Enhancing Mathematical Problem Solving for Students with Disabilities

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This article focuses on the research program on mathematical problem solving conducted by the Center on Accelerating Student Learning (CASL). First, a subset of CASL themes, illustrated in the mathematical problem-solving studies, is highlighted. Then, the theoretical underpinnings of the mathematical problem-solving intervention methods are described, along with a brief overview of prior research. Next, the problem-solving dependent measures are presented, with explanation of how these measures explore a range of transfer distances that extend to real-life problem-solving situations. Then, to illustrate the research program, 3 intervention studies are described. The article concludes with recommendations for practice and future study.

Approximately 5% of the school-age population experiences mathematics disability (Badian, 1983; Gross-Tsur, Manor, & Shalev, 1996; Kosc, 1974). Despite its prevalence, math disability has been the focus of less systematic study than has reading disability (cf. Rasanen & Ahonen, 1995). This relative neglect is unfortunate, because mathematics skill is important for school success and accounts for employment, income, and work productivity even after intelligence and reading have been controlled for (Rivera-Batiz, 1992).

Despite this relative neglect, important work on math disability has been accomplished over the past 20 years. The literature describes the functional arithmetic difficulties of students with math disability and demonstrates how cognitive deficits are associated with the development of arithmetic cognition (see Geary, 1993, for a review). This literature has, nevertheless, focused disproportionately on the acquisition of basic facts. When problem solving has been studied (e.g., Jordan & Hanich, 2000), it has been confined largely to one-step word problems involving addition and subtraction number facts. In schools, such problems are relegated to the first- and second-grade curriculum. Moreover, such problems fail to represent real-world mathematical problem solving (MPS). For these reasons, it is difficult to make generalizations from the math disability literature to math disability as it occurs in schools and to the kinds of mathematical competence required in the real world. Therefore, a focus on mathematics competence beyond arithmetic and arithmetic story problems is required to understand, prevent, and remediate math disability in its many forms.

One reason for a disproportionate focus on basic facts and simple arithmetic story problems is belief in a hierarchical math sequence, where mastery of fundamental skills is prerequisite to instruction on complex MPS. Unfortunately, this thinking leaves most math-disabled students without instruction on complex problems until late in their schooling experience, and some math-disabled students never achieve sufficient mastery of prerequisite skills to permit any attention to the kind of MPS required in real life. At the same time, for typically progressing students, the traditional school curriculum introduces complex MPS as early as the third grade.

Consequently, the Center on Accelerating Student Learning (CASL) adopted the goal of providing math-disabled students instruction on MPS in the third grade, the same time as the school curriculum expands its focus to include complex problem solving for typically developing children. We were interested in studying the effects that an explicit approach to teaching MPS would have on children with math disability and on their nondisabled classmates. This focus on complex content, even when students may not have already mastered some relevant lower order skills, is a theme that has permeated CASL's work. It is reflected in Joanna Williams's focus on comprehension of expository text for low-performing and reading-disabled second and third graders (e.g., Wilder & Williams, 2001); in Steve Graham and Karen Harris's focus on different genres of written expression for at-risk and writing-disabled second and third graders (e.g., Graham & Harris, 1997); and in Doug Fuchs's focus on comprehension strategies for the full range of the population of first graders (D. Fuchs & Fuchs, 1999).

CASL's work on MPS also illustrates three additional themes important to CASL's "centeredness." First, we conceptualized MPS as a transfer challenge, whereby students solve problems they have never seen before and whereby

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problem difficulty is operationalized in terms of the degree of difference between novel problems and those used for instruction. This focus on transfer was an organizing CASL theme, evident both in the explicit instructional methods for developing transfer among students with disabilities and in the range of dependent measures employed across the CASL sites. A second theme is CASL's focus on the feasibility of its intervention methods for real-school application. Toward that end, CASL's research program on MPS was conducted in real classrooms, primarily using whole-class instruction that incorporated children with disabilities. The studies also relied on scripted manuals, designed with input from participating teachers, to facilitate transportability of the intervention for broader teacher use. Finally, CASL's math work illustrates CASL's emphasis on multiple instructional components, designed to address the multiple difficulties children with disabilities experience. In this article, we illustrate this CASL theme by showing how we borrowed Karen Harris and Steve Graham's self-regulation methods (e.g., Graham & Harris, 1997) for application to MPS.

In this article, we summarize CASL's research program on MPS. We begin by describing the theoretical underpinnings of our intervention methods, summarizing similarly conceptualized intervention research, and explaining how the CASL research program expands those earlier studies. Then, we describe the MPS tasks representing the domain CASL intervention targeted for improvement, explaining how those measures explore a range of transfer distances, including real-life problem-solving situations. The heart of this article follows, with a description of three intervention studies designed to enhance performance on those problem-solving tasks. We conclude with some recommendations for practice and for future study.

Our Approach to MPS Instruction

Conceptual Underpinnings

We conceptualized MPS as a transfer challenge, which requires students to apply knowledge, skills, and strategies to novel problems (cf. Bransford & Schwartz, 1999; R. E. Mayer, Quilici, & Moreno, 1999). Accomplishing this form of transfer can be especially difficult for primary-grade children (Durnin, Perrone, & MacKay, 1997) and for students with disabilities (White, 1984). Schema construction theory is a framework for conceptualizing how MPS is achieved. The theory postulates that MPS occurs with the development of schemas by which students group problems into types that require the same solution (Chi, Feltovich, & Glaser, 1981; Gick & Holyoake, 1983; R. E. Mayer, 1992; Quilici & Mayer, 1996). The broader the schema, the greater the probability that students will recognize connections between familiar and novel problems so they will know when to apply the solution methods they have mastered (Gick & Holyoak, 1983). When this happens, MPS—and the transfer it requires—occurs.

