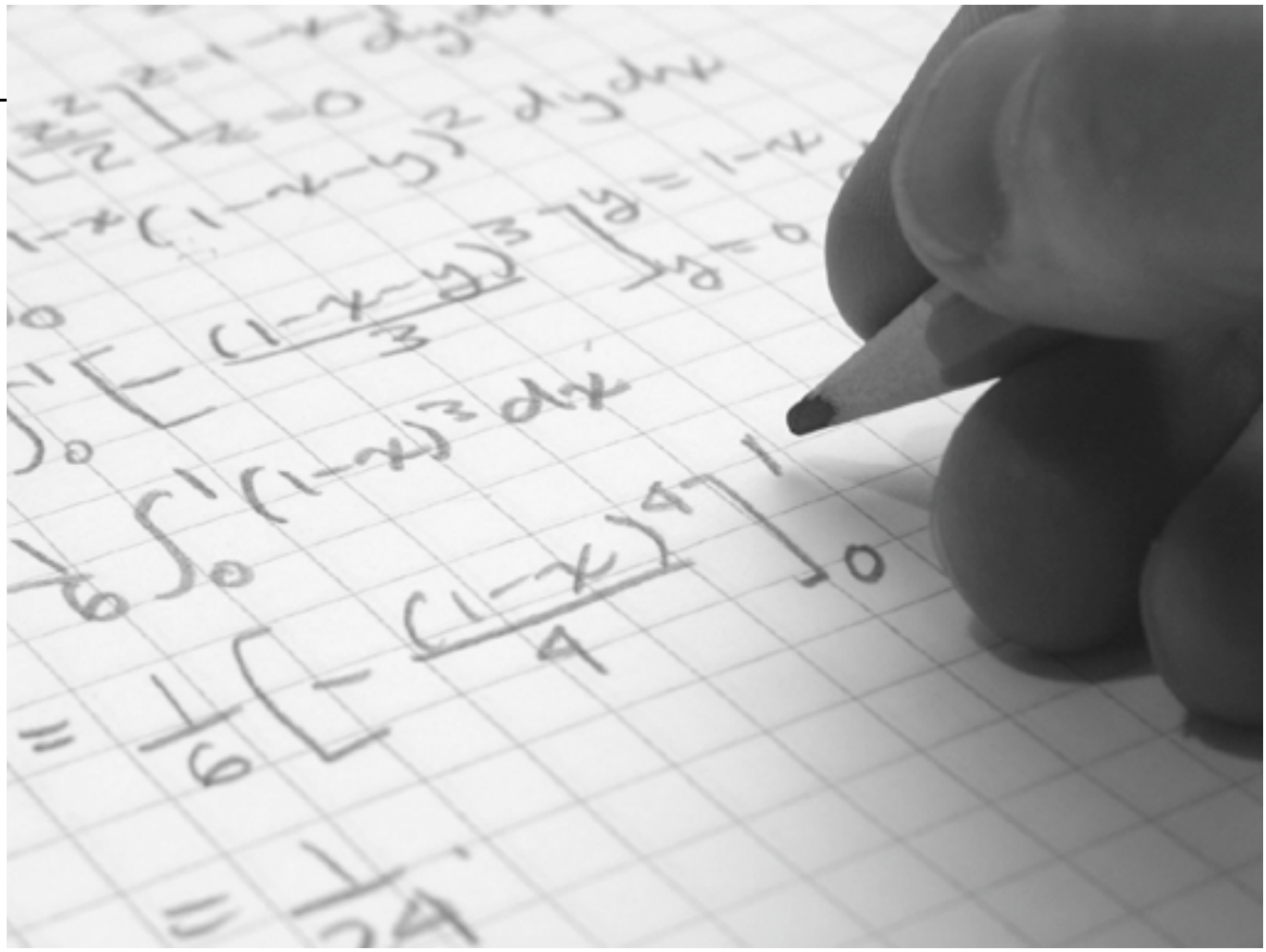
Mathematically Gifted Students:

How Can We Meet Their Needs?

Today, as usual, Mrs. Johnson began her 3rd-grade math class by reading aloud a thinking puzzle: Charlie, the dog, was tied to a 2-meter rope. His favorite ball was lying in the grass at least 10 yards away from him. He managed to grasp the ball easily. How did he manage to do this?

