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Conducting a Teaching Experiment With a Gifted Student

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In this study, the teaching experiment methodology is used to observe firsthand a gifted student's mathematical learning and reasoning. A series of teaching experiments was conducted with 1 gifted and 1 average 7th-grade student to investigate how the gifted student's mathematical concepts and operation constructions differed from those of the average student. The teaching experiment approach provides opportunities for gifted and average students to be challenged by immersion in various advanced mathematical topics. The data analysis provides evidence that the gifted student was more adept at applying mathematical ideas to unfamiliar problems. As a result of being able to see mathematical patterns and to think abstractly, the gifted student was able to use analytical, deductive, and inductive reasoning to solve problems in more flexible and creative ways than the average student.

The growing popularity of educational programs tailored to the special needs of gifted students makes it especially important that educational research findings be used to support the rationale for providing such programs. One of the major challenges those in gifted education face is convincing policymakers of the need for specialized personnel and differentiated learning models to serve gifted students (Gallagher, 1997; Renzulli, 1982; Renzulli & Reis, 1998) by challenging the hackneyed idea that "gifted students can make it on their own." Communication of related research findings must create an understanding as to *why* traditional teaching methods in regular classrooms are inadequate for serving the needs of gifted students (Park, 1989; Westberg, Archambault, Dobyns, & Salvin, 1993).

Although mathematics is generally considered a strand in the theory of intelligence (Gardner, 1999; Sternberg, 1985), the nature of being mathematically gifted and how the needs of mathematically gifted students can be met are relatively unexplored areas. Thus far, research studies have demonstrated the need for gifted students to have access to advanced mathematical content (Johnson & Sher, 1997) and exposure to authentic and challenging mathematics problems (Johnson, 1993; Kolitch & Brody, 1992). However, mathematics curricula and instructional modifications made for gifted students are often inappropriate because of the highly repetitive nature of the courses and their lack of depth (Johnson & Sher; Kolitch & Brody; Park, 1989; Westberg et al., 1993). Thus, there is a strong need for research about the kinds of educational experiences that should be provided for mathematically gifted students, as well as research into the use of technological tools that could effectively and appropriately enhance instruction.

Methodology

By conducting a series of teaching experiments, the author sought to explore individual differences between a gifted student and an average student in terms of their abstract reasoning abilities. In the early 1980s, the term *teaching experiment* was used to describe a research technique designed to help mathematics educators develop a greater understanding of students' mathematical constructions. This form of inquiry has since become a popular way of doing research in mathematics education. The teaching experiment was originally used to connect the practices of mathematics education research to teaching mathematics (Steffe & Thompson, 2000), and the implementation of well-developed teaching experiments enables mathematics educators to elicit evidence of students' mathematical learning and reasoning processes and to construct models that explain their responses and mathematical thinking.

Teaching experiments consist of recording and analyzing a number of teaching episodes in which the analysis of the previous session(s) is used to guide the next teaching episode. During a teaching experiment, the mathematical reasoning of the students is the focus of the researcher's attention, just as it is in a clinical interview. However, this method differs from classical clinical interviews in that teaching sessions are organized as learning situations (Steffe & D'Ambrosio, 1996) in which the students are encouraged to formulate and explain their reasoning. The process requires that the researcher ask probing questions to elicit information about students' mathematical reasoning. The nature of the process, from a teaching perspective, requires the researcher to find ways of interacting with the students that will encourage them to modify their current thinking. Contrary to the situation in clinical interviews, an acceptable outcome of the teaching experiment is for students to modify their thinking (Lesh & Kelly, 2000).

Two 7th-grade students participated in this study. One student had qualified for and had been participating in gifted classes. The other student did not qualify for gifted placement and was in average-level classes. The activities were selected to allow observation of ways in which the gifted and the average student might differ from each other regarding the following characteristics: (1) the level of interest in studying mathematics, (2) the depth of their mathematical understanding, and (3) the pace at which they learn.

