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Technology is evolving and now some secondary classrooms see students using computer algebra systems (CAS), however, Zbiek (2003) points out that 'graphical representations seem to abound in CAS-using classrooms' (p. 208) and 'many [research] studies of the use of CAS depend highly on the graphing component of the tool' (p. 210). Kissane (2001) suggests that with the personal experience afforded by a graphing calculator the nature of learning 'if carefully structured by the teacher, can provide pupils with important insights about functions and their graphical representations' (p. 40).

Burrill et al. in their 2002 review note that the core research finding is that the type and extent of gains in student learning in the presence of handheld graphing technology 'are a function, not simply of the presence of the graphing technology, but of how the technology is used in the teaching of mathematics' (p. i). They concluded that 'specific issues regarding the effective use of handheld graphing technology in the classroom have not yet been adequately addressed' (p. ii). This paper describes approaches used by senior secondary students using a Texas Instrument (TI) 83 graphing calculator during a small research study to determine a global view of a difficult function. Implications of these approaches for teaching in a graphing calculator environment are presented and discussed.

The study

An understanding of what constitutes a complete graph of a function has always been important for senior secondary students (Leinhardt, Zaslavsky, & Stein, 1990; van der Kooij, 2001, Zaslavsky, 1997). With the mandating of graphing calculator use in schools, expertise in using a graphing calculator to find a global view of the graphical representation has become essential for senior secondary mathematics students studying functions (Anderson, Bloom, Mueller, & Pedler, 1999). A case study was undertaken to explore student understanding from the perspective of how graphing calculators are used during a task to demonstrate understanding of functions.

The five pairs of students, two from Year 12 and three from Year 11, selected for this study came from a mathematics classroom where the students were expected to use graphing calculator intelligently, to consider when to use it appropriately and when using it to contemplate alternative methods for any given problem. The use of graphing calculators was integrated throughout their mathematics course. The students were selected because they were expected to be able to solve the task as their strategies, not their success rate, were of primary interest. For more details of methodology see Brown (2003).

There are, however, some difficulties in observing students' use of graphing calculators (Williams, 1993). It is not easy to see the screen as the student uses the calculator and even more difficult to identify keystrokes used to generate these windows. In the study

uIt function

described, a View Screen was used to project the calculator screen. The subsequent videotaping of this projection went some way to overcoming this difficulty. The record of the results of students' actions and videotaping of graphing calculator screen outputs supplemented audio recordings, observational notes, and students' scripts allowing new insight into student understandings to be gained.

The literature reports that students' experiences continue to include mainly homogeneous scaling systems for axes as evidenced by most textbooks using equally scaled axes the majority of the time (Cavanagh & Mitchelmore, 2000; van der Kooij, 2001; Mitchelmore & Cavanagh, 2000). Students pay little attention to, and place minimal importance on, scale and often treat graphs and axes as separate and independent entities (Cavanagh & Mitchelmore, 2000; Dunham & Osborne, 1991; Mitchelmore & Cavanagh 2000). Graphing calculators 'allow students to specify a viewing window or rectangle when drawing graphs' (Demana & Waits, 1990, p. 215). Both students and teachers must accept and understand that the shape of a graph is dependent on the viewing rectangle within which the graph is viewed and they 'must learn to choose viewing rectangles that give good pictures as they explore' functions and search for a "complete' picture of the behaviour of graphs [that is] a graph that shows all the relevant behaviour' (Demana & Waits, p. 216). The continual use of identical scales can lead to students making incorrect generalisations about functions. Van der Kooij (2001) and others suggest that these difficulties are not caused by the use of graphing calculators

per se, rather that the increased repertoire that students are now expected to experience due to the use of graphing technologies includes more realistic graphs.

Different views of functions

Before the task undertaken by the student pairs in the study is presented, consider the following: how would you or your students respond to each of the following views of functions as displayed in the following graphing calculator outputs (Figure 1)?