In an effort to promote the MPS of primary-grade children with math disability, we have relied on schema construction theory to design a treatment that teaches transfer explicitly. As conceptualized by Cooper and Sweller (1987), three variables contribute to problem-solving transfer. Students must (a) master rules for problem solution, (b) develop categories for sorting problems that require similar solutions, and (c) be aware that novel problems are related to previously solved problems. Research has substantiated the importance of the first variable, mastering rules for problem solution (e.g., Sweller & Cooper, 1985). As students master problem-solution rules, they allocate less working memory to the details of the solution and instead devote cognitive resources to identify connections between novel and familiar problems and to plan work.

Less is known about how to effect Cooper and Sweller's (1987) second and third variables, which are central to the role schemas play in transfer. To gain insight into this role, Cooper and Sweller questioned eighth graders as they worked novel algebra problems. The researchers coded responses in terms of whether statements reflected schemas (e.g., when faced with a new problem, students reported thinking about how an earlier problem had been solved) and demonstrated that schemas strongly influence performance on problems that fall within the boundaries of those schemas. They also noted, however, that students' schemas were disappointingly narrow. The challenge in effecting the transfer involved in MPS, of course, is to help learners develop broad schemas.

With respect to Cooper and Sweller's (1987) third variable, prior work reveals the importance of triggering awareness of the connections between training and transfer tasks. Research (Asch, 1969; Catrambone & Holyoak, 1989; Gick & Holyoak, 1980; Keane, 1988; Ross, 1989) shows that performance increases when participants are cued to anticipate similarities across tasks. To achieve the transfer involved in MPS, however, it is necessary to go beyond cuing by an external agent. Methods are needed to activate independent searches for connections between novel and familiar tasks.

Prior Research

Unfortunately, schema induction has proven difficult to achieve (e.g., Bransford & Schwartz, 1999; Cooper & Sweller, 1987; R. E. Mayer et al., 1999). Some work has examined how examples can be used to induce schemas. Quilici and Mayer (1996), for example, demonstrated that college students who independently studied statistics problems grouped by problem type (*t* test vs. chi square vs. correlation) sorted subsequent problems on the basis of structure, rather than surface features, more than students who studied without examples. They also showed that schema induction was strengthened through the use of structure- rather than surface-emphasizing examples. The pattern was stronger for lower ability students, who may tend to focus more on surface features than do higher ability peers unless primed to do otherwise. Of course, among these college students, low ability was operationalized with a mathematics SAT score below 575, a definition with limited application to the public school–age population.

In related work conducted with elementary school children, Chen (1999) examined how the variability of sample problems facilitates not only schema induction but also problem solution accuracy and its relation to schema development. Results illustrated how variant procedural features were more likely than invariant procedural features to result in schema induction and problem-solving transfer, but this time effects were mediated by age: Older children (10–12 years old) were more successful than 8- or 9-year-olds at extracting schemas from the sample problems and at solving subsequent problems.

As Quilici and Mayer's (1996) and Chen's (1999) studies illustrate, important questions remain about how to promote MPS among low-performing and younger students. Both studies, as is the case for much of the research on schema induction, relied on single-session interventions without explicit instruction to prompt students' schema construction. As Quilici and Mayer concluded, further study is warranted to examine whether explicit instruction and structured practice in schema-inducing activities, rather than independent study of examples, might strengthen effects.

Some research has explored the potential of teacher-directed schema-inducing instruction as a method for promoting MPS. For example, in a small-group tutoring study conducted with low-achieving students in Grades 2 through 6, Jitendra et al. (1998) tested a two-step intervention that combined schema-induction methods with the use of diagrams. In the first step, designed to induce schemas, students categorized the arithmetic word problem as a change, group, or compare problem type; in the second step, students used a diagram representing the relevant problem type to assist in problem solution. The effects of this multicomponent intervention were statistically significant, with an effect size of 0.45 on students' accuracy in solving arithmetic word problems, suggesting the potential for schema construction theory to guide the development of teacher-directed instruction on MPS. Of course, this study does not provide the basis for separating the effects of schema-induction methods from the use of diagrams. So, in the first CASL study presented later in this article, our goal was to separate the effects of schema-based transfer instruction for promoting MPS. Before presenting this study, however, we provide some information on the MPS tasks used in our series of CASL studies.

Complex MPS: The CASL Tasks

To identify tasks with which to study MPS, we relied on schema construction theory to configure our measure along a continuum of (a) contextual realism and (b) familiarity with the problems used for instruction. We predicted that as problems became (a) more similar to those found in real life and (b) concurrently less similar to those for instruction, it would become more difficult for students to identify novel problems as problems for which they had learned solutions. Accordingly, we studied MPS in terms of three transfer distances: immediate transfer, near transfer, and far transfer. Across measures, the problems became more similar to those found in real life and less similar to those used for instruction. The degree of novelty varied from measure to measure, but no problem on any of the measures had been used during instruction.

Each measure had two alternate forms; the problems in both forms required the same operations and presented text with the same number or length of words. The immediate-transfer alternate forms incorporated the same numbers, as did the near-transfer alternate forms. The far-transfer alternate forms used similar numbers. (In half the classes in each condition, we used Form A at pretest and Form B at posttest; in the other half, forms were reversed.) The problems on all three measures represented the four problem types (described later) constituting the focus of instruction; so, students had been taught methods to solve all problems. For all three measures, scores awarded credit for parts of work reflecting four dimensions: conceptual integrity, computational accuracy, communicative skills about the math, and problem-solving ability.

See Figure 1 for a sample problem used for instruction, along with a sample problem incorporated into each transfer measure. For the 10 problems on the immediate-transfer measure, the only source of novelty was the cover story; otherwise, the problems resembled those used for instruction. On the near-transfer measure, each problem incorporated a novel cover story as well as a novel superficial feature, which made the problem look more different from the problems used for instruction. This increased the novelty of the problem, increased the problem's relation to real life, and increased transfer distance. The far-transfer measure introduced multiple sources of novelty and was designed to mirror a real-life problem-solving task. Multiple sources of novelty were incorporated by (a) combining all problem types addressed in the 16-week intervention; (b) simultaneously varying all types of superficial feature changes covered in the 16-week intervention; (c) incorporating irrelevant text and numbers; and (d) assessing application of six additional skills from the district curriculum. Also, to decrease association with experimental treatment, the measure was formatted to look like a commercial test.

