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Learning to read in mathematics classrooms

Reading and the mathematics classroom

Teacher: So you need to read the question and understand it. If we cannot understand the question, then we can't answer it. OK. And there's no point in sitting there going, "Oh well, I can't understand the question," and just keep letting it pass by. You need to ask questions, if you don't understand.

The frustration of this teacher at her Year 10 students' lack of engagement with what they were reading is evident in this quotation. The Macquarie Dictionary (Delbridge, 1982) states that "to read" means "to observe and apprehend the meaning of (something written, printed, etc.)". However, from this class, in Term 1 of 2003, it was clear that if the meaning was not easily identifiable in the text, then many students gave up. As a result, they either failed to do the activity or they pestered the teacher or another student to give them the information and so did not learn how to retrieve the meaning themselves.

Other research has suggested that this class was not alone in failing to gain meaning from factual texts. McDonald and Thornley (2004) suggested that New Zealand students entering high school were likely to have difficulty extracting relevant information from material which was abstract and had few visual supports. In the USA, Demana, Schoen and Waits (1993) found that junior high school students were not expected "to make global or qualitative interpretations of graphs" (p.19). Mathematics students need to retrieve information from written words and also from diagrams and graphs to understand mathematical concepts and to apply the information to a range of different problem situations. Without this, students' reading cannot become a useful part of the learning process.

The model

As a result of our concerns about students' lack of reading skills, we designed and introduced the *Read–Think–Do (x2)* model which is shown in Figure 1. It uses the ideas of Michael Halliday on how the context of the situation affects both the choice of language and also how it can be interpreted (Halliday & Hasan, 1985). Any piece of writing is influenced by the author's background, the purpose for the writing and whether the material is in a textbook or written on the blackboard. We assumed that if students understood the context of situation in which something was written, they would be better able to interpret the piece of writing. The *Read, Think, Do (x2)* is a cyclic model providing questions for students to answer in order to help make their reading part of the learning process. The cycle begins with the *Read* section at the top and moves clockwise around the model. However, getting to the final *Do* section may mean that students have to go back over some of the previous sections.

