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The Impact of Virtual Manipulatives in Online Learning on 4th Grade Students' Learning

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This study aims to examine the impact of mathematics teaching supported by virtual manipulatives in an online learning environment on the academic achievement of primary school 4th grade students. In the study using a mixed method, experimental (n=24) and control (n=24) groups were established. While the experimental group was taught online with virtual manipulatives based on the Realistic Mathematics Education approach, traditional methods were employed for the control group. Mathematics academic achievement test and semi-structured interview forms were used as data collection tools. While the quantitative data were analyzed using independent samples t-test, the qualitative data were analyzed by the descriptive analysis method. The findings indicated that the academic achievement of the students in the experimental group increased significantly compared to the control group ($p<0.05$). In addition, the experimental group students reported that the virtual manipulatives used in the online environment enhanced their conceptual understanding, increased their motivation and facilitated their mathematics learning. As a result, it has been revealed that teaching practices using virtual manipulatives and real-life examples in the online learning environment make positive contributions to students' mathematics learning, although some students require additional support.

Keywords: Virtual manipulatives, online learning, Realistic Mathematics Teaching, computer-aided teaching.

INTRODUCTION

Piaget, who has significantly contributed to our understanding of human cognitive development and is one of the theorists most influential in mathematics education, has asserted that students, particularly young ones, learn most effectively through concrete activities by viewing learning from a cognitive perspective (Olkun and Uçar, 2014). The National Council of Teachers of Mathematics (NCTM, 2000)) recommends the use of concrete materials to represent mathematical ideas in its Principles and Standards for School Mathematics. The term 'concrete materials' is often used interchangeably with 'mathematical manipulatives' in the mathematics education literature. The concepts of "manipulative," "object," or "model" can also refer to these materials (Demir and Gün, 2023). Manipulatives are defined as physical objects that students and teachers can utilize to visualize and explore mathematical concepts (Van de Walle, 2013). Discussions on mathematical manipulatives encompass pictorial representations (Sowell, 1989) and can include both physical and virtual manipulatives. Moyer (2001) defines concrete objects as those employed by students to conceptualize an abstract mathematical idea. A theory of learning that involves a progression from the concrete to the abstract has been proposed by both Piaget and Bruner (Quigley, 2021). Any technological tool used in mathematics lessons can only be expected to support meaningful learning if it is integrated into the learning process within the framework of appropriate pedagogical principles (Baki, 2018). Computer-aided teaching, where education and technology form a cohesive whole, provides qualified instruction in mathematics courses where abstract concepts predominate. NCTM (2000) identified the principle of technology as one of the six principles of quality mathematics education in Principles and Standards for School Mathematics. This principle states: "Technology has an important place in the process of learning and teaching mathematics. Technology enhances students' learning by positively impacting the mathematics taught." (NCTM 2000, p. 24). Technology facilitates meaningful learning by helping to clarify abstract expressions (Efendioğlu, 2015). A wealth of content has been produced to support the learning-teaching process in the computer environment. One of the learning tools developed in this context is virtual manipulatives (VMs) (Reiten, 2020). VMs can serve as counterparts to real physical models (Moyer-Packenham & Bolyard, 2016). VMs can assist students in learning meaningfully about the intricacies of mathematical concepts. It is crucial for students to analyze and synthesize abstract mathematical concepts metacognitively. For primary school students who are in the concrete stage, VMs offer an opportunity for detailed learning.

When examining the literature, numerous studies indicate that the use of VMs in the learning-teaching process enhances students' academic success in mathematics (Mutluoğlu & Erdoğan 2021; Samioglu & Siniksaran, 2016; Çakıroğlu, 2014). Furthermore, literature comparing the effects of using VMs versus various learning materials on students' academic achievement reveals no significant differences between them (Hawkins, 2007; Reimer and Moyer-Packenham, 2005). The quality of the prepared VMs is also crucial for effective learning. Mutluoğlu & Erdoğan (2021) demonstrated that the VMs they developed positively contributed to students' academic success in geometry. Altıparmak & Çiftçi (2018) conducted an experimental study investigating the effectiveness of the computer-aided realistic mathematics education approach. In the study, it was observed that computer-aided models led the experimental group students to learn the concepts more meaningfully. Aqda et al. (2011) conducted a study investigating the effect of computer-aided instruction on students' creativity in mathematics. In the study, it was found that the experimental group in which computer-aided instruction was applied showed a significant increase in creativity scores compared to the control group in which the traditional method was applied. Erdoğan (2018) conducted a study investigating the effect of mathematics teaching based on realistic mathematics education on academic achievement, permanence and reflective thinking skills. In the study, it was found that the experimental group in which mathematics teaching based on realistic mathematics education was applied showed a significant increase in academic achievement, permanence and reflective thinking skills compared to the control group in which the traditional method was applied. Park et al. (2022) reviewed the literature on interventions using virtual manipulatives to improve the math performance of K-12 students with learning disabilities. A total of 19 single-case design studies (16 peer-reviewed articles and three dissertations) were selected based on specific criteria. Overall, these studies found that students with learning disabilities demonstrated mathematical improvement after receiving interventions that included virtual manipulatives.

