel, & Uerkwitz, 1996). Ownership often fosters increased persistence in problem solving (Prenzel, 1992) and this is another potential outcome of MEAs.

Finally, MEAs require students to create mathematical models for successful solutions to problems. The creation of models by students is one of the most powerful mathematical activities in which a student may engage (Glas, 2002). Additionally, modeling may often be neglected in school systems due to its complexity and unfamiliarity to students and teachers. Creating mathematical models through MEAs provides a venue for engaging students in precollege mathematical thinking, which may not be accomplished with other curricular approaches (Lesh et al., 2000). Indeed, as a result of rapid acceleration, gifted students may view mathematics as a series of discrete subjects (e.g., algebra has no relation to statistics). Creating mathematical models serves to illustrate the interconnectedness of mathematics (Lesh et al.). Glas lists four educational outcomes achieved by modeling in the mathematics classroom. Models and modeling help students (a) recognize the interconnectedness inside and outside of mathematics, (b) recognize various perspectives on a domain of knowledge, (c) be creative in mathematical thought, and (d) view mathematics in a practical and applicable way.

A rich discussion of models and the modeling process can be found Lesh and Doerr's (2003) *Beyond Constructivism: Models and Modeling Perspectives on Mathematics Problem Solving, Learning, and Teaching*. In the book, Lesh and Doerr have outlined numerous assets of models and modeling and have presented a compelling argument for incorporating them into the math curriculum. MEAs may be the most available method for infusing models and modeling into the mathematics curriculum. With the curricular and learning components of MEAs described, the final section of the paper will complete the picture by showing how MEAs can be used as tools to identify individuals creatively gifted in mathematics.

Creativity and its Relationship to MEAs

MEAs lend themselves to being solved creatively and they serve as an exemplar of how to identify creatively gifted mathematicians. The first goal of MEAs is to get students to create mathematical models to solve complex mathematical tasks (Lesh & Doerr, 2003). A principal difference in algorithmic problems and problem solving is that problem-solving tasks rely on the solver(s) to create and then to implement the process for success (Hiebert et. al, 1997). Conversely, algorithmic problems rely on the solver(s) to implement the process just illustrated by the instructor.

Creativity is at the heart of MEAs, and it plays a significant role in student success in mathematics. With respect to math solutions, diversity in thinking is a process that is paramount to the successful development of models (Lesh & Doerr, 2003). However, the significance of creativity in school mathematics may be minimized because it is not formally assessed on standardized tests, which purport to thoroughly measure mathematical learning. MEAs can act to fill the assessment void created by standardized tests, and they provide performance-based assessment of the ability of students to generate creative mathematical ideas in response to a well-structured problem.

Why Should MEAs Be Used to Identify Students Creatively Gifted in Mathematics?

By third grade, students already are "predisposed to thinking in different ways" (Lamon, 2003, p. 438). This is one premise for the design of MEAs. An objective of MEAs is to provide rich information for students' thinking. If only one method to solve each MEA existed, then investigating students' thinking would be a pointless task because multiple solutions would not exist. No consideration for process would exist because the product would be the only consideration. Furthermore, MEAs would be relegated to the status of algorithmic problems such as those found on worksheets and many standardized tests. MEAs are different from other problem-solving tasks because the process is the product and creativity is a major emphasis in solving MEAs.

Practically speaking, curricular resources do not exist to identify students considered to be creatively gifted in mathematics. Curricula such as Creative Problem Solving, Math Olympiad, Mathcounts, and so forth may foster creative thinking by engaging students in mathematical problem solving. The intent of such curricula is to foster creative thinking, but they were not necessarily created with identification in mind. MEAs may be viewed as a tool to identify individuals with a high amount of creativity and tacit knowledge in mathematics through the use of the Quality Assurance Guide (Lesh et al., 2000), in coordination with the Ways of Thinking Sheet (Chamberlin, 2004).

