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Empirical Research

Learning Number Notations – Comparison of a Sign-Value and Place-Value System

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Supplementary Materials: Code, Data, Materials, Preregistration [see Index of Supplementary Materials]

Abstract

Although numbers are universal, there are great differences between languages and cultures in terms of how they are represented. Numerical notation can influence number processing. Two well-known types of notational systems are sign-value, such as the Roman numeral system, and place-value systems, such as the Indo-Arabic numeral system. What is involved in learning each system? Here we report a study that investigated adults' abilities to implicitly learn an artificially created sign-value or place-value system. We asked if they could perform symbolic comparison and ordering tasks using the novel symbol system. We found adults could learn the ordinal meaning of symbols within either system and were able to extend the system to symbols not encountered during training. There was a relative advantage of the sign-value system over the place-value system for expressions encountered during the training, but also for expressions that had not previously been encountered. These results shed light on how easily the structure of place-value and sign-value systems can be learned.

Keywords

artificial symbol learning, place-value system, sign-value system, symbolic comparison task

Non-Technical Summary

Background

There are different ways of representing numbers. In some representation systems the location of a symbol within an expression changes the symbol's meaning, for example '1' in 12 means something different to '1' in 2134. These are known as place-value systems. Other representation systems, such as (additive) Roman numerals, do not have this property: the 'X' in XXI has the same meaning as the X in CXVI. These are known as sign-value systems.

Why was this study done?

We wanted to compare how easy it is to implicitly learn the structure of novel numerical systems, and whether this differs between place-value and sign-value systems.

What did the researchers do and find?

We asked 204 adults to learn a novel numerical system by showing them ordered sequences of expressions. Half the participants learned a sign-value system, half learned a place-value system. We then explored how well they understood the ordinal meaning of expressions from the system by asking them to compare the values of different expressions. This comparison task included expressions that they had seen during the learning phase, but also novel expressions which they had to deduce the meaning of, based on the structure of the system. We found that participants were surprisingly good at inferring the structure of both systems, and that there was a small advantage for the sign-value system over the place-value system.

What do these findings mean?

The findings suggest that numerical structure can be inferred from relatively little exposure to novel symbol systems, which gives us some hints about how early number learning might work.

Highlights

- The ordinal meaning of novel artificial number symbols within both sign-value and place-value systems can be learned with relatively little exposure.
- Participants in the current study could learn the structure of both artificial sign-value and place-value systems.
- Sign-value systems might be easier to learn and use in simple numerical tasks than place-value systems.

Numerals are everywhere and come in a variety of formats. They can be represented as number words, Indo-Arabic digits, or as Roman numerals. They are almost always part of some kind of numeral system, often with both written and verbal notations or representations. Despite their omnipresence, there are substantial differences across languages and cultures in how these notational systems are constructed.

Different notational systems have existed for thousands of years. Most numeral systems have distinct structural properties (Bender & Beller, 2013a), are usually made up of a small set of individual symbols for representing the basic numbers, and also contain some compositional rules for the larger numbers (Bender & Beller, 2013b, 2018). Two common types of notational systems are i) sign-value systems (e.g., the Roman numeral system), in which the values of a series of numerical symbols, when combined together, represent the given number, and ii) place-value systems (e.g., the Indo-Arabic numeral system), in which the value of a numeral is determined by the positions of the symbols constituting the given numeral. Nevertheless, more fine-grained classifications of numeral systems have also been proposed (Chrisomalis, 2010; Widom & Schlimm, 2012; Zhang & Norman, 1995).

Sign-value systems are often considered the simplest notational system (Zhang & Norman, 1995). In the most simplest form, a one-dimensional system, increasing set sizes are represented by using the same symbol repeatedly, in a cumulative fashion (e.g., one line for one, two lines for two, etc.; Bender & Beller, 2013b; Zhang & Norman, 1995). However, not all sign-value systems are one-dimensional but may have multiple dimensions. For example, a main power dimension, a sub-base dimension, and a sub-power dimension (e.g., (1×1)×1D systems, such as the Roman numeral system; Zhang & Norman, 1995). In contrast to this are place-value or positional systems, which are often considered more complex and are more common today (Chrisomalis, 2010; Gill & Dixit, 2008; Ifrah, 2000; Latif et al., 2011). To use a place-value system correctly, it is crucial to understand the structure, the compositional rules and principles (e.g., cardinality, ordinality) of the system. This is because the value of a numeral depends on multiple factors: the base of the system, the value of the symbol and the position of the symbol in relation to other symbols. The role of the position of a symbol is a crucial difference between place-value and sign-value systems. Numerals in the simplest form of sign-value systems are usually written in hierarchical order from left to right (i.e., largest numeral to the left, e.g., 16 in Roman numerals is *XVI,* rather than *XIV*, *VXI*, *VIX*, *IXV* or *IVX*). This is, however, a convention rather than a rule and is not essential unless subtraction forms such as the later introduced Roman *IV* are used (i.e., VXI can still be understood as 16, although this is not the conventional form). In place-value systems, however, this rule is essential (273 is not the same as 372).

Although sign-value and place-value systems are often said to be informationally equivalent, the amount of cognitive resources needed to use each system is debated, especially with regards to carrying out mathematical computations. Place-value systems are considered highly efficient (Dehaene, 1997; Krajcsi & Szabó, 2012; Zhang & Norman, 1995), yet some have argued that addition and subtraction in a sign-value system may be easier than in a place-value system: addition simply requires counting all symbols and concatenating these (i.e., using simplification rules, such as *IIIII* = *V* for Roman numerals) and subtraction only requires cancelling out symbols (Anderson, 1971; Bidwell, 1967). Multiplication, in contrast, has been argued to be more difficult in sign-value systems, as it can become cumbersome and complex due to the large amount of individual symbols (Anderson, 1971; Detlefsen et al., 1976; Schlimm & Neth, 2008), even though far fewer multiplication facts have to be remembered (as there are fewer single symbols) compared to typical place-value systems (Anderson, 1971; Schlimm & Neth, 2008). Sign-value systems have been very popular in the past, even when place-value systems were available (Chrisomalis, 2010; Ifrah, 2000), and children still get taught about them (e.g., the Roman system). Although used less frequently nowadays, they can still be found in some everyday contexts, for example on clocks and buildings for date inscriptions, for numbering initial book pages, and for monarchs. Furthermore, much of the research into the differences between numerical systems is based on theoretical analyses, computations and descriptions of the different systems and how these work. Studies are generally not based on direct empirical data from participants or real-world cases, as there “simply is no such evidence” (Chrisomalis, 2020, p. 67) to show how, for example, Roman numerals were actually used to perform basic arithmetic. It is therefore difficult to determine whether one system is superior to another, especially in terms of the cognitive resources needed to use each system, but more importantly, it is unknown how different systems are learned and how they affect numerical processing.

One way to investigate this is by using artificial learning paradigms. These have been previously used in various domains, such as for language and grammar learning (e.g., Bulgarelli & Weiss, 2021; Crespo & Kaushanskaya, 2022; Gillis et al., 2022; Milne et al., 2018; Schiff et al., 2021), to investigate different aspects of and influencing factors on learning. Artificial grammar learning (AGL) paradigms are rule induction paradigms that measure a participant’s ability to implicitly extract regularities from sequential stimuli (Schiff et al., 2021). In AGL paradigms, stimuli are usually created from a finite-state rule system and these stimuli sequences, which typically vary in length from three to six symbols, are passively viewed by participants in the learning phase. Importantly, participants are not told about the existence of the rules governing the formation of the stimuli sequences. In the test phase, participants complete a grammatical judgement task which assesses their ability to classify novel stimuli as either grammatical or non-grammatical, in other words whether the stimuli follow the governing rule or violate the rule. Importantly, although participants are unaware of the underling rule of the system, they are still able to classify novel stimuli as grammatical or ungrammatical at rates significantly above chance. AGL tasks have been used in different modalities (e.g., visual: Pavlidou & Williams, 2014; auditory: Loui et al., 2010; Kahta & Schiff, 2016) and in different populations: neurotypical adults (e.g., Chang & Knowlton, 2004; Reber, 1967, 1989, 1993; Schiff et al., 2017; Westphal-Fitch et al., 2018), adults with amnesia (e.g., Knowlton et al., 1992; Meulemans & Van der Linden, 2003), adults with developmental disorders (e.g., Don et al., 2003; Kahta & Schiff, 2016; Schiff et al., 2017), both typically developing children and children with various developmental disorders (e.g., Crespo & Kaushanskaya, 2022; Don et al., 2003; Gillis et al., 2022; Pavlidou & Williams, 2014; Pavlidou et al., 2009, 2010), infants (e.g., Gomez & Gerken, 1999; Marcus et al., 1999) and even non-human primates (e.g., Milne et al., 2018). Important here is that these studies provide evidence that learning does not need to be explicit and does not necessarily result from conscious hypothesis testing (Seger, 1997), but instead learning is incidental and can happen implicitly.