However, some students with learning difficulties did not show any improvement. The researchers also noted that further investigation is necessary to validate the current findings. In her meta-analytical study, Taşpınar-Şener (2023) explored the impact of technology use in mathematics courses on academic achievement. Studies published between 2019 and 2023 were reviewed by scanning the Web of Science (WoS) and Scopus databases, resulting in 14 studies that met the specified criteria for inclusion in the meta-analysis. A total of 22 effect sizes were calculated, encompassing 1,065 samples. Upon examining the results of the meta-analysis conducted with a random effects model, it was determined that the overall effect of technology on academic achievement was moderate (EF=0.605). Among all the studies included in the meta-analysis, only one study reported a negative effect size, while the others yielded positive values. The study with a negative value is based on the observation that students in the control group were trained through experiential learning without technology. The results from the analysis of the mixed effects model did not indicate a significant difference between the effect sizes regarding application time and grade level. The impact of technology on academic achievement in mathematics courses varied significantly based on the sample size. Consequently, as the number of samples increases, the effect value also rises. In this study, relevant research involving technology-supported applications was assessed within a general framework. It is evident that there is a need for numerous studies that explore the impact of technology across various dimensions and factors.

Mathematics is a crucial discipline that permeates all aspects of daily life and fosters the development of essential skills such as problem-solving, critical thinking, and logical reasoning. It is a field rich in abstract concepts, which can pose challenges for primary school students in grasping these ideas (Baki, 2002). Unfortunately, mathematics education is often viewed as a daunting and abstract experience for learners. Specifically, comprehending abstract concepts and applying them to real-life situations presents a significant hurdle for students. This scenario can foster negative attitudes toward mathematics and hinder academic achievement. Concepts like fractions, which are commonly encountered in everyday life yet challenging for students due to their abstract nature, can serve as a major barrier in the learning process (Işıksal, 2006).

In mathematics education, the challenges students face in grasping abstract concepts and developing problem-solving skills have prompted educators to seek new and effective teaching methods. In this context, the influence of computer-aided mathematical models on learning has become a focal point of educational research in recent years. Mathematics encompasses abstract concepts for students, particularly at the elementary school level. Computer-aided models visualize and embody these concepts, facilitating easier comprehension for students. Virtual manipulatives allow students to actively engage in the learning process. By interacting with manipulatives, adjusting various variables, and analyzing outcomes, students can gain a deeper understanding of concepts. As previously mentioned, current literature suggests that further studies are necessary in this area. This has been a significant motivation for us to conduct this study. In our research, we adopted the Realistic Mathematics Education (RME) approach, linking mathematical concepts to real-life examples with the aid of virtual manipulatives. Realistic Mathematics Education (RME) is an approach that emphasizes the teaching of mathematical concepts within concrete contexts and aims to help students connect their learning with real-life examples. RME employs various manipulatives to facilitate students' problem-solving and understanding of new information based on their individual experiences. This method uses concrete materials and technological tools (such as virtual manipulatives) to enhance students' comprehension of abstract mathematical concepts (Gravemeijer, 1998). The advantages of RME include increasing student motivation, deepening conceptual understanding, and developing mathematical thinking. Additionally, this approach provides active learning opportunities during instruction, allowing students to improve their problem-solving skills (Van den Heuvel-Panhuizen, 2000). In this context, the adoption of RME as an effective mathematics teaching strategy plays an essential role in enhancing students' mathematical success. The purpose of this study is to examine the effects of virtual manipulatives on student learning processes within an online learning environment framed by RME and to investigate students' opinions about the learning experiences created with virtual manipulatives.

METHOD

Research Model

This research incorporates both quantitative and qualitative designs. In instances where quantitative methods alone fall short, studies that utilize qualitative methods to enhance the research are considered mixed-method studies. Greene, Krayder, and Mayer (2005) define the mixed-method approach as the integration of two or more analysis or data collection tools within a single study. Creswell (2008) asserts that combining quantitative and qualitative research methods in mixed-method research, rather than employing them separately, offers greater clarity regarding the research's problems and questions.

Participants

The research study group consists of two branches selected from the 4th grade classes of a public primary school in Izmir during the 2020-2021 academic year. The selection of the study group was conducted using the group matching method. Group matching is a technique employed in research involving pre-existing groups when random assignment of subjects to groups is not feasible or practical. It is important to note that the average scores of the groups included in the research are selected to be equivalent (Büyüköztürk et al., 2017). The school where the research took place has four 4th grade classes. Academic achievement tests were administered to all four classes. This test served to measure academic achievement, the dependent variable of the study. Subsequently, based on the test results, two branches that were equivalent in terms of performance were chosen. One of these branches was designated as a random control group, while the other was assigned as a random experimental group. This ensured that there was no significant difference between the pre-test results of the study groups and that the groups were comparable in terms of performance. The data obtained from the preliminary academic achievement test of the experimental and control group students are presented in Table 1 below. According to Table 1, the p-value is 0.08. Thus, based on the pre-test results, there was no significant difference between the mathematics academic achievement scores of the experimental and control group students (p=0.081 > 0.05).

Table 1

Independent Samples t-test Analysis results for the Pre-academic achievement Test of experimental and Control Group Students

	N	Average	Std. Deviation	t	P
Experimental Group	24	0,3542	0,17069	-1,783	0,081
Control Group	24	0,2674	0,16663		

Data collection tools

Mathematics Academic Achievement Test

Mathematics Academic Achievement Test; 2018 Mathematics Course Curriculum published by the Ministry of National Education of the Republic of Turkey (Ministry of National Education, 2018) in the book 4th grade "division with natural numbers", "fractions" and "operations with fractions" were developed in line with the ideas of 3 mathematics education experts for 8 outcomes related to the sub-learning areas. The achievements in the Academic Achievement Test and the number of questions related to the achievements are given in Table 2 below.

