REDIMAT
Journal of Research in Mathematics Education

Hipatia Press
www.hipatiapress.com

Journal of Research in Mathematics Education
Volume 14, Issue 1, 24th February, 2025, Pages 1–29

http://dx.doi.org/10.17583/redimat.15471

Development of Didactic Analysis Competence in Prospective Mathematics Teachers

Bethzabe Cotrado[1], María Burgos[2], Pablo Beltrán-Pellicer[3], & Alfredo Carlos Castro[1]

1) *Universidad Nacional del Altiplano,* Peru
2) *Universidad de Granada,* Spain
3) *Universidad de Zaragoza,* Spain

Abstract

In this article, we describe the implementation and results of a formative experience with prospective mathematics teachers, focused on developing the competence for didactic-mathematical analysis of curriculum materials, specifically student workbooks related to probability. The research design follows a methodological approach typical of design-based research, utilizing content analysis to examine participants' responses. The study was conducted with 16 Peruvian students preparing to become mathematics teachers at the National University of the Altiplano. The responses from the prospective teachers revealed deficiencies in their common content knowledge. They also encountered difficulties in distinguishing mathematical practices and recognizing the mathematical objects involved in the study process, especially propositions and their respective arguments. Furthermore, they struggled to differentiate between intuitive, classical, and frequentist meanings of probability. To improve these outcomes, it is necessary to reinforce didactic-mathematical knowledge regarding probability.

Keywords

Teacher education, curriculum materials, probability, ontosemiotic analysis

To cite this article: Cotrado, B., Burgos, M, Beltrán-Pellicer, P., & Castro, A. C. (2025). Development of didactic analysis competence in prospective mathematics teachers. *Journal of Research in Mathematics Education, 14*(1), pp. 1-29. http://dx.doi.org/10.17583/redimat.15471
Corresponding author(s): Bethzabe Cotrado
Contact address: bcotrado@unap.edu.pe

REDIMAT Journal of Research in Mathematics Education

Hipatia Press www.hipatiapress.com

Journal of Research in Mathematics Education
Volumen 14, Número 1, 24 de febrero de 2025, Páginas 1–29

http://dx.doi.org/10.17583/redimat.15471

Desarrollo de la Competencia de Análisis Didáctico en Profesorado de Matemáticas en Formación Inicial

Bethzabe Cotrado[1], María Burgos[2], Pablo Beltrán-Pellicer[3], y Alfredo Carlos Castro[1]

1) *Universidad Nacional del Altiplano,* Perú
2) *Universidad de Granada,* España
3) *Universidad de Zaragoza,* España

Resumen

En este artículo describimos la implementación y los resultados de una experiencia formativa con futuros profesores de matemáticas, centrada en el desarrollo de la competencia de análisis didáctico-matemático de materiales curriculares, específicamente cuadernos de trabajo del estudiante sobre probabilidad. El diseño de la investigación sigue un enfoque metodológico típico de la investigación basada en el diseño, utilizando el análisis de contenido para examinar las respuestas de los participantes. El estudio se llevó a cabo con 16 estudiantes peruanos que se preparan para ser profesores de Matemáticas en la Universidad Nacional del Altiplano. Las respuestas de los futuros profesores revelaron deficiencias en su conocimiento común del contenido. También encontraron dificultades para distinguir las prácticas matemáticas y reconocer los objetos matemáticos involucrados en el proceso de estudio, especialmente las proposiciones y sus respectivos argumentos. Además, tuvieron problemas para diferenciar entre los significados intuitivo, clásico y frecuencial de la probabilidad. Para mejorar estos resultados, se concluye que es necesario reforzar el conocimiento didáctico-matemático con respecto a la probabilidad.

Palabras clave

Formación de profesores, materiales curriculares, probabilidad, enfoque ontosemiótico

Cómo citar este artículo: Cotrado, B., Burgos, M., Beltrán-Pellicer, P. y Castro, A. C. (2025). Desarrollo de la Competencia de Análisis Didáctico en Profesorado de Matemáticas en Formación Inicial. *Journal of Research in Mathematics Education, 14*(1), pp. 1-29. http://dx.doi.org/10.17583/redimat.15471

Correspondencia Autores(s): Bethzabe Cotrado
Dirección de contacto: bcotrado@unap.edu.pe

The work of a mathematics teacher is a complex activity that requires mastery of various types of knowledge and competencies (Chapman, 2014). They must possess mathematical knowledge that allows them to solve the problems outlined in the curriculum at the educational level where they teach. Additionally, they need specialized knowledge about the subject matter, the transformations required for the teaching and learning processes, as well as psychological and pedagogical factors, among others, which influence these processes.

When implementing a specific instructional process, a teacher should be able to interpret the information in the curriculum materials based on their ability to assist students in achieving the learning objectives set forth in the study programs and curriculum guidelines (Thompson, 2014). Textbooks, teacher manuals, student workbooks, and other resources, serve as tools that support teachers in making educational decisions, acting as sources of learning and a means of interaction with students (Pepin & Gueudet, 2018; Remillard & Kim, 2020). Therefore, research in mathematics education emphasizes the need for the analysis of such resources to be one of the competencies included in teacher training (Burgos et al., 2020; Lloyd & Behm, 2005; Yang & Liu, 2019). These actions in teacher training are significant for the teachers' own learning because the analysis of materials requires a deep dive into the characteristics related to mathematical objects, their representations, and relationships, as well as knowledge about the teaching and learning of mathematics, prompting teachers to question some of their own beliefs (Lloyd & Behm, 2005).

In response to this demand, prior research (Breda et al., 2017; Font et al., 2010; Giacomone et al., 2018; Pino-Fan et al., 2023; Pino-Fan et al., 2018; Pochulu et al., 2016) suggests the use of tools from the Ontosemiotic Approach (OSA) to mathematical knowledge and instruction (Godino et al., 2007) to develop in teachers the specific competence of didactic analysis of instructional processes. This competence involves, among other aspects, the teacher's ability to describe and explain the mathematical practices at play when solving problems and studying the intended mathematical content (Burgos et al., 2019; Burgos & Godino, 2021; Giacomone et al., 2018; Godino et al., 2017). Recognizing the mathematical practices carried out when solving tasks in curriculum materials, as well as the mathematical objects and processes involved, allows the teacher to become aware of potential learning conflicts, assess them, and make the necessary adjustments to address their limitations (Yang & Liu, 2019).

Much of the literature on curriculum materials has focused on textbooks, neglecting other components such as student workbooks. The importance of student workbooks lies in that they constitute tools: a) for practice, providing students with structured and sequenced tasks or activities that challenge and expand their conceptual understanding of the content and skills developed in class or in textbooks; b) for assessment, as they include tasks covering a variety of content with different levels of complexity using answers, solutions, or rubrics, and c) for monitoring, allowing the determination of how much curriculum content is covered and at what levels of cognitive demand (Hoadley & Galant, 2016). In addition to these characteristics, in contexts such as Peruvian secondary education, which is the context of this research, administrations provide students with workbooks updated according to current curriculum regulations, while textbooks have not yet been revised, so workbooks allow for the implementation of curriculum regulations in teaching practice. However, we have not found in previous literature any research aimed at developing prospective teachers' competence in the didactic analysis of student workbooks, which motivates the study presented in this paper. Workbooks differ from textbooks in that the latter include "explanatory" configurations (that is, they introduce definitions, procedures, etc.,) that do not appear in workbooks. Tasks in textbooks are, to some extent, determined by these prior configurations, as they aim to reinforce the knowledge introduced earlier. In this study, the distributed workbooks are not accompanied by a textbook or a teacher's guide, so it is expected that the teacher understands and interprets the mathematical elements required in the tasks included. In this sense, although the proposed tasks may be similar to those found in other textbooks, the material analysed here involves greater complexity. Since the mathematical content included in these workbooks is very extensive, so it is relevant to focus on a specific content, in our case, probability.