Within the mathematical cognition domain, these artificial learning paradigms have also been used, especially to test how adults might attach semantic or numerical meaning to abstract, novel symbols and what factors might influence such symbol learning (e.g., Bennett et al., 2019; Krajcsi et al., 2016; Krajcsi & Kojouharova, 2017; Krajcsi & Szabó, 2012; Lyons & Ansari, 2009; Lyons & Beilock, 2009; Merkley et al., 2016; Merkley & Scerif, 2015; Tzelgov et al., 2000; Wege et al., 2020; Weiers et al., 2023; Zhao et al., 2012). To assess how participants can learn the meaning of novel numerical symbols, their performance on a symbolic comparison task is often measured. This task, which involves judging which of two presented stimuli (normally two dot-arrays or Arabic digits) is numerically larger, is typically used in the mathematical cognition literature to determine how accurately someone can discriminate between two numerosities. In terms of artificial symbol learning studies, results generally show that adults can perform symbolic comparison tasks with artificial symbols after being trained to learn symbol-symbol associations (e.g., Tzelgov et al., 2000; Weiers et al., 2023) or symbol-magnitude associations (e.g., Lyons & Ansari, 2009; Lyons & Beilock, 2009; Merkley & Scerif, 2015; Weiers et al., 2023). These studies involved single-digit artificial number symbols and therefore did not consider multi-digit number systems.

As there is very little empirical data involving participants learning number systems, claims about the cognitive efficiency of different number systems, as described above, are often based on analyses of the systems, and not on direct empirical comparisons. We therefore tested participants' ability to implicitly learn different systems with the aim of comparing the two types of systems. To the best of our knowledge, only one previous study has directly compared two different artificial symbol systems. Krajcsi and Szabó (2012) designed an artificial sign-value and place-value system using base-4 to avoid interference with the Indo-Arabic base-10 system. Participants first learned to identify all novel symbols and were then explicitly told how each notational system works and that a different base was used. Participants then completed comparison and addition tasks using the learned symbols (i.e., no new symbols or untrained symbols were introduced) in both notations. The authors found a relative advantage of their sign-value system over their place-value system, whereby participants better compared and added novel symbols in sign-value notation. The authors provided explicit instructions about how each novel system works, but it is not yet clear what effect the notation has on learning symbols without explicit instructions about the rules of the system, and on simple processing, especially of new (i.e., untrained) symbols. A complementary follow-up, and the aim of the current study, is therefore to test and directly contrast the relative difficulty with which a novel sign-value and place-value system can be learned and then used in simple tasks. Specifically, we are interest in the implicit learning of the underlying structure of different notational systems (i.e., whether participants can really learn the structure and thus compositional rules of each system), whether the structure of one system is learned more easily than another, and if participants' learning generalises beyond the initially learned artificial symbols.

In contrast to Krajcsi and Szabó, who provided explicit instructions about the structure of each system, in the current study, we focus on participants' ability to implicitly learn the structure. While explicit instruction is important and may be useful for learning, a lot of learning happens implicitly. Within the domain of numbers, children can generally recite the count sequence long before they are explicitly taught about place-value. They have therefore already acquired some implicit knowledge about numbers before formal instruction about the structure of the number system. For example, in the national mathematics curriculum of England, pupils are only formally taught about place-value in Year 2 (age 6-7; Department for Education, 2021). However, prior to this, in Year 1 (age 5-6), they are expected to be proficient in counting and recognising single- and multi-digit numbers. Implicit learning is therefore common, and, as previous studies have shown (e.g., AGL studies mentioned above), is a viable way to learn the rules of some systems.

In the current study, we therefore investigated adults' ability to implicitly learn the underlying structure of an artificially created sign-value or place-value system using a base-3 structure (to avoid the confound of familiarity with base-10). Each system was composed of three basic individual *symbols* (like the single digit numerals in the Indo-Arabic place-value system) and these symbols can be combined according to the compositional rules of the system to form an *expression* (similar to multi-digit numerals in the Indo-Arabic place-value system). After providing training about the ordering of the first twelve expressions, we tested participants' ability to complete a simple symbolic comparison and ordering task using the expressions of the system. We also tested whether they could accurately compare untrained symbols.

Our first question of interest is therefore whether individuals can learn the structure of sign-value and place-value symbol systems from a small number of expressions without explicit instruction about the systems' structures or their expressions. Following on from this, our second question concerned whether participants learned the structure of the system in the sense that they are able to generalise beyond learned expressions, rather than simply remembering the order that expressions were presented in during training. We therefore tested participants' ability to compare expressions not encountered during training. Lastly, by using two different notational systems, we asked and aimed to investigate whether the underlying structure of one system can be learned implicitly more easily than the other.

Experiment 1

Participants

This online study was hosted by Gorilla (https://gorilla.sc/). A sample comprising 204 adults was recruited via Prolific (https://www.prolific.co/), which gives 90% power to detect an effect of d = 0.456 in an independent samples t-test. Participants were required to be between 18 – 60 years and to have normal or corrected-to-normal vision. The study was approved by the Loughborough University Ethics Approvals (Human Participants) Sub-Committee. All participants gave explicit consent before taking part and received £6 as an inconvenience allowance once the study was completed.

Design

Participants were randomly assigned to one of two training groups (sign-value, place-value) and one of three expressions sets (i.e., sets consisted of the same symbols, but in different orders; see Table 1). The experiment lasted approximately 45 minutes. Participants could take regular, short breaks throughout.

Stimuli

There were three base symbols, which when combined according to the rules of the symbol system formed all expressions of that system. All expressions of the sign-value system followed an additive base-3 sign-value structure, and all expressions of the place-value system followed a base-3 place-value structure (see Table 1). Equal numbers of participants were randomly assigned to each expression set.

The artificial symbols were created using KLaTeXFormula. All symbols (and thus expressions) were presented in black font on a white background.

Pre-Training Phase

Participants first gave informed consent and then answered basic demographic questions. Following this they were presented with the instructions for the training phase.

Training Phase

Participants passively viewed the first twelve expressions of the assigned symbol system (i.e., the first half of the expressions depicted in Table 1) as an ordered sequence. Expressions were presented one by one (500ms each) in the middle of the screen. Participants were told that each expression represented a particular quantity and instructed to learn the order of expressions. Each sequence was preceded by a fixation cross (800ms). The training was split into ten blocks, each block contained the sequence six times, thus the sequence was seen 60 times in total, as was each individual expression. Example video recordings of one block of the training phase can be found online (Weiers et al., 2024S-c).

Test Phase

Participants completed two test tasks in fixed order: first the symbolic comparison task, then the global ordering task.

Symbolic Comparison Task

Participants saw two expressions simultaneously appearing side by side and indicated which represented the larger quantity. If participants thought the expression displayed on the left represented the larger quantity, they pressed the 'x' key, otherwise the 'm' key. Before each trial a fixation cross (500ms) appeared. Trials were presented until a response was given, but for a maximum of three seconds. The next trial was started once a response was given. The order of expression pairs was random. There were 528 trials in total. Half (n = 264) contained seen expressions (i.e., expressions 1 – 12 in Table 1) and half contained unseen expressions (i.e., the next 12 expressions of the sequence; expressions 13 – 24). Each seen expression was paired with every other seen expression twice, and each unseen expression was paired with every other unseen expression twice. The presentation side, and thus response key, was counterbalanced (i.e., two comparisons per display side). Each trial was seen four times in total. There were 12 blocks of trials.