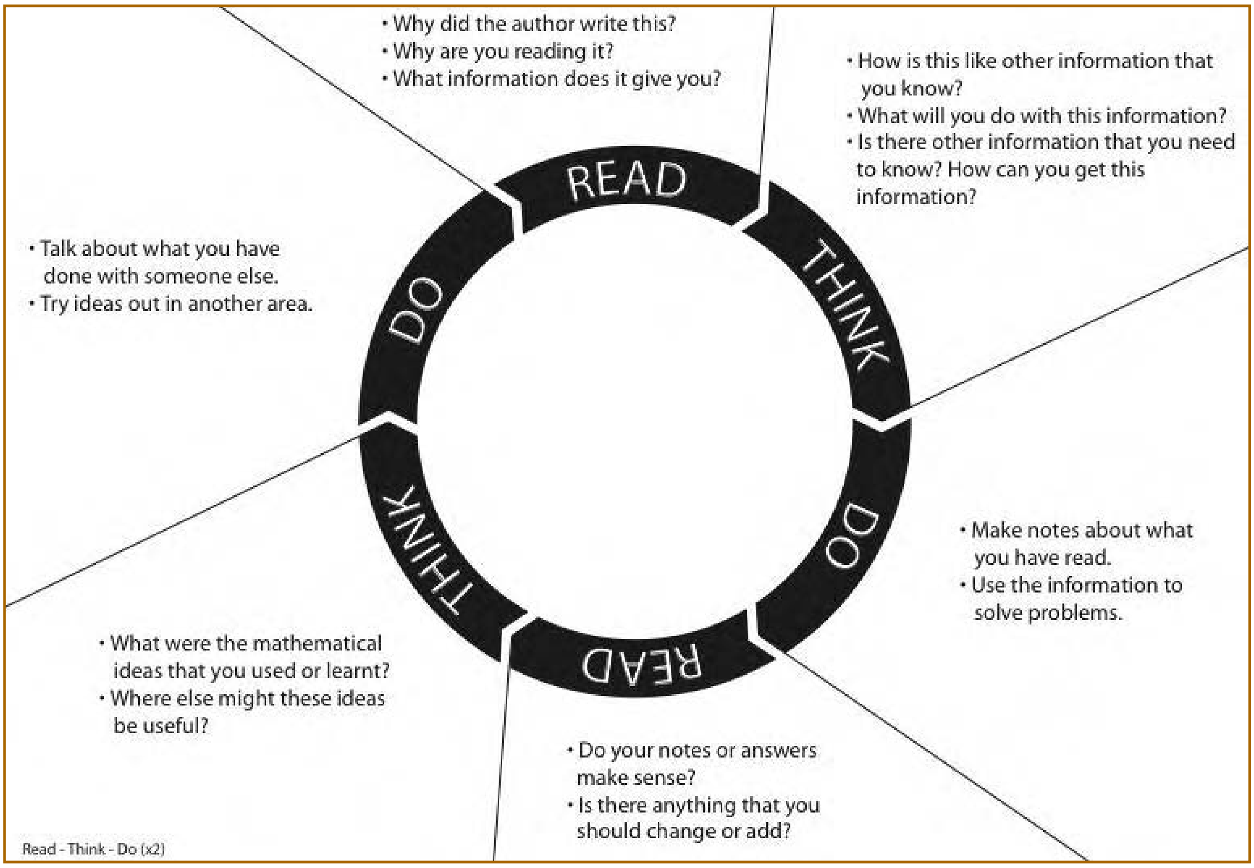


Figure 1. The Read–Think–Do (x2) model.

For the model to be useful, the teacher had to show how it could be used and to scaffold students into using it. This paper goes through each of the stages and uses extracts from classroom audiotapes and interviews with students. It shows why the model was needed by looking at conversations about written problems and diagrams and how the classroom teacher made students aware of the model. Encouraging students to change the way they approach their reading does not happen quickly. By the end of 2003, some students were incorporating some of the stages of the model into their reading, but this had required repeated reinforcement by the teacher.

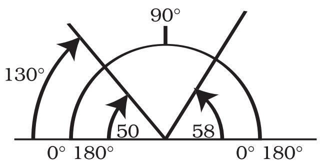
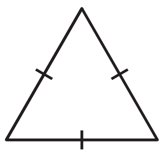
Read

- Why did the author write this?
- Why are you reading it?
- What information does it give you?

Students are asked to read material for a range of different reasons. During the last three terms of 2003, a copy was made of all the material written on the whiteboard for one or two lessons a week. As this material included examples from exercises, tests and text book references, as well as the teacher's definitions of mathematical terms and explanations, it was a fairly representative sample of the material that students had to read. There were five main reasons for having students read material. These reasons, together with an example of each type, are shown in Table 1.

In Table 1, it can be seen that there is not only a range of materials to be read but that they serve several purposes. In questionnaires completed at the beginning of the project, students were asked, "What things do you read in maths lessons or for maths homework?". They were then asked, "Why are you asked to read these?". Students wrote, "to help us learn" or "because it helps us in life". These general responses suggest that students were not very clear about the different purposes for reading. It is more likely that students will gain more meaning if they know why they are reading a piece of writing or diagram.

Table 1. The different types of material to be read and their purposes.

Example	***Why did the author write this?***	***Why are you reading it?***	***What information does it give you?***
$x + 4$ (Any) number plus 4	Explanation or definition of a mathematical concept.	To understand and make use of this mathematical concept.	Information on how this mathematical concept relates to other concepts.
Mean = $\frac{7+8+10+11+12+14+15+16}{8}$	To supply information on when and how to determine a mathematical skill (in this case means).	To learn when and how to use a mathematical skill.	Shows how to recognise a situation which requires the application of the skill, as well as how to apply the skills.
	To show possible problems or misunderstandings that students have in reading a protractor.	So that possible misunderstandings or misapplications of mathematical concepts or skills can be avoided.	Awareness of the common errors made in retrieving meaning from material.
	To visually present an equilateral triangle using conventional symbols.	To use this information to understand a concept or problem.	Necessary information for solving problems or understanding concepts.
$a + a + a =$ ___ a	To provide opportunities to practise using a mathematical skill.	To learn how to apply mathematical skills so that it becomes an automatic process.	Information about the situation which requires the application of these skills.