The limited existing studies on the treatment of probability content in the curriculum and textbooks highlight deficiencies. For instance, they reveal a bias towards scenarios involving games of chance, insufficiently representative and balanced situations, a lack of experimental and simulation-based scenarios utilizing manipulatives or software, among other issues. These deficiencies hinder the development of adequate probabilistic literacy (Cotrado et al., 2022; Vásquez & Alsina, 2015).

The objective of this research is to describe and analyse the results of implementing a formative experience with prospective Peruvian mathematics teachers aimed at developing their competence in didactic-mathematical analysis of curricular materials in probability. Specifically, we focus on identifying practices, objects, and meanings of probability considered in student workbooks. We consider this type of analysis essential for trainee teachers to recognize deficiencies in the material that may require action decisions on their part to ensure adequate teaching of probability.

Theoretical Framework

The research is based on the model of Didactic-Mathematical Knowledge and Competencies (the DMKC model) for mathematics teachers developed by Godino et al. (2017). This model is supported by the system of theoretical tools established in the OSA framework (Godino, 2024). Below, we describe the key tools that will be fundamental in our research.

Pragmatic Meaning and Ontosemiotic Configuration

From the perspective of the OSA (Godino, 2024), the systematic analysis of instructional processes, whether planned, anticipated, or implemented, requires an understanding of the mathematical practices involved in the content and the identification of the network of objects and processes that these practices mobilize. These analyses are built upon the notions of pragmatic meaning and ontosemiotic configuration. These tools have been employed in teacher education, utilizing various strategies and within different mathematical contexts (Burgos et al., 2020; Burgos & Godino, 2021; Burgos et al., 2018; Giacomone et al., 2018).

In the OSA, the notion of *mathematical practice* serves as the starting point for the analysis of mathematical activity (Font et al., 2013). Mathematical practices, or systems of mathematical practices, involve various types of *mathematical objects* as actions carried out by a subject to solve problems, communicate and/or generalize their solutions. Mathematical objects that are involved in and emerge from systems of mathematical practices are interrelated, forming ontosemiotic configurations of practices, objects, and processes. In the OSA, a *mathematical process* is any sequence of actions carried out over a certain period to achieve an objective, typically the solution of problem situations or the communication of their solutions. Mathematical objects such as languages, problems, concept-definitions, propositions, procedures, and arguments emerge from mathematical practices through their respective mathematical processes of communication, problematization, definition, enunciation, algorithmization, and argumentation. Other processes like modeling or problem-solving can be understood as mega-processes, as they involve one or more of the aforementioned processes. Practices and processes have many aspects in common (concatenation, time, etc.), which is why they are sometimes confused. However, they have enough differences to not be identified as the same. In the OSA, a distinction is made between practice, procedure, and process. Being a mathematical object means participating in some way in mathematical practice. Thus, at first glance, anything that can be 'individualized' in mathematics will be considered an object. The analysis of mathematical activity reveals a first type of objects that intervene in mathematical practices problems, concept-definitions, propositions, etc.—which we will refer to here as primary objects. Primary objects are related to each other and form configurations, which can be defined as networks of objects that participate in systems of practices and emerge from them.

Font et al. (2013) develop an ontology of mathematical objects, their different types, the configurations they form, their ways of being in mathematical practices, their forms of existence, etc. Based on these considerations, Font et al. (2013) explain how mathematical practices can produce a referent that, implicitly or explicitly, is considered a mathematical object, and that appears to be independent of the language used to describe it (called a secondary object, which in this article would be probability). In other words, this object would be the content to which, explicitly or not, the pair—mathematical practices and the configuration of primary objects that activate them—globally refers. Put another way, a configuration of primary objects is considered as definitions, representations, properties of a secondary object that is independent of these primary objects. In turn, a secondary object can only be made operational through the use of a configuration of primary objects. Given this symbiosis between primary and secondary objects, we will use the term 'object' and will only distinguish between primary and secondary when necessary.

Since a mathematical object is conceived as emerging from the practices performed by an institution (person) associated with a field of problems in which the object is involved, the institution (person) meaning of the object is determined by the institution (person) practices associated with the field of problems from which the object arises in a given moment and context (Godino et al., 2007). In particular, we will mean the *reference meaning* as the system of practices used as a reference to elaborate the intended meaning. In a specific educational institution, this reference meaning will be part of the holistic meaning of the mathematical object. Determining this overall meaning requires conducting a historical-epistemological study on the origin and evolution of the object in question, as well as considering the diversity of contexts in which this object is used (Godino et al., 2007, p.3).

Reference Meanings of Probability

The OSA considers that mathematical objects emerge from practices, which entails their complexity (Font et al., 2013; Rondero & Font, 2015), understood as a multiplicity of meanings. From this principle, it follows that this complexity must be considered, as much as possible, in the design and redesign of didactic sequences. For this reason, various studies have been conducted using OSA as theoretical framework to deepen the understanding of the multiplicity of meanings of different mathematical objects and to explore students' and teachers' comprehension of this complexity (Burgos et al., 2018, 2020; Calle et al., 2021, 2023).

The reference meanings of probability considered in current secondary education curriculum programs are intuitive, subjective, frequentist, classical, and axiomatic (Batanero et al., 2016; Beltrán-Pellicer et al., 2018). Each of these meanings has specific differences, not only in the definition of probability itself but also in the objects and processes involved in the practices used to solve or model various specific real-world problems or phenomena (Batanero et al., 2016). Intuitive meaning corresponds to the intuitive ideas that children may have about uncertainty and the everyday use of terms stemming from experiences and contexts related to random phenomena. Subjective meaning develops the concept of probability as a degree of belief based on personal judgment that can be revised based on an individual's knowledge and experience. The first mathematical definition of probability is associated with the classical meaning. It considers probability as the ratio of the number of favourable outcomes to the total number of possible outcomes, provided that all elementary events are equally likely. This definition is valid only for sample spaces with a finite number of equiprobable elementary events and gave rise to Laplace's rule and the calculation of probability in situations involving games of chance, where combinatorial reasoning is often applied (Batanero et al., 2016). In the frequentist meaning, probability is defined as the hypothetical value toward which the relative frequency of an event stabilizes when the experiment is repeated a large number of times. Compared to the classical approach, it has the disadvantage that the true value of probability is never actually calculated, i.e., it is only estimated through relative frequency, which can lead to confusion with probability (Batanero et al., 2016). However, it has the advantage of being applicable to experiments with non-equally likely events. Finally, the axiomatic theory addresses the problem of organizing and structuring the other partial meanings of probability and allows for the development of all known results at the time regarding probability calculations. While some textbooks, typically at the end of secondary education, incorporate Kolmogorov's axioms, the axiomatic meaning is too formal and is generally recommended for university-level study (Batanero et al., 2016).

Each meaning of probability entails different systems of practices and mathematical objects that must be considered together and integrated into the processes of teaching and learning probability. In works by Batanero (2005) and Gómez (2014), for every meaning, the field of problems from which it arises is described, as well as the concepts, languages, properties, procedures, and arguments involved in the mathematical practices that address them. Specifically, the analysis of the Peruvian curriculum program conducted by Cotrado et al.