Nathan's hand flew into the air just microseconds after his teacher finished posing the question. While his classmates were pondering the problem, Nathan had already formulated the answer. Surprising even Mrs. Johnson, Nathan immediately found the lateral thinking puzzle required little effort and absolutely no math. While other students were converting meters to yards, moving decimal points, and drawing pictures, Nathan realized that the other end of the rope was not attached to anything; the dog merely had a 2-meter rope tied to his collar, but was not tethered at all.

by
Jennifer K. Rotigel
and
Susan Fello



Whether Nathan had heard the "riddle" before or whether he surmised that mathematics couldn't solve the scenario remains a mystery. Nevertheless, most teachers have had similar experiences with children who are talented in mathematics and strong in logical reasoning. Unfortunately, many programs for gifted children are inadequate and poorly designed (Heid, 1983), leaving classroom teachers to struggle to meet the needs of gifted children effectively. What resources are available for these students? What assessment tools are appropriate? Do these children need acceleration or enrichment? How can we meet their needs when there are so many other demands on teachers' time? This article will address these and other questions in an attempt to shed some light on the difficult issues of challenging and nurturing children who demonstrate talent in the field of mathematics.

Characteristics of the Gifted Math Student

Whether math problems require computation skills, problem-solving strategies, inferential thinking skills, or deductive reasoning, mathematically talented students are often able to discern answers with unusual speed and accuracy. Mathematically gifted students are able to see relationships among topics, concepts, and ideas without the intervention of formal instruction specifically geared to that particular content (Heid, 1983). Due to their intuitive understanding of mathematical function and processes, they may skip over steps and be unable to explain how they arrived at the correct answer to a problem (Greenes, 1981).

For example, Mariah, an energetic, 6th-grade prealgebra student, often seems disinterested during her hour-long math class, as she doodles and appears to be preoccupied. While the teacher demonstrates the steps required for calculating the correct answer to $4b + 11 = 2b + 23$, Mariah leafs through her history folder. After all, she can solve these linear algebraic equations in just one step. Like many gifted students, she barely listens to the teacher's directions, does not write page numbers in her assignment book, and does not make eye contact with the teacher. Mariah views practicing step-by-step processes as a waste of time when solutions can be found by just looking at the problem.

Students who are talented in mathematics often demonstrate an uneven pat-

tern of mathematical understanding and development, since some are much stronger in concept development than they are in computation (Rotigel, 2000; Sheffield, 1994). Gifted math students often want to know more about the "hows" and "whys" of mathematical ideas than the computational "how-to" processes (Sheffield). Since these children often prefer to learn all they can about a particular mathematical idea before leaving it for new concepts, a more expansive approach to mathematics based upon student interest may avoid the frustration that occurs when the regular classroom schedule demands that it is time to move on to another topic. A more linear approach to mathematics is often a better match for gifted children instead of the spiral curricula often found in textbook series and followed by classroom teachers. For example, when the topic of decimals is introduced, children with mathematical talent can be allowed to delve much further into the topic, learning practical applications for decimals and the connections between decimals and other mathematical topics.

Many of these students' gifted characteristics emerge during the preschool years. Bailey, a mathematically precocious 5-year-old, understands that numbers have patterns and relationships to real life. While watching a series of movie previews at the local theater, she can skillfully decide which new releases will occur before or after she turns 6 simply by noting their release dates in the upcoming year. Parents of preschoolers may report that their child demonstrates an unusual interest in mathematical concepts and particularly enjoys games involving numbers. At an early age, some gifted students note relationships between products and prices in the grocery store, the passage of time, changes in weather temperatures, and measurements of distances. Parents of these "number sense gurus" are fascinated by their children's precociousness, but are often unaware of the significance or relevance of these early mathematical discoveries.

By the time these emergent mathematical geniuses arrive for their first formal math lessons in kindergarten, they may have already established their own unique theories of number sense, sequences and patterns, problem solving, and computational strategies. Too frequently, the teachers following the curriculum merely touch on many math concepts, failing to recognize and nurture young mathematicians (Pletan, Robinson, Berninger, & Abbott, 1995). Formal instruction in elementary school classrooms often lacks challenge for the gifted learner since courses in regular classrooms sometimes have a relatively narrow range of topics, minimal investigation of concepts, repeated drill and practice, and yearly repetition. The basic mathematical concepts that are presented in kindergarten and 1st grade can be a particular problem for children who have already mastered number recognition, one-to-one correspondence, and counting. Recent studies indicate that few instructional adaptations are made to accommodate these young learners' needs (Archambault, Westberg, Brown, Hallmark, Emmons, & Zhang, 1993). Students gifted in mathematical thinking and problem solving need greater depth and breadth of topics and open-ended opportunities for solving more complex problems (Sheffield, 1994).