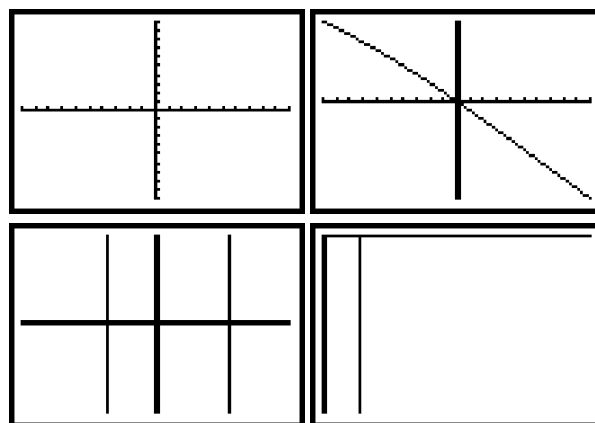


Figure 1. Graphing calculator views of a function.

What mathematical and graphing calculator knowledge would you or your students use given each of the views above in order to facilitate the determination of a global view of the functions under consideration? Would you and your students use the same or different knowledge? How do we explicitly teach this mathematical and graphing calculator knowledge? What learning situations can we provide for our students that allow them to consider

applying a range of mathematical and graphing calculator knowledge to ensure they have myriad options so that when a situation occurs where their preferred or usual application of knowledge is ineffective they have other choices available?

The task

The task was adapted from Binder (1995) and involved students working cooperatively to produce a sketch of the complete graph of a difficult cubic function. Before reading any further use your graphing calculator to help find a complete graph of the following function. Take note of the mathematical and graphing calculator knowledge used in your solution process.

Using a graphing calculator, sketch completely the graph of $y = x^3 - 19x^2 - 1992x - 92$. Show all important features.

To successfully undertake the task students need to understand the relationship between the algebraic, graphical, and numerical representations of a function and the information each provides about this function. They need to understand what constitutes a complete graphical representation of any function and the possible shapes of cubic functions. They also need a good working knowledge of the graphing calculator, in particular its features dedicated to altering the viewing window and to identification of key features of a function. Any degree of success will demonstrate some of the students' understanding about functions and their representations.

How do students apply their mathematical knowledge and graphing calculator knowledge to determine a global view of a difficult function?

Students applied a range of mathematical and graphing calculator knowledge in their search for a global view of the function. This mathematical knowledge included the possible shapes of cubic functions, more specific

knowledge informed by the algebraic representation of the function including information regarding the shape (e.g. consideration of the effect of the coefficient of x^3), the coordinates of the y intercept, the existence of at least one x intercept, and the domain of the function being all real numbers. Graphing calculator knowledge included directly altering the WINDOW settings (each of the maximum and minimum x , the maximum and minimum y ; and the scale marks on each axis), use of ZOOM menu items including Zoom Standard (which sets the window at $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$), Zoom Out (which zooms out by a (default) factor of four on a point specified by the user), and Zoom Fit (which selects y values that include the maximum and minimum y values of the function in the domain specified); and use of the Equation Solver. The use of graphing calculator features involved physical or both physical and cognitive actions, for example, pressing GRAPH or selecting Zoom 6: Standard Window, respectively. The cognitive nature of the latter action is apparent in that the students selected a known viewing domain and range.

At times a combination of mathematical and graphing calculator knowledge was applied. This included using knowledge of the coordinates of the y intercept to inform the WINDOW settings to find an appropriate view efficiently, knowledge of the existence of an x intercept and use of the Equation Solver, knowledge of the domain of the function in the selection of Zoom Fit or Zoom Out, or using the current graphing calculator view linked with a range of mathematical knowledge to inform subsequent actions.

All five pairs eventually solved the task. The knowledge and choices of one of the Year 12 pairs enabled them to apply their mathematical and graphing calculator knowledge in a way that ensured that their solution process quickly became routine. None of the other pairs of students consistently applied their mathematical and graphing calculator knowledge in conjunction with sensible choices of actions. One aspect of graphing calculator knowledge that was applied by all pairs was adjusting the WINDOW settings. All pairs undertook this action in their initial search for a global view of the graphical representation of the function, however only when combined

with mathematical knowledge and additional graphing calculator knowledge did this lead to the solution process becoming routine or potentially routine.