Table 2

Academic Achievement Test Outcomes and Number of Questions

Gains	Number of Questions
M.4.1.5.7. Determines the value that is not given in one of the two mathematical expressions with equality between them and explains that equality is achieved	1
M.4.1.5.8. Explain the operations that must be performed in order for two mathematical expressions that do not have equality between them to be equal	1
M.4.1.6.1. Recognizes simple, compound, and integer fractions and illustrates them with models	3
M.4.1.6.2. Compares and arranges unit fractions	1
M.4.1.6.3. Determines a specified simple fraction of a multiplicity	1
M.4.1.6.4. Compares up to three fractions with equal denominators	2
M.4.1.7.1. Adds and subtracts denominators with equal fractions	2
M.4.1.7.2. Solve problems that require addition and subtraction with fractions	1

The achievement test, consisting of 12 questions, was applied to 48 students before it was applied in the experimental and control groups, and Cronbach's alpha value was calculated for reliability analysis. While evaluating the achievement test, 1 point was given for correct answers and 0 points for incorrect answers. As a result of the reliability analysis performed in the SPSS 25.0 program, the Cronbach's alpha value of the test was found to be 0.83. If the value found as a result of the analysis is $0.80 < R2 < 1.00$, it is stated that the test is of high reliability (Tavşancıl 2006). According to the results of the analysis, there was no need to remove questions from the test because the reliability of the test was high.

Semi-Structured Interview Form

The semi-structured interview technique is slightly more flexible than the structured interview technique. Semi-structured interviews offer researchers a flexible approach and provide them with the opportunity to obtain in-depth information through predetermined questions (Galletta, 2013). The semi-structured interview technique provides convenience for researchers due to the fact that the questions are planned in advance and is more systematic. It is a very suitable research technique for pedagogy studies (Yıldırım and Şimşek, 2016). The interview questions used in the study were prepared based on the purpose of the study. This approach aims to collect comprehensive data that is compatible with research questions (Creswell & Poth, 2018). The main themes of the interview guide are the efficiency of the teaching environment prepared with virtual manipulatives, the difficulties experienced and the views on the use of virtual manipulatives. Thus, it is planned to reach the general experiences of the experimental group students about the teaching environment. The interviews are scheduled to last around 30 minutes in a setting where participants can feel at ease. This duration was established by considering the attention span during interviews with children (Irwin & Johnson, 2005). The following tools were utilized in the data collection process:

1. **Semi-Structured Interview Form**: A form that included key themes and associated questions was utilized, allowing for flexibility based on the interview's progression.
2. **Voice Recorder**: To ensure the interviews were preserved completely and accurately, audio recordings were made with the parents' consent.
3. **Participant Information Form**: A brief form was utilized to gather demographic information about the students and their overall attitudes toward the mathematics course.

Experimental Procedures

The application of the study carried out on the experimental and control groups was completed in 6 weeks. For the experimental and control group students, the teaching process related to 8 achievements belonging to the sub-learning areas of “division with natural numbers”, “fractions” and “operations with fractions” in the fourth grade mathematics curriculum of primary school lasted for 6 weeks. In Table 3, the procedures and achievements performed for the experimental and control groups for 6 weeks are indicated separately on a weekly basis.

Table 3

Weekly Procedures Performed on the Experimental and Control Groups

Week	Mathematics outcomes	Operations performed in the experimental group	Operations performed in the control group
1st Week	-	Pre-test	Pre-test
2nd Week	M.4.1.5.7 M.4.1.5.8	Students' ideas were discussed, leading to generalizations supported by real-life examples, scaled activities, prepared scale materials, and virtual manipulatives.	Definitions were established on the subject. Images and examples in the book were analyzed. Activities included in the textbook were performed.
3rd Week	M.4.1.6.1 M.4.1.6.2	Modeling studies of simple, compound, and integer fractions were conducted using real-life examples, activities, and virtual manipulatives.	The definitions of simple, compound and integer fractions were written in the notebook, the examples in the textbook were examined, and sorting activities were carried out.
4th Week	M.4.1.6.3 M.4.1.6.4	Using real-life problems and virtual manipulatives, students discussed their approaches to determining a specific simple fraction of a multiplicity. Comparisons were facilitated using manipulatives for fractions with equal denominators.	The activities in the textbook were conducted using brainstorming and a question-and-answer technique.
5th Week	M.4.1.7.1 M.4.1.7.2	Fraction problems involving addition and subtraction, designed around real-life situations, were solved using activities and visual models.	The problems in the textbook and those written in the notebook were solved together with the students.
6th Week	-	Post-test Semi-structured interview	Post-test

Teaching environment implemented for the experimental group

In the study, a teaching environment was created for the experimental group using virtual manipulatives within the online learning setting, based on the Realistic Mathematics Education (RME) approach. To effectively implement the Realistic Mathematics Education approach, it was deemed appropriate for the instruction to be conducted by the researcher, and it progressed as follows: First, the teacher introduces a problem scenario from everyday life to the students. This problem is designed to capture the students' attention and motivate them to engage actively. For instance, "Ali, who works in a cake shop, sells slices of cake to customers. Customers are curious about how many slices the cake has and the size of each slice. How can Ali respond to the customers' inquiry?" Next, students strive to comprehend the presented problem and devise solutions. At this stage, with the teacher's guidance, students use virtual manipulatives to clarify the problem and uncover solutions. For example, they can represent the slices of cake using a virtual cake model and ascertain the quantity in each slice. Subsequently, students share and discuss their solutions with peers. By listening to students' explanations, the teacher facilitates and enhances their conceptual understanding. Based on the students' solution processes, the teacher explores mathematical concepts (such as fractions, division, ratios, etc.) in greater depth. Virtual manipulatives aid in concretizing and elucidating these concepts. Finally, students generalize by applying the mathematical concepts they have learned to similar problem situations. Virtual manipulatives enable students to utilize concepts in various contexts. For instance, students can reinforce the concept of fractions by using slices of chocolate, pizza, or cake. Additionally, students' progress in the learning process is evaluated through online activities, tasks, and projects. The teacher observes students' conceptual understanding and problem-solving abilities. In this context, virtual manipulatives assist students in grasping mathematical concepts by embodying them and relating them to everyday life. They also promote students' active participation and self-regulated learning.