Think

- How is this like other information that you know?
- What will you do with this information?
- Is there other information that you need to know? How can you get that information?

Mathematical texts use language and diagrams in ways slightly different from those used in other subjects. Part of learning to read mathematics involves learning the common ways of presenting written materials, including diagrams such as graphs. An example of this is that actions are often represented as nouns, rather than as verbs, which is the case in most English writing. The following is an extract from one of the lessons that were audio-taped. In it, the teacher modelled her thinking process for the problem which was written on a worksheet that the class was working through together: “The product of two numbers is 51. If one of the numbers is 3, what is the other number?”

Teacher: Seventeen. Three seventeens are fifty-one. We need to know that the product, it means multiplying two numbers, OK. So we know what the answer is and we know that one of the numbers is three, so three times what is fifty-one?

...

Student: So how do you do that?

Teacher: But this one though, in this case, we’ve already been given the answer. It says, the product of two numbers is fifty-one. So it told us two numbers had been multiplied to give fifty-one.

One of those numbers is three, so it's three times something equals fifty-one. Can you see that?

The teacher showed that the key to this problem was recognising that the noun, "the product," represented an action, that of multiplying, which could be undone. Using nouns to represent mathematical processes allows many ideas to be linked together into a succinct logical whole (Meaney, 2005). For students to be able to do this problem by themselves, they need to understand that actions are nouns. They then need to remember that "product" is the noun which describes the action of multiplication. If they can make a link to this knowledge, then they will be able to do the division needed to solve the problem.

Do

- Make notes about what you have read.
- Use the information to solve problems.

Most students in this class were happy to launch into solving a problem. Researchers such as Kintsch (1998) have suggested that students focus on a few key words to determine a solution strategy. Slowing down the selection of a solution strategy by taking notes or making drawings can help students identify a more appropriate one than when they rush into making this choice. The following comes from a student working out the answer to the question, "David counted eight posts along the side of the road. If the posts were twenty metres apart, how far was it from the first post to the last post?".

Tamsin: OK. This one.
Student: One hundred and sixty metres.
Tamsin: Why?
Student: Because it's eight times twenty.
Tamsin: OK. Do you want to draw that for me?
Student: How do you mean?
Tamsin: Draw a picture of those eight posts with the twenty metres between them. [pause]

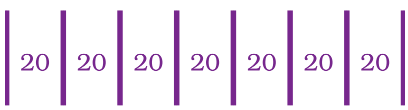


Figure 2. Student's drawing.

Tamsin: OK. Now count.
Student: A hundred and forty.
Tamsin: OK. Why do you think there's that difference?
Student: Because there's eight posts, but there's only seven gaps.
Tamsin: OK. Do you think that sometimes drawing would help interpret the information? Because sometimes when you read it...
Student: Um, yep.
Tamsin: Do you ever draw?
Student: No.

For many students, the aim of each lesson is to complete all the exercises that have been set as quickly as possibly. As a result, even when they have skills in their repertoire such as drawing, they may not use them. The doing part of reading a problem needs students to try out different ways of working with the information that they have read. A more effective approach for helping students use their reading to support their learning may be to set fewer exercises and to give more time for discussion over the different ways for solving them.

Read

- Do your notes or answers make sense?
- Is there anything that you should change or add?

Many mathematics teachers have students to copy down notes with the expectation that they will be a useful resource. Of the fifteen students who completed the first questionnaire, only six of them said that they read their notes or books to help them learn. All other respondents felt that they only read problems or questions. As the year progressed, some students stated that they did use their notes.

Tamsin: Do you want to show me some of the things which you have in your exercise book that you have to read in maths and tell me what they are.
Student 1: I don't know. Median and that and at the back that's [where] the glossary is. That's the glossary and it tells me what the... some facts are.
Tamsin: When would you look at your glossary?

Student 1: When I was doing questions about anything to do with… when I don't know what to do.
Tamsin: So for your quiz or test or…?
Student 1: Look lots of the questions up there, when I don't know what product means.

