Establishing a MATHEMATICS-ENRICHMENT COMMUNITY in a multi-school environment

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Background

Trinity College is a low-fee independent school in Gawler, South Australia. The college has an enrolment of 3500 students of varied socio-economic mix, drawn from the northern suburbs of Adelaide. Trinity has recently been restructured into four R–10 schools and one senior school for Years 11–13. The schools share a common mathematics curriculum.

A growing need for mathematics enrichment has been identified in the middle schools (Years 6–10). Last year, materials from the Australian Mathematics Trust were trialled. A small group of enthusiastic Year 6 students worked their way through the "Newton Series" mathematics enrichment program provided by the AMT.

The three students involved were extracted from their regular class for one lesson per week for two terms. They were a happy team and showed obvious enjoyment in working on the enrichment materials. The group worked in a quiet room of the library with technology and whiteboards at hand. The materials from the AMT helped me provide a structured enrichment program for the students which blended collaborative and independent work.

Concurrent with this pilot scheme, discussions took place among staff on the relative merits of acceleration and enrichment. Some staff attended a workshop presented by Dr Anthony Gardiner in April 2004 at which the benefits of enrichment were outlined. The outcome of the staff discussions coupled with the success of the pilot scheme was that enrichment was favoured over acceleration as the means of catering for the needs of bright, enthusiastic mathematics students.

The enrichment plan

Having successfully trialled the materials from the AMT, the question of how to implement their use across the college was considered. At the beginning of this year, samples of the various levels of materials from the AMT were examined along with guidelines of suitability for different year levels.

After discussing the situation with the Dean of Studies, I was invited to attend a meeting with the Heads of Teaching and Learning of the four R-10 schools. At the meeting, I presented the proposal to embark on a college-wide program of mathematics enrichment. I outlined the procedures of the program and displayed samples of the materials.

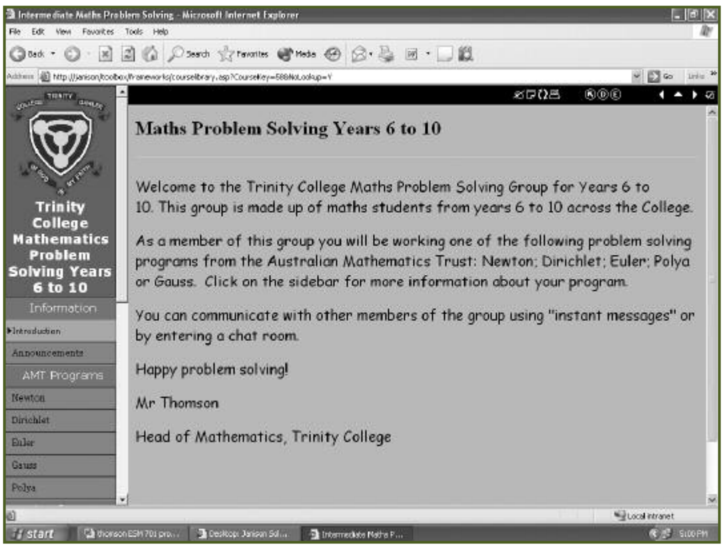
It was proposed that students from Years 6–10

across the college would be identified as being suitable for involvement in mathematics enrichment. The suggested criteria for suitability were that students were capable of staying a lesson ahead and that they were enthusiastic about mathematics. “Giftedness” was not an explicit criterion but realistic choices would have to be made in selecting students. Communication with parents would take place. I would be the overall administrator of the program for the college.

Each student was to be equipped with the student notes booklet from an appropriate level of the AMT mathematics enrichment materials, and a student problem booklet. The student notes booklet displays problem solving techniques to students and provides them with problems and exercises to try for themselves. Collaboration with peers, and teacher input, are encouraged where possible. Problems from the student problem booklet were to be tackled independently, and marked using the guidelines provided in the teacher reference notes.

The program would run through terms 2 and 3 and it was suggested that students remain in their peer class groups while working on the materials. (Apart from any other considerations, it would not have been possible to provide staff to supervise extraction groups.) It was recognised that students could experience isolation. It was therefore suggested that:

- Wherever possible small groups of students from the same class would be encouraged to work together on the student notes booklet at times scheduled by the class teacher
- A key “mathemaperson” (someone who specialises in mathematics in some way) would be sought from each school community who would be willing to be an occasional source of advice to participants and their teachers
- A website for learners and teachers would be established to facilitate discussions across the college normally precluded by the “tyranny of distance”.