Virtual manipulatives are designed to enable students to visualize and explore mathematical concepts. These manipulatives, presented as animations, transform abstract mathematical ideas into concrete and dynamic forms. By engaging with these tools, students can follow the step-by-step progression of concepts and understand each phase. The animations are crafted to capture students' attention and enhance their comprehension. Animated and interactive visuals encourage active student participation and facilitate their conceptual understanding. Moreover, the animations provide immediate feedback, allowing students to quickly identify misunderstandings and concentrate on the concept. With all these features, virtual manipulatives can help students learn abstract mathematical concepts in a more meaningful and enduring manner. Screenshots of some virtual manipulatives utilized in the research are displayed in the Figures 1,2 and 3 below. These manipulatives were created as animations and appeared on the screen in a specific sequence, allowing students to observe the stages related to the concept.

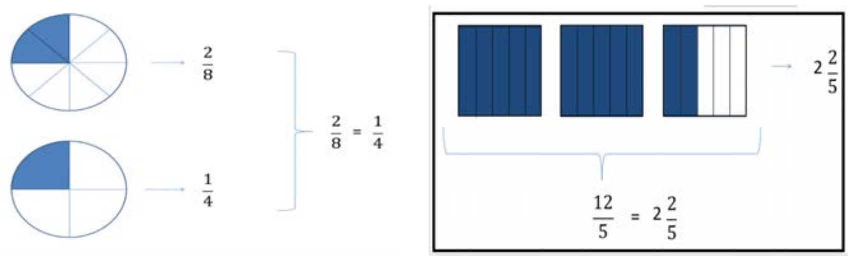
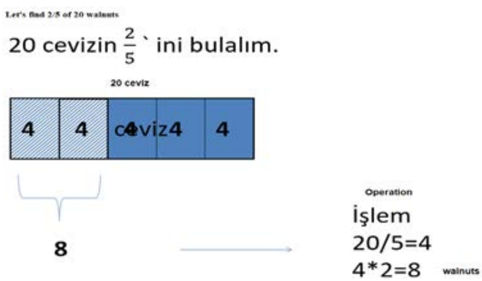


Figure 1. Recognizes simple, compound, and integer fractions and illustrates them with models. (M.4.1.6.1.)



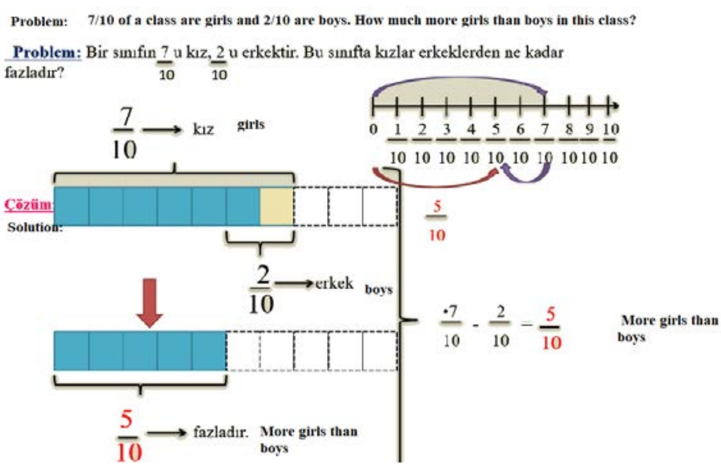


Figure 2. Determines a specified simple fraction of a multiplicity. (M.4.1.6.3.)

Figure 3. Solves problems that require addition and subtraction with fractions. (M.4.1.6.3.)

Teaching environment implemented for the control group

In the research, an online teaching environment based on the constructivist learning approach was established for the control group. This teaching environment aimed to provide students with active participation opportunities and encouraged them to construct their own learning. The decision to have the research conducted by the students' own teachers in the control group was made to offer a learning experience closer to the natural classroom environment and to assess the effects of the applied approach in alignment with the real classroom dynamics.

Textbooks played a significant role in this process. The teacher aimed to increase student engagement by adapting the activities and content from the textbooks to the online environment. For instance, in mathematics lessons, the visual materials from the textbooks were transferred to the online platform. This allowed students to concretize and better understand abstract mathematical concepts, thereby enhancing their participation and comprehension. Additionally, problem-solving activities from the textbooks were conducted online. Figure 4 displays an image from the textbook.