The Quality Assurance Guide (Lesh et al., 2000; see Appendix A) is a rubric used by teachers to rate the comprehensiveness of students' solutions. Creativity was not an initial emphasis of this instrument, but the instrument includes a strong emphasis on the "usefulness" of the model developed, and usefulness is one aspect of the definition of applied mathematical creativity. In addition, the instrument could be modified quite easily to include a second column focusing on the novelty of student solutions. Also, it can be adapted by superimposing creativity on each of the three sections outlined in the introduction. For instance, creativity can be measured with respect to the development of a conceptual tool, the satisfying of the client's needs and purposes, and the usefulness of the solution. When using the Quality Assurance Guide, students are provided with a score of 1 to 5, with 5 being regarded as an exemplary answer and 1 being regarded as an answer that is completely unacceptable. Usually, the most creative solutions receive the highest scores because high amounts of creativity are required to develop level 5 solutions. For example, models that both fit the data and generalize to other situations would elicit a score of 5.

In addition, teachers may use the Ways of Thinking Sheet (Chamberlin, 2004) in Appendix B in order to document (unique) mathematical strategies created by students, and when possible, excerpts from student work should be included. Ways of Thinking Sheets are imperative because MEAs have been specifically designed to identify and nurture mathematically creative students (Zawojewski, Lesh, & English, 2003). As with the quality assurance guide, these sheets were initially designed for general use with MEAs. However, they lend themselves to assessing creativity better than any other instrument created so far for assessment use with MEAs.

Parts of MEAs are left intentionally ambiguous to enable students and groups to interpret meanings. In turn, students' interpretations of meanings significantly impact the way they solve problems. Interpreting words literally may lead to fairly straightforward responses. However, interpreting parts of the problem in unconventional ways may lend itself to more creative answers than might otherwise be attained with a conventional interpretation of terminology.

As an example of an ambiguous definition, in the problem *On-Time Arrival* (Chamberlin & Chamberlin, 2001), students are asked to identify which airline is most likely to be on time. To successfully solve this problem, students analyze 30 days of data from five airlines and they often use one or more combinations of central tendency (mean, median, or mode), range, or standard deviation. Anecdotal information suggests that students who interpret *on time* to mean something other than zero minutes late often arrive at models that have a higher degree of creativity than those who confine their interpretation of on time to only mean zero minutes late. Individuals who interpret on time to mean literally zero minutes late often only use a frequency count, such as mode, to identify the most on-time airline, which trivializes the problem. Alternatively, students may simply find the mean of all arrivals and select the mean closest to zero. Thus, by restricting the definition of on time, the task becomes a very simplistic one that does not lend itself to a creative solution. Simply implementing an algorithm solves the problem, but not satisfactorily nor creatively. Practically speaking, zero to 10 minutes late is on-time in airline jargon due to variations in the official time.

Similarly, in the *O'Hare Airport Problem* (Zawojewski & Lesh, 1999), students are asked to design a model to get to the airport from a north suburb of Chicago for various times of the day, thus accounting for various traffic patterns. The specific question asks, "What is the best way to get from here, a north suburb of Chicago, to O'Hare Airport?" As Zawojewski and colleagues (2003) have pointed out, "The term *best* was left undefined in this problem because a goal was to have students investigate processes involved in problem formulation, information interpretation, and trial solution evaluation" (p. 348).

Thus, the term best was used intentionally because it is ambiguous and it fosters creativity. It is likely that the students who limit and closely define the term often have models of limited creativity. To the contrary, students who creatively define the word best are more likely to have creative models. Creative students may use multiple modes or intelligences to solve problems, while students with limited creativity may be seeking the "quick-fix" solution.

Authentic math learning should replace the showing of algorithms as a means to math instruction (NCTM, 2000). MEAs enable teachers to closely investigate student thinking through three avenues. First, teachers may follow students' thinking as they are creating the model. While doing MEAs, teachers act as a metacognitive coach by posing questions to students. Hence, teachers can be apprised of students' thinking as it unfolds. Second, in each MEA, students are asked to document their model in written form (Lesh et al., 2000). This written form enables teachers to engage in a detailed analysis of students' thinking at a later time. Third, students present their results to the class in a formal presentation called debriefing and peers are provided the opportunity to ask questions of each presenter. Thus, teachers can investigate students' thinking as they are engaging in thought, when they document it, and when they present the model.

Limitations of MEAs

As with any curricular approach, MEAs have limitations. For instance, the interplay between Ways of Thinking Sheets (Chamberlin, 2004) and the Quality Assurance Guide (Lesh et al., 2000) has not been empirically researched. While the instruments have provided detailed information about students in field tests, they have not been used in harmony and creativity was not a major focus when they were initially created. Validation studies of these instruments for use in measuring mathematical creativity and identifying creatively gifted mathematicians are needed.