Challenges for School Districts

Misunderstandings regarding the nature of giftedness and talent abound, and busy teachers and administrators are sometimes at a loss to know how to nurture and challenge children whose abilities belie their age-based grade-level placement. However, according to the Principles and Standards for School Mathematics (National Council of Teachers of Mathematics, 2000, p. 13), these students must be supported so that they too have an opportunity to reach their mathematical potential. All too often, the regular curriculum is insufficient in depth, breadth, and pace to meet the needs of the gifted child (Wolfle, 1986). In addition, the recent emphasis on state standardized testing programs has increased the use of basic skills instruction and drill in an attempt to assure that all students will be successful on these tests (Moon, Brighton, & Callahan, 2002). A great deal of research supports the conclusion that gifted students need to use advanced materials and curricula if they are to reach their potential (Reis, Westberg, Kulikowich, & Purcell, 1998; VanTassel-Baska, 1995, 1998a).

Most educational settings do not adequately address and meet the needs of gifted students, and most teachers make only a few minor modifications to the curriculum when attempting to teach them (Archambault et al., 1993). Planning for 12 years of mathematics instruction for all students has sent many administrators and district curriculum specialists scrambling for the latest research on best practices, and the notion of meeting the needs of mathematically advanced students adds complexity to the task. The dilemma of choosing between acceleration to an advanced math class placement and providing planned enrichment activities within the regular classroom setting plagues math coordinators, curriculum specialists, superintendents, and parents.

The Critical Role of Assessment

According to the Principles and Standards for School Mathematics (NCTM, 2000, p. 23), assessment and

instruction should be integrated so that assessment provides information for the teacher to use to make instructional decisions. Children who demonstrate high achievement in mathematics should be carefully evaluated to determine the extent of their talent and provide a profile of their strengths and weaknesses (Lupkowski-Shoplik, Sayler, & Assouline, 1994). Grade-level achievement test results can be somewhat helpful, for if a student scores extremely well, it may indicate that mathematics is an area of strength for him or her.

If a child scores at or above the 95th percentile on a grade-level achievement test, it is possible that the test did not have enough items of appropriate difficulty for the child, thus the score may not indicate his or her true level of understanding. In this case, it may be necessary to administer an above-level test that will contain more items of higher difficulty. Julian Stanley pioneered this concept in 1971 when he began the talent searches (Stanley & Benbow, 1986), and 30 years worth of research have shown that out-of-level testing is a valuable tool for determining the level of programmatic modification that is needed for a gifted student.

Although a variety of standardized tests may be used for out-of-level testing, research has demonstrated the effectiveness of the EXPLORE test to identify talented elementary school students (Colangelo, Assouline, & Lu, 1994; Rotigel, 2000; Rotigel & Lupkowski-Shoplik, 1999). The EXPLORE (American College Testing Program, 1997) was developed for 8th-grade students and therefore contains a sufficient number of higher level items to allow students to demonstrate their proficiency more fully. According to ACT, the EXPLORE is directly related to student educational progress and includes a large number of complex problem-solving items and fewer measures of narrow skills. Many of the regional talent searches around the country offer EXPLORE testing in mathematics, as well as other subject areas.

No evaluation should be based simply on standardized testing, however. Teacher observations, classroom-based assessment, daily performance, and social and emotional needs must be included in the evaluation. It is important that data be gathered and analyzed by a multidisciplinary team of educators who are able to make and carry out educational recommendations for the student.

Program Evaluation and Curricular Collaboration

From the administrative perspective, a needs assessment could be conducted with all math teachers to determine each teacher's individual perceptions, teaching methods, and curricular successes or problems. An investigation of repeated topics, overlapping concepts, and ineffective activities could identify weaknesses in the math program. In order to meet students' needs, many concepts and topics in the curriculum could be compacted (Reis et al., 1998). For example, ratios and proportions could be coupled with simple fractional portions of sets. Some measurement or temperature concepts could be integrated into other curricular areas like science, thus allowing for more enrichment lessons in visual logic, inferential thinking, and deductive reasoning. According to VanTassel-Baska (1998b), efficient use of time is an important consideration in the development of talent.