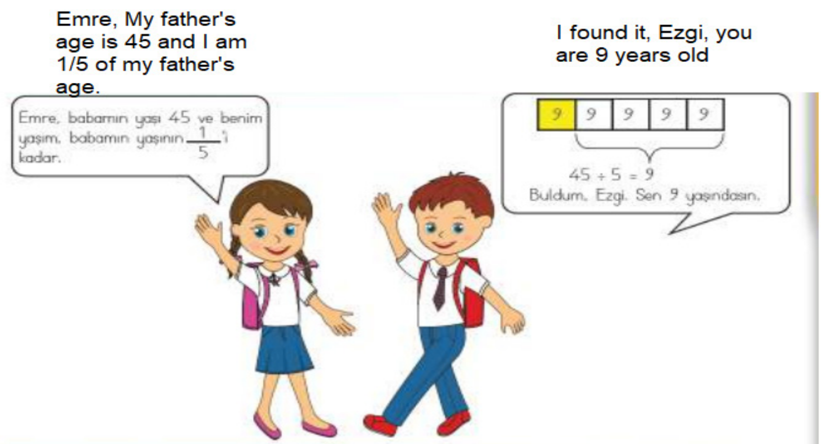


Figure 4.
An image from the textbook (Determines a specified simple fraction of a multiplicity. (M.4.1.6.3.)

Despite the limitation that the lessons in the experimental group were conducted by the researcher while those in the control group were conducted by the students' own teachers, equal duration, content scope, and assessment methods were meticulously applied in both groups to balance the educational process. Ensuring equal educational opportunities for both experimental and control group students was adopted as a fundamental principle.

Data Analysis

In the mathematics academic achievement test, 1 score was coded for the correct answers to the questions and 0 points were coded for the wrong answers. Normality analysis of the data collected from the pre- and post-tests of mathematics academic achievement for both the experimental and control group students was conducted using the Shapiro-Wilk test. According to this test, it was observed that the data showed normal distribution. Since the data showed normal distribution, the analysis of the data was carried out by independent sample t-test, which is one of the parametric tests. The significance level was taken as $p<0.05$.

In this study, 22 students from the experimental group participated in the interviews and the interviews were carried out via Zoom and audio and video recordings were taken. Descriptive analysis method was used to analyze the experiences and opinions of the 4th grade students participating in the study about virtual manipulatives used in online learning environments. Descriptive analysis is a qualitative data analysis approach that involves summarizing and interpreting the obtained data according to predetermined themes (Yıldırım & Şimşek, 2016).

In the analysis of the data obtained from the research, the process of creating the defined themes was systematically examined based on studies in the literature. The theoretical foundations of the themes are as follows: Moyer's (2001) foundational work revealed that concrete objects are utilized by students to conceptualize abstract mathematical ideas, which forms the basis of the theme "general experience with virtual manipulatives" in our study. Moyer-Packenham & Suh (2012)'s study investigated the impact of virtual manipulatives on different achievement groups in technology-supported mathematics learning, followed by the meta-analysis conducted by Moyer-Packenham & Westenskow (2013), which demonstrated a moderate positive effect of virtual manipulatives on student success. These findings provide the basis for the theme "impact on learning." The theme regarding the opinions on the use of manipulatives is based on Quigley (2021)'s study examining teachers' beliefs and practices regarding the use of concrete materials in the classroom and Çakıroğlu's (2014) comparative study of project-based learning environments enriched with virtual manipulatives. All these studies were brought together within the framework of Piaget's theory of learning from concrete to abstract and the principles and standards for the use of technology in mathematics education as outlined by NCTM (2000), thereby establishing the theoretical and empirical foundations of the three main themes that constitute the analysis framework of our research.

In line with these theoretical foundations, the data analysis was carried out in several stages: First, the data obtained from the semi-structured interviews were systematically organized according to the established thematic framework and compiled meaningfully to form a coherent whole. In this process, direct quotes that best reflect the student opinions were carefully selected. Subsequently, the organized data were clearly defined under each theme in an understandable language, supported by quotes that reflected students' perspectives. In the final stage, the relationships among the identified findings were analyzed, causal connections were established, and comparisons were made among different student opinions to interpret the findings within the framework of the research questions.

To enhance the reliability of the data analysis, the interviews conducted with 24 students were coded as S1, S2, S3, ... S24 and analyzed by two independent researchers. The inter-coder agreement percentage calculated using the formula suggested by Miles and Huberman (1994) was found to be 87%. In the descriptive analysis process, the data obtained from the semi-structured interviews were systematically organized according to the established thematic framework and thoroughly examined under each theme. Direct quotes that best reflected students' views on the teaching practice enriched with virtual manipulatives and real-life examples were specifically identified, and the findings were supported by these quotes. In the final phase of the research, the relationships among the identified findings were analyzed, causal links were established, and comparisons were made between different student opinions to interpret the findings in relation to the research questions (Yıldırım and Şimşek, 2016).

RESULTS

In this section, the findings from the analysis of data obtained through the mathematics academic achievement test and semi-structured interviews are presented. According to Table 1, there was no significant difference between the pre-academic achievement tests of the experimental and control group students. The results of the difference between the pre- and post-academic achievement tests of the experimental group students, where computer-aided teaching was implemented, are given in Table 4.

Table 4

Independent Samples t-test Analysis Results of the Pre- and Post-academic Achievement Tests for the Experimental Group Students

	N	Average	Std. Deviation	t	P
Pre-Test	24	0,3542	0,17069	-3,636	0,001
Final Test	24	0,5868	0,26293		

According to the analysis results, the average score of the experimental group students in the pre-test was 0.3542, while the average score in the post-test was 0.5868. A statistically significant difference (p=0.001<0.05) was found when comparing the results of the independent sample t-test analysis of the pre- and post-academic achievement tests of the experimental group students. These results indicate that the teaching environment, which included virtual manipulatives prepared for the experimental group, helped the students improve their success. The results of the difference between the pre- and post-academic achievement tests of the control group students, to whom the constructivist approach was applied, are given in

Table 5

Independent Samples t-test Analysis Results of the Pre- and Post-academic Achievement Tests for the Control Group Students

	N	Average	Std. Deviation	t	P
Pre-Test	24	0,2674	0,16663	-0,952	0,346
Final Test	24	0,3264	0,25409		

According to Table 5, the pre-test average score of the control group students was 0.2674 and the post-test average score was 0.3264. There was no statistically significant difference (p=0.346>0.05) when the results of the independent sample t-test analysis of the pre- and post-academic achievment test of the control group students were compared.