Table 1

Sets of Expressions for Each of the Training Groups

	Sign-value systems			Place-value systems		
Expression	a)	b)	c)	a)	b)	c)
1	⊨	⋔	ㅅ	⊨	⋔	ㅅ
2	⊨⊨	⋔⋔	ㅅㅅ	⋔	ㅅ	⊨
3	⋔	ㅅ	⊨	ㅅ	⊨	⋔
4	⋔⊨	ㅅ⋔	⊨ㅅ	⊨⊨	⋔⋔	ㅅㅅ
5	⋔⊨⊨	ㅅ⋔⋔	⊨ㅅㅅ	⊨⋔	⋔ㅅ	ㅅ⊨
6	⋔⋔	ㅅㅅ	⊨⊨	⊨ㅅ	⋔⊨	ㅅ⋔
7	⋔⋔⊨	ㅅㅅ⋔	⊨⊨ㅅ	⋔⊨	ㅅ⋔	⊨ㅅ
8	⋔⋔⊨⊨	ㅅㅅ⋔⋔	⊨⊨ㅅㅅ	⋔⋔	ㅅㅅ	⊨⊨
9	ㅅ	⊨	⋔	⋔ㅅ	ㅅ⊨	⊨⋔
10	ㅅ⊨	⊨⋔	⋔ㅅ	ㅅ⊨	⊨⋔	⋔ㅅ
11	ㅅ⊨⊨	⊨⋔⋔	⋔ㅅㅅ	ㅅ⋔	⊨ㅅ	⋔⊨
12	ㅅ⋔	⊨ㅅ	⋔⊨	ㅅㅅ	⊨⊨	⋔⋔
13	ㅅ⋔⊨	⊨ㅅ⋔	⋔⊨ㅅ	⊨⊨⊨	⋔⋔⋔	ㅅㅅㅅ
14	ㅅ⋔⊨⊨	⊨ㅅ⋔⋔	⋔⊨ㅅㅅ	⊨⊨⋔	⋔⋔ㅅ	ㅅㅅ⊨
15	ㅅ⋔⋔	⊨ㅅㅅ	⋔⊨⊨	⊨⊨ㅅ	⋔⋔⊨	ㅅㅅ⋔
16	ㅅ⋔⋔⊨	⊨ㅅㅅ⋔	⋔⊨⊨ㅅ	⊨⋔⊨	⋔ㅅ⋔	ㅅ⊨ㅅ
17	ㅅ⋔⋔⊨⊨	⊨ㅅㅅ⋔⋔	⋔⊨⊨ㅅㅅ	⊨⋔⋔	⋔ㅅㅅ	ㅅ⊨⊨
18	ㅅㅅ	⊨⊨	⋔⋔	⊨⋔ㅅ	⋔ㅅ⊨	ㅅ⊨⋔
19	ㅅㅅ⊨	⊨⊨⋔	⋔⋔ㅅ	⊨ㅅ⊨	⋔⊨⋔	ㅅ⋔ㅅ
20	ㅅㅅ⊨⊨	⊨⊨⋔⋔	⋔⋔ㅅㅅ	⊨ㅅ⋔	⋔⊨ㅅ	ㅅ⋔⊨
21	ㅅㅅ⋔	⊨⊨ㅅ	⋔⋔⊨	⊨ㅅㅅ	⋔⊨⊨	ㅅ⋔⋔
22	ㅅㅅ⋔⊨	⊨⊨ㅅ⋔	⋔⋔⊨ㅅ	⋔⊨⊨	ㅅ⋔⋔	⊨ㅅㅅ
23	ㅅㅅ⋔⊨⊨	⊨⊨ㅅ⋔⋔	⋔⋔⊨ㅅㅅ	⋔⊨⋔	ㅅ⋔ㅅ	⊨ㅅ⊨
24	ㅅㅅ⋔⋔	⊨⊨ㅅㅅ	⋔⋔⊨⊨	⋔⊨ㅅ	ㅅ⋔⊨	⊨ㅅ⋔

Global Ordering Task

Participants simultaneously saw all twelve expressions from the training phase randomly distributed across the screen and were required to determine the order from smallest to largest. Participants could review their chosen order before submitting it. Participants' accuracies were scored based on the number of expressions placed in their respective correct positions in the sequence, whereby a score of 1 was given for each expression if it was in the correct position, otherwise a score of 0 was given. The average across all expressions was then taken to indicate the accuracy on this task.

Upon completion of this task, participants were debriefed and re-directed to Prolific to receive payment.

Data Analysis

Before data collection, the study research questions, sample size, exclusion criteria and analysis plan were preregistered (Weiers et al., 2021S). All experiment files can be found online (Weiers et al., 2023S). Descriptive statistics were calculated for demographics as well as reaction times (RTs) and accuracies. Inferential statistics were calculated for both RTs and accuracies. Analyses were conducted using JASP (JASP Team, 2018). The main variables of interest were accuracy, and median RT on correct trials. The alpha level for the analyses was set to .05. Significant main effects of the omnibus tests were further investigated with post-hoc analyses.

Results

All data analysis files can be found online (Weiers et al., 2024S-a). Preregistered analyses, including pre-processing, descriptive statistics and inferential statistics are presented first. Secondary preregistered analyses, and exploratory analyses (not preregistered), are then presented.

Preregistered Analyses

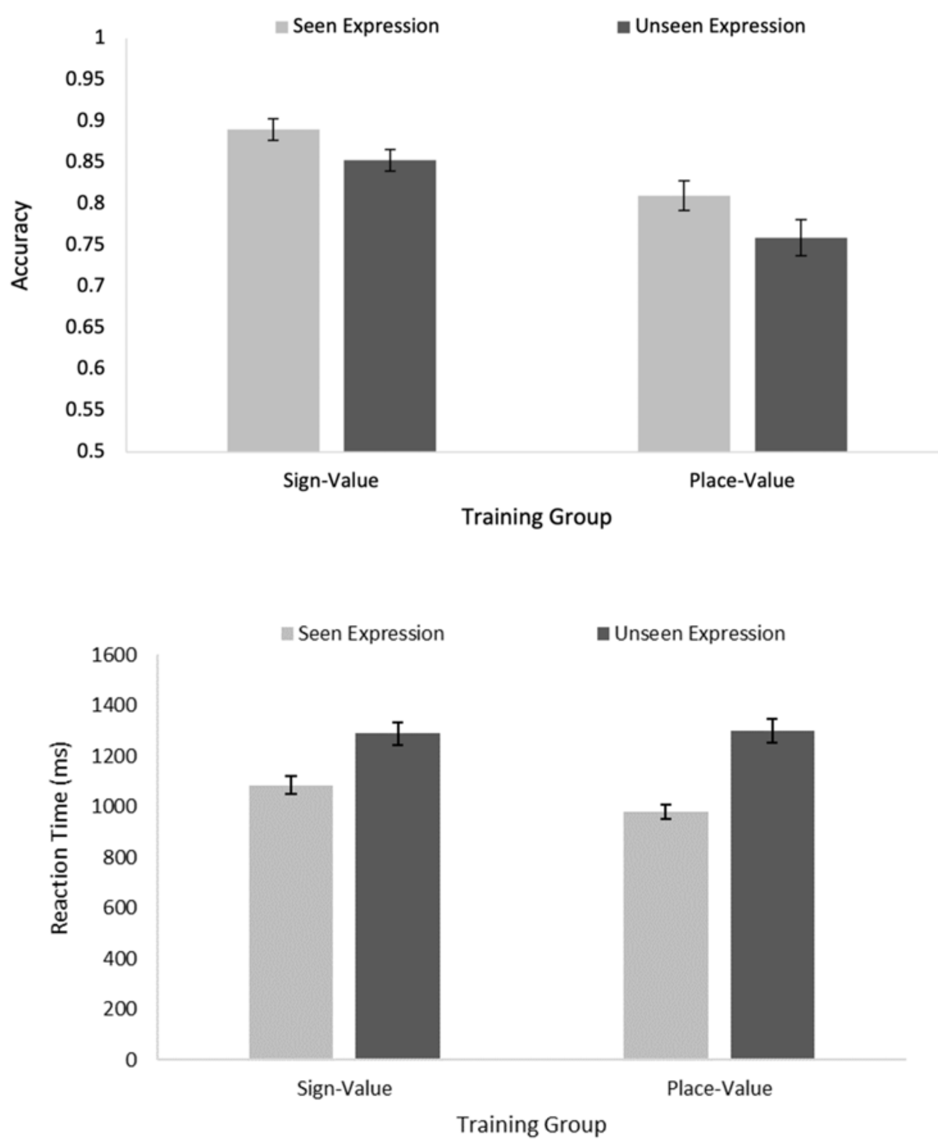
204 participants (102 per group), aged between 18 – 57 (M = 26.94, SD = 7.49), completed the experiment and provided full data sets for the analyses.