Challenges for Teachers

Differentiation of Instruction

Once a sufficient foundation of information is gathered, an individualized plan can be established for each mathematically gifted student. It then becomes the responsibility of classroom teachers to implement the program. The plan may include enrichment experiences; differentiation of instruction, including pretesting and compacting the curriculum; flexible cluster grouping by topic or mathematics achievement; grade skipping in math; mentoring; and increased use of technology. The decision regarding which level of intervention is necessary should be based upon the evaluation. Highly gifted students may require more intense modification such as grade skipping in mathematics. Rather than choosing one method over the other, research indicates that a combination of these approaches makes for a stronger program for divergent math thinkers (Stanley & Benbow, 1986). Daily ongoing assessment, teacher observations, achievement tests, and above-level testing can all be helpful in determining the type of program that will best meet the needs of each gifted child.

Defined by Tomlinson (1995), differentiated instruction is "the consistent use of a variety of instructional approaches to modify content, process, and/or products in response to the learning readiness and interest of academically diverse students." Teachers must add components to each lesson and modify the content for the high-ability students, as well as for those who need remediation. For example, a lesson on calculating the area of polygons might include just the basic formula for most students, but should provide various real-world applications of calculating area for gifted learners. The increased complexity of the problems should require higher order thinking skills and provide opportunities for open-ended responses. Effective differentiation of instruction is very different from the unfortunate practice of simply assigning

20 problems to the gifted child while the remainder of the class is given only 10. "More of the same piled higher" is inappropriate and may lead children to conceal their abilities in order to avoid the extra, unnecessary work.

The task of differentiating each lesson requires accessing additional resources, planning for small-group interaction, and perhaps even modifying lessons during delivery (Tomlinson, 1995). In most instructional settings, the mathematical understanding and performance of the students is diverse, so classroom teachers plan their instruction with a myriad of learners and learning styles as the focus. Using a pretesting component in the math program allows for the identification of attained skills, strategies, and achieved concepts prior to the beginning of a new unit. For the gifted population, this helps eliminate the repetition from year to year in the mathematics curriculum. Pretesting and compacting the curriculum allows for a diagnostic approach for planning the teacher's instruction and allows educators to have a more accurate account of the skills and concepts students have mastered and those they have yet to be introduced to or need to strengthen. This process provides the foundation for effective differentiation of instruction, as each student should receive instruction based upon his or her identified instructional level (Howley, 2002). As noted by Winebrenner (2003), gifted students whose programs have been compacted can spend time working on their differentiated activities while their classmates are preparing for state assessments.

Enrichment and Grouping

It is certainly possible to meet the needs of some gifted students simply by enriching and modifying the existing mathematics curriculum. Enrichment is designed to expose students to a variety of topics related to those of the regular education program and to allow for further investigation of them.

In cases where math students are grouped according to their understanding of mathematical concepts and ideas, teachers can cover concepts at an appropriate pace for the group. Pretesting and compacting of the curriculum are helpful here, as they allow the group of mathematically talented students to be appropriately challenged. In addition, a classroom of homogeneously grouped gifted students would enable a teacher to apply the Enrichment Triad Model (Renzulli, 1977). For example, once students have mastered basic algebraic concepts, additional learning opportunities would be provided in the area of divergent thinking, individual projects, and group activities that would connect those algebraic concepts with real-world events and scenarios.

Acceleration and Technology

Since many mathematically talented students have already mastered the basic skills, the enrichment activities and advanced projects their teachers have planned may not provide sufficient challenge. Indeed, for many talented youngsters, some degree of acceleration is needed based on their demonstrated achievement and ability. Simply working in the highest math class at their grade level may not meet gifted learners' needs, regardless of how well the teacher has differentiated the lessons. Gifted students may have already acquired the content and concepts presented in these classes, so acceleration to a math class at a higher grade level may be the most viable option. However, Lewis (2002) cautioned that acceleration should not be done unless it also meets the studen-

Helpful Web Sites

The following Web sites provide additional information regarding gifted students, their characteristics, and current research in the field. Administrators, teachers, parents, and paraprofessionals may find these sites worthwhile.

National Association for Gifted Children (NAGC)
http://www.nagc.org
Educators will benefit from the research, journals, conventions, and a nice collection of publications in the bookstore area.

American Association for Gifted Children (AAGC)
http://www.aagc.org
AAGC includes information that is especially helpful for parents and teachers of young gifted children.

National Research Center on the Gifted and Talented, University of Connecticut
http://www.gifted.uconn.edu
Provides details on the Schoolwide Enrichment Model, graduate programs, and many resources.