The Heads of Teaching and Learning accepted the proposal and agreed to provide the necessary funding as, clearly, the proposal addressed needs that were arising within their schools. (Some parents had already requested acceleration or enrichment for their children; teachers and time-tablers were eager to find appropriate structured materials and solutions that could be provided on an ongoing basis.)

Prior to the meeting, I drew up a suggested list of students for the program. This list was distributed to the Heads of Teaching of Learning of the four R–10 schools. The Heads of Teaching of Learning consulted class teachers and mathematics teachers, and some names were added to or subtracted from my list. Eventually, a list of over 100 students was decided upon. Some Year 5s were included. Parents were consulted and the project was set to go ahead.

Key “mathemapersons” have since been found for three of the schools and a parent is likely to fulfil this role in the fourth school. I proceeded to order a range of different levels of material from the Australian Mathematics Trust. These materials have now arrived and the program is under way.

Using skills I acquired through professional development provided by the IT department of the college, I set up a website on the college intranet to facilitate communication across the college and the dissemination of information. The website can be accessed by registered users via the World Wide Web. Students have been registered on the site and have user names and passwords. The students will be able to access the site from school or at home by entering http://janison.trinity.sa.edu.au into their browser (see screen capture).

The website provides the students with information and, if they are online together, allows them to communicate with one another using “instant messages” or via a chat room called the “Maths Café”. Discussions can also be conducted and “threaded” according to users or subjects. Teachers have higher rights of access than students on the website. For example, teachers will be able to send announcements to all students or to those from a specific location.

Enthusiasm is growing for the project and one school has already agreed to staff an extra-curricular elective for their enrichment students. It is has even been suggested that students from the four schools could meet at "pizza and problem parties"! The program will be monitored closely and the progress of the participants will provide some interesting longitudinal information.

Analysis of the plan

The process of establishing this mathematics-enrichment community, and its outcomes, can be analysed in relation to the six components of teaching mathematics successfully outlined by J. Mousley (2005). These components are listed below:

- Building for understanding
- Engaging
- Organising for learning
- Communicating
- Problem solving
- Nurturing

Building for understanding

"Building for understanding" involves using suitable approaches to develop understanding in the students. In the process of teaching mathematics, the teacher may choose to foster instrumental understanding or relational understanding (Skemp, 1976). Relational understanding demands that a student is not only competent in applying a method to a problem but can also comprehend the reasons why the method works. By contrast, instrumental understanding requires only that the student can implement the method successfully. Relational understanding can be carried from one task to another. This makes it easier to remember than instrumental understanding which requires a new "recipe" for each new problem area.

In opting for a program of enrichment as opposed to acceleration, a choice has been made to seek depth of understanding rather than to move quickly on to the program for the next year level. This depth of understanding aligns well with relational understanding. In delving further into an area of mathematics, the number of connections made between varied content areas increases. This is exemplified in the student notes booklet of the "Euler Enrichment Stage" in a section that deals with figurate numbers. The dot patterns of counting numbers, triangular numbers, square numbers and pentagonal numbers connect into ideas associated with sequences and algebra. (Evans & Henry 2004).

The choice of enrichment over acceleration is one that prioritises understanding, specifically relational understanding, above all else. Perkins and Blythe (1994) advocate this strategy of "putting understanding up front". They note that "understanding is a matter of being able to do a variety of thought-demanding things with a topic." The enrichment materials from AMT provide this variety.

A by product of fostering relational understanding in students is the growth of intrinsic motivation (Skemp, 1976). This arises due to the rewards of satisfaction that the students experience when they gain depth of understanding (Skemp, 1976). In the mathematics enrichment plan for Trinity a connection can therefore be made between promoting depth of understanding and engaging students.

Engaging

"Engaging" students involves allowing them some control over their own learning as well as providing suitable activities to motivate them and capture their interest (Cross, 2004). The enrichment plan also caters for this since it allows for a degree of autonomous learning. Although students will remain with their peer class and complete work that meets the criteria of the regular coursework, they have the opportunity to take some control of their learning by engaging with the enrichment materials.