(2022) reveals a greater representation of mathematical objects (concepts, procedures, propositions) related to the classical and frequentist meanings of probability, to the detriment of the intuitive meaning. The problem-situations considered lead to recognizing the conditions defining a random situation, expressing the value (decimal or percentages) of probability as more or less likely, determining the sample space, calculating the probability of events using Laplace's rule or by calculating their relative frequency, interpreting information from texts involving probabilistic situations, or drawing conclusions about the probability of event occurrences. Each of these meanings allows for the resolution of different types of tasks; therefore, if the goal is for students to become competent in solving a variety of problems, particularly extra-mathematical ones where the mathematical object of probability plays a determining role, it is necessary for students to have a well-connected network of partial meanings.

Teacher Knowledge and Competencies Model in the OSA

The DMKC model (Godino et al., 2017) developed within the OSA framework can serve as a basis for guiding the training of mathematics teachers. This model acknowledges that a teacher should have a *common mathematical knowledge* related to the educational level where they teach and *expanded knowledge* that enables them to connect it to higher levels. Furthermore, as mathematical content is brought into play, the teacher must possess *didactic-mathematical knowledge* of the various facets involved in the educational process. In particular, teachers should have the necessary knowledge to recognize, on the one hand, the different meanings (understood as systems of practices) of the corresponding content and their interconnection, and on the other hand, the diversity of objects and processes involved (ontosemiotic configuration) for the different meanings. Additionally, teachers should be able to use this knowledge competently in the processes of didactic design.

In the DMKC model, it is considered that the two key competencies of a mathematics teacher are mathematical competence and the *competence of didactic analysis and intervention*, which essentially involves "designing, applying, and evaluating learning sequences for oneself and others, using techniques of didactic analysis and quality criteria, to establish cycles of planning, implementation, evaluation, and propose improvements" (Breda et al., 2017, p. 1897). This overall competence of didactic analysis and intervention of the mathematics teacher is articulated through five sub-competencies, associated with the OSA conceptual and methodological tools: competence in the analysis of global meanings, competence in ontosemiotic analysis of practices, competence in the management of didactic configurations and trajectories, competence in normative analysis, and competence in the analysis of didactic suitability (Godino et al., 2017). In your work, the focus is on the sub-competencies of the analysis of global meanings and ontosemiotic analysis.

In the *competence of global meaning analysis*, the focus is on "identifying problem situations that provide partial meanings or senses to the mathematical objects or topics under study, and the operational and discursive practices that must be employed in their resolution" (Godino et al., 2017, p. 99). This competence allows teachers to answer questions such as: What are the meanings of the mathematical objects involved in the study of the intended content? How are they interconnected? On the other hand, the *competence of ontosemiotic analysis of mathematical practices* enables teachers to identify the objects and processes involved in the mathematical practices necessary for solving problem situations. This recognition facilitates "anticipating potential and effective learning conflicts, evaluating students' mathematical competencies, and identifying objects (concepts, propositions, procedures, arguments) that need to be remembered and institutionalized at the appropriate moments in the study processes" (Godino et al., 2017, p. 94).

In line with the work of Pino-Fan et al. (2023), we consider the following levels of development for the competence in didactic-mathematical analysis (onto-semiotic analysis of practices and global meanings):

L0 The teacher identifies some evident mathematical elements: languages, procedures, or definitions of certain concepts used in the analysed practices, without recognizing the partial meanings involved.
L1 The teacher analyses mathematical practices, recognizing most types of mathematical objects involved, as well as partial meanings, based on their experience but without considering any theoretical-methodological analysis tool.
L2 The teacher uses theoretical tools, particularly the onto-semiotic configuration, to analyse mathematical practices and recognize the network of emerging mathematical objects but does not use them to differentiate meanings.
L3 At this level, the teacher is familiar with the onto-semiotic configuration and uses it as a tool to analyse mathematical practices. Additionally, based on this analysis and in relation to the context, the teacher identifies the different meanings involved.

Existing Research

Studies that specifically address teachers' mathematical and didactic-mathematical knowledge about probability reveal deficiencies that may hinder its effective teaching (Batanero & Álvarez-Arroyo, 2024; Vásquez & Alsina, 2015). In this section, we summarize the results of studies that have explicitly examined the competence for didactic analysis as part of their research with teachers.

Contreras (2011) examined 183 prospective primary teachers in relation to a task involving the calculation of simple, compound, and conditional probabilities, finding that although they managed to identify some mathematical objects, they did not recognize all those necessary to solve the problem. The participants had no difficulty recognizing language types, with definition-concepts being the most frequently identified mathematical objects, followed by procedures, while properties and arguments went unnoticed. Nonetheless, overall results were poor and highly variable among students.

Mohamed (2012) assessed the specialized knowledge of 31 groups of prospective teachers on fair play and sample space, revealing that only a small group was able to correctly identify key concepts such as fraction comparison, proportionality, and randomness. Concepts such as expected value and combinatorial reasoning were not identified, highlighting deficiencies in their understanding of these topics.

Vásquez and Alsina (2015) applied a questionnaire to assess the didactic-mathematical knowledge of 93 in-service Chilean teachers for teaching probability. Results showed a limited performance level across different knowledge types involved, particularly regarding the notion of a sure event, the calculation and comparison of probabilities of elementary events, and understanding event independence. Significant difficulties were found in identifying concepts or properties involved in situations requiring probability calculation and comparison or sample space analysis.

On the other hand, studies by Batanero et al. (2021), Gómez (2014), and Parraguez et al. (2017) focus on understanding different meanings of probability. Gómez (2014) analysed the probability knowledge of 81 prospective primary education teachers, noting that while they have an adequate understanding of the classical meaning of probability and can recognize fundamental concepts such as probability, sample space, and fair play, they face limitations with the frequentist meaning, struggle with understanding variability in small samples, and exhibit equiprobability bias. Results from Parraguez et al. (2017), with a group of 60 prospective primary education teachers, and Batanero et al. (2021), with 139 secondary education prospective teachers, highlight their difficulties in linking the classical probability concept to the relative frequencies of an experiment due to challenges in understanding the notion of convergence.

In our research, we follow the line of studies by Burgos et al. (2018) and Giacomone et al. (2018), which use the onto-semiotic configuration tool to develop the competence for didactic analysis in prospective teachers for tasks involving proportionality or diagrammatic visualization and reasoning, respectively, guiding them in the identification of practices, objects, and meanings.

Methodology

The research problem is the design, implementation, and evaluation of a formative action with prospective secondary education teachers to develop their competence in didactic-mathematical analysis about probability, through the identification of practices, objects, and meanings of probability considered in student workbooks. Therefore, the methodological approach follows the typical phases of design-based research, as proposed by Godino et al. (2014) within the OSA framework: 1) preliminary study; 2) design of the didactic trajectory (selection of problems, sequencing, and a priori analysis); 3) implementation of the didactic trajectory (observation of interactions between individuals and assessment of achieved learning); 4) evaluation or retrospective analysis derived from the contrast between what was planned in the design and what was observed in the implementation. The study is essentially qualitative, as it collects and analyses information based on the actions of prospective teachers in a real classroom context. Content analysis methodology (Cohen et al., 2011) is used to examine class recording transcripts and participant response protocols[1].

Research Context, Participants, and Data Collection

The formative experience was conducted with a group of 16 students from the Secondary Education Program specializing in Mathematics, Physics, Computer Science, and Informatics at the National University of the Altiplano in Peru. This program spans 10 academic semesters (5 years). The participating prospective teachers (referred to as PT) were in their fourth semester, taking the Descriptive Statistics course through the virtual platform LAURASIA. The PTs had only studied probability in secondary education and had not covered probability topics during their university training. The course planning included synchronous sessions via Google Meet and asynchronous activities to provide study materials and assignments (Classroom). During the workshop's implementation, which consisted of three virtual synchronous sessions, each lasting two hours, all 16 participants were present in the first session, while 13 participants attended the second and third sessions (three students left the course). The instructor responsible for managing the workshop also serves as a researcher.