The results of the difference between the final academic achievement tests of the experimental and control group students are given in Table 6.

Table 6

Independent Samples t-test Analysis Results of the Final Academic Achievement Test of the Experimental and Control Group Students

	N	Average	Std. Deviation	t	P
Experimental Group	24	0,5868	0,26293	-3,489	0,001
Control Group	24	0,3264	0,25409		

Based on the analysis results, the mean score of the experimental group in the post-test was 0.5868, while the pre-test mean score of the control group was 0.3264. Comparing the mean scores of the experimental and control group students on the final academic achievement test revealed a statistically significant difference (p= 0.001<0.05) between them.

Following the study, the results from the semi-structured interviews with the experimental group students about virtual manipulatives are presented below.

Insights into the experience of the learning environment supported by virtual manipulatives in the online environment

In Table 7, the students' opinions about the experience of the learning environment supported by manipulatives in the online environment are categorized as positive and negative.

Table 7

Student Views on the Efficiency of the Learning Environment

	REASON	STUDENT	NUMBER OF VIEWS	
POSITIVE	1. Having a quiet and suitable environment	S1, S18, S20	3	56
	2. Interesting visuals	Ö2, Ö6, Ö10, Ö11, Ö13, Ö16	6	
	3. Helps to understand concepts	Ö2, Ö8, Ö9, Ö16, Ö17	5	
	4. Motivating to ensure active participation in classes	Ö3, Ö5, Ö8, Ö9, Ö12, Ö14, Ö15, Ö17	8	
	5. Manipulatives to be in line with real life	Ö2, Ö4, Ö6, Ö8, Ö9, Ö11, Ö12, Ö13, Ö14, Ö15, Ö16, Ö18, Ö19, Ö20	14	
	6. Feedback from the teacher, helping to understand the wrong places	S2, S8, S3	3	
	7. Helping to develop mathematical ability	S3, S11	2	
	8. Increased interest in mathematics	Ö1, Ö3, Ö5, Ö9, Ö11, Ö15, Ö18, Ö20, Ö22	9	
	9. Make the lesson fun	S1, S2, S3, S9	4	
NEGATIVE	1. Inability to focus.	S21, S22	2	9
	2. Problems with the internet connection and interruption of the lesson	S7	1	
	3. Low motivation due to not being able to answer some of the questions asked	S19	1	
	4. Being disturbed by the noise	S4	1	
	5. Difficulty of the subject and inability to focus	S19, S21, S7, S10	4	

Table 7 shows positive and negative student opinions on the efficiency of the learning environment. According to Table 7, 56 positive and 9 negative opinions were identified. In the positive category, the most common situation is when 14 students talk about manipulatives being compatible with real life. The most common negative situation in Table 7 is that 4 students mentioned the difficulty of the subject and stated their inability to focus.

The opinions of some students who look positively at the educational process are given below.

S1: "Yes, it was useful. For me, it was like one of those distance learning classes. I love math class and I enjoy attending classes."

S2: "It's productive. Because, for example, when our teacher asks us questions, he asks us fun. We also respond with fun. That's why we can understand."

S3: "Yes. I do the homework given by the teacher and so on. I'm working. I try to attend live classes as much as I can so that I can learn the subjects."

S5: "I understand better. I'm more involved in classes and I'm interested in math."

S6: "Actually, I love the live lesson. It helps me understand that the manipulatives used in the class are things I know."

S9: "Animations and models related to fraction problems caused me to learn better."

S14: "I was interested in the questions we did in the lessons and examples such as dividing the cake. I understood the topic well. It came easy."

S3: "Yes, it was like our usual classes, but it was more fun. I can do some of the fraction problems I struggle with better by drawing models like in the lessons."S9: "Yes, after the lesson with you, the fraction questions came easier. I love math, the classes were like our regular classes."

S16: "The lessons are going well because the visuals related to the subject help me understand."

S9: "Yes, the fraction questions were easier after the lesson we had with you. I love mathematics; the lessons were like our normal lessons."

S17: "I would like to thank you very much for these lessons, both on behalf of my friends and myself. These lessons were a little different for me than other lessons. We don't do such practices very often. Our teacher mostly opens tests or covers the new topic from our textbook. Our teacher opens such questions every now and then. We solved different questions."

As can be understood from the students' views, the main reason why they went through the process efficiently is their individual and teacher's efforts. Students who constantly participated in the lesson, had a sense of responsibility, did their homework and studies, and had the ability to organize their own learning went through the process better.

The statements of some students who gave negative feedback on the learning process are given below.

S4: "It didn't go well. There's a lot of noise when everyone is talking at the same time. I have a headache. I was very distracted. I already got low in the exams. That's why it wasn't productive."

S19: "So it was kind of bad for me. This is the 4th. The classroom is the best study period of elementary school. He was kind of bad at that. Our topics were heavy. You understand better when you are face-to-face. It's happening with your friends or something. Now you only see your friends on one screen. I think it's bad. It's better to come to school and work there."

S21: "I can't understand anything. It's getting better face-to-face. It's getting very bad online. I can't give myself."