Three Illustrative CASL Studies on MPS

Over the course of 5 years of study, we developed an intervention to enhance the capacity of students with math disability to do the kind of MPS reflected in our three problem-solving measures. Given space limitations, we sum-

Problem Used for Instruction on Problem-Solution Rules

For the holidays, Anita will buy 1 CD for each of her friends. CDs come in bags with 3 CDs in each bag. How many bags will she buy for her 13 friends?

Immediate-Transfer Measure Sample Item (with Novel Cover Story)

You want to buy some balloons. Balloons come in bags with 6 balloons in each bag. How many bags of balloons do you need to buy to get 17 balloons?

Near-Transfer Measure Sample Item (Different Question)

You need hair clips. Hair clips come in bags of 4 for $1. They also come in bags of 7 for $2. To save money, should you buy hair clips in bags of 4 or in bags of 7?

Far-Transfer Measure (Problem Situation with One of Four Questions)

Class Pet

Your class decides to buy some parakeets as the class pets. You'll have to buy the birds, a cage, swings, and food dishes for the cage, and enough parakeet food to last for the rest of the year. Your teacher told you that you can also buy other things for the pet center in your classroom.

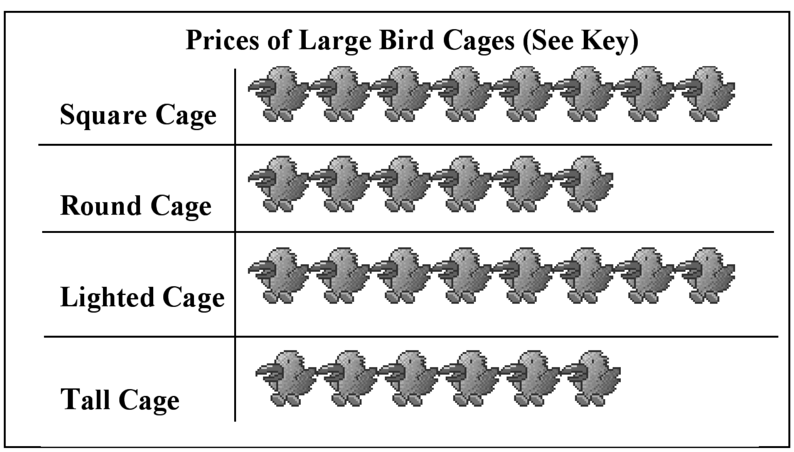
After talking to the pet shop owner, your class decides to buy four parakeets, a large cage, three bird swings, and four food dishes for the cage. Your teacher figures out that you'll need eight pounds of food to last for the rest of the year.

Your class raised one hundred nine dollars from a bake sale and ninety-five dollars from the book fair to buy the birds and supplies. Sixty-two people came to the book fair.

Prices of Birds and Supplies

1 Parakeet $14
1 Bird Swing $8
Bird Food
(5 pound can) $6
1 Food Dish $2

Key: Each  **means $10.**
All cages on sale for 1/2 the price on the chart.



How much money will the students spend on the birds, the cage, the swings, the food, and the food dishes for the cage? Show all your work.

FIGURE 1. Items illustrating novelty in Transfer-1, -2, and -3 Measures using the "Buying Bags" (Step-Up Function) problem type.

marize three studies within the program of research we conducted on this intervention: One study examined the potential for whole-class instruction rooted in schema-based theory to effect better outcomes; the second explored how self-regulated learning strategies might be used to strengthen that whole-class schema-based instruction; and the third investigated the efficacy of schema-based instruction when it was delivered in small-group tutoring.

Study 1: The Potential of Schema-Based Instruction

Our Question and Study Groups. Our research question involved pinpointing the effects of schema-based instruction on third-grade students with math disability and their typically developing classmates. To answer this question, we (L. S. Fuchs et al., 2003b) randomly assigned 24 third-grade classes to four groups for 16 weeks of treatment.

Regardless of condition, all students received the same base treatment, which comprised a basal text and the district curriculum. The basal text was *Math Advantage* (Burton & Maletsky, 1999). The district curriculum required teachers to address the same skills each week of the year. We chose our four problem types from this curriculum to ensure that the control group had instruction relevant to the study outcomes. The four problem types were "shopping list" problems (e.g., Joe needs supplies for the science project. He needs 2 batteries, 3 wires, and 1 board. Batteries cost $4 each, wires cost $2 each, and boards cost $6 each. How much money does he need to buy supplies?), "half" problems (e.g., Marcy will buy 14 baseball cards. She'll give her brother half the cards. How many cards will Marcy have?), "buying bags" problems (e.g., Jose needs 32 party hats for his party. Party hats come in bags of 4. How many bags of party hats does Jose need?), and "pictograph" problems (e.g., Mary keeps track of the number of chores she does on this chart [pictograph is shown with label: Each picture stands for three chores]. She also took her grandmother to the market 3 times last week. How many chores has Mary done?).

Six classrooms were assigned randomly to the control condition (Group 1), which represented conventional classroom MPS instruction. To guide instruction relevant to the four problem types, control teachers relied primarily on the basal. Instruction addressed one problem type at a time (as did the experimental treatments). It focused on the concepts underlying the problem type and taught a prescribed set of problem-solution rules, with explicit steps for arriving at solutions for the problems.

The other three conditions involved experimenter-designed instruction, which was explicit and incorporated worked examples and dyadic practice. Among the remaining 18 classrooms, 6 were assigned randomly to experimenter-designed instruction on problem-solution rules, without any explicit attempt to develop broader schemas to enhance MPS. This experimenter-designed problem-solution instruction (Group 2) incorporated 32 sessions, divided into five 3-week units, with two lessons per week (and two cumulative review sessions after winter break). Each lesson lasted from 25 to 40 minutes. Two units were dedicated to shopping-list problems; one unit was allocated to buying-bag problems, one to half problems, and one to pictograph problems.