Three 70-minute sessions, which included mathematical tasks and interviews about problem-solving strategies, were conducted with the participants. The first two sessions were held a week apart, while the last session was conducted 2 months later. In the first two episodes, both students worked in a group setting, but the final session was conducted with each student individually. The author transcribed and analyzed all data taken in the three interview sessions. He used an interpretive approach (Packer & Mergendoller, 1989) in the analysis, focusing on the details and meaning of the actions and utterances of the students, as well as those of the researcher in the study sessions.

The author extended the traditional use of the term *analysis* in this study to include descriptions and interpretations (Wolcott, 1994) and to develop plausible relationships (Creswell, 1998). The planned learning activities included problems that would require explicit mathematical reasoning by the students. Through a responsive and intuitive interaction during teaching periods, the results of the study revealed information about the students' abstract reasoning abilities. The major aim was to investigate how the two students' abstract reasoning abilities, as well as their attitudes toward mathematics, differed from one another in the context of posed mathematics problems.

The First Session

In the first teaching session, Steven (the gifted student) and Tony (the average student) were asked to find the sum of the interior angles of a pentagon. This question was asked without any indication of a particular method the students should follow, although they were instructed to work individually. When it was necessary to encourage the students to modify their thinking, the interviewer posed questions intended only to guide them in finding a solution. For example, at one point, the interviewer suggested that the students determine the sum of the interior angles of a triangle; upon getting a correct answer, the interviewer asked about the sum of the interior angles of a square. With the aim of helping the students develop recursive mathematical reasoning, which is basically the ability to use previous results to derive the next result, *why* and *what if* questions were posed to help in the development of methods to find a solution. This approach led to productive interactions and spontaneous contributions by both students, as well as evidence of the difference in their ability to think abstractly.

Tony: Um . . . It [the sum of the measures of interior angles of a square is 360 degrees] is a rule, like the sum of the interior angles of a triangle is 180. I just knew that.

Steven: *(interrupting)* A square can be divided into two triangles, and you know in each triangle the sum of interior angles is 180. So, the sum of interior angles of a square must be 360.

Interviewer: What about the sum of the measures of the interior angles of a pentagon?

Before coming up with the answer, Steven drew a pentagon and dissected it into three triangles without showing any sign of hesitation, whereas Tony was still thinking.

Steven: 540 degrees.

Interviewer: Is there any pattern?

Tony: What do you mean?

At this point, the interviewer realized that, because Tony did not seem to be able to relate the information that had been developed about a triangle's angles to the angles of a related figure (the square), he did not seem to have well-developed recursive reasoning.

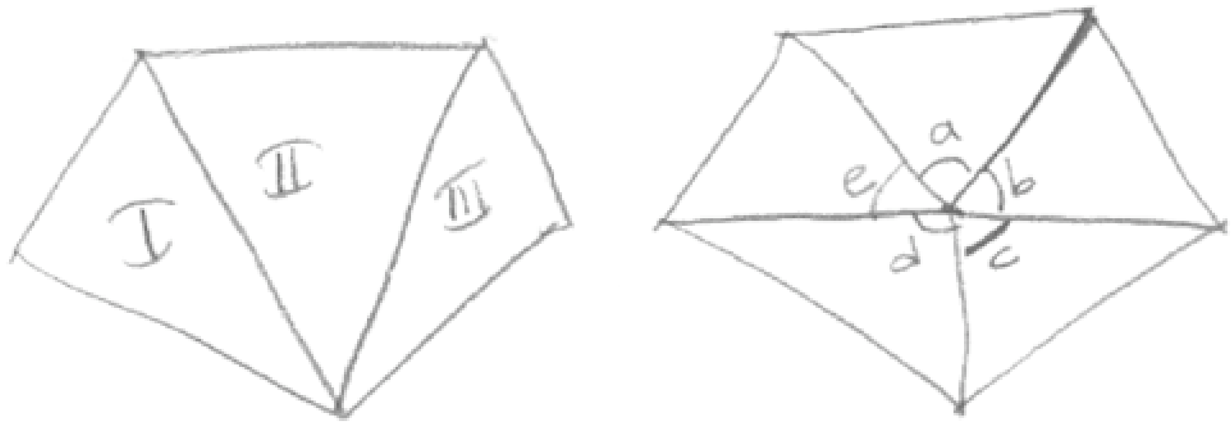

Figure 1. Steven's first and second pentagons

Interviewer: Is there any relationship between the type of polygon and the sum of the measures of its interior angles?