Symbolic Comparison Task — Figure 1 shows the mean accuracies and RTs for each group on the symbolic comparison task, separately for trials containing seen (i.e., expressions encountered during the training phase) and unseen expressions (i.e., expressions not encountered during the training phase). As the Shapiro-Wilk test indicated a violation of the assumption of normality, Wilcoxon signed-rank tests were used. These indicated that both groups performed significantly above chance (chance = .5) and with high levels of accuracy on trials containing seen expressions (sign-value: $W = 5253$, $p < .001$; place-value: $W = 5027.5$, $p < .001$) and unseen expressions (sign-value: $W = 5253$, $p < .001$; place-value: $W = 4799$, $p < .001$). This answers our first two questions of interest that participants in both groups implicitly learned the structure of each system from a small number of expressions without explicit instruction, and that they were also able to generalise beyond the expressions they were originally trained on.[1]

1) It should be noted that seen trials (expressions 1 – 12) represented numbers of lower numerical value than unseen trials (expressions 13 – 24), which meant that the numerical ratio of seen and unseen comparison trials differed (mean ratio seen trials = 0.5, mean ratio unseen trials = 0.796). Given that ratio might influence performance, we selected a subset of trials (n = 164) where seen and unseen trials were closely matched for ratio. A 2 (training group: sign-value; place-value) × 2 (expression type: seen; unseen) mixed ANOVA revealed a significant main effect of expression type on accuracy, $F(1, 202) = 4.123$, $p = .044$, $h_p^2 = 0.020$, whereby participants were significantly more accurate on unseen expressions (M = .803, SD = 0.193) than on seen expressions (M = .792, SD = 0.198). This may, however, be because participants took longer to complete these trials (see below). There was a significant main effect of training group, $F(1, 202) = 8.331$, $p = .004$, $h_p^2 = 0.040$, whereby the sign-value group (M = .835, SD = 0.156) performed significantly better than the place-value group (M = .760, SD = 0.216). The training group × expression type interaction was not significant, $F(1, 202) = 1.041$, $p = .309$. Using RT data for the same trial subset revealed a significant main effect of expression type, $F(1, 202) = 106.617$, $p < .001$, $h_p^2 = 0.345$, whereby participants were significantly slower on unseen expressions (M = 1306.783, SD = 496.718) than on seen expressions (M = 1153.065, SD = 383.681). The main effect of training group was not significant, $F < 1$, but the group × expression type interaction was, $F(1, 202) = 40.177$, $p < .001$, $h_p^2 = 0.166$. To follow up on the significant interaction, paired samples t-tests comparing seen to unseen expressions for each group separately were conducted. These revealed that the sign-value group was significantly slower on unseen expressions (M = 1243.457, SD = 428.651) than on seen expressions (M = 1184.102, SD = 388.957), $W = 1237$, $p < .001$. The place-value group was also significantly slower on unseen expressions (M = 1370.108, SD = 551.486) than on seen expressions (M = 1122.028, SD = 377.685), $W = 271$, $p < .001$. The place-value group showed significantly greater slowing (M = 248.080, SD = 267.569) than the sign-value group (M = 59.355, SD = 137.224), $t(150.693) = 6.339$, $p < .001$, $d = 0.888$.

Figure 1

Accuracy Rates (Top) and Reaction Times (Bottom) for Both Training Groups on the Symbolic Comparison Task, Split by Trials Containing Seen and Unseen Expressions



Note. Error bars indicate ± 1 standard error of the mean.

To investigate our third question of interest, if one system was learned more easily than the other, we tested for group differences using independent samples *t*-tests on the accuracy and RT data separately. When Levene's test of equality of variances was violated, Welch's *t*-test was used. On seen expressions, the sign-value group was more accurate (M = .89, SD = 0.129) and slower (M = 1084.32, SD = 357.336) than the place-value group (accuracy: M = .81, SD = 0.18; $t(182.965)$ = 3.633, $p < .001$, d = 0.509; RT: M = 981.921, SD = 288.692; $t(202)$ = 2.251, p = .025, d = 0.315). On unseen expressions, the sign-value group (M = .853, SD = 0.127) was more accurate than the place-value group (M = .759, SD = 0.226), $t(158.68)$ = 3.666, $p < .001$, d = 0.513), but there was no significant difference between the groups in RT ($t(202)$ = 0.174, p = .862). This suggests that the structure of a sign-value system may be learned implicitly more easily.

Heat maps showing accuracy rates for each group on each of the comparisons of the symbolic comparison task are shown in Figure 2 (Figure 2 (top) for symbols encountered during training and Figure 2 (bottom) for untrained symbols).

Figure 2

Heatmaps of Accuracy Rates of All Trials Involving Trained Expressions 1-12 (Top) and Untrained Expressions 13-24 (Bottom) for the Sign-Value and Place-Value Group

Place-Value \ Sign-Value	1	2	3	4	5	6	7	8	9	10	11	12
1		0.956	0.904	0.968	0.98	0.966	0.983	0.973	0.922	0.975	0.99	0.971
2	0.838		0.772	0.885	0.971	0.897	0.966	0.975	0.821	0.904	0.968	0.926
3	0.848	0.784		0.946	0.978	0.985	0.983	0.98	0.912	0.958	0.966	0.971
4	0.917	0.593	0.537		0.973	0.868	0.978	0.98	0.799	0.882	0.961	0.907
5	0.917	0.787	0.623	0.833		0.689	0.877	0.966	0.75	0.792	0.865	0.797
6	0.934	0.775	0.772	0.816	0.789		0.949	0.971	0.748	0.816	0.919	0.868
7	0.887	0.914	0.662	0.828	0.792	0.662		0.951	0.672	0.728	0.814	0.787
8	0.897	0.936	0.681	0.843	0.809	0.659	0.792		0.672	0.691	0.684	0.691
9	0.887	0.895	0.804	0.853	0.828	0.787	0.814	0.667		0.944	0.936	0.961
10	0.922	0.897	0.868	0.828	0.801	0.76	0.777	0.738	0.657		0.953	0.841
11	0.885	0.919	0.895	0.836	0.824	0.794	0.804	0.779	0.745	0.757		0.681
12	0.88	0.897	0.926	0.85	0.838	0.838	0.814	0.819	0.779	0.843	0.821	

Place-Value \ Sign-Value	13	14	15	16	17	18	19	20	21	22	23	24
13		0.941	0.838	0.956	0.973	0.787	0.875	0.919	0.895	0.971	0.973	0.973
14	0.833		0.561	0.833	0.946	0.699	0.735	0.838	0.735	0.833	0.958	0.895
15	0.836	0.777		0.907	0.956	0.745	0.77	0.843	0.836	0.914	0.912	0.975
16	0.824	0.748	0.645		0.936	0.713	0.725	0.797	0.74	0.86	0.9	0.831
17	0.855	0.828	0.647	0.794		0.674	0.659	0.699	0.669	0.73	0.782	0.708
18	0.848	0.809	0.777	0.784	0.674		0.924	0.951	0.924	0.941	0.946	0.939
19	0.824	0.804	0.713	0.777	0.73	0.701		0.949	0.865	0.941	0.963	0.926
20	0.836	0.838	0.792	0.799	0.74	0.745	0.748		0.662	0.853	0.944	0.885
21	0.841	0.831	0.821	0.806	0.784	0.814	0.821	0.767		0.953	0.958	0.968
22	0.816	0.77	0.703	0.775	0.713	0.64	0.662	0.632	0.613		0.941	0.858
23	0.833	0.799	0.706	0.797	0.701	0.706	0.679	0.642	0.605	0.775		0.608
24	0.831	0.804	0.792	0.779	0.801	0.745	0.738	0.703	0.627	0.784	0.73	

Note. Accuracy rates for the sign-value group are displayed above the diagonal and accuracy rates for the place-value group are displayed below the diagonal.