Hoagies Gifted Education Page
http://www.hoagiesgifted.com
This extensive listing of conferences, resources of every kind, articles, and support for parents and teachers is a comprehensive "first stop" for everyone.

GT World
http://www.gtworld.org
GT World is designed primarily for parents and includes e-mail lists and lots of links to resources.

The following Web sites are designed for student use. The sites are easily accessible and provide information on math topics, problem-solving activities, and everyday mathematical applications. Students can explore a variety of mathematical concepts and strategies.

Math Forum
http://www.mathforum.org
Includes problems of the week and a teacher exchange

A+ Math
http://www.aplusmath.com
This site for children includes a homework helper and math games.

GoMath
http://www.gomath.com
Students enjoy the automated math solutions and an SAT preparation area.

Mathlab
http://www.mathlab.com
A hands-on geometry site.

How Stuff Works
http://www.howstuffworks.com
A popular Web site for children that includes information on all sorts of topics.

The 24 Game
http://www.24game.com
An online opportunity for children of all ages to play The 24 Game.

t's affective needs, which is sometimes difficult to determine.

Although accelerating a student to a higher grade-level class can present logistical problems for teachers and administrators, it is important to match not only the mathematical content with the learner's needs, but also to provide an appropriate pace of instruction for his or her rate of acquisition. Acceleration may be the only way to accomplish this. Classroom experience and research demonstrate that, even though they may be younger, children who are exceptionally talented in mathematics will learn material much more quickly and with fewer repetitions than the regular curriculum allows (Sowell, 1993). Thus, the repetitions of a spiral curriculum become redundant and mundane to a gifted math student. On the other hand, gifted students in accelerated classes have opportunities to work with advanced concepts, in-depth topic investigations, and problems with real-world applicability.

Many advances in technology can assist the classroom teacher in meeting the learning needs of gifted math students by providing opportunities to explore complex problems and mathematical ideas (NCTM, 2000, p. 14). Readily accessible classroom computers, supervised access to the Internet, and appropriate software programs offer opportunities for gifted students to advance at their own rate.

A Conclusion That's Outside the Box

Teachers sometimes experience frustration when gifted children can arrive at correct answers through nontraditional methodologies or when some of their questions are far beyond the scope of the lesson at hand. Since gifted students can often interpret, predict, and analyze mathematical situations and problems better and faster than their teachers, a significantly different instructional approach may be necessary. Successful teachers of gifted learners adapt their teaching strategies to accommodate the students' unusual thinking strategies and methodologies. Unfortunately, inexperienced or untrained teachers sometimes make provisions for the gifted by assigning them enrichment worksheets, independent projects, or reports on famous mathematicians. Quantity, in this case, does not always equal quality (Greenes & Mode, 1999; Wolfle, 1986).

Being sensitive and aware of the unique characteristics of gifted students enables teachers to set more realistic expectations in the classroom. Teachers need to be confident in their own mathematical knowledge and teaching abilities in order to accept the divergent thinking abilities of their gifted students. In Mariah's case, traditional expectations of step-by-step problem solving with paper-and-pencil assignments may not be appropriate. When gifted students are able to arrive at the correct answer by following an unmarked thinking path, teachers should acknowledge this creative, divergent problem-solving strategy and not reprimand the student through missed points or a lowered grade because of not following more traditional techniques. Teachers should adapt the content where appropriate, condense the concepts where applicable, alter the pace of content acquisition, and allow for open-ended, multiple solutions to problems.

Meeting the needs of each learner is the goal of every teacher, and each grade level has its own unique challenges. Whether it is the preschooler, Bailey, who can often make math connections without formal instruction; Nathan, the 3rd grader with the quirky thinking style; or Mariah, the prealgebra student who "sees" the answer without doing the work, each student thinks "outside the box." Accessing all available resources, using a variety of assessment tools, and choosing appropriate placements for each student are all aspects of meeting the individual needs of each learner. Being aware and sensitive to the unique characteristics of gifted learners will assist teachers in providing a myriad of opportunities for growth in mathematical reasoning and problem solving. GCT

continued on page 65

continued from page 51

References

American College Testing Program (ACT). (1997). *EXPLORE technical manual*. Iowa City, IA: Author.

Archambault, F. X., Westberg, K. L., Brown, S. W., Hallmark, B. W., Emmons, C. L., & Zhang, W. (1993). *Regular classroom practices with gifted students: Results of a national survey of classroom teachers*. Storrs: The National Research Center on the Gifted and Talented, University of Connecticut.