In every session, the problems used for instruction varied only cover stories and quantities. A poster listing the steps of the solution method was displayed in the classroom. In the first session, teachers addressed the underlying concepts inherent in the problem type, presented a worked example, and, as they referred to the poster, explained how each step of the solution method had been applied in the example. Students responded frequently to questions. After reviewing the concepts and presenting several worked examples in this way, teachers presented partially worked examples while students applied steps of the solution. Students then completed one to four problems in dyads, with stronger students helping weaker ones solve problems and check work with answer keys. Remaining sessions were structured similarly, with a greater proportion of time spent on partially worked examples and dyadic practice.

Another six classrooms were assigned to experimenter-designed schema-based instruction, with shortened problem-solution instruction (Group 3). In this condition, we matched the number of instructional sessions to the number of sessions in (Group 2's) experimenter-designed instruction on problem-solution rules. To add a focus on schema-based transfer instruction, while maintaining only 32 sessions, we shortened the number of sessions on problem-solution rules to 22, adding 10 sessions of teacher-directed instruction on schema induction.

Teacher-directed instruction on schema induction incorporated 12 sessions and taught children how four transfer features can make a problem for which a solution method is known look unfamiliar. Each unit incorporated two schema-induction sessions related to the unit's problem type. The problems used for instruction in these 10 sessions differed from those used for problem-solution instruction in that they varied cover stories and quantities as well as one transfer feature per problem.

With schema-based instruction, teachers first taught that *transfer* means to *move:* Just as we transfer (move) to a different school, we can transfer (move) skills we learn to new situations. Teachers also presented examples of how children transfer skills; for example, children learn to drink from a toddler cup, then move this skill to a real cup, then move this skill to a glass, then move this skill to a soda bottle. Other examples were presented from everyday life, and students volunteered examples. Math examples were included (e.g., we learn to add two-digit horizontal problems; then move this skill to two-digit vertical problems; then move this skill to three-digit problems; then move this skill to the supermarket checkout line).

After discussing the meaning of *transfer,* teachers taught the four transfer features that change a problem without altering its type or solution. That is, a familiar problem type, for which a solution is known, can be formatted so that the problem looks novel, can use unfamiliar vocabulary, can pose a different question, or can be a small part of a bigger problem. A poster, "Transfer: Four Ways Problems Change," was displayed. Teachers explained the poster, illustrating each transfer feature with a worked example. They gradually moved to partially worked examples. Then, students worked in pairs to apply the solution method to problems that varied transfer features. Finally, homework was assigned. Teachers also reminded students to search novel problems for transfer features to identify familiar problem types and apply the solutions they knew.

Finally, to test the power of the full treatment, we randomly assigned another six classrooms to experimenter-designed schema-based instruction, with full problem-solution instruction (Group 4). This condition incorporated the full set of 32 sessions of teacher-directed instruction on problem solution rules plus the 10 sessions of teacher-directed instruction on schema induction, for a total of 42 experimenter-designed instructional sessions.

What We Found. Results supported the utility of the experimenter-designed problem-solution instruction. On the immediate transfer test, which required students to solve the four problem types when fresh cover stories constituted the only source of novelty, the two groups that received more of the experimenter-designed problem-solution treatment improved significantly and substantially more than the group that received less of the experimenter-designed problem-solution treatment (effect sizes were between 1.49 and 2.08). This provided evidence for the importance of Cooper and Sweller's (1987) first transfer-inducing variable, mastering rules for problem solution (as operationalized by our experimenter-designed problem-solution treatment).

Of course, the extent of transfer required on the immediate-transfer task was minimal because problems varied from the content of the problem-solution treatment only in terms of cover stories. So the effectiveness of the experimenter-designed problem-solution treatment is illustrated better with findings concerning the near-transfer measure, which not only varied cover stories but also manipulated, for each problem, one additional transfer feature that made problems appear more novel. Results indicated that, although this near-transfer measure was aligned more closely with the schema-based transfer conditions than with the problem-solution treatment, all three experimental conditions, including the experimental group that did not receive the schema-based transfer treatment, again improved significantly and substantially more than the control group (effect sizes were between 2.11 and 2.25). Moreover, the combined condition (problem-solution instruction plus schema-based transfer instruction) with the full set of problem-solution sessions persuasively outgrew the combined condition with fewer problem-solution sessions (effect size = 1.29).

In addition to supporting the effectiveness of the problem-solution treatment, the near-transfer results also provided evidence for the utility of the schema-based transfer treatment, which was designed to broaden the schemas by which students group problems requiring the same solution methods. Support for this schema-based transfer treatment was found on the near-transfer measure, in the dramatically superior growth of the combined treatment (schema-based instruction plus the full set of problem-solution lessons) versus the problem-solution condition alone without schema-based transfer instruction (effect size = 1.45). Both conditions had received all of the problem-solution lessons; the key difference was the provision of the schema-based transfer instruction. Although the simple addition of experimenter-controlled instructional time in the full problem-solution plus schema-based transfer condition might explain this effect, this explanation is unpersuasive in light of improvement on the immediate-transfer measure, where the contrast between these two conditions was not statistically significant (effect size = 0.19).

At the same time, the most convincing measure of learning in this study was the far-transfer test, our real-world problem-solving measure, which posed questions with the greatest degree of novelty: an unfamiliar cover story, simultaneous manipulation of all four transfer features taught, and inclusion of irrelevant information, as well as incorporation of additional problem structures and content taught in the district's curriculum. Also, we minimized extraneous cuing by formatting the far-transfer measure to resemble commercial achievement tests and using unfamiliar testers. Results on this far-transfer measure further substantiated the additive effect of schema-based transfer instruction: Both groups that received schema-based transfer instruction impressively outgrew the control group (effect sizes were between 1.01 and 1.16). Moreover, the problem-solution condition, which did not receive the schema-based transfer treatment, did not outgrow the control group (effect size = 0.39).

We also examined interactions between study condition and the mathematics grade-level status with which students began the study. We found no significant interaction; so, effects were not mediated by students' prior achievement histories. This is notable in light of previous work indicating that transfer is more difficult to effect among low-achieving students (Cooper & Sweller, 1987; L. S. Fuchs et al., 1999; D. P. Mayer, 1998; Woodward & Baxter, 1997).