Global Ordering Task — Both training groups performed significantly above chance (chance = .083) on the global ordering task (sign-value: W = 5015, p < .001; place-value: W = 5032, p < .001). An independent samples t-test showed that the sign-value group (M = .700, SD = 0.418) and place-value group (M = .647, SD = 0.426) did not differ significantly in performance, $t(202)$ = 0.899, p = .370. This confirmed that participants from both groups successfully learned the order of the expressions.

Exploratory Analyses

The preregistered analyses appear to demonstrate that participants learned the structure of the number systems because they could perform with reasonably high accuracy on the untrained symbols. However, participants may have instead succeeded on the comparison task by consistently using superficial strategies. Similarly, the difference between groups could also be explained by use of a superficial strategy rather than differences in ease of learning the structure of the number system. A series of exploratory analyses was therefore run to investigate whether these strategies were consistently used by participants to solve the symbolic comparison task.

Individual Expression Length — The length of an expression, i.e., how many symbols it is composed of, can give an indication about the expressions' magnitude, which may be used to make judgements. In a sign-value system, not all three symbol expressions are necessarily larger than all two symbol expressions, however, in a place-value system, all three symbol expressions are larger than all two symbol expressions. We therefore tested whether participants consistently based their judgements on this. The sign-value group was significantly more accurate ($t(101) = 7.885$, $p < .001$, $d = 0.781$) and faster ($t(101) = 2.960$, $p = .004$, $d = 0.293$) when the longer expression represented the larger quantity (accuracy: $M = .954$, $SD = 0.054$; RT: $M = 1166.844$, $SD = 401.715$) than when the longer expression represented the smaller quantity (accuracy: $M = .789$, $SD = 0.226$; RT: $M = 1222.785$, $SD = 397.676$). In place-value systems, this works slightly differently due to the nature of the system. The place-value group was also significantly more accurate ($W = 1734.5$, $p = .003$) and faster ($W = 59$, $p < .001$) on trials where the longer expression was larger (accuracy: $M = .834$, $SD = 0.147$; RT: $M = 1122.686$, $SD = 371.304$), than when the expressions were of the same length (accuracy: $M = .772$, $SD = 0.227$; RT: $M = 1208.768$, $SD = 423.375$). Thus, both groups were better and faster on trials on which the strategy worked than on trials where this strategy did not work.

Importantly though, both groups still performed significantly above chance on trials for which this strategy did not work (sign-value: $M = .716$, $SD = 0.309$, $t(101) = 7.043$, $p < .001$; place-value: $M = .772$, $SD = 0.227$, $t(101) = 12.089$, $p < .001$). Thus, although participants may have used this strategy on some of the trials, participants did not consistently use this strategy throughout the experiment and thus, the use of this strategy alone cannot explain overall performance patterns.

Leftmost Symbol Comparison — In both notational systems, the leftmost symbol always carries the highest value. Thus, magnitude judgements could be made based on solely comparing the leftmost symbols of two expressions. If participants were consistently using this strategy, we would expect a large difference in accuracy and RT between trials where the leftmost symbol was identical and trials where it was not. For the sign-value group, a paired-samples *t*-tests revealed no significant difference in accuracy between trials on which the leftmost symbols were the same and trials on which the leftmost symbols differed ($p = .917$), but participants were significantly slower on trials on which the leftmost symbols were the same ($M = 1249.689$, $SD = 427.150$), than when these differed ($M = 1058.570$, $SD = 348.218$; $t(101) = 13.942$, $p < .001$, $d = 1.380$). The place-value group was significantly more accurate and slower on trials on which the leftmost symbols were the same (accuracy: $M = 0.784$, $SD = 0.232$; RT: $M = 1308.760$, $SD = 476.746$), than when these differed (accuracy: $M = 0.762$, $SD = 0.232$, $t(101) = 2.478$, $p = .015$, $d = 0.245$; RT: $M = 1134.281$, $SD = 449.663$, $t(101) = 6.588$, $p < .001$, $d = 0.652$). These results indicate no large differences in accuracy or RT between trials on which this strategy works versus those on which it does not, for either training group. Thus, although there is some indidcation that the place-value group, but not the sign-value group, may have used this strategy on some of the trials, neither group used this strategy consistently throught out experiment, meaning this cannot explain the performance differences.

Total Number of Symbols on Screen — The expressions' length might affect the RTs of the training groups differently. During the comparison task, the maximum total number of symbols simultaneously presented varied between two and six for the place-value group and between two and ten for the sign-value group. We expected longer RTs if more symbols were simultaneously presented due to increased processing times, and this may be stronger for the sign-value group as their total number of symbols visible was larger than those of the place-value group.

Separate repeated measures ANOVAs revealed that accuracy tended to decline (place-value: $F(1.619, 163.522) = 8.249$, $p < .001$, $\eta_p^2 = 0.076$; sign-value: $F(2.067, 208.744) = 23.232$, $p < .001$, $\eta_p^2 = 0.187$) and RT tended to increase (place-value:

F(1.927, 186.894) = 124.191, p < .001, η_p^2 = 0.561; sign-value: F(2.591, 248.759) = 89.612, p < .001, η_p^2 = 0.483) with increasing number of symbols visible. This suggests similar performance patterns for both groups. Using only trials on which both groups saw the same number of symbols (i.e., 2, 3, 4 and 6 symbols in total), a 2 (training group) × 4 (total number of symbols visible) repeated measures ANOVA on accuracy revealed a significant main effect of total number of symbols, F(1.816, 366.863) = 22.991, p < .001, η_p^2 = 0.102, and a significant main effect of training group, F(11, 202) = 15.258, p < .001, η_p^2 = 0.070, but no significant interaction, F < 1. Using RT, there was a significant main effect of total number of symbols visible, F(1.820, 360.390) = 170.517, p < .001, η_p^2 = 0.463, and a significant interaction, F(1.820, 360.390) = 8.502, p < .001, η_p^2 = 0.041, but no main effect of training group, F < 1. Therefore, these results replicated the main analyses and differences in the total number of symbols visible cannot explain the overall group difference in accuracy.

To summarise these exploratory analyses, we cannot rule out that some participants may have used some of these superficial strategies for some of the trials of the comparison task. However, the groups' above chance performance and the differences between the training groups cannot be solely attributed to the use of any of these strategies. Instead these seem to stem from the differences in the ease of learning the structure of the respective number systems.

Discussion

The main aim of this study was to investigate whether a novel artificial sign-value and place-value system can be learned implicitly and whether one system can be learned more easily than the other. Furthermore, we aimed to investigate to what extent participants understood expressions not encountered during training. Using an artificial symbol learning paradigm, we showed that the structure of both a sign-value and a place-value symbol system can be learned implicitly with relatively little exposure and can be generalised beyond trained expressions.

We found that participants in the sign-value group were more accurate in determining which of two expressions (both seen and unseen) was greater in magnitude than participants in the place-value group. This suggests participants in the sign-value group could better learn and infer the structure and understand the individual expressions of this system compared to the place-value group. We also tested for various superficial strategies and confounds which may have been used to solve the task rather than learning the structure of the system, however, none of these could explain the overall group differences or overall above-chance performance.

Both training regimes equally allowed participants to learn and recreate the order of the expressions on the order task. The lack of group difference on this task is perhaps unsurprising as it only tested learning of the trained items and did not test participants' knowledge of the system structure.

Experiment 2

Rationale

It is possible that participants in Experiment 1 simply memorised the order in which expressions were given to them, irrespective of the underlying structure. To rule out this possibility, we conducted a second experiment using a group that was trained to learn the same number of expressions, yet there was no structure or any underlying system that the expressions were part of.

If participants in Experiment 1 had simply memorised the ordered sequence of expressions during training, we would expect the current group to show similar level of performance on the symbolic comparison and global ordering task. If, however, participants in Experiment 1 inferred the structure of the systems, then we would expect the current group to perform worse on seen and especially on unseen expressions, as there would be no information to infer their ordinal meaning from trained expressions.

Participants

This experiment was also hosted on Gorilla (https://gorilla.sc/). Experimental files (Weiers et al., 2023S) and data analysis files can both be found online (Weiers et al., 2024S-b). To match the group sizes from Experiment 1, we recruited an additional 102 participants serving as a control group, which we called the random-value group. The eligibility criteria, set up and procedure were identical as in Experiment 1.