The glossary was a collection of definitions based on the students' own descriptions. When a new term came up, the term was discussed in class and a joint meaning negotiated. Students then copied this into the back of their exercise books so that it was easily located. Often definitions were accompanied with an example, which also came from the students. Although the definitions originated from the students themselves, they did need to be encouraged to use them.

Teacher: Right. Now yesterday, we started to look at loci. Can anybody tell me what a locus was?
Student 1: A bracket.
Teacher: If you can't remember, look up your note book. You should have written the definition of what a locus is.
Student 2: A locus is
Teacher: Yeah.
Student 2: It's…
Teacher: You wrote it [laugh].
Student 2: I can't read my writing. Quite simply, it is all the points equal distance from something.
Teacher: Mmm.
Student 2: Like a dog on a chain and how far it can go out.

The first student's inappropriate response, triggered the teacher to remind the students that they had a resource to draw upon in their notes. Having the next student look at his notes meant that not only did he have to read his own handwriting, but he also had to make sense of it. An important component in writing notes or a solution to a problem is for them to make sense not only to the writer but also to others who might be reading them. An understanding of the needs of an audience is something than some students need to learn (Meaney, 2002). Encouraging students to review and use their own notes, is likely to contribute to them being able to study from them at home.

Think

- What were the mathematical ideas that you used or learnt?
- Where else might these ideas be useful?

Many students seem to believe that once they have solved a problem or written their notes that their opportunities for learning are over. In fact, the reading process needs to be thought of as extending beyond just gaining meaning from text to integrating that meaning into the learning process. The following extract shows how reflecting on an answer can lead to further learning. Figure 3 shows the problem that the students were answering.

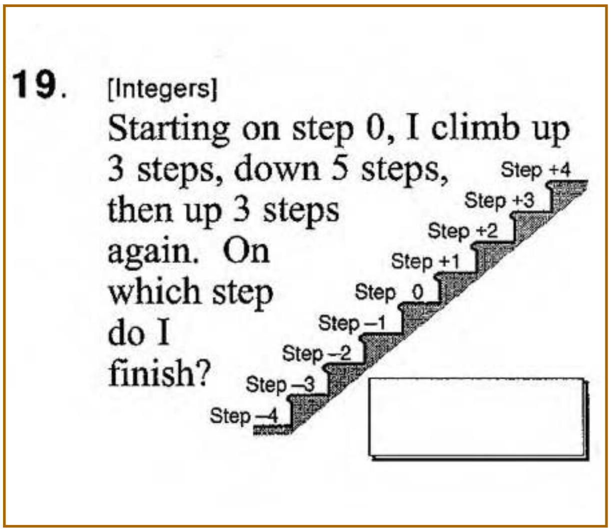


Figure 3. Integer problem.

Tamsin: So what was the important information in there that you were…
Student: Yeah, the information what you get was what step would there be, would you move to, to get to the end.
Tamsin: Mmm-hmm.
Student: All of those things that you have to do.
Tamsin: OK. Now if you didn't have that diagram, would you be able to work it out?
Student: No. Don't think so.
Tamsin: Would you have been able to draw something like that yourself?
Student: Oh yeah, you could if you drawed something like that.

The important part of this exchange was the reflection on the use of diagrams as an aid to solving the problem. Southwell (1993) has suggested that reflection supports the learning process because it "leads to synthesis, validation, integration and appropriation of knowledge" (p. 481). Therefore, reflection integrated into reading and this is most likely to occur when students have plenty of time to respond to challenging problems. The purpose of having students do a whole set of similar problems is to make the solution automatic. However, it can lead to students racing through the problems believing that there is only one way to solve them. The result is a reduction in students' thinking about what they are reading and what they are learning.

Do
- Talk about what you have done with someone else.
- Try these ideas out in another area.

Reflection on learning through reading needs also to include making connections to other situations. This can often come by discussing with others the ideas raised in reading. The following exchange occurred in a lesson on scale drawings. The students, with the teacher, using a scale factor of 1:80, had already worked out the length of a car given in a diagram. The length gained from their calculation caused them to comment on how much longer it seemed than an ordinary car. They decided that the car was probably a Jaguar. The next question was about the diameter of the tyre.