One Year 11 pair, for instance, after initially zooming out twice saw the view shown in Figure 2. From their dialogue and their adjustment of both the y minimum and maximum values, it can be inferred that this pair were unaware of the section of the graph coincident with the y axis. The steepness of the lines did not stop their using their mathematical knowledge to infer that there must be a turning point in the viewing domain under observation, however. As they were unsure of the type of turning point, they sensibly adjusted both endpoints of their viewing range. These changes resulted in a maximum turning point becoming visible in the viewing window (Figure 3), albeit probably not quite where they expected.

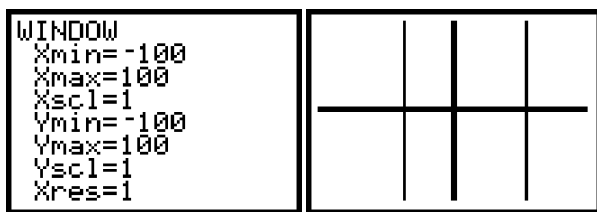


Figure 2. The vertical line effect as seen after zooming out twice from the standard window

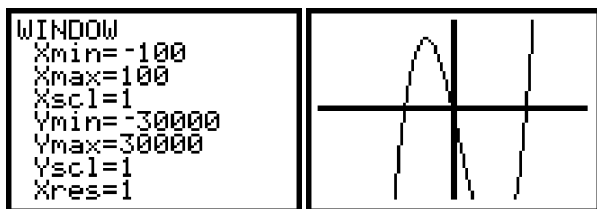


Figure 3. The almost global view resulting from informed alterations to the WINDOW settings.

In contrast, application of mathematical and graphing calculator knowledge by a second Year 11 pair was apparent from their use of the y intercept to inform their decision to alter the WINDOW settings. By selecting a viewing range that included the y ordinate of the y intercept, this pair could reasonably expect to see a view of the graph intersecting the y axis in the resulting viewing window. The resulting view (Figure 4), however, showed one part of the graph coincident with the y axis, a fact the students may not have realised, as this was most probably their first experience of this situation. However, neither student explicitly commented on the lack of a visible y

intercept and this lack of application of mathematical knowledge hindered a potentially routine solution.

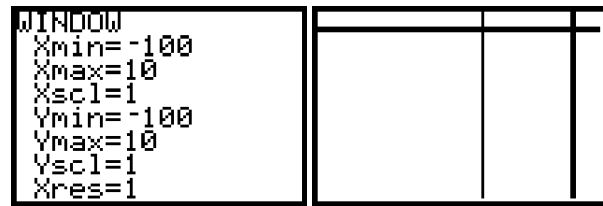


Figure 4. No y intercept apparent.

All pairs undertook actions that demonstrated they had a clear mental image of the function for which they were searching and the possible position of the output of the graphing calculator relative to this (see Figure 5), but at times their actions were not consistent with this, notably when confronted with difficult views of the function (see Figure 6). The actions of some pairs suggested that they temporarily did not have a clear idea which section of the function was currently the focus for the viewing window. However, a view that proved difficult for one pair was not necessarily difficult for another pair.

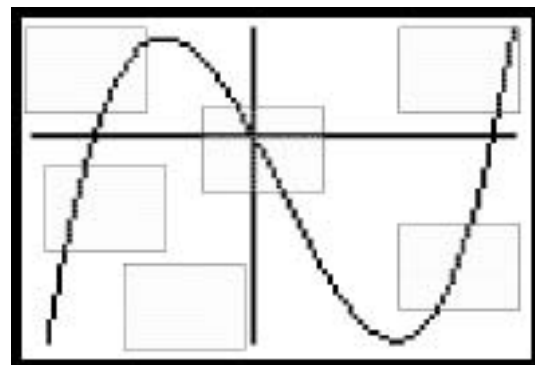


Figure 5. A selection of calculator outputs positioned relative to a global view.