Stimuli

24 expressions were randomly selected from all expressions (excluding duplicates) that were either part of the sign- or place-value system in Experiment 1. These were then randomly ordered and split into two sets of 12 expressions. The first set containing expressions used during the training phase and the second set containing those not seen during training and only during the comparison task. There was no underlying structure to the expression sets (see Table 2).

Results

Surprisingly, a Wilcoxon signed-rank test revealed that the random-value group performed significantly above chance on both seen (M = .526, SD = 0.073; W = 3372, p < .001) and unseen expressions (M = .530, SD = 0.051; W = 4010.5, p < .001) of the symbolic comparison task. However, accuracy was only slightly above 50% and was substantially below that of the sign-value (M = .871, SD = 0.126; t = 17.814, p < .001) and the place-value group (M = .785, SD = 0.199; t = 13.311, p < .001) in Experiment 1.

A paired samples t-test on seen versus unseen expressions on both accuracy and RT revealed no significant difference in accuracy (W = 2124.5, p = .094), but a significant difference in RT (W = 1459.5, p < .001) whereby participants were significantly faster on seen (M = 1082.674, SD = 674.765) than unseen expressions (M = 1110.790, SD = 625.748) (see Figure 3).

A one-sample t-test on accuracy on the global ordering task revealed that the group was significantly above chance (W = 3798, p < .001). Yet, their mean accuracy (M = .120, SD = 0.137) shows that they placed, on average, 1.44 expressions (out of 12) in their correct positions (compared to 8.4 expressions for the sign-value and 7.76 expressions for the place-value group in Experiment 1). This suggests that this group was unable to re-create the global order of the expressions.

Discussion

These results show that this group performed only just above chance, and similarly on both the seen and unseen expressions. These results seem to show that participants could make some inferences, for example based on the ordinal associations between the expressions, in order for them to be above chance.

In comparison to both groups in Experiment 1, the random-value group performed much worse on the symbolic comparison task than both the sign- and place-value groups overall, as well as on both seen and unseen expressions. Furthermore, the random-value group performed much worse on the global ordering task than the sign- and place-value groups. This indicates an advantage of learning expressions that are part of a structured system compared to just learning an ordered sequence of expressions.

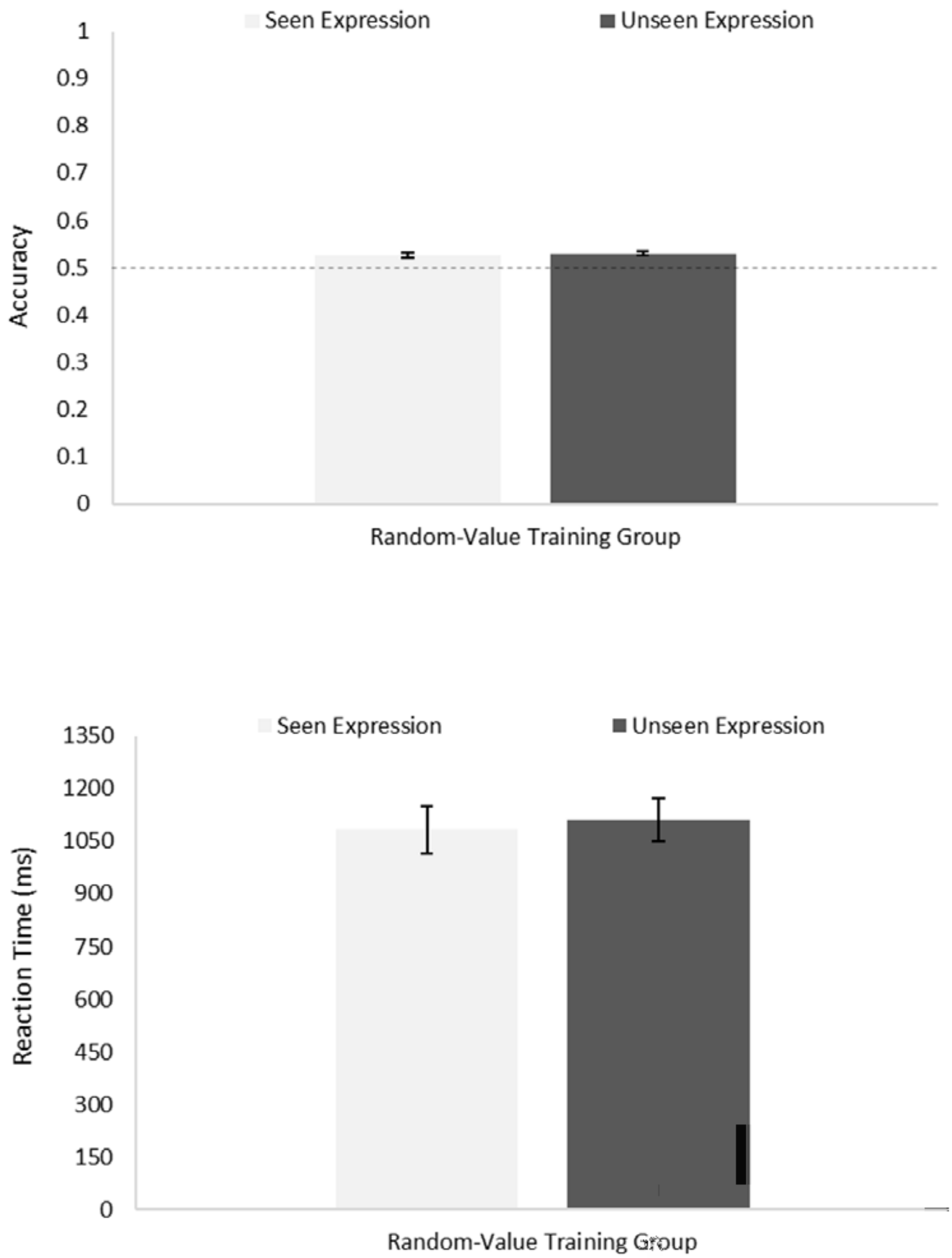
Table 2

Sets of Expressions for the Random-Value Training Group

	Random-value systems		
Expression	a)	b)	c)
1	⊨人⊨	⋔⊨⋔	人⋔人
2	⋔人	人⊨	⊨⋔
3	⊨人	⋔⊨	人⋔
4	人人⋔⋔	⊨⊨人人	⋔⋔⊨⊨
5	⊨	⋔	人
6	人人⊨	⊨⊨⋔	⋔⋔人
7	人人⋔	⊨⊨人	⋔⋔⊨
8	⋔⋔	人人	⊨⊨
9	人⊨⊨	⊨⋔⋔	⋔人人
10	⊨⋔⊨	⋔人⋔	人⊨人
11	⋔	人	⊨
12	⊨人人	⋔⊨⊨	人⋔⋔
13	⋔⊨⊨	人⋔⋔	⊨人人
14	人人	⊨⊨	⋔⋔
15	⊨⊨人	⋔⋔⊨	人人⋔
16	⊨人⋔	⋔⊨人	人⋔⊨
17	人⋔⋔	⊨人人	⋔⊨⊨
18	⋔⊨⋔	人⋔人	⊨人⊨
19	人人⋔⊨⊨	⊨⊨人⋔⋔	⋔⋔⊨人人
20	⊨⊨⋔	⋔⋔人	人人⊨
21	人⋔	⊨人	⋔⊨
22	⋔⋔⊨⊨	⊨⊨⋔⋔	⋔⋔人人
23	⊨⋔	⋔人	人⊨
24	人⋔⋔⊨	⊨人人⋔	⋔⊨⊨人

Figure 3

Accuracy Rates (Top) and Reaction Times (Bottom) for the Random-Value Training Group on the Symbolic Comparison Task, Split by Trials Containing Seen and Unseen Expressions



Note. Error bars indicate ± 1 standard error of the mean. The dashed line indicates chance-level.

General Discussion

We have shown that individuals can implicitly learn the ordinal meaning of, and make accurate numerical judgments about, expressions that are part of two different artificially created notational systems. We first consider what our results allow us to infer about participants' ability to learn the structure of a notational system and then discuss potential implications of our findings.