The concept of team learning and the development of team skills are argued to be beneficial dimensions of a mathematics teaching program. (Vale, 1999) Certainly, the benefits of team skills were evident in the pilot program at Trinity. There is scope in the enrichment plan for teamwork to develop in two forms: in enrichment groups of two or more students interacting face-to-face; and in the virtual classroom on the website.

Research conducted by Diezmann et al. indicates that collaboration will be preferred amongst very able students providing the tasks are sufficiently challenging (Diezmann et al., 2001). There is also evidence to suggest that students who work collaboratively will outperform students who work in individual circumstances (Barron, 2000). Given that able students are spread across the four schools at Trinity, it is quite possible that some of them will never meet until they enter the senior school. The use of the website may prove to be a valuable communicative tool for these students.

Additionally, Chen (1999) outlines the benefits of investigations in mathematics education and found that through an investigative approach:

1) Students experience meaningful learning
2) Students become involved in their own learning process and
3) Students interacted with each other (Chen, 1999, p. 5).

It is hoped that the investigations provided by way of the mathematically designed problems in the AMT enrichment materials will achieve similar results.

Organising for learning

"Organising for learning" involves providing a clear direction for lessons and communicating this direction to the students. An intended feature of the enrichment plan is that it will provide an organised, coherent structure for mathematics enrichment across the college. Until now, pockets of enthusiasm from class teachers have produced inventive and engaging materials for students to work on. Inevitably, because of pressures on staff time, the enrichment outcomes have been ad hoc. Increasingly, acceleration has been chosen as an expedient measure.

The plan has addressed the need for organising for learning and created an appropriate learning environment. The learning environment is multifaceted. It is geographically divided and yet united by common materials and the use of the internet. In addition, and perhaps most significantly, the environment is problem-based. A beneficial feature of the plan from a problem-based perspective is that there are no rigid restrictions imposed as far as the timetable is concerned (Mousley, 1996). This does not negate the existence of an underlying order in the structure of the program and in the planning of activities. Rather it is intended that there will be a socially-useful order underpinning engaging activity in the realm that lies between traditional predictable mathematics education and radical constructivism (Mousley, 1996).

Communicating

"Communicating" involves allowing students and teachers to interact in a variety of modes. It requires the establishment of structures to facilitate collaboration and two-way communication between students and teacher. It is difficult to predict how much communication will take place between students and their teachers. This will no doubt vary from teacher to teacher according to time constraints, the demands of the rest of the class, and importantly the beliefs held by the teachers about mathematics teaching. Kath Cross emphasises the significance of this aspect in saying, "…what the teacher thinks about mathematics and the purpose of mathematics teaching are crucial elements" (Cross, 2004, p. 4).

Hopefully the teachers will see beyond the functionality of mathematics and seek to instil in students a full appreciation of its worth. Ideally the teachers will engage the students with appropriate questioning techniques and allow them to explain their understanding verbally and in written reflections. It is also hoped that the students will experience the text of the enrichment materials to be dialogical and inviting of interaction, and not simply a monological delivery of new concepts.

The plan does address other aspects of communication more definitely. The connecting of students into suitable groups associated with the varied materials from the AAMT and the provision of the website demonstrate that communication is highly valued. Steps have been taken in the planning process to establish a community that will be able to function in a communicative manner. There is an aim to develop the kind of community described by Goos et al. who assert that: "Recent research in mathematics education has conceptualised mathematics teaching and learning as an inherently social and communicative activity" (Goos et al., 1997, p. 1).

Problem solving

"Problem solving" involves encouraging students to make personal connections with mathematical activities. It includes "activities such as risk-taking, challenging, exploring, investigating, thinking, asking, and posing." It is no surprise that social and communicative activity is more likely to take place over problem solving than over routine skill work. Further, treating mathematics as essentially a problem solving activity is considered to be an effective classroom practice to raise standards in mathematics (Jones et al., 2000). It is therefore worth examining in more detail the skills that students would do well to develop if they are to become able problem solvers.