Another concern is that MEAs are largely available online at this point and it is difficult for classroom teachers to create MEAs without some training. Classroom teachers may read example activities online, but they may not understand how to write these activities. Moreover, interpretation of student thinking is the sole responsibility of the teacher (until the assessment instruments are formalized with respect to creativity).

Areas for Future Research

The areas outlined in the limitations are an excellent place to start with areas for future research. Reliable and valid instruments would precipitate greater trust in using the instruments and therefore provide more accurate data for teachers. Creativity assessment instruments need to be teacher-friendly and tips for implementation should be provided for teachers. In addition, a definition of creativity as it relates to mathematics may need to be reformulated given the complexity of nonroutine mathematical tasks. Perhaps the greatest challenge in reformulating the definition will come in establishing a definition that is proven empirically.

MEAs appear to hold great promise for identifying and developing the type of applied mathematical creativity needed in fields like engineering, computer programming, and economics. It would be interesting to conduct longitudinal research on middle school students who seem particularly talented in working with MEAs to determine their educational and career paths. It would also be interesting to compare creative performances on MEA activities with scores on generic, divergent thinking measures to see if the two are correlated. There are many possible avenues for future research on how to identify and develop applied mathematical creativity using activities like MEAs. It is hoped that this article will stimulate interest in research on mathematical creativity in general, and MEAs in particular.

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Appendix A
Quality Assurance Guide

This Quality Assurance Guide is designed to help teachers (and students) evaluate the products that are developed in response to Model-Eliciting Activities with the following characteristics: (a) the goal is to develop a conceptual tool, (b) the client's purposes are known and met, and (c) the tool must be sharable with other people and must be useful in situations where the data are different than those specified in the problem.

Performance Level	**How useful is the product**?	**What might the client say?**
Level one: Requires Redirection	The product is on the wrong track. Working longer or harder won't work. The students may require some additional feedback from the teacher.	*"Start over. This won't work. Think about it differently. Use different ideas or procedures."*
Level two: Requires Major Extensions or Refinements	The product is a good start toward meeting the client's needs, but a lot more work is needed to respond to all of the issues.	*"You're on the right track, but this still needs a lot more work before it'll be in a form that's useful."*
Level three: Requires Only Minor Editing	The product is nearly ready to be used. It still needs a few small modifications, additions, or refinements.	*"Hmmm, this is close to what I need. You just need to add or change a few small things."*
Level four: Useful for this Specific Data Given	No changes will be needed to meets the immediate needs of the client.	*"Ahhh, this will work well as it is. I won't even need to do any editing."*
Level five: Sharable or Reusable	The tool not only works for the immediate situation, but it also would be easy for others to modify and use it in similar situations.	*"Excellent, this tool will be easy for me to modify or use in other similar situations— when the data are slightly different."*

Note. From "Principles for Developing Thought-Revealing Activities for Students and Teachers," by R. Lesh, M. Hoover, B. Hole, A. Kelly, and T. Post, in *Handbook of Research in Mathematics and Science Education* (p. 145), by A. Kelly and R. Lesh (Eds.), 2000, Mahwah, NJ: Lawrence Erlbaum and Associates. Copyright ©2000 by Laurence Erlbaum and Associates. Reprinted with permission.

Appendix B
Ways of Thinking Sheet

	Description of Strategy and Examples of Students' Work	Mathematics: Concepts covered	Effectiveness in Creativity
Thinking Strategy #1			
Thinking Strategy #2			
Thinking Strategy #3			
Thinking Strategy #4			

The Ways of Thinking Sheet, initially created by Chamberlin (2004), has been adapted to measure creativity. In the initial iteration of this document, teachers considered many components in assessing students' ways of thinking in addition to creativity. For instance, teachers initially discussed the ease of use of the model with other clients and how well it met clients' needs. This instrument has been adapted to focus on creativity. By documenting the mathematics used (e.g., the use of a nonstandard solution to one that lends itself to being solved by a standard algorithm), teachers may measure the effectiveness and creativity of the solution. Only minor changes have been made to the sheet so teachers may use it to cater their assessment needs.