What Evidence Is There to Suggest Participants Learned the Structure of Each System?

Our results revealed that both the sign-value and place-value groups performed well above (and significantly above) chance on a symbolic comparison task, suggesting successful learning of the system and the individual expressions of the system. Importantly, participants were not only above chance, and with reasonably high levels of accuracy, on trained expressions, but also on those not encountered during training. This suggests participants did not learn the systems superficially, or simply memorise the order of expressions from the training, but that they must have implicitly learned the structure of the system too. To further check that participants did not simply memorise the sequence, we conducted a second experiment with a random-value group who learned just an ordered sequence of expressions which were not part of any underlying structured system. This random-value group performed considerably worse on the symbolic comparison task than the other groups on both seen and unseen expressions. Furthermore, they performed much worse on the global ordering task too, only placing 1.44 expressions on average in the correct position in the sequence. This suggests a qualitative difference between the learning of a structured system versus learning an ordered sequence of expressions without any underlying structure or system.

One key difference between the current study and previous studies is that we did not provide explicit instructions about the compositional rules of each system to participants, but instead investigated whether participants can implicitly learn the structure of different notational systems. This is similar to studies that have used artificial grammar learning (AGL) paradigms: we also created our symbols from a finite-state rule system, expressions varied in length from one to five, expressions were presented as an ordered sequence which was passively viewed, and participants were at no point told about the existence of the rules governing the formation of expressions. Our test phase was also similar in that participants completed a simple comparison task which assessed their ability to judge which of the stimuli represented the numerically larger one, based on the structure and rules of the system. Importantly, our results are in line with AGL studies as participants were able to make such comparisons at rates significantly above chance, and not only for those expressions that were learned during the training phase, but also those never encountered before, but which also followed the rules of the system. This suggests that participants were able to implicitly learn the underlying, and to them unknown, structure of both a novel sign-value and place-value system, but also that these results are generalisable to other rule-governed (and therefore most notational number) systems as well as other, similar, simple processing tasks. Taken together, these results indicate that participants in the sign-value and place-value groups did not simply memorise the ordered sequence of expressions, but rather inferred the structure of the system, as we would have otherwise expected similar performances of all groups, including the random-value group, on all tasks.

Why Might a Sign-Value System Be Learned More Easily?

In Experiment 1 we found a relative advantage of the sign-value group over the place-value group, suggesting that it may be easier for individuals to learn the expressions and infer the structure of a sign-value system. We ruled out the possibility that superficial strategies and confounds, such as making judgements based on the length of the expressions, the leftmost symbols of the to-be compared expressions, or the total number of symbols on screen, could explain the group differences we found on the symbolic comparison task. We also do not have any reason to believe that participants' previous knowledge about different notational systems, their cultural background or languages spoken, influenced their ability to learn our artificially created sign-value or place-value system, as we recruited a large, heterogenous sample with no restrictions with respect to these factors. Furthermore, we designed the two training conditions to be as similar as possible using novel symbols different to the usual notations (Arabic digits) and using a likely unfamiliar base factor. Additionally, both training groups learned the same number of new symbols. It therefore appears most likely that the relative advantage of the sign-value group was due to the structure of that system, and not due to participant factors or other confounding factors associated with the system.

Other properties of each system could potentially also explain why the structure of one system may be learned more easily than of another. Previous research has assessed some of the (dis)advantages of different numerical notations, especially comparing place-value and sign-value systems, though most do not provide direct empirical learning data to support their claims and conclusions (Chrisomalis, 2020). As there is no direct evidence of how Roman numerals were actually used at the time, most research evaluating the merits of different systems, has therefore focussed on various structural properties of each system, such as sign count (i.e., the number of individual symbols needed to form any expression of the system), extent (i.e., the highest possible expression expressible without needing a new symbol) and conciseness (i.e., the length of the expression in a given system) (Bender et al., 2015; Chrisomalis, 2010, 2020; but see Schlimm & Neth, 2008; and Lengnink & Schlimm, 2010).

In our work, sign count was kept constant as all participants learned the same number of individual symbols and were only tested on expressions containing those symbols. Extent was not directly applicable as we used a finite set of expressions that participants were tested on (though, given the base-3 structure, new symbols would be needed in order to represent expressions representing higher numerical values). In terms of conciseness, it is difficult to compare these systems as such a comparison should control for factors such as "the relative frequency of particular numerical expressions in each written tradition" (Chrisomalis, 2020, p. 60). Given our systems are artificially created this is not possible, however the average length of the expressions in the sign-value notation (2.92 symbols) was slightly higher than in place-value notation (2.38 symbols). Chrisomalis (2020) highlighted another factor, called size ordering, that should be considered when comparing notational systems, which has however not received much attention in the previous literature. Size ordering refers to comparing the length of an expression with its numerical size. Place-value notations are well size-ordered compared to sign-value systems, as three-symbol expressions will always represent higher numerical values than two-symbol expressions, whereas this is not the case in sign-value notation. Furthermore, one could also use the leftmost symbol to judge the numerical size if two expressions have the same length in a place-value system. One would thus expect a relative advantage of the place-value system over the sign-value system. We evaluated both these properties in the current study and found that these could not explain the group differences we observed.

Generally, sign-value systems are considered less concise than place-value systems (Chrisomalis, 2010, 2020), yet if a more concise system would be more advantageous, in our case easier to learn and use, then we would expect to see a relative advantage of the place-value system over the sign-value system, however this was not the case. The relative advantage of the sign-value system therefore suggests that participants can pick out such a structure more readily, which in turn suggests that sign-value systems may in fact be simpler and easier to learn than place-value system. Although not tested here, it also seems plausible that this would be the case even more so if explicit instruction had been provided during the training phase.

What Does Our Study Add?

Unlike some earlier studies, we did not investigate the computational efficiency or how well each notation can be used to perform more complex numerical processing tasks, such as arithmetic. Instead, we were interested in more simple processes and whether individuals can implicitly learn the structure of a sign-value and place-value system, and understand expressions not encountered during training, and whether there is a performance difference depending on which system was learned, in other words whether one system is learned and used more easily than the other. Our results indicate that i) participants can infer enough of the structures of different artificial symbol systems without explicit instructions about the compositional rules of the systems to ii) generalise the rules of the systems and apply these to expressions not seen during training and to iii) complete a simple numerical comparison task using the newly learned expressions, as well as other expressions, of the system, and that iv) a sign-value system might be easier to learn and use in those simple tasks compared to a place-value system.

As far as we know, only one previous study (Krajcsi & Szabó, 2012) has directly compared an artificial sign-value and place-value system. While they mostly focussed on how numbers are represented and used in more complex numerical computations such as addition, we focussed on participants' ability to first learn the expressions and infer the structure of an artificial sign-value and place-value system and then on participants' performance on simple numerical comparison and ordering tasks using the newly learned systems. Furthermore, there were some important methodological differences. First, our participants were randomly assigned to either the place-value or sign-value condition, whereas participants of Krajcsi and Szabó learned and completed the test tasks in both notations. We used a base-3 system without the digit zero (of the form discussed by Schlimm (2012); see also Boute, 2000; Forslund, 1995), whereas Krajcsi and Szabó used base-4 in most of their experiments (base-3 in Experiment 4) and included the digit zero[2]. Nevertheless, given that we only used a simple comparison task and not a more complex addition task like Krajcsi and Szabó, it is unlikely that using a zeroless system made our artificial place-value system more difficult for participants than if an artificial place-value system with zero had been used. Furthermore, how participants were trained to learn the systems and their expressions differed. Krajcsi and Szabó's training phase was split into two stages. In the first stage participants learned the new symbols by associating these with their equivalent Arabic digit, i.e., participants learned that Ł means 0, Ө means 1, Ð means 2, И means 3, Я means 4, and Ч means 16. Feedback was given. In the second stage, the new notation was introduced. That is, participants were explicitly told how the notational system works and that base-4 was used. Participants completed some practice trials and after each incorrect response, the system and how the numbers should be interpreted were explained, ensuring participants understood the system. In contrast, in our training phase participants were only presented with the ascending order of the first twelve expressions of the system. They were not given any quantity information about any of the expressions, and expressions were not paired with their Arabic numeral equivalent. The systems to which the expressions belonged were not explained at any point and participants did not know that base-3 was used. We also did not provide any feedback and participants were not given any practice trials. While we do not expect that all these differences would lead to substantial changes in the results (for example whether a base-3 or base-4 is used), other differences, such as giving explicit instructions and feedback versus implicit learning, may potentially lead to changes in results, although as we and previous literature have shown, implicit learning is not uncommon and possible without problems.