Data collection instruments include recordings of the class sessions through Google Meet, the instructor's notes, and written response protocols from the participants during the synchronous and asynchronous sessions of the workshop.

Implemented Sessions and Teaching Resources

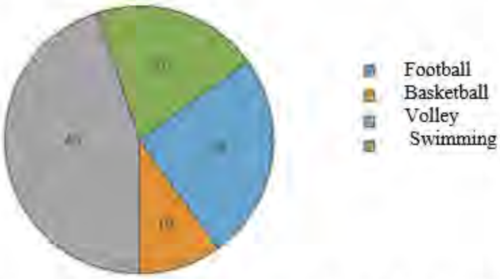
The formative workshop was organized into three virtual synchronous sessions, each lasting two hours, which included theoretical-practical training and group discussions. In these sessions, asynchronous activities were also provided to be completed by the participants as a complement to the two hours of synchronous work. These activities involved reading documents and solving tasks that are part of the participants' final assessment.

Session 1: Initial Exploration of Mathematical Practices and Objects in Probability Tasks

The session began with the resolution of three selected problems from the assessment section of worksheets 9 (initial diagnostic problems 1 and 2) and 13 (initial diagnostic problem 3) that deal with probability in the Mathematics Problem-Solving Workbook for Secondary Education. Problem 1 (Figure 1) presents a simple random experiment described through a pie chart, where the events are not equiprobable.

Figure 1
Initial Diagnostic Problem 1

The chart represents the population of 100 students from a sports academy and the disciplines they practice.



If a student is randomly selected on any given day, calculate the probability that they do not practice basketball.

Source. MINEDU, 2019a, p. 129. Authors’ translation.

The graph (Figure 1) represents the population of 100 students of a sports academy and the disciplines they practice. The PTs must calculate the probability that a student randomly chosen on any given day does not practice basketball.

The context of problem 2 involves a compound random experiment consisting of three simple experiments (flipping a coin three times): “Problem 2. A coin is tossed three times. What is the probability of getting "heads" exactly twice?” (MINEDU, 2019a, p. 129). In problem 3, a compound random experiment is presented:

> Problem 3. There are 200 workers in a company, 100 of whom are men, and the rest are women. The workers who read the magazine "La Estación" are 30 men and 35 women. If an employee is randomly selected, calculate the probability that: (a) He is male and does not read "The Station" magazine. (b) That he reads the magazine "La Estación" (MINEDU, 2019b, p. 182).

Once the participants solved these problems, the session continued with the initial exploration of the personal meanings of the PTs regarding the nature of mathematical objects and their ability to identify these objects in mathematical practices. To do this, the participants individually described and listed the practices they used to solve problem 1 and identified the concepts, symbols, graphs, or tables used, mentioning any difficulties encountered if applicable. Subsequently, they shared and presented their answers in class, comparing them with the analysis of the solution to problem 1 proposed by the instructor. Before concluding the session, the PTs were instructed to read the article by Batanero (2005) concerning pragmatic meanings of probability, and based on this, create a summary table proposing examples of problem situations associated with the different meanings of probability considered. This reading was part of the asynchronous activity, providing flexibility to participants to respond according to their availability.

Session 2: Pragmatic Meanings and Ontosemiotic Configurations in Probability

The session began with the following questions:

- What types of probability meanings does the reading propose?
- What characteristics should a problem have to relate to a specific partial meaning of probability?
- What meanings of probability do the problems you solved in the previous session correspond to?

The intention was to engage the participants in a reflection on the different meanings of probability and how they could be characterized based on the network of mathematical objects emerging in associated practices. After a detailed explanation by the instructor about the problem situations and elements that characterize the various meanings of probability, as well as their progressive inclusion in the teaching of this content in secondary education, the participants were asked to work individually to respond to the following instructions:

1. Analyse the problem-situation 2, describing its solution process.
2. Identify the mathematical objects involved and relate them to a specific meaning of probability.
3. Mention any difficulties encountered.

The written responses were shared in class. Then, the instructor presented the a priori analysis of problem 2 (MINEDU, 2019a, p. 129), using the ontosemiotic configuration tool. After breaking down the solution into elementary practices (units of analysis), the use and intentionality of each of them were identified, along with the mathematical objects involved. At the end of the session, participants were assigned an individual asynchronous task to analyse Situation A, which appears alongside its solution in worksheet 9 of the problem-solving workbook for first-degree problems (Figure 2). In this case, the ontosemiotic analysis was not conducted on their practices but on the solution proposed by the author of the curricular material, using the same type of table employed by the instructor when presenting the ontosemiotic configuration.

In addition, they were asked to link situation A to one of the meanings of probability and to specify the difficulties they encountered in relating the situation to the meanings of probability and in identifying the mathematical objects.

Figure 2

Situation A Proposed in the Testing Section of the First Grade Worksheet 9

Significant situation A

A die is rolled once. Based on this, determine whether each event is certain, impossible, or probable.

Event A: Rolling an even number.

Event B: Rolling a composite number greater than 4.

Event C: Rolling a prime number greater than 5.

Event D: Rolling a number less than 10.

Resolution

The sample space (Ω) is the set of all possible outcomes of a random experiment. Therefore, we first determine the sample space (Ω), which is all possible outcomes when rolling a die.

Ω = {1, 2, 3, 4, 5, 6}

An event is a subset of the sample space consisting of the outcomes of the experiment. So, we list the possibilities for each event:

Event A, rolling an even number: A = {2, 4, 6}

Event B, rolling a composite number greater than 4: B = { 6 }

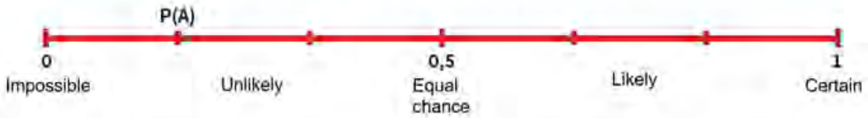
Event C, rolling a prime number greater than 5: C = { }

Event D, rolling a number less than 10: D = {1, 2, 3, 4, 5, 6}

Then, we calculate the probability of each event using the Laplace's rule:

$$P(A) = \frac{Number\ of\ favorable\ outcomes\ for\ event\ A}{Number\ of\ possible\ outcomes}$$

The results of the probabilities can also be represented on a number line:



Event A of rolling an even number is probable because: P(A) = 3/6, then P(A) = 0.5 To express the probability in percentages, we multiply by 100%. P(A) = 0.5 × 100%, then P(A) = 50% This means that there are 3 (favorable outcomes) out of 6 (possible outcomes), a 50% chance of rolling an even number when rolling a die.

Event B of rolling a composite number greater than 4 is unlikely because: P(B) = 1/6 = 0.166..., then P(B) = 0.1666... × 100%, that is, P(B) = 16.666...% This implies: rolling a composite number greater than 4, when rolling a die once, is unlikely.

Source. MINEDU, 2019a, p. 120-121. Authors' translation.

Session 3. Sharing and Proposal of Assessment Tasks

In this session, the participants presented the results of applying the ontosemiotic configuration tool to conduct a detailed analysis of the mathematical practices described in Situation A (Figure 2). Subsequently, similar to the previous sessions, the instructor shared and explained the a priori analysis of Situation A, allowing the participants to compare their responses and discussing the challenges encountered. At the end of the third session, they were asked to individually complete two assessment tasks.