What about students with math disability? Among these children, effects were least supportive for the combined condition that incorporated only the partial set of experimenter-designed problem-solution lessons. In that condition, 60% to 80% of students with math disability (depending on outcome measure) failed to progress more than the control group of students with math disability. This unacceptable rate of unresponsiveness reveals the need to develop a strong foundation in problem-solution rules before instruction designed to promote transfer can contribute to learning. For the other treatments, which incorporated the full set of lessons designed to teach problem-solution methods, findings were more encouraging. On immediate transfer, effects were smaller than for nondisabled students (where effect sizes ranged between 1.49 and 2.08). However, effects for the math disability sample were respectably strong (effect sizes were between 0.66 and 1.78). So explicit instruction, with strong reliance on worked examples and peer mediation, was effective in helping students with math disability master problem-solution rules, when novel word problems (i.e., new cover stories) were presented in the exact same format in which they had been taught.

As problems became increasingly novel from those practiced during instruction, however, the discrepancy of effects between students with and without math disability grew. For near transfer, effect sizes for nondisabled students who received schema-based transfer instruction ranged between 2.11 and 2.25; for students with math disability, the effect, while moderate, was dramatically lower (0.45 in the transfer condition with the full set of solution lessons). And on far transfer, where the problem situation approximated real-world problem solving, there was essentially no effect for students with math disability (effect size = 0.07), whereas nondisabled students clearly transferred their knowledge to the real-life problem-solving situation (effect sizes were between 1.01 and 1.16).

From this database, we realized the need to strengthen the power of the intervention to address the needs of students with math disability. Our next step in the research program, therefore, was to add an instructional component to boost the power of the treatment. We added self-regulation to the intervention, while examining its separate contribution to the combined intervention package.

Study 2: The Contribution of Self-Regulated Learning Strategies

Our Purpose and Study Groups. To strengthen the power of the schema-based intervention for enhancing MPS, we worked collaboratively with Karen Harris and Steve Graham to add a treatment component that relied on the Graham/Harris self-regulated learning strategies procedures (e.g., Graham & Harris, 1997). It made conceptual sense to incorporate a self-regulated learning strategies component within MPS intervention, because MPS requires metacognition (i.e., decision-making processes that regulate the selection of various forms of knowledge; Zimmerman, 1989). Also, metacognition is a critical process for self-regulation, whereby self-regulated learners set goals, self-monitor, and self-evaluate their performance (cf. Zimmerman, 1990). So, one approach for strengthening the metacognitive component of our problem-solving treatment was to incorporate the self-regulated learning strategies of goal setting and self-monitoring.

We were also interested in whether the contribution of self-regulated learning strategies would vary as a function of students' achievement histories. This seemed plausible because early work showed that children with cognitive deficiencies experienced difficulty determining how well they used strategies (Borkowski & Buechel, 1983) and failed to make accurate competency assessments (Licht & Kistner, 1986). As Schunk (1996) suggested, because average and high achievers assess their progress more reliably than remedial students, self-regulated learning strategies effects may be weaker for low achievers. Our design addressed this possibility by separating effects for students with varying achievement histories, including those with math disability.

In Study 2, we (L. S. Fuchs et al., 2003a) employed an experimental design to separate the effects of self-regulated learning strategies, including goal setting and self-evaluation, on MPS for third-grade children with varying achievement histories. We investigated effects on the Study 1 MPS tasks and on self-regulated learning processes, as assessed with a student questionnaire tapping self-efficacy, goal orientation, self-monitoring, and effort. Treatment duration again was 16 weeks.

Our study conditions were as follows. First, we employed a control group, as in Study 1, to reflect conventional classroom instruction on mathematical problems that included the same four problem types (we also included a 3-week introductory unit on basic MPS information). Our second condition was a schema-based transfer treatment, which also incorporated problem-solution instruction, as in Study 1's Group 4 (i.e., with 42 sessions). We did incorporate four enhancements to this condition, based on lessons we learned during the Study 1 implementation:

1. We added a 3-week introductory unit on basic MPS information (making sure answers make sense; lining up numbers from text to perform math operations; checking computation; labeling work with words, monetary signs, and mathematics symbols); this unit was also added to the control condition.
2. We dedicated one (instead of two) unit to the shopping-list problem type.
3. At the end of each session, students completed one problem independently and checked work against an answer key.
4. Students were assigned a homework problem, which they returned the next morning to the classroom homework collector (a competent classmate).

Our self-regulated learning strategies treatment incorporated these same schema-based instruction methods, but it also layered onto that schema-based instruction the following self-regulated learning strategies features (guided by our collaborators, Karen Harris and Steve Graham). First, after students finished working the independent problem of each session, they also scored their independent problem using an answer key specific to the unit's problem structure, which provided credit for the process of the work and the accuracy of the answer. Second, students charted their daily scores on an individual thermometer that went from 0 to the maximum score for that problem type. For each unit, students kept their chart in a personal folder. The chart showed 4 or 5 consecutive thermometers, one for each of the last 4 or 5 sessions of the unit. Third, at the beginning of the next session, students inspected their charts, were reminded that they wanted to beat their previous score(s) (or again achieve the maximum score), and set a goal to beat their highest score on that day's independent problem. Fourth, using an answer key, students scored homework prior to submitting it to the homework collector. Fifth, at the beginning of Sessions 1, 3, 4, 5, and 6 of each unit, students reported to the class examples of how they had transferred the unit's problem structure to another part of the school day or used it outside of school. The sixth activity involved a class graph, on which the teacher recorded (a) the number of students who had submitted homework to the homework collector and (b) the number of pairs reporting a transfer event. In these ways, self-regulated learning strategies incorporated goal setting and self-assessment referenced to the content of instructional sessions, including problem-solution rules instruction and transfer instruction.