Mayer classifies problem solving skills into three areas: cognitive, metacognitive, and motivational (Mayer, 1998). Cognitive skills are considered to be the basic skills which are required to reach a solution. When working on non-routine problems, these basic skills alone, however, are insufficient for the successful problem solver. The problem solver needs to be able to transfer cognitive skills to apply them to novel situations (Mayer, 1998). This ability can be referred to as "metacognitive" or as a metaskill (Mayer, 1998)

Beyond skill and metaskill, Mayer identifies a third factor which is required for successful problem solving. This third factor addresses the motivational dimension of the situation and is described by Mayer as the problem solver's will. The three components of cognitive skill, metaskill and motivation are deemed to be essential in problem solving and so teaching mathematics successfully will encapsulate all three.

Comparing these ideas about problem solving with the content of the materials from AMT produces some interesting results. The "Newton" student notes booklet begins with a story about children who become trapped in a strange castle — a castle of problems — which is a metaphor for the problem solving experience.

The characters in the story are required to solve problems in order to find their way out of the castle. In the process of doing this, the characters model the skills of problem solving to the reader. They display cognitive skills in reading and diagramming, and control the application of these methods using their metaskill. Using their imagination they seek solutions by methods such as reading a code backwards and translating information in a poem into an interior plan of the castle.

In addition to these cognitive and metacognitive skills, the characters also model the motivational attributes of successful problem solvers. Despite being trapped in the Castle of Problems they make a vow to be persistent and not allow themselves be overcome by anxiety: "Let's make a pact. No matter what happens, we stay calm, think positively and don't give up!" (Henry et al., 2004, p. 2).

Nurturing

"Nurturing" concerns the development of the student-teacher relationship. Teaching and learning is viewed as a reciprocal activity that takes place between expert and novice. It would seem reasonable to expect that an enrichment plan of the sort described would necessarily enhance any mathematics curriculum. However, it has been suggested that the benefits of such a plan will result in the nurturing of both students and teachers. The students' mathematical learning will improve, and the teachers will develop professionally. These nurturing effects will significantly augment the educational benefits for the students. Gervasoni posits this phenomenon: "It can be expected that increasing the professional knowledge of teachers and improving the curriculum in which the children engage will increase children's mathematical learning." (Gervasoni, 2002, p. 176).

The enrichment project at Trinity will involve teachers of students from Years 5–10. It will therefore be interesting to observe any variations in their responses. The teachers of students from Years 5–7 will be class teachers who teach the same class of students for a variety of subjects. The teachers of the Years 8–10 students will predominantly be mathematics teachers. In other institutions these two groups of teachers might be classified as primary and secondary teachers. They may well have contrasting beliefs about successful teaching and learning in mathematics. The nature of mathematics, motivational techniques, pedagogy and reasons why students succeed or fail are areas where differences may be apparent (Archer, 1999).

Professional development, an obvious feature of nurturing, could prove fruitful if this combined group of teachers is challenged to use a problem-based approach to mathematics teaching. There would be scope for teachers to discuss and exchange ideas, so that differences in their values and beliefs may be articulated and explored. This interaction would increase the value of any program of professional development and enhance the nurturing effect (Archer, 1999, p. 1).

Conclusion

A plan for establishing a mathematics enrichment community in a multi-school environment has been outlined. Using materials from the Australian Mathematics Trust, the program will be implemented during terms 2 and 3, 2005, and will involve students from Years 5–10 from four of the schools at Trinity College. Students will remain in their peer classes. Where possible groups of students will be encouraged to meet and all the students will be connected together as registered users of a website specially constructed for their use.

There is justifiable optimism for the successful implementation of the project. The plan has been analysed in relation to six components of successful mathematics teaching as described by Mousley (2005): building for understanding; engaging; organising for learning; communicating; problem solving; and nurturing. The findings of the analysis suggest that the problem-based approach of the materials coupled with the communicative facilities will benefit the learning community.

Preliminary findings

Some interesting preliminary findings are beginning to emerge following the establishment of the mathematics enrichment community. Reports from teachers at the 4 schools indicate that levels of engagement with the enrichment materials have been high. Several clusters of students have been gathered together in the schools. Students have enjoyed taking a collaborative approach. The website tracking system shows that students have been accessing the programmes and the supplementary materials on the website. The chat rooms have been used sparingly, however. An online forum approach will be emphasised next year.

Significantly, students have sustained their interest at a high level in situations where teachers have been able to provide regular weekly mentoring sessions. In view of this, it is planned that next year a time allocation will be provided for this, possibly with the aid of associate teachers (tertiary mathematics students).

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