Figure 3

Situation B Included in the Testing Section of Worksheet 9 of First Grade

Significant situation B

Two dice are simultaneously rolled once. Determine:

a. How many elements does the respective sample space have?

b. If we add the values of the results of both dice, which sum is more likely to occur?

c. What is the probability of obtaining that sum?

Resolution

a. We draw a two-way table and write down all possible results when rolling the two dice:

	⚀	⚁	⚂	⚃	⚄	⚅
⚀	1;1	1;2	1;3	1;4	1;5	1;6
⚁	2;1	2;2	2;3	2;4	2;5	2;6
⚂	3;1	3;2	3;3	3;4	3;5	3;6
⚃	4;1	4;2	4;3	4;4	4;5	4;6
⚄	5;1	5;2	5;3	5;4	5;5	5;6
⚅	6;1	6;2	6;3	6;4	6;5	6;6

Since there are 6 rows and 6 columns, our sample space will have: 6 × 6 = 36 possible outcomes.

b. To answer the second question, we need to know the sum of the possible values when rolling both dice. To do this, we write down the results of the sum in the two-way table.

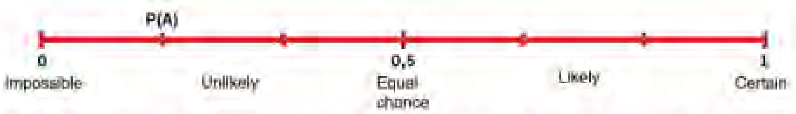
	⚀	⚁	⚂	⚃	⚄	⚅
⚀	2	3	4	5	6	7
⚁	3	4	5	6	7	8
⚂	4	5	6	7	8	9
⚃	5	6	7	8	9	10
⚄	6	7	8	9	10	11
⚅	7	8	9	10	11	12

We observe that equal sums are represented in the same colour. Therefore, the most likely sum is 7 because it is the value that repeats the most, which is found in the yellow diagonal.

c. We consider A as the event: "the sum of the values obtained when rolling the dice is 7". Finally, we determine the probability of A applying Laplace's rule:

$$P(A) = \frac{Number\ of\ favorable\ outcomes\ for\ event\ A}{Number\ of\ possible\ outcomes}$$

The probability of obtaining that sum as 7 is: P(A) = 6/36 = 1/6 = 0.1666...



It is unlikely to roll a sum of 7 when rolling two dice. However, it is the most likely compared to the other sums.

Source. MINEDU, 2019a, p. 122-123. Authors' translation.

In the first assessment task, participants were required to perform an analysis using the ontosemiotic configuration tool of the solution provided by the instructor for problem 3. In the second task, they should identify the meaning, develop the ontosemiotic configuration, and recognize possible errors or conflicts in the solution provided by the author of the workbooks for Situation B (Figure 3) concerning the simultaneous roll of two dice. To promote independent work and responsibility for submission, participants had one week to submit their analysis report through the virtual platform.

Results and Analysis

Although the formative experience is aimed at developing competence in analysing meanings and ontosemiotic analysis, it's important to consider the shortcomings in the common knowledge of probability among the participants as a potential obstacle to progress in this competence. It's also of interest in this regard to understand their initial ideas about mathematical practices and objects.

Initial Assessment of Common Content Knowledge

It is observed that of the 16 participants, two did not solve the diagnostic problem 1, and 14 provided partially correct answers since they correctly obtained the value 0.9 but did not justify how they calculated it or only made use of Laplace's rule. Furthermore, an incorrect use of Laplace's rule as a problem-solving strategy is noted. Participants did not consider the distribution of absolute frequencies and assume that the four possible outcomes of the random experiment forming the sample space are equiprobable, thus displaying an equiprobability bias (Lecoutre, 1992). This bias appeared in studies like those of Mohamed (2012), Gómez (2014), Parraguez et al. (2017), and Batanero et al. (2021).

Regarding problem 2, errors in probability calculations resulted from an incorrect enumeration of the sample space and the inappropriate use of counting schemes to calculate the number of favourable and possible cases (Gómez, 2014; Mohamed, 2012). In other cases, participants correctly identified all possible outcomes but make mistakes in counting favourable cases because they considered two different outcomes as identical, or vice versa, erroneously thinking that two distinct results are the same. This type of combinatorial error was identified by Batanero et al. (1997) as a confusion of the type of object.

None of the participants solved problem 3 correctly. The common error observed in this statement is the confusion of conditional probability with compound probability (Contreras, 2011; Estrada & Díaz, 2007). That is, the PT incorrectly restricted the possible outcomes of the random experiment, considering only all the men instead of the entire sample.

All these limitations, as also noted in previous research, suggest the need to enhance the training of PTs in probability (Batanero et al., 2021; Parraguez et al., 2017; Vásquez & Alsina, 2015). As we show below, the participants themselves highlight these shortcomings in common knowledge when asked about the difficulties encountered while solving the task.

Development of the Ontosemiotic Analysis Competence

The study of the responses provided by the PTs the analysis of meanings and ontosemiotic configuration allows us to observe the difficulties in understanding the requirement of the tasks, the achievements reached, and the possibilities offered by the situations presented in the training workshop sessions.

Initial Exploration of Mathematical Practices and Objects

During the first session, the PTs were not clear about the nature of mathematical practices and primary mathematical objects, as observed in Giacomone et al. (2018). When asked to describe or distinguish mathematical practices and identify mathematical objects (primarily focusing on concepts and languages, which we expected they would be more familiar with) in the solutions or responses given to problem 1, eight participants did not respond, three did not distinguish the sequences of practices but mentioned some mathematical objects, and five attempted to sequence mathematical practices based on their use and intentionality. In this case, they endeavoured to describe mathematical practices through their intentionality based on Polya's generic problem-solving strategies. For example, they used expressions such as "identifying the problem data," "verifying the problem," "reading the problem," "gathering data," "representing data, and calculation." Overall, in this initial session, none of the 16 PTs distinguished elementary practices in describing the mathematical activity carried out to solve the task. This demonstrates the initial difficulties of the participants in distinguishing and sequencing the units of elementary practices.

After describing the mathematical practices, the participants were required to identify some mathematical objects, such as concepts or languages. In this case, it is observed that only four of them partially and somewhat hesitantly ("The concepts will be, the ones I am using in addition or percentages, that part I don't understand," PT3) recognized the following concepts: fractions, total cases, favourable cases, percentages, probability, and population.

The identification of concepts by the PTs during the sharing phase also allowed us to recognize deficiencies in their knowledge of probability, which they often associated solely with the classical approach and Laplace's rule. For example, PT6 wrote, "uses the concept of probability 'favourable cases/possible cases'," and similarly, PT3 commented:

> Regarding percentages and probabilities, I always use it as a formula, you could say, on the top of the fraction is what I want or what the problem asks us for, and at the bottom is the total, in this case, on top, it asks for those who do not play basketball.

Additionally, participants were asked to identify types of linguistic representations used in mathematical practices. Most of them indicated "none" or left the response blank. Only one PT recognized the pie chart ("none except for the pie chart that was given, PT4"), and another identified procedural elements as languages ("sum of data, division of total data," PT7). The initial difficulty of the participants in recognizing concepts and languages used in probability mathematical practices would be associated with their lack of knowledge of these mathematical objects.

Finally, participants were required to mention the complications they have encountered during problem-solving. Four PTs pointed out little familiarity with the type of problem, difficulties in understanding the problem statement, or were unsure about which procedure was suitable to solve it.