What We Found. Results strengthen previous work (L. S. Fuchs et al., 2003b) showing that MPS may be improved with schema-based transfer instruction. On the immediate- and near-transfer problem-solving measures, students in the schema-based transfer treatment reliably outgrew those in the control group. Effect sizes were large, regardless of students' initial achievement status (1.91 and 1.98 for high achieving, 1.22 and 1.78 for average achieving, and 1.24 and 1.83 for low achieving) and similar to those reported by L. S. Fuchs et al. (2003b). On far transfer, however, effects did not reliably favor the schema-based transfer treatment over the control group, as Fuchs et al. had found (even though effect sizes were moderate to large: 0.47 for high-achieving, 0.54 for average-achieving, and 0.69 for low-achieving).

As already mentioned, Study 1 effects on the far-transfer measure indicated a need to enhance the strength of the problem-solving transfer treatment, and self-regulated learning strategies represented an avenue to accomplish that goal because of their capacity to strengthen the metacognitive value of the problem-solving transfer treatment and to increase perseverance in the face of challenge (Zimmerman, 1995). In fact, the combination of the schema-based transfer instruction and self-regulated learning strategies promoted reliably stronger improvement compared with the control group. Effect sizes exceeded 2 standard deviations on immediate transfer, ranged from 1.81 to 2.40 on near transfer, and fell between 0.81 and 1.17 on far transfer. So whereas the schema-based transfer instruction alone failed to promote reliable effects on the far-transfer measure (the most novel, and therefore truest, measure of MPS in our line of work), the combination of schema-based transfer and self-regulated learning strategies succeeded in effecting this challenging outcome.

Of course, the study design also permitted us to estimate the specific contribution of self-regulated learning strategies by comparing the improvement of students who received the schema-based transfer treatment combined with self-regulated learning strategies with those who received the schema-based transfer treatment alone. Results were mixed. On immediate transfer, the contribution of self-regulated learning strategies was evident. Children in the combined treatment reliably outgrew those in the problem-solving treatment without self-regulated learning strategies. Although the interaction between condition and students' initial achievement status was not significant, effect sizes were larger for high and average achievers than for low achievers. This suggests the possibility of differential efficacy for self-regulated learning strategies, which Schunk (1996) hypothesized on the basis of research showing that low-performing students may not monitor their performance accurately (Borkowski & Buechel, 1983; Licht & Kistner, 1986). Moreover, this suggestive pattern on the immediate-transfer measure was evident on the near-transfer task, for which interaction between condition and initial achievement status was significant: Although high-achieving students in the combined treatment reliably outgrew those in the schema-based transfer treatment alone, with an effect size exceeding 1 standard deviation, the growth of the average- and low-achieving students was not statistically significant, with moderate effect sizes of 0.55 and 0.35. Finally, on the far-transfer measure, distinctions between the two experimental treatments were unreliable and small for each achievement group, with effect sizes ranging between 0.12 and 0.25. So, as transfer demands increased across the range of problem-solving measures, the specific contribution of self-regulated learning strategies became less clear.

Consequently, on the one hand, the combined treatment with self-regulated learning strategies promoted far transfer when the problem-solving transfer treatment alone failed to effect this challenging outcome. On the other hand, the specific contribution of self-regulated learning strategies, as revealed by comparing the two experimental groups, was clear only on immediate transfer and, for high-achieving students, on near transfer. It is therefore instructive to examine findings on the student questionnaire, which tapped self-regulated learning strategies processes. Results indicated that the explanation for the superior growth of the combined treatment may in fact reside with self-regulated learning strategies. On three of four questions assessing self-efficacy, goal orientation, self-monitoring, and effort, students in the combined treatment scored better than those in the problem-solving transfer treatment without self-regulated learning strategies (and better than those in the control group). For "I learned a lot about MPS this year," an index of self-efficacy, the effect sizes comparing the combined treatment with the problem-solving transfer treatment without self-regulated learning strategies was 0.92. For "When I do math, I think about whether my work is getting better," a question designed to tap goal orientation and self-monitoring, the effect size was 1.20. Moreover, student effort was greater in the combined condition with self-regulated learning strategies, with students in the combined treatment agreeing more strongly with the statement "I worked hard this year so I could get better in math," compared with students in the problem-solving treatment alone (effect size = 1.35). In this way, self-regulated learning strategies may have provided the key mechanism by which the effects of the combined treatment were realized.

What about students with math disability? Both treatment groups grew comparable amounts on immediate transfer and improved more than the control group. Effect sizes were large: for transfer versus control, 1.07; for transfer with self-regulated learning strategies versus control, 1.43. Moreover, although effects for the combined treatment on measures with greater transfer challenge failed to achieve statistical significance for the small sample of students with math disability, the effect sizes of 0.95 on near transfer and 0.58 on far transfer are notable. In fact, the effect size for the combined treatment of 0.58, with lessons delivered to the whole class, was almost identical to the effect sizes reported on the same far-transfer measure for small-group tutoring that incorporated the problem-solving transfer treatment without self-regulated learning strategies (L. S. Fuchs, Fuchs, Hamlett, & Appleton, 2002). So, even for this lowest achieving group of students, who may experience difficulty setting realistic goals (Robbins & Harway, 1977; Tollefson, Tracy, Johnsen, Buenning, & Farmer, 1982) and monitoring performance accurately (e.g., Borkowski & Buechel, 1983; Licht & Kistner, 1986), the promise of self-regulated learning strategies is strong.

Study 3: Delivering Schema-Based Transfer Instruction in Small Groups

Another strategy for strengthening the effects of schema-based instruction is to intensify instruction by relying on small-group tutoring rather than whole-class sessions. We (L. S. Fuchs et al., 2002) examined the effectiveness of schema-based instruction for students with math disability when that treatment was conducted in small groups of two to four students. The lessons were identical to those used in Study 1's experimenter-designed schema-based instruction with the full set of problem-solution instruction sessions (i.e., Group 4 with 42 sessions, no self-regulated learning strategies), with one exception. Only one unit was dedicated to shopping-list problems, and the 3-week introductory unit on basic MPS information was included instead (it was also included for all four conditions).