Colangelo, N., Assouline, S. G., & Lu, W. (1994). Using EXPLORE as an above-level instrument in the search for elementary student talent. In N. Colangelo, S. G. Assouline, & D. Ambroson (Eds.), *Talent development II: Proceedings from the 1993 H. B. and Jocelyn Wallace National Research Symposium on Talent Development* (pp. 281–297). Dayton: Ohio Psychology Press.

Greenes, C. (1981, February). Identifying the gifted student in mathematics. *Arithmetic Teacher*, 14–17.

Greenes, C., & Mode, M. (1999). Empowering teachers to discover, challenge, and support students with mathematical promise. In L. J. Sheffield (Ed.), *Developing mathematically promising students* (pp. 121–132). Reston, VA: National Council of Teachers of Mathematics.

Heid, M. K. (1983). Characteristics and special needs of the gifted student in mathematics. *Mathematics Teacher, 76,* 221–226.

Howley, A. (2002). The progress of gifted students in a rural district that emphasized acceleration strategies. *Roeper Review, 24*, 158–160.

Lewis, G. (2002). Alternatives to acceleration for the highly gifted child. *Roeper Review, 24*, 130–134.

Lupkowski-Shoplik, A. E., Sayler, M. F., & Assouline, S. G. (1994). Mathematics achievement of talented elementary students: Basic concepts vs. computation. In N. Colangelo, S. G. Assouline, & D. Ambroson (Eds.), *Talent development II: Proceedings from the 1993 Henry B. and Jocelyn Wallace National Research Symposium on Talent Development* (pp. 409–414). Dayton: Ohio Psychology Press.

Moon, T., Brighton, C., & Callahan, C. M. (2002). State standardized testing programs: Friend or foe of gifted education? *Roeper Review, 25*, 49–61.

National Council for Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics*. Reston, VA.: Author.

Pletan, M. D., Robinson, N. M., Berninger, V. W., & Abbott, R.D. (1995). Parents' observations of kindergarteners who are advanced in mathematical reasoning. *Journal for the Education of the Gifted, 19*, 30–44.

Reis, S. M., Westberg, K. L., Kulikowich, J. M., & Purcell, J. H. (1998). Curriculum compacting and achievement test scores: What does the research say? *Gifted Child Quarterly, 42*, 123–129.

Renzulli, J. (1977). *The enrichment triad model: A guide for developing defensible programs for the gifted and talented*. Mansfield Center, CT: Creative Learning Press.

Rotigel, J. V. (2000) *Exceptional mathematical talent: Comparing achievement in concepts and computation*. Unpublished doctoral dissertation, Indiana University of Pennsylvania.

Rotigel, J. V., & Lupkowski-Shoplik, A. (1999). Using talent searches to identify and meet the educational needs of mathematically talented youngsters. *School Science and Mathematics, 99,* 330–337.

Sheffield, L. J. (1994). *The development of gifted and talented mathematics students and the National Council of Teachers of Mathematics Standards* (Report No. RBDM 9404). Storrs: National Research Center on the Gifted and Talented, University of Connecticut. (ERIC Document Reproduction Service No. ED388011).

Sowell, E. J. (1993). Programs for mathematically gifted students: A review of empirical research. *Gifted Child Quarterly, 37*, 124–129.

Stanley, J. C., & Benbow, C. P. (1986). Youths who reason exceptionally well mathematically. In R. J. Sternberg & J. E. Davidson (Eds.), *Conceptions of giftedness* (pp. 362–387). New York: Cambridge University Press.

Tomlinson, C. A. (1995). Deciding to differentiate instruction in middle school: One school's journey. *Gifted Child Quarterly, 39*, 77–87.

VanTassel-Baska, J. (1995). The development of talent through curriculum. *Roeper Review, 18*, 98–102.

VanTassel-Baska, J. (1998a). *Excellence in educating gifted and talented learners* (3rd ed.). Denver: Love.

VanTassel-Baska, J. (1998b). The development of academic talent. *Phi Delta Kappan, 79*, 760–764.

Winebrenner, S. (2003). Teaching strategies for twice-exceptional students. *Intervention in School and Clinic, 38*, 131–137.

Wolfle, J. A. (1986). Enriching the mathematics program for middle school gifted students. *Roeper Review, 9*, 81–85.

Author Note

The authors would like to thank Michael Bosse and Jacquie Gentile for their helpful comments on an earlier version of this paper.