Meanings of Probability and Ontosemiotic Analysis: Initial Progress

Following a reflection on the meanings of probability and their associated mathematical objects, PTs were required to discuss their identification of the sequence of mathematical practices and mathematical objects. This identification should encompass not only concept-definitions and languages but also procedures, propositions, and arguments involved in solving problem 2, connecting it with one of the meanings of probability. The results showed that five participants did not complete the task, and of the eight who did, four did not manage to sequence the practices properly, while the other four only partially described the elementary units. This observation is consistent with findings from Burgos et al. (2018), where only half of the participants in their study failed to distinguish practice units within the resolution sequence or provided limited configurations.

In this session, we saw a slight improvement compared to the results from the first session regarding the identification of concept-definitions and languages (the only types of mathematical objects considered in the initial task as an introduction). While analysing problem 2, five PTs appropriately recognized graphical, symbolic, and numeric languages. Another three mentioned them, referring to how they were used in mathematical practices. For example, PT7 stated, "I use the [tree diagram] method to solve probabilities."

Recognition of the mathematical object "concept-definition" continued to be problematic in this session. None of the PTs mentioned the concept-definitions of compound random experiment, sample space, possible outcomes, and favourable outcomes, which, however, are recurrent concepts in this task. Only five participants recognized the concept-definitions of probability, event, percentages, and fractions, which appear explicitly in their solutions. However, when they did, they often described these concepts as the mathematical practices that involve them or the intention behind these practices. For example, PT1 mentioned, "concepts: multiplying fractions," and PT7 stated, "the concepts I used were first to identify the data in the problem and the probability," where it seems that they understood "data from the problem" as a mathematical concept-definition.

PTs had difficulty recognizing procedures in their practices. In fact, 11 of them did not respond, and two assimilated procedures with the practices in the sequencing (see Figure 4).

Figure 4

Object Identification by PT1

A coin is flipped three times. What is the probability of getting "heads" exactly twice?

Describe the procedure

- The coin has 2 sides
- The probability of getting heads: 1/2
- The probability of getting heads 1 time: 1/2*1/3=1/6
- 2 times: 1/6+1/6=2/6=1/3

Identify the concepts: Probability, fraction, multiplying fractions.

The languages are: symbolic

The procedures have already been mentioned

Propositions:

Arguments: generalization

Note. Authors' translation.

Only PT12 identified "the probability of getting only heads is 3/8" as a proposition. Similarly, regarding the mathematical object argument, only one PT mentioned it in his report, and he did it incorrectly. This is the case of PT1 who, as observed in Figure 4, identified the (object) argument as the process of generalization. Therefore, despite the training received on mathematical practices and objects involved in probability tasks, the participants still faced challenges in sequencing elementary mathematical practices and identifying the objects of procedures, propositions, and arguments. However, there were some small achievements in recognizing concepts and languages.

Regarding the meaning of probability, six PTs managed to relate this task to the classical meaning, but they did not justify it based on equiprobability conditions or the finiteness of the sample space, although one of them attributed it to the context of gambling. In some cases, they exhibit a confused idea about the nature of the meaning of probability, as seen in PT3's response:

> In the types of probability meanings, I put that there is a lower probability of this process occurring since it is less than 50%... I put it as classical.

Finally, the participants were asked to mention any difficulties they encountered while working on the task. In this case, only three PTs indicated troubles in understanding the problem they had to solve. None of them referred to specific difficulties regarding the identification of mathematical objects.

Analysis of situation A

At the beginning of the final session, the PTs were asked to discuss the analysis of Situation A (Figure 2), using the ontosemiotic configuration tool. It was expected that the training received and providing them with a means to sequence, identify intentionality, and recognize objects in elementary practices would improve the analysis of a situation, moreover, resolved by the author of the textbook (expert solution).

In general, there was some improvement in the sequencing of elementary practices and the identification of mathematical objects (especially in languages, concept-definitions, and procedures) when analysing the solution provided in the workbook. Specifically, 12 PT managed to divide the solution into three or four units of analysis. Additionally, the 13 PTs who analysed this task correctly recognized the types of languages used in mathematical practices (symbolic and numeric most frequently, with natural language being mentioned less frequently).

All the participants successfully recognized some concept-definitions. In this case, as in the studies by Gómez (2014) and Vásquez y Alsina (2015), the most frequently mentioned concepts were probability, event, sample space, favourable cases, and possible cases, while the less recognized ones included random situation, likely event, certain event, unlikely event, impossible event, and possible outcomes. However, three of them incorrectly identified Laplace's rule as a concept rather than a property. This confusion may be due to the usual overrepresentation of the classical meaning in curricular materials, which can lead to an assimilation of concept, property, and procedure (Cotrado et al., 2022; Gómez, 2014; Gómez et al., 2015). Recognizing Laplace's rule as a concept rather than a property leads PTs to not reflect on the conditions that justify the use of this property.

They also successfully identified some of the procedures considered in the a priori analysis. For example, 11PTs identified the procedure "construct or list the sample space" and "calculate the probability using Laplace's rule," the latter in line with what was observed in Mohamed (2012). However, arithmetic procedures or conversion between fractions and decimal numbers went unnoticed. For example, none of the PTs mentioned the procedure for reducing fractions and expressing the probability value as a decimal, although one mentioned "expressing probability as percentages."

Although the participants recognized most of the concept-definitions, languages, and some procedures involved in Situation A, they still struggled to recognize propositions and arguments, as seen in the studies by Giacomone et al. (2018) and Burgos et al. (2018). Only two participants (PT1, PT13) partially identified a proposition and its associated argument, while two others (PT5, PT7) mentioned the proposition as the intention or requirement of the practice ("rolling a die once, determining the events"). For example, PT1 indicated the proposition "The number of favourable cases in event A is 3" and as an argument "because in the 6 events of the sample space, only 3 meet what event A requires-." Also, PT13 identified the proposition "the probability value ranges from 0 to 1" and the argument "Laplace's rule." In other cases, generic arguments are mentioned before the propositions and are not relevant.

In this sense, it can be concluded that, although there are achievements in identifying the objects of concepts-definition, languages, and procedures, PTs continue to have difficulties in recognizing propositions and arguments.

Situation A was incorrectly related to the intuitive meaning of probability by six PTs, four of them associated it with the classical meaning, one with the classical and intuitive meaning, and two did not respond to this question. Except for PT8, who justified that the associated meaning was classical because it was a game of chance, the others did not provide reasoning for their responses. They also did not do so during the discussion, despite questions from the instructor, although in some cases, they mentioned their difficulties regarding this. In this regard, seven participants mentioned having conflicts in recognizing mathematical objects ("To be honest, I didn't have many difficulties, just a little trouble recognizing mathematical objects because studying mathematical objects is something new to me," PT4), three in relating the situation to a meaning of probability; two referred to difficulties in understanding the procedure of the solved situation, and four did not record responses.

Difficulties in identifying the meanings of probability implied in the proposed situations, especially in distinguishing between the intuitive and classical meanings, were not only observed in the analysis of PTs written answers, but also during the sharing. For instance, in the group discussion, PT5 mentioned that although they initially thought the meaning was intuitive because "the intuitive had events, sure, possible, impossible," they later began to have doubts if it was the classical meaning "because there were favourable and possible cases."

Final Assessment Tasks

While systematic assessment of participants' progress was carried out throughout the workshop, Problem 3 and Situation B (both including an expert solution, in the first case provided by the instructor and, in the second case included in the textbook, Figure 3) are used as final evaluation instruments to determine the degree of ontosemiotic analysis competence achieved with the training action.

Results of the Ontosemiotic Analysis of the Expert Solution to Problem 3

The ontosemiotic analysis of the solution to item 3, facilitated by the instructor, was carried out by 10 PTs As a result, it was observed that seven of them distinguished sequences of mathematical practices into two or three units of analysis, while the other three did not differentiated them. Regarding the identification of objects, eight participants correctly identified verbal and symbolic-numerical languages in different elementary units, but they encountered difficulties in recognizing tabular language (identified this way by only one PT), which half of them referred to as graphical language (see Figure 5).