Steven: Yes! The sum of interior angles of a polygon is multiply 180 by the number you get when you subtract 2 from the number of sides of the polygon. I mean, subtracting 2 from the total number of sides of the polygon.

Steven's explanation is evidence of his recursive reasoning, relating the simpler result to a later and more complex problem, thereby enabling him to derive the correct result.

Tony: Cool! For a triangle [3 – 2] means 1 x 180 and for a pentagon [5 – 2] means 3 x 180.

This teaching episode was designed to help students generate a hypothesis based on thoughtful rationale and prior observations. The interviewer's probing questions let the students take the lead in the discovery of a solution and were formulated to them develop competence in doing mathematics by enhancing their ability to transfer learning from one mathematical context to another. Until this point, the interviewer had observed some differences between the two students' logical abstract reasoning abilities, exemplified by the discussion about the pattern Steven observed in determining the sum of the angles of the pentagon. However, Steven's next remark indicated the magnitude of the gap.

Steven: Actually, there is another way to come up with the same formula.

Interviewer: What do you mean?

Steven drew another pentagon, put a point inside it, and drew line segments from the point to each of the vertices of the pentagon, labeling each of the central angles: a, b, c, d, and e (see Figure 1).

Steven: We get five triangles. Adding their [the triangles] interior angles is 5 x 180. That's 900. We need to subtract 360 from that.

Tony: Why?

Steven: (He demonstrates using his drawing) We counted five angles (angles a, b, c, d, and e). Those aren't the interior angles of the pentagon. These are. (Steven points to the labeled angles in his sketch.) And a + b + c + d + e = 360. So, the sum of the interior angles of a pentagon is 540, just like we found before.

In contrast to Steven's enthusiasm, Tony was resistant to expressing his ideas or giving reasons to support his work. While the interviewer was trying to ensure that the problem-solving session did not degenerate into a guessing game, Tony tried to check Steven's conjecture for several polygons. The fact that Tony insisted on checking the formula for a square after he had already checked it for a rectangle revealed a gap in Tony's knowledge of basic plane figures.

The Second Session

In the second teaching session, the students were asked to compare radicals. After they were asked to find which of these two numbers was greater, the sum of $\sqrt{10} + \sqrt{17}$ or $\sqrt{53}$, Tony's initial response was to ask to use his calculator. During an ensuing conversation, which provided evidence of Tony's misconceptions of the mathematics involved in radicals, the researcher realized that Tony did not understand the value of making educated guesses in problem solving. After students worked on the problem for 20 minutes, Tony was frustrated and decided not to work on it any longer. While Tony was not persistent and did not feel confident about himself as a learner, Steven continued to work on the problem, obviously finding the problem intrinsically interesting and enjoyable. With excitement, he declared the sum $\sqrt{10} + \sqrt{17}$ of was greater.

While Tony used his graphing calculator in his effort to find a solution and accepted the graphing calculator's output without any reservation or further exploration, Steven's method for solving the problem was based on a geometrical analysis of the problem. Steven's approach came as a surprise because the interviewer had not expected to see either student solve the problem by implementing geometrical methods. Steven was confident he had the right idea and stated his solution eloquently, but he was not happy with his drawings and later spent almost 2 hours of his time in front of the computer to create the figure using Geometer's Sketch Pad software (see Figure 2).