DISCUSSION AND CONCLUSIONS

In this study, the impact of mathematics teaching supported by virtual manipulatives on the academic achievement of 4th-grade students was examined. The research results indicated that the online teaching environment, where the Realistic Mathematics Education (RME) approach supported by virtual manipulatives is applied, is more effective in enhancing students' academic achievement in mathematics compared to online teaching using textbooks according to the constructivist teaching philosophy.

A significant difference was found between the pre-test and post-test scores of the experimental group students. This result shows that teaching supported by virtual manipulatives is effective in increasing students' mathematics achievement. In their meta-analysis, Moyer-Packenham and Westenskow (2013) found that virtual manipulatives had a moderate positive effect on student achievement. This parallels the work of Moyer-Packenham and Suh (2012) and Reimer and Moyer (2005). These researchers have also found that virtual manipulatives increase students' understanding of math concepts and their achievement. The embodiment power of virtual manipulatives was effective in the study. Virtual manipulatives visualize abstract mathematical concepts and make them interactive, making these concepts easier for students to understand. This is especially important for elementary school students who are at the stage of concrete thinking. Virtual manipulatives allow students to actively participate in the learning process.

Students can learn concepts in more depth by interacting with manipulatives, manipulating different variables, and analyzing results. This provides a more effective learning experience than passive learning. In the research, virtual manipulatives were used by integrating them with real-life problems. This, in accordance with the basic principles of the RME approach, enabled students to relate mathematical concepts to daily life and to realize meaningful learning. In the study, virtual manipulatives were used continuously throughout the teaching and students were made to do modeling studies. This consistency has enabled students to better assimilate concepts and increase their achievement. The significant difference observed in the experimental group emphasizes the effectiveness of virtual manipulatives and reveals the importance of alternative teaching methods.

The fact that there was no significant difference between the pre-test and post-test scores of the control group raises questions about the effectiveness of the teaching method applied to this group. Studies with similar results in the literature show that traditional methods may be insufficient to increase student achievement under certain conditions. For example, in the meta-analytical study by Taşpınar-Şener (2023), it was found that technology-supported applications are more effective than traditional methods. The lack of significant difference between the pre- and post-test scores of the control group in our study may be due to several factors. Traditional methods may not be effective enough in teaching some mathematics subjects, may be insufficient to attract students' attention and increase their motivation, may struggle to meet the needs of students with different learning styles, and their effectiveness may have decreased in the distance education process. However, this result does not mean that traditional methods are completely ineffective. In the literature, there are studies showing that traditional teaching methods applied to the control group are also effective in increasing student achievement. For example, in the studies by Çakır (2013), Uysal and Sönmez (2021), and Altıparmak and Çiftçi (2018), it was stated that there was a statistically significant difference between the pre-test and post-test scores of the control group. These studies concluded that the current teaching approach applied to the control group also helped to increase the academic achievement of the students.

When the final academic achievement tests of the experimental and control group students were compared, a significant difference in favor of the experimental group was observed. This result indicates that the teaching method applied to the experimental group positively affects student achievement. Similar findings were reported in the studies of Reimer and Moyer (2005), Suh and Moyer-Packenham (2007), Bolyard and Moyer-Packenham (2012), Hwang et al. (2009), Çakıroğlu (2014), Mutluoğlu and Erdoğan (2021), Demirdögen and Kaçar (2010), and Laurens et al. (2018).

In the experimental group, teaching was conducted using the realistic mathematics education (RME) approach and computer-aided virtual manipulatives. Virtual manipulatives offer a visual and interactive environment for embodying abstract mathematical concepts. They provide an interactive learning environment in which students actively participate. By engaging with manipulatives, students can make their own discoveries, find solutions through trial and error, and take an active role in the learning process. This promotes more permanent learning and deeper understanding of concepts. Virtual manipulatives allow students to learn at their own pace and make their own discoveries. Students can use manipulatives as many times as they want, try different options, and develop their own solutions. This improves students' independence and problem-solving skills. Virtual manipulatives allow students to make mistakes and learn from them. When students make a wrong transaction with a manipulative, they can immediately go back, see their mistake, and try the right way. This allows students not to be afraid of their mistakes, to take risks, and to approach the learning process more positively. Virtual manipulatives increase students' motivation by making learning more fun and engaging. A visually rich, interactive, and gamified environment allows students to participate more actively in the learning process and have a greater interest in the subject. Virtual manipulatives are accessible at any time and place. Using tablets, computers, or smartphones, students can work with virtual manipulatives in the classroom, at home, or anywhere else. This expands learning opportunities and allows students to learn at their own pace and preferences.

In the control group, a traditional teaching method was used in constructivist format, adhering to textbooks. Textbook-based teaching often encourages a passive approach to learning. Students receive information mostly passively and limit their own exploration, trial and error, or active participation. This makes it difficult for students to understand the concepts in depth and learn permanently. The lack of visual and interactive elements can cause students to have difficulty embodying and visualizing concepts. Teaching based on textbooks can reduce students' interest and motivation. A monotonous and passive learning environment makes it difficult for students to actively participate in the learning process and maintain interest in the subject. Textbook instruction can provide limited opportunities for students to make mistakes and learn from them. Students often learn the right answers and solutions directly from books and may miss out on the opportunity to learn from their own mistakes. In textbook-based teaching, teachers may be limited in their ability to provide individual guidance to students and adapt to their learning needs. Teachers often focus on the content in the textbook and may not have the time and opportunity to provide individual support to students.