Nine participants recognized explicit concepts in this task, such as probability, randomness, favourable cases, and possible cases. However, concepts like population (Figure 5), event, sample space, double, marginal and conditional frequency were mentioned very rarely. Similar to Contreras (2011), the participants did not identify the concept of compound probability in the solution (PT1 did not indicate simple probability nor compound probability in the last elementary practice) They also mentioned as concept-definitions essential elements in the information provided by the problem (e.g., "total workers" instead of possible cases).

Figure 5

Ontosemiotic Analysis of the Expert Solution to Problem 3 (PT1)

<table>
<tr><th>Intentionality</th><th>Sequence of elementary practices</th><th>Mathematical objects</th></tr>
<tr><td>Problem statement</td><td>Calculate the probability of:
• Being male and not reading "The Station" magazine.
• Reading the magazine "La Estación".</td><td>• Meaning: Frequency meaning of probability
• Language: verbal
• Concepts-definitions: probability, chance</td></tr>
<tr><td>Identify the data</td><td>Identify all the data of the problem:
• Total population: 200 workers
• [Number of] Men: 100
• [Number of] Women: 100
• [Number of] Those who read the magazine "La Estación": 30 men and 35 women.</td><td>• Language: symbolic, numeric
• Concepts-definitions: population</td></tr>
<tr><td>Build a table</td><td>Make a table where we place the data.
<table>
<tr><th>Company</th><th>Men</th><th>Women</th><th>Total</th></tr>
<tr><td>Read La Estación</td><td>30</td><td>35</td><td>65</td></tr>
<tr><td>Do not read la estación</td><td>70</td><td>65</td><td>135</td></tr>
<tr><td></td><td>100</td><td>100</td><td>200</td></tr>
</table></td><td>• Language: graphic.
• Procedure: Place the identified data in the table.
• Concepts-definitions: Frequency of data</td></tr>
<tr><td>Answer the questions by applying Laplace's Rule</td><td>We apply Laplace's rule using the above data.
To answer statement (a)
$P(\text{Be man} \cap \text{Do not read } Estación) = \frac{70}{200} = 0{,}35.$
To answer statement (b)
$P(Red\ the\ magazine) = \frac{65}{200} = 0{,}325$</td><td>• Language: symbolic, numerical (fractional and decimal).
• Procedure: applying Laplace's Rule.
• Concept- Definitions: possible and favourable cases, decimals.
• Proposition: The probability of being male and not reading the magazine is 0.35. The probability that they read the magazine is 0.325.
• Arguments: graphical support</td></tr>
</table>

Note. Authors' translation.

The mathematical object "procedure" was recognized in various practices by seven participants. In line with Contreras' study (2011) and like previous tasks, most referred to the method of calculating probability using the "application of Laplace's rule." In the practice unit "we build a contingency table, place the variables, and distribute the compound, marginal and total absolute frequencies in each cell," only two participants partially mentioned procedures like "building a two-way table" and "transferring data." In this case, participants indicated arithmetic calculations as a procedure, which had not been mentioned in previous analyses.

Participants continued to face difficulty in recognizing arguments and propositions (Burgos et al., 2018; Contreras, 2011; Giacomone et al., 2018; Gómez, 2014). Only three correctly pointed out some propositions in the given solution, for example, "the probability of being a man and not reading the magazine is 0.35" (PT1 in Figure 5, PT4). In this case, they associated it with an explicit argument, for example, "Because out of 200, only 70 are men and do not read" (PT4), although not always appropriately, for instance, "Argument: graphical support" (PT1), referring to the information included in the two-way table (Figure 5). This was the only argument indicated by the participants ("table support to verify the data," PT5), in addition to the one based on Laplace's rule (PT3). The rest of the participants confused the proposition with what the problem asks, indicated propositions related to the two-way table, or did not provide responses.

Lastly, it's worth mentioning that three participants related this task to the frequentist meaning, and two to the classical meaning, but, as in previous tasks, they did not provide justifications. In particular, as shown in Figure 5, despite associating mathematical practices with frequentist meaning, PT1 indicated that Laplace's Rule was applied to determine the probabilities required in questions (a) and (b). Regarding difficulties, only PT13 quoted having difficulties identifying mathematical objects, while the rest did not indicate any or maintained their initial responses (Session 1) regarding the comprehension of the problem itself.

Results of the Analysis of Situation B Prepared by the Prospective Teachers

We observed that out of the 12 participants who analysed Situation B, nine broke down the sequences of mathematical practices into two or three units of analysis, while three did not distinguish elemental practices. Although 11 PT adequately recognized the presence of language of different types: verbal, symbolic and numerical, only two participants identified the graphic or tabular language involved in the two-way table. At this point, participants are starting to more successfully recognize emerging concept-definitions in mathematical practices, with the most frequently mentioned one being "sample space" and "event". This progress in identifying concept-definitions following the training aligns with previous observations by Giacomone et al. (2018) and Burgos et al. (2018). However, they still do not consider "possible outcome" and "probable outcome," or, as shown in Figure 6, do not recognize "compound probability," "random variable," "distribution," and "mode" as concepts involved in Situation B. However, similar to previous tasks, four PTs persisted in recognizing the Laplace's Rule as a concept-definition, rather than a property. In this task, as in Contreras' study (2011), the PTs s recognized concepts as elements that describe the problem: colour, die.

The mathematical object "procedure" was recognized by seven participants, primarily in the practice unit "We determine the probability of A applying the Laplace's rule." The most frequently mentioned procedure was "calculation of probability using Laplace's rule" (in all cases), followed by "building/interpreting a two-way table" (four PTs) and "listing the sample space" (three PTs) (see Figure 6). Two PTs indicated "finding the sum" or arithmetic calculations as procedures involved in determining the number of possible cases or in applying Laplace's rule as procedures. Other participants did not mention any type of procedure or continue to incorrectly identify a procedure with the intention or use of mathematical practices. For example, they pointed: "identifying the given data regarding how to write the data obtained in the two-way table and number line" (FP5); "identifying how much the sample space will be" (FP1); "using the table method to solve" (FP7).

Although no participant to correctly identified all the propositions and arguments involved in situation B, and they continued to confuse propositions with questions posed in the problem ("How many elements does the respective sample space have?"; "Which sum is more likely to occur?"), there was some progress in the identification of these objects. As shown in Figure 6, some PTs started to correctly recognize propositions and their associated arguments specifically for elementary practice units. As for the object "argument," other participants, like FP5, stated that the argument "is supported by the table."

Figure 6

Excerpt from the Ontosemiotic configuration for Situation B (PT4)

Elaboration of the two-way table **(Answering the second question)**	Construction of the double-entry chart with the summed pairs, i.e., the results of the sum of the dice.  The most probable sum is 7, since it is the one that is most repeated.	**Language**: Graphic (double entry table) **Procedure**: Construct a two-way table where the possible results of both sides are added up. **Definition**: two-way table, probability, events.
Application of Laplace's rule **(Answering the third question)**	With the application of Laplace's rule, it is easy to find the probability of the sum of the dice is 7 Probability of an event = $\frac{Favourable\ cases}{possible\ cases}$ Probability that when throwing 2 dice, the sum of these is 7 is:	**Language:** Symbolic numeric, fractional, and decimal. **Concepts**: Probability, fractions, events, favourable cases, and percentages. **Procedure**: Calculate the probability of the occurrence of an event, using the rule of event, using Laplace's rule. **Proposition**: The probability that when throwing the dice 7 are added together is 16.67%. **Argument**: Because out of 36 possible results (sample space), only 6 result in a sum equal to 7.