To create a stringent test of efficacy, we assessed the contribution of this treatment to computer-assisted practice on real-world problem-solving tasks, in which students actually had intensive, guided, direct practice on the alternate forms of the study's far-transfer task. Students were randomly assigned to schema-based transfer tutoring (or not) and to computer-assisted practice (or not). Random assignment created four conditions: schema-based transfer tutoring, computer-assisted practice, schema-based transfer tutoring plus computer-assisted practice, and control, all of which received the same mathematics curriculum from which the four problem types had been selected for Study 1. As with L. S. Fuchs et al. (2003b), tutoring incorporated explicit instruction (Carnine, 1997) and peer-mediated practice (L. S. Fuchs, Fuchs, Hamlett, et al., 1997). Computer-assisted practice included incorporated guided feedback with motivational scoring.

Six special education teachers agreed to have their fourth-grade students with math disability participate. Teachers nominated 62 students who met two criteria. First, their standard scores on an individually administered intelligence test were 90 or above. Second, their special education teachers reported that they had math disability. To these 62 students, we administered a test of computational fluency to identify students (n = 40) who scored at least 1.5 standard deviations below a regional normative sample.

Stratifying so that each condition was represented approximately equally for each teacher, we randomly assigned students, with 10 students in each of four conditions: problem-solving tutoring with computer-assisted practice, computer-assisted practice, problem-solving tutoring, and control. Groups were comparable on students' sex, free/reduced-rate lunch status, race, and problematic classroom behavior and on computational fluency, math applications, and arithmetic story problems.

Results showed that schema-based transfer tutoring, which provided students with explicit instruction on rules for problem solution and schema-based transfer instruction in small groups, did differentially promote MPS growth among students with math disability. Significantly, this growth was manifested on the full range of measures, although with smaller effects on the real-world problem-solving, far-transfer measure.

On the immediate-transfer and near-transfer measures, tutoring produced statistically significant improvement compared with a control condition in which students had received a 3-week instructional unit on word problems. On the immediate-transfer measure, effect sizes comparing the tutoring condition with this control group exceeded 2 standard deviations and clearly were in the same range as those documented in the earlier study for nondisabled students (2.11–2.25). On the near-transfer measure, effect sizes were reliable and large (0.88)—nearly double those revealed for students with math disability when lessons were delivered in whole-class arrangement (0.45).

In addition, although results for the real-world problem-solving, far-transfer measure were not statistically significant, the effect size exceeded half a standard deviation. This figure approximated the effect size of 0.63 associated with the computer-assisted practice condition, even though the computer-assisted group spent all of its experimental time practicing problems analogous to the real-world problem-solving task. Moreover, the effect size of 0.61 for students with math disability when lessons were delivered in small groups was substantially larger than the figure of 0.07 for students with math disability when lessons were delivered in whole-class arrangement. Across the three tasks, therefore, results support the value of small-group instruction (Elbaum, Vaughn, Hughes, & Moody, 2000), where opportunities to respond, to seek clarification, and to obtain guided feedback are substantially greater than in whole-class instruction. Of course, this effect size of 0.61 for students with math disability, when lessons were provided in small groups, still pales in comparison to effect sizes exceeding 1 standard deviation for nondisabled students when lessons were delivered in large groups.

This study provided the basis for some optimism and some caution. On the one hand, results documented the efficacy of explicit instruction on problem solutions and schema-based transfer features when that instruction is delivered within the context of small groups and paralleled findings for students without disabilities, at three points on the achievement continuum, when the problem-solving program was delivered in large-class format (L. S. Fuchs et al., 2003b). As documented by Fuchs et al., the effectiveness of the schema-based transfer program resides in both components: Instruction on rules for problem solution explains growth on the immediate-transfer measure; the explicit transfer component explains growth on the near- and far-transfer measures. Of course, tutoring across both the problem-solution rules and the transfer components was explicit (Carnine, 1997) with peer mediation (L. S. Fuchs, Fuchs, Hamlett, et al., 1997) and small-group tutoring (Elbaum et al., 2000)—instructional features with proven efficacy for promoting reading. The study extends previous work by documenting effects on MPS, a curricular area that has received relatively little attention, especially for students with math disability, and where previous work indicates that outcomes are difficult to effect among low-achieving students.

Clearly, findings provide the basis for an important message to practitioners: Teachers can improve the MPS performance of students with math disability. On the other hand, effects on far transfer, although respectable, were lower than those effected for nondisabled students. So, additional work is warranted to identify how to enhance outcomes for students with math disability.

It is also interesting to consider the effects of computer-assisted practice, as operationalized in this study. Our software provided intensive, instructional feedback with motivational scoring. Results on real-world problem solving (i.e., the far-transfer measure), although not achieving statistical significance, did produce scores that exceeded the control group by a notable 0.63 standard deviation. This might have been expected given that computer-assisted practice was conducted on tasks directly paralleling the real-world problem-solving measure. More notable was a similar effect size of 0.60 on immediate transfer, along with a small effect size of 0.31 on near transfer, suggesting that some “downward” transfer occurred. Together, these effect sizes provide the basis for additional research on computer-assisted practice designed to guide students toward enhanced problem solving. Future research should incorporate greater power, not only with larger samples but also with software that enhances the instructional value of the practice provided. We have undertaken this effort because of the strong appeal of improved MPS without the need for expensive adult guidance.

It is nevertheless interesting to note that with the explicit, small-group, peer-mediated instruction on problem-solution rules and schema-based transfer in place, our computer-assisted practice condition provided no added value. On immediate and near transfer, the comparison between the tutoring treatments with and without computer-assisted practice revealed small effect sizes favoring the tutoring without computer-assisted practice. On far transfer, which paralleled the very tasks students practiced via computer, the effect size favoring tutoring plus computer-assisted practice was a disappointingly low 0.14. From a research design perspective, this lends credence to the value of the schema-based transfer tutoring treatment because the combined treatment, which failed to effect better growth, incorporated twice the amount of problem-solving instructional time. Substantively, results are bolstered by the fact that, on immediate and near transfer, direct contrasts between the computer and the schema-based transfer tutoring treatments reliably favored tutoring: Effect sizes comparing tutoring plus computer-assisted practice versus computer-assisted practice alone were 1.27 and 0.93; for tutoring without computer-assisted practice versus computer-assisted practice alone, 1.50 and 1.02. So, as revealed with the software used in this study, computer-guided practice failed to provide a meaningful substitute for or addition to carefully formulated adult tutoring. Perhaps software designed to provide better elaborated instruction would effect better outcomes, or it might be used in conjunction with problem-solving tutoring to reduce the amount of teacher time (an expensive resource) required to enhance MPS.