Teacher:	Right, now what's the next thing it says. We want the diameter of a tyre. The diameter goes right across, so you've got to measure the tyre, we want this dimension across here.
Student:	I got 9, yeah 9.
Teacher:	OK. So we've got, I'm saying 9 millimetres, times it by 80, what do we get, how many millimetres is that?
Shane:	720.
Teacher:	720 millimetres, which is 72 centimetres. OK, so a 72 centimetre tyre?
Student:	That's quite a big tyre.
Student:	OK, maybe to go fast.
Teacher:	OK, so…
Student:	Um, how big are tyres?
Teacher:	That's 72 centimetres across.
Student:	Um, divide by 2.5 and…
Teacher:	Oh, do you know tyres in inches?
Student:	Yeah. That's how they go…
Student:	How many inches is 72 centimetres?
Student:	It's 28 inches.
Teacher:	Let's see, I'll just get somebody to go out and measure a tyre off a car… OK, Peter, can you go and measure, just go down there to a car and see if you can measure a diameter, and we'll just compare…
Student:	Go for the biggest wheel.
Student:	He should get three and then find out what the average is.
Teacher:	Sorry?
Student:	He should do like three tyres and then find…
Teacher:	Find the average, well he could, but… we're just wanting to get an idea of how much this may be out by. It does look like a Jaguar though, it could be a big tyre.
[pause]	
Student:	What was it?
Student:	I got 62.
Teacher:	That's what um, Philip has just mentioned, 62 centimetres, so it's 10 centimetres bigger, the diameter.
Student:	What kind of car was he measuring though?

It was quite clear that the diameter of the tyre made the students even more sceptical about whether the diagram represented a real car. In the discussion, students brought in a variety of other mathematical information, such as converting to inches and getting the mean tyre diameter of several tyres to determine if the car in the diagram was close to reality. They read the result of their calculation with scepticism as they brought to it not only what they knew about cars but a range of other mathematical ideas. They then went on to determine a more appropriate ratio. Opportunities need to be made available for students to question what they read and relate it to what they already know mathematically and from their lived experiences.

Conclusion

Reading in mathematics classrooms needs to be an active part of the learning process. If it continues to be viewed as a passive way to gain information, then its benefit to the learning process will also continue to be under-utilised. The *Read–Think–Do (x2)* model can be a support to students to become more active readers. However, teachers need to show students how to make use of the model and to encourage them to do so. By having the model on a wall in the classroom, teachers can be reminded to ask questions as their students are reading. Changing students' reading habits will need constant reinforcement and will not occur quickly.

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Diversions solutions

Idea I

A.

1. $\frac{1}{2}+\frac{1}{4}$
2. $\frac{1}{2}+\frac{1}{10}$
3. $\frac{1}{2}+\frac{1}{3}$
4. $\frac{1}{4}+\frac{1}{8}$
5. $\frac{1}{2}+\frac{1}{8}$
6. $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$

B.

1. $\frac{1}{4}+\frac{1}{12}$
2. $\frac{1}{6}+\frac{1}{30}$
3. $\frac{1}{7}+\frac{1}{42}$
4. $\frac{1}{8}+\frac{1}{56}$
5. $\frac{1}{9}+\frac{1}{72}$
6. $\frac{1}{10}+\frac{1}{90}$
7. $\frac{1}{9}+\frac{1}{10}+\frac{1}{90}$
8. $\frac{1}{3}+\frac{1}{9}$
9. $\frac{1}{2}+\frac{1}{18}$
10. $\frac{1}{2}+\frac{1}{9}+\frac{1}{10}+\frac{1}{18}+\frac{1}{90}$

Idea II

For $c = 6$,

a	b	c
16	16	8
24	12	8
40	10	8
72	9	8

a	b	c
12	12	6
15	10	6
18	9	6
24	8	6
42	7	6

In general, a surprising connection that the values of a are

$$\frac{b\times c}{b-c}$$

so for any value of c, let b take on values from $2c-1$ down to $c+1$, discarding any non-integral results to the calculation of

$$\frac{b\times c}{b-c}$$

There may be other ways of stating this!