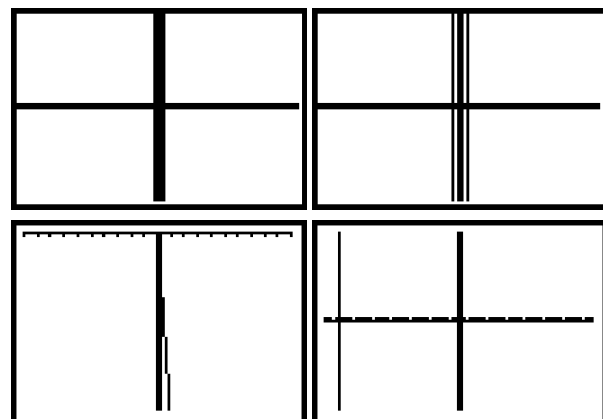


Figure 6. Difficult views of the function.

Evidence from this study supports the view that the combined application of mathematical

and graphing calculator knowledge is more efficient and effective in the determination of a global view of a difficult function. Such a finding is in keeping with views expressed by Anderson et al. (1999, p. 491). The solution process of all students in this study unequivocally demonstrated their understanding that the graphing calculator screen presents only a portion of the graphical representation of a function, not an understanding shared by many students according to Dunham and Osborne (1991), Goldenberg (1987), and others. In addition they all had the ability to adjust the current view presented by the graphing calculator, albeit varied in efficiency and effectiveness, to find an appropriate window for the function.

Two implications for teaching arising from these findings involve students' developing a correspondence between the output of the calculator and their mental image of the archetypical function under consideration and the use of the automatic range scaling feature for a given domain. Firstly, in order for students to apply their mathematical and graphing calculator knowledge to determine a global view of a difficult function, they need learning experiences that develop their understanding of positioning the specific output of the calculator with the global view being considered including the effect of scale and shape (Billings & Klanderma, 2000; Goldenberg, 1987; van der Kooij, 2001). Mathematical knowledge is essential for this understanding. Secondly, the use of an automatic range scaling feature for a given domain, for instance, Zoom Fit on the TI-83 or Autoscale on the HP38G, can facilitate the determining of a global view of a difficult function, however, its use does not necessarily imply mathematical understanding. Teachers and other mathematics educators need to consider the importance of students developing a mental image of a function and being able to position various outputs with this image. They may also need to consider the question: Is this mental image, and the positioning relative to this image, important for mathematical understanding, or is it simply a means that can be facilitated with the use of this calculator feature?

For students, an interpretation of these findings is that learning activities are required

whereby the opportunity is provided for them to develop and improve their mental images of functions, to make connections between transformations and changes of scale, and to resolve the confusion between these two. This understanding includes shape being an artefact of scale and challenging confusion between geometric transformations and a scale change. Both of these have already been identified as matters of concern by researchers such as Demana and Waits (1990), Dunham and Osborne (1991), and Goldenberg (1987).

Future research questions arising from these findings include: (a) If students are provided with learning opportunities to develop an understanding of automatic range scaling features for a given domain and their applications, does this help, hinder, or have no impact on mathematical understanding of the positioning of particular graphing calculator output relative to the complete graph and (b) What learning experiences best facilitate the development by students of how a particular graphing calculator viewing window output may relate to the global view of the function?

Further investigation is needed in order to determine the mental images held by students. Figure 5 could be used to help determine this. For each WINDOW shown in Figure 5, what would a sensible response action be, both from the ZOOM menu, or if restricted to adjustment of WINDOW settings? This could begin to address the question: What understandings do students demonstrate of the position of any particular graphing calculator view of a function relative to the global view of that function?

Other functions that students can spend valuable learning time searching for a global view are $y = 0.1x^3 - 0.4x^2 - 16x - 31$ and $y = x^4 - x^2 - 11$ (the second of these was suggested to me at the 2003 AAMT Biennial Conference). The latter function allows the benefits of using the TABLE function to be demonstrated. Beginning with the table set up to display an initial value of -3 and having an increment of 1 should result in all students asking the question 'is this graphical view showing all the global features of the function?'

I would be interested in any other interesting functions people have to share.

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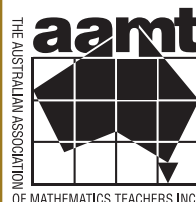
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