Conclusions

These three studies illustrate CASL’s research program on MPS. On the basis of this program of research, we draw five major conclusions about how to enhance MPS among students with math disability. Before identifying these implications for practice, we note that, in keeping with the CASL focus on feasibility and teacher friendliness, our interventions are scripted in the form of teachers’ manuals. The hope is that teachers can use these manuals to enhance MPS in their own classrooms. Also, we hope that, in becoming familiar with the instructional methods reflected in our manuals, teachers can apply our instructional principles to design more effective instruction on other problem types included in their curriculum.

Our first conclusion is that students with disabilities who are as young as 8 or 9 years can profit from instruction on MPS, even when their foundational math skills are poor. A major CASL goal was to explore the potential for enhancing performance on complex skills for students with disabilities, who typically have few opportunities for expanding higher order skills until late, if at all, in their schooling. With the CASL research program, we have documented that important gains on MPS can be achieved for this group of low-performing students. This should encourage teachers to include a focus on MPS for their students with math disability, even as they continue to teach foundational skills.

A second conclusion is that a strong foundation in rules for problem solution is a necessary component of (not a prerequisite to) MPS instruction. This means that children must master problem-solution methods on problems with low transfer demands (i.e., identically worded problems that only vary cover stories). This is necessary for MPS, where the challenge is to recognize problem types with greater transfer distance. The need for mastery of problem solutions has been substantiated in earlier research (e.g., Sweller & Cooper, 1985), showing that as students master problem-solution rules, they allocate less working memory to the details of the solution and instead devote cognitive resources to identify connections between novel and familiar problems and to plan their work. Teachers should ensure student mastery of problem-solution methods before teaching children about how transfer features can make familiar problem types difficult to recognize.

Our third conclusion about enhancing MPS is more noteworthy. It involves the need for explicit instruction on transfer, designed to increase awareness of the connections between novel and familiar problems. Toward that end, we relied on schema construction theory to build our intervention, which was designed to broaden the categories by which students group problems that require the same solution methods. The three experiments we presented in this article illustrate the efficacy of such explicit, schema-based instruction for promoting transfer among nondisabled students; effect sizes were substantial on the range of transfer measures examined. Moreover, our studies show how this approach also promotes improvement for students with math disability. Of course, among students with math disability, results are more impressive and reliable for near- than for far-transfer measures. Additional work is required to identify strategies for increasing the magnitude and range of problem-solving effects. In our subsequent studies within this research programs, we have examined the effects of two additional strategies. We added practice on sorting problems, where students classify problems with increasing transfer distance into the problem types for which they have learned solutions (without having to actually solve those problems). This has produced moderate benefits (L. S. Fuchs, Fuchs, Prentice, et al., 2004). A second approach, for which effects are stronger, is to increase the number of transfer features on which instruction occurs, so that the connections to real-life problem-solving situations become more transparent (L. S. Fuchs, Fuchs, Finelli, Courey, & Hamlett, 2004). The additional transfer features are irrelevant information, combining problem types, and combining transfer features. One of CASL's major themes was a focus on transfer. The MPS strand of the CASL research program illustrates how transfer can be achieved with an explicit and systematic instructional approach.

Based on the CASL research program, a fourth conclusion about how to enhance MPS for students with disabilities is that combining instructional components can provide important benefits. In Study 2, we illustrated how self-regulated learning strategies were combined with explicit instruction and peer mediation to provide a clear advantage in outcomes for students with disabilities.

Our final conclusion concerns service delivery issues. Our research program illustrates how outcomes can improve for students with math disability in solving challenging math problems when instruction is delivered in whole-class arrangement—and how impressive benefits also accrue for nondisabled classmates. At the same time, our research provides evidence about the value of small-group tutoring. Anecdotally, tutors reported some important process advantages of small-group tutoring, which may be central to the effects we observed. These included the capacity for teachers to increase on-task behavior and to monitor student response so that immediate corrective action can occur to correct faulty or incomplete understanding.

In terms of future research, we offer four directions. First, with respect to service delivery, it seems timely, in light of the popularity of multitiered instructional systems and response to intervention as a conceptual basis for identifying learning disabilities (e.g., Vaughn & Fuchs, 2003), to explore tiers of whole-class and small-group tutoring arrangements for preventing and identifying math disability. A second direction for future study is to study how combinations of instructional components, in addition to self-regulated learning strategies, may promote MPS. Third, given evidence that MPS represents a productive instructional target for children as early as third grade, it is important to explore the possibility of extending a schema-based instructional paradigm to the even younger children in second grade. Finally, it may be instructive to study the cognitive features associated with poor response to otherwise effective MPS instruction, which could help researchers identify other features of effective MPS instruction. It could also lead to the development of assessments for identifying children who respond poorly so that we can intervene even earlier with the goal of promoting better long-term outcomes.

Finally, study is required of the effects of MPS intervention for subgroups of students with math disability, with and without reading disability. This focus is important because research suggests that unique patterns of functional (e.g., Hanich, Jordan, Kaplan, & Dick, 2001; Jordan & Hanich, 2000; Jordan & Montani, 1997) and verbal and visual-perceptual (e.g., McLean & Hitch, 1999; Rourke & Finlayson, 1978; Siegel & Ryan, 1989) competence underlie math disability alone and math disability with reading disability. Because of the semantic challenges associated with MPS, regardless of whether problems are accessed through reading or listening, the verbal performance deficits of comorbid students along with more pervasive disruptions of language (Rourke, 1993) may render MPS a more difficult outcome to effect for students with comorbidity. Future research should identify effective practices for bringing about this outcome for students with serious difficulty in reading and math.

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