Despite these methodological differences, our results are consistent with those found by Krajcsi and Szabó (2012), as in both studies participants were better able to compare artificial number symbols in sign-value notation compared to place-value notation. This contrasts with the longstanding view that place-value systems are generally superior to sign-value systems.

Our study is the first to investigate whether participants can generalise ordinal meaning from novel artificial expressions learned during a short training phase to new expressions which are part of the same notational system, but have not been previously encountered. Although both the sign-value and the place-value training group in our study were able to extrapolate the ordinal meaning from the initial sequence of expressions of the system to expressions occurring later in the sequence, the sign-value group was significantly better at this. This further adds to our evidence suggesting that sign-value systems may be learned more easily compared to place-value systems.

In line with Krajcsi and Szabó (2012), we also suggest that the relative advantage of our sign-value system is not because of differential influences of participants' previous knowledge of other numerical systems. Additionally, although there are differences between the two systems in terms of the composition of the systems' expressions, we demonstrated that different strategies which might be more advantageous to one group than the other could not explain the group differences and thus the superior performance of the sign-value group.

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2) The familiar decimal system contains zero to represent an empty space, a placeholder representing the absence of a number. For example, in 203, the zero indicates that the tens position is empty, and that therefore there are no tens in this number. Many have argued that the availability of zero is crucial for positional systems, however this need not be the case. The idea of not needing zero is not new and dates back as early as the 1940s, with various researchers proposing decimal systems without zero (e.g., Foster, 1947; Forslund, 1995, Boute, 2000; Schlimm, 2012). Although at first, a system without zero may seem more difficult to the reader, this is likely to be because of unfamiliarity. In fact, a system without zero can be used just as well for any arithmetic computations as the standard decimal system. All numbers from 1 to 9 can be retained and used in the same way, the only change is adding a symbol to represent the digit ten. A system without zero therefore does not use the digits from 0 to b-1, but instead from 1 to b (where b is the base of the system). For example, the symbols of a decimal system without zero could be 1, 2, 3, 4, 5, 6, 7, 8, 9, and H (whereby H now represents 10). As in the standard decimal system, after H a new position is added with the lowest symbol (i.e., 1), so the number that follows would be 11 (as in the standard decimal system). This is then followed by 12, 13, 14 etc. up to 19, which is then followed by 1H (1 ten and H ones, the equivalent of 20 in traditional notation) and then 21, 22, 23 and so on. The number 100 would be represented by 9H (9 tens and H ones) and is followed be H1, H2, H3 and so on. Thus, only numbers containing zero in our standard decimal system are represented differently in a system without zero, and all other numbers remain the same and have the same numerical values (see also Foster, 1947; Forslund, 1995; Boute, 2000; and Schlimm, 2012 for more in-depth descriptions of zeroless systems). To the best of our knowledge, only one model, the natural multi-power number representation by Krajcsi and Szabó (2012), accounts for why place-value systems may be harder to understand than sign-value systems. This model can represent "numbers as specific number of objects, in which some type of objects may represent items, while other types of objects can represent groups or higher powers" (Krajcsi & Szabó, 2012, p. 13) and a sign-value system structure is closer to the proposed than a place-value structure. While a place-value system without zero may seem to be more difficult than the regular decimal system, especially for more advanced computations than simple comparisons, and less systematic, "the internal logic is just as systematic as in our system" (Schlimm, 2012, p. 3) and it may simply be less familiar. Given that in the current study only a simple comparison task was used, we do not believe that using a zeroless system made it any more difficult for participants in the place-value condition than if we had not used this.

In sum, our results add to the body of literature investigating implicit learning, but also more specifically to those investigating differences between different notational number systems. Our results highlight the effects of notation on the learning of artificial number systems and on simple number processing. Given that we used novel artificial symbols which were part of structured notational systems, we believe that these results also generalise to other, similar notational systems (e.g., a place-value system containing zero, a place-value or sign-value system with a different base, a sign-value using the subtraction form, a different form of sign-value system (like the Mayan system)), or even a completely novel, structured notational system (but not an unstructured system). Additionally, our results also revealed a relative advantage for learning a sign-value system over a place-value system, which fits with the historical view of the popularity of sign-value systems, even when place-value systems were available too (Chrisomalis, 2010, 2020; Ifrah, 2000).

There are various possibilities for future research that could lead on from the current study. We only employed an ordinal training condition, thus we cannot draw any conclusions about adults' ability to learn different types of artificial symbol systems if different types of information were to be provided about the expressions (e.g., magnitude information). Future studies could employ different training regimes, such as a magnitude or ordinal with magnitude training regime, to investigate possible differences in the learning of artificial symbol systems.

Furthermore, since all participants in the current study received the full ordinal sequence of the first 12 expressions of the assigned symbol system, it is perhaps somewhat unsurprising that participants were successful when asked to re-create the overall order of these 12 expressions. Although the current study tested if participants could compare expressions that came later in the sequence and which they had not encounter during the training phase, to further test their understanding of the structure of the symbol system, it would be beneficial to also test their ability to order expressions not encountered previously and to generate novel expressions occurring later in the sequence.

In a similar vein, future research could investigate whether either of these artificial symbol systems could be used for arithmetic computations and whether one type of system might be easier for this, given that previous research has suggested that place-value systems may be more efficient for mathematical computations.

Overall, this study demonstrated that the ordinal meaning of novel artificial number symbols that are part of either a sign-value or place-value system can be learned with relatively little exposure. Participants in the current study were not only able to learn the structure of both systems, but also learned the ordinal meaning of, and relations between, the individual expressions that are part of these systems. Furthermore, a sign-value system might be easier to learn and use in simple numerical tasks than a place-value system as participants in the current study were better able to infer the ordinal meaning of expressions that are part of a sign-value system as opposed to those that are part of a place-value system.

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Competing Interests: The authors declare that there is no conflict of interest.

Ethics Statement: The reported empirical work has been carried out in accordance with the University's ethical principles and standards and was approved by the Loughborough University Ethics Approvals (Human Participants) Sub-Committee. All participants gave explicit consent before taking part.

Data Availability: All data associated with Experiment 1 and Experiment 2 of this manuscript are publicly available (see Weiers et al., 2024S-a, 2024S-b).

Supplementary Materials

The Supplementary Materials contain the following items:

- The preregistration for the study research questions, sample size, exclusion criteria and analysis plan (Weiers et al., 2021S)
- All research data associated with the experiments of the study:

- Experiment 1 (Weiers et al., 2024S-a)
- Experiment 2 (Weiers et al., 2024S-b)

- Additional materials:
  - Experimental files for Experiment 1 and 2 (Weiers et al., 2023S)
  - Example video recordings of one block of the training phase (Weiers et al., 2024S-c)

Index of Supplementary Materials

Weiers, H., Gilmore, C., & Inglis, M. (2021S). *Learning artificial symbol systems with ordinal training (ID #76122)* [Preregistration]. AsPredicted. https://aspredicted.org/y48c-35hk.pdf

Weiers, H., Gilmore, C., & Inglis, M. (2023S). *Artificial symbol systems comparison* [Experimental files]. GORILLA. https://app.gorilla.sc/openmaterials/712859

Weiers, H., Gilmore, C., & Inglis, M. (2024S-a). *Dataset 1 for Learning number notations – Comparison of a sign-value and place-value system* [Research data]. Loughborough University Research Repository. https://doi.org/10.17028/rd.lboro.24624954

Weiers, H., Gilmore, C., & Inglis, M. (2024S-b). *Dataset 2 for Learning number notations – Comparison of a sign-value and place-value system* [Research data]. Loughborough University Research Repository. https://doi.org/10.17028/rd.lboro.24624963

Weiers, H., Gilmore, C., & Inglis, M. (2024S-c). *Training phase videos for Learning number notations – Comparison of a sign-value and place-value system* [Example video recordings]. Loughborough University Research Repository. https://doi.org/10.17028/rd.lboro.25795636

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