Steven's approach to the problem not only demonstrated the fact he had above-average ability and creativity in mathematics, but also that he was able to use his mathematical knowledge with flexibility and creativity (Ervynck, 1991; Renzulli, 1983). Steven was persistent in solving a difficult and complex problem; in addition to this determination to find the solution, he was able to understand and apply mathematical ideas swiftly, see mathematical patterns, think abstractly, transfer mathematical concepts to an unfamiliar situation, and use analytical, deductive, and inductive reasoning strategies both flexibly and creatively (Ervynck; Holton & Gaffney, 1994; Miller, 1990). Access to technological tools provided inspiration and an independent learning environment for Steven in his exploration of this complex and interesting problem.

The Third Session

In the last session, which was conducted 2 months after the second session, the students were individually asked questions related to their perceptions of the nature of mathematics, mathematics learning, and the teaching experiment in which they had participated. Tony's negative attitude toward doing mathematics was rooted in his perception of mathematics as a set of tricks for coming up with the right answer, and he viewed his role as a memorizer of all kinds of tricks. He also expressed a negative attitude toward the teaching experiment.

Tony: I know I don't like math, but when I'm working in class . . . I can actually stand it for [a] few minutes. Um

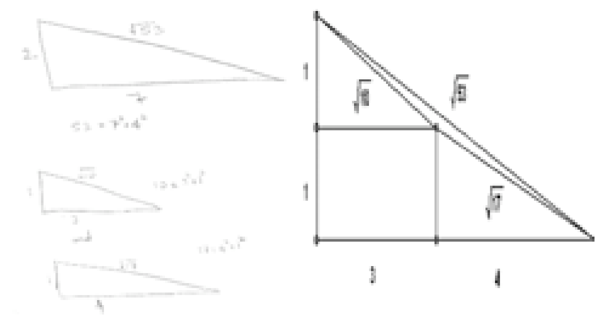

Figure 2. Steven's first trial and his computer construction

. . . I liked to have you there to help me. But . . . if you cannot solve [the] problem in 10 minutes, you have to move . . . cannot get the correct answer.

On the other hand, Steven expressed a positive attitude toward the teaching experiment and mathematics in general. He remembered what questions had been asked 2 months before. Moreover, he said he had continued to work on the problems during the week after the teaching experiment was conducted and had found a third method for finding the sum of the measures of the interior angles of a pentagon. After demonstrating his three different approaches to that problem (see Figure 3), he asked if there might be another way to solve it. The interviewer demonstrated how ancient Babylonians and Greeks found solutions to such problems, reinforcing Steven's awareness of the nature of mathematics, its role in society, and the importance of mathematics as an instrument of learning.

When the interviewer asked Tony and Steven about what sort of animal could be used to describe mathematics, their responses revealed that they also had very different perceptions about the nature of mathematics. While Tony's response was "snake," Steven's response was the "chameleon." In the followup questions, it was revealed that Tony's response was rooted in his fear and lack of self-confidence in doing mathematics, whereas Steven's response was rooted in his observation of abundant applications of mathematics in daily life.

Conclusions, Recommendations, and Pedagogical Implications

In the analysis of the data, it became apparent that each student's feeling of self-efficacy was a strong predictor of his