The RME approach and computer-aided virtual manipulatives applied in the experimental group positively affected the students' mathematics learning and increased their academic success. This finding emphasizes the importance of teaching methods based on concretization and association with daily life in mathematics education.

As a result of the interviews, the majority of the experimental group students (56 opinions) stated that the learning supported by virtual manipulatives in the online learning environment benefited them. The students mentioned the positive aspects such as having a quiet and appropriate environment, visuals being interesting, helping to understand concepts, ensuring active participation in the lesson, manipulative activities being compatible with real life, feedback from the teacher being helpful, improving mathematics ability, increasing interest in mathematics and making the lesson fun. These findings can be interpreted as virtual manipulatives that support students' conceptual understanding, ensure their active participation in the lesson, and increase their interest in mathematics. The findings are consistent with similar results obtained by Satsaniet al. (2018), Zacharias& Olympiou(2011), and Jaakkola. et al. (2011).

On the other hand, some of the students (9 opinions) stated that the learning environment was not productive. These students mentioned negative situations such as difficulty focusing, internet connection problems, low motivation, noise and difficulty in the subject. These findings show that some students experience technical problems, distraction, and conceptual difficulties during the distance education process. Similar results were also found by Satsani et al. (2018), Zacharias & Olympiou (2011), and Jaakkola et al. (2011). These studies presented findings indicating that students experienced distractions, technical issues, and conceptual difficulties during distance education processes. These findings generally examine the effects of both virtual and traditional teaching methods on students' learning experiences while also highlighting the challenges associated with integrating virtual manipulatives into education.

In the study, it was found that most students did not experience difficulties with fractions, and that the use of virtual manipulatives along with real-life examples aided their understanding of the concepts. Students reported that they learned more effectively and solved problems more easily when they could visualize the concepts of fractions. This finding indicates that virtual manipulatives play a significant role in making abstract concepts more comprehensible and in enhancing students' mathematical skills.

This conclusion aligns with the effectiveness of virtual manipulatives in educational contexts highlighted in the work by Moyer-Packenham and Suh (2012). Similarly, Moyer-Packenham and Westenskow (2013) emphasized that virtual manipulatives are essential tools for grasping mathematical concepts. Nevertheless, the fact that some students (4 students) still struggled with fractions and required additional support is consistent with the challenges noted in the research by Pantziara and Philippou (2012). These findings underscore the importance of various supportive strategies in teaching fraction concepts. Therefore, effectively employing virtual manipulatives and real-life examples is crucial for enhancing students' mathematics learning experiences. According to Farra et al. (2024), the use of manipulatives in the learning process enhances students' understanding of fraction concepts more effectively. These findings support the existing literature, which highlights a strong correlation between the use of concrete manipulatives and the learning of mathematical concepts.

In the conducted research, a large majority of the students (22 students) expressed that the teaching practice was beneficial for them and helped them to understand the subjects better. The students highlighted that the practice was different from other lessons, providing a more enjoyable and engaging atmosphere. These findings suggest that the teaching practices utilizing virtual manipulatives and real-life examples positively impact students' learning in mathematics. Similar contributions of virtual manipulatives to students' understanding of mathematical concepts were also noted by Farra et al. (2024), as highlighted by Moyer-Packenham and Westenskow (2013).

However, some students reported that the application was not different from other courses and even worse, pointing out issues such as difficulties in maintaining focus, technical problems, and challenges with the subject matter during the distance education process. Siller and Ahmad (2024) emphasized the positive effects of combining concrete and virtual manipulatives while also acknowledging that students faced various challenges in their learning processes. This indicates that some students encounter difficulties within the distance education environment, suggesting the need for additional support.

As a result, the majority of students positively evaluated the teaching practice using virtual manipulatives and real-life examples. Students stated that this practice supported their conceptual understanding, increased their interest in the lesson, and facilitated their learning of mathematics. However, some students have faced various difficulties during the distance learning process. These findings show that teaching practices using virtual manipulatives and real-life examples contribute positively to students' mathematics learning, but some students need more support in the distance education environment.

This study demonstrates the positive effects of virtual manipulatives on the mathematics learning of fourth-grade students. The students in the experimental group showed a statistically significant increase in academic achievement when taught using virtual manipulatives ($p<0.05$). Students provided positive feedback regarding the depth of concepts, increased motivation, and ease of learning associated with virtual manipulatives. However, the fact that some students expressed a need for additional support during remote learning suggests that careful consideration is necessary for effectively implementing this teaching approach on a larger scale. The findings underscore the importance of integrating virtual manipulatives into mathematics education and encourage future research to examine their effects and adaptability across different student groups. Nonetheless, the study has limitations, such as a small sample size and the application within a specific school context, which may restrict generalizability. Future research should aim to investigate how these methods yield results among different socioeconomic groups and broader student populations. Furthermore, it is known that the Realistic Mathematics Education (RME) approach enables students to better understand mathematical concepts related to everyday life. In this context, it is suggested that greater emphasis be placed on the widespread application of this approach in primary mathematics education. As for practical recommendations, developing teaching materials to promote the integration of virtual and concrete manipulatives and enhancing strategies for teacher training are essential. Additionally, gaining more insights into students' challenges and needs will contribute to the individualization of teaching strategies.

DECLARATIONS

The authors declare no conflicts of interest. No funding was received for conducting this study. Necessary approvals were obtained from Ege University Scientific Research and Publication Ethics Board All human subjects were consented for their participation in the study. Following approved procedures from the Ministry of Education Research Ethics, the researchers provided comprehensive information to the study participants regarding all aspects of the current research. The authors declare they have no financial interests.

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