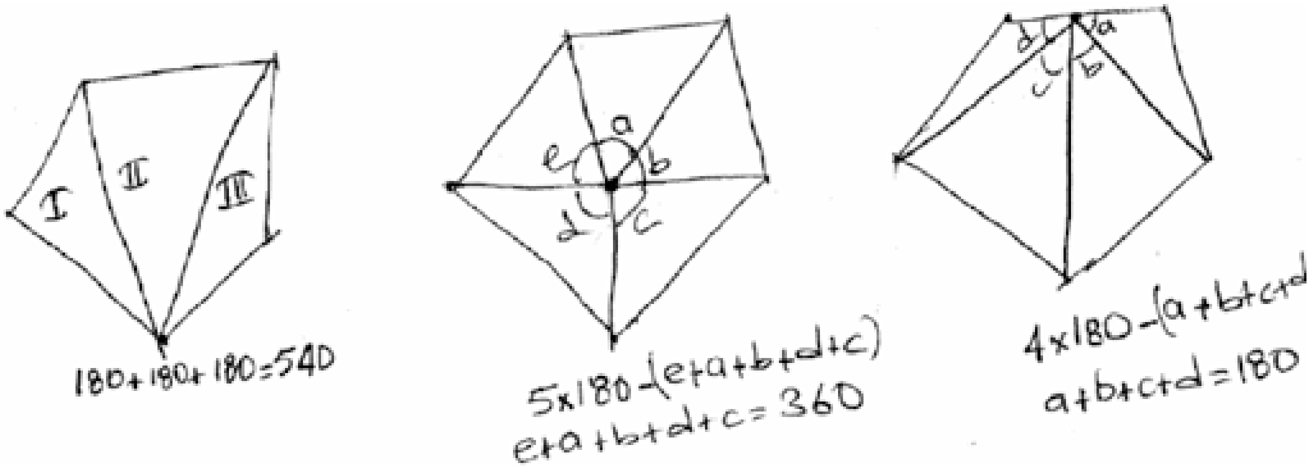

Figure 3. Steven's demonstration of his three different approaches

mathematical performance. In private conversation, Steven viewed himself as a creator of mathematics, while Tony's image of mathematics was limited to its being the most difficult class in his schedule. During the problem-solving sessions, Steven approached each problem confidently, solved the problems correctly, and successfully used recursive and explicit reasoning to construct generalizations and to develop mathematical conjectures. Tony, on the other hand, often seemed hesitant, and although he was able to find correct values for particular cases, he exhibited difficulty in constructing generalizations, did not use recursive reasoning, and was unable to formulate mathematical conjectures. This may be due in part to his focus on finding an answer, rather than trying to understand the essential mathematical processes and ideas that are involved (or required) in problem solving.

Their different approaches might be viewed as an indication that Tony and Steven's thinking styles were different in terms of global and local thinking, as well as their abilities of abstract thinking (Wilmot & Thornton, 1989). Steven was more creative in his ability to invent unexpected, original solutions and was able to see his results as useful and adaptive. This particular finding supports educational psychology studies linking creativity with the ability to make abstractions and generalizations in complex problem-solving situations (Frensch & Sternberg, 1992; Sternberg, 1985). Furthermore, the research presents some evidence for allowing mathematically gifted students who have a well-developed abstract reasoning ability to move to advanced mathematics classes (Kolitch & Brody, 1992; Steinberg, Sleeman, & Ktorza, 1990).

Steven showed high levels of task commitment and creativity, and he was capable of learning more complex mathematical ideas than Tony was. In addition, Steven was more adept at distinguishing between important and unimportant information in the problem-solving situations. He exhibited greater facility in applying mathematical ideas quickly to unfamiliar problems. As a result of being able to see mathematical patterns, use multiple representations, and think abstractly, Steven was able to use analytical, deductive, and inductive reasoning to solve problems in flexible and creative ways.

This teaching experiment suggests that gifted students may benefit from following a differentiated curriculum that provides greater depth, varied mathematics topics, authentic and open-ended problems, and an accelerated pace. The gifted student in this study differed from the average student in the following abilities: ability to formulate mathematics problems, flexibility and creativity in problem-solving strategies, fluency in mathematical skills, originality in the construction of mathematical conjectures, the ability to use multiple representations, and the ability to make formal generalizations for mathematical patterns.

Although teaching experiments in mathematics education, by definition, focus on the learning attributes and thought processes of a limited number of students at a time, it is an approach that provides an in-depth look into some of the ways students perceive mathematical problem-solving situations, how they think about possible methods of finding solutions, and how they use related concepts in formulating solutions. The findings of this study may not allow generalizations, but they do shed some light on why gifted students need to follow an enriched mathematics curriculum (Gallagher, 1997; Westberg et al., 1993; Winebrenner, 1992) that can provide exposure to mathematical ideas in a greater depth and breadth with richer and more varied instructional methods than are usually found in traditional mathematics curricula. Implications for further research might include exploring the possibility that the integration of more complex and authentic problems into existing mathematics curricula could also be considered as an avenue for enhancing gifted students' creativity and mathematical reasoning.

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