Authors' translation.

Five participants related this task to the classical meaning, two of them said it corresponds to both the frequentist and classical meanings, and one related it to the intuitive meaning. As had already happened in the analysis of Situation A, none of them managed to justify why they relate it to these meanings. Regarding difficulties, two mentioned that they still found the elaboration of the onto-semiotic configuration challenging, but in this case, none of them mentioned having had difficulties in relating the task to the meanings of probability (even though four of them did not respond to that instruction).

Conclusion

Throughout this work, we have described the design, implementation, and results of a formative experience with prospective Peruvian mathematics teachers aimed at fostering the competence of didactic-mathematical analysis of student workbooks in probability. Since teachers often rely on curriculum materials as the institutional knowledge that must be taught and learned, it seems necessary to ensure that PTs are competent in critically analysing the content that underpins the management and use of such materials (Godino et al., 2017; Shawer, 2017). This competence can only be achieved with a deep mathematical and didactic knowledge of the intended teaching object, allowing for a microscopic analysis of the sequence of operational, discursive, and normative practices proposed by the author to ensure learning by potential students.

Distinguishing between meanings and identifying the objects involved in mathematical practices is a challenge for PTs. However, it is a competence that will allow them to understand the progression of learning, manage the necessary institutionalization processes, and assess students' mathematical competencies.

The results of the initial exploration, as seen in previous research (Contreras, 2011; Gómez, 2014; Mohamed, 2012; Vásquez & Alsina, 2015), showed a deficient common knowledge of probability among PTs. Their responses exhibited equiprobability bias (Lecoutre, 1992), incorrect enumeration of the sample space, inappropriate use of counting schemes (tree diagram) to calculate the number of favourable and possible outcomes, confusion between conditional and compound probability (Contreras, 2011; Estrada & Díaz, 2007), and calculation errors.

In addition to the limitations posed by a deficient mathematical knowledge for the progress of the didactic analysis competence, we found that PTs participating in the training experience did not have a clear understanding of the nature and function of mathematical objects. Concerning procedures, they often confused them with the mathematical practices themselves or their intentions, and they couldn't distinguish arguments from propositions, which sometimes appeared in interrogative form, referring to the problem statements (Burgos & Godino, 2021; Burgos et al., 2018). In this regard, as noted by Font et al. (2013), although the onto-semiotic configuration tool is clear a priori, applying it to analyse mathematical activity can be complex because primary mathematical objects (conventional rules) appear as descriptions of secondary mathematical objects. Thus, a procedure or a property may present itself as the definition of a concept in the mathematical practices included in curricular materials. Nonetheless, we believe that the training provided, the sharing of experiences, and discussions of the answers given in each class helped PTs improve their analytical competence. They managed to distinguish and sequence the units of elementary mathematical practices, began to recognize concepts and languages, and, to some extent, the various pragmatic meanings of probability put into play in different proposed tasks.

A significant progression within the levels of development for the competence in didactic-mathematical analysis was observed among the PTs throughout the sessions. In the first session, out of the 16 PTs who participated, only four were able to partially recognize some mathematical objects, placing them at level L0. For the second session, in which situations 2 and A were addressed asynchronously and individually, the group was reduced to 13 participants. Among them, five PTs demonstrated progress by partially recognizing elemental units and some meanings based on their experience. Thus, they reached level L1, with an improved understanding of sequencing elementary practices and identifying mathematical objects, such as concepts, languages, and procedures. In the third and final session, the training received, experience-sharing, discussions on responses, and the use of the ontosemiotic configuration tool promoted a notable advancement in didactic analysis competency. Of the 13 participants, more than half managed to distinguish and sequence units of elementary mathematical practices, appropriately identified concepts, languages, procedures, and partially recognized the various pragmatic meanings of probability within the proposed tasks. However, none were able to justify why they related these meanings. This process allowed them to reach level L2.

The results of the experience highlight the challenges presented by these types of activities for both PTs and educators. Nevertheless, the recognition of meanings, the description of practices, and the identification of objects are key elements in training teachers to implement mathematics study processes that promote students' mathematical competence. Our results are consistent with those obtained in Rubio (2012), which highlight that the teacher's competence in analysing mathematical practices, objects, and processes is a "deep knowledge" that enables the evaluation and development of their students' mathematical competence, However, its implementation in professional training is not without ambiguities As Godino (2024) indicate, teaching a mathematical content may be compromised if teachers do not recognize the nature and role of the objects involved in the mathematical practices associated with the problem-solving domain: problem situations are the origin of activity; arguments justify procedures and propositions connecting concepts; languages are the ostensive part of concepts, propositions, and procedures while also contributing to the elaboration of arguments.

Since a clear limitation of our study is that the sample size may restrict the generalizability of the results obtained, further research cycles will be necessary. In future experiences, it is necessary, first and foremost, to reinforce mathematical knowledge related to the content involved, in our case, probability. More space should be dedicated to reflection on a wider variety of problem situations that allow achieving an adequate level of competence in the analysis of meanings and onto-semiotic analysis of mathematical practices.

Acknowledgments

This work is part of the research project PID2022-139748NB-100 founded by MICIU/AEI/10.13039/501100011033/ and FEDER, UE, with support from the research group FQM-126 (Junta de Andalucía, Spain) and from Group S60_23R - Investigación en Educación Matemática (Government of Aragón).

Notes

[1] The research complies with the ethical principles established in the guidelines of the Institutional Research Ethics Committee of the Universidad Nacional del Altiplano. The consent report of the participants is available. Our research guarantees the strict confidentiality of the information provided by each participant and preserves their anonymity.

References

Batanero, C. (2005). Significados de la probabilidad en la educación secundaria [Meanings of Probability in Secondary Education]. *Revista Latinoamericana de Investigación en Matemática Educativa, 8*(3), 247-263.

Batanero, C. y Álvarez-Arroyo, R. (2024). Teaching and learning of probability. *ZDM - Mathematics Education*, *56*(1), 5–17. https://doi.org/10.1007/s11858-023-01511-5

Batanero, C., Begué, N., Álvarez-Arroyo, R., & Valenzuela-Ruiz, S. M. (2021). Prospective mathematics teachers understanding of classical and frequentist probability. *Mathematics, 9*(19), 2526. https://doi.org/10.3390/math9192526

Batanero, C., Chernoff, E. J., Engel, J., Lee, H. S., & Sánchez, E. (2016). *Research on teaching and learning probability*. Springer

Batanero, C., Navarro-Pelayo, V., & Godino, J. D. (1997). Effect of the implicit combinatorial model on combinatorial reasoning in secondary school pupils. *Educational Studies in Mathematics, 32*(2), 181-199. https://doi.org/10.1023/A:1002954428327

Beltrán-Pellicer, P., Godino, J. D., & Giacomone, B. (2018). Elaboración de indicadores específicos de idoneidad didáctica en probabilidad: aplicación para la reflexión sobre la práctica docente [Elaboration of Specific Didactical Suitability Criteria in Probability: Application for Reflection on the Teaching Practice]. *Bolema: Boletim de Educação Matemática, 32*(61), 526-548. https://doi.org/10.1590/1980-4415v32n61a11

Breda, A., Pino-Fan, L. R., & Font, V. (2017). Meta Didactic-Mathematical Knowledge of Teachers: Criteria for The Reflection and Assessment on Teaching Practice. *EURASIA Journal of Mathematics, Science and Technology Education, 13*, 1893-1918. https://doi.org/10.12973/eurasia.2017.01207a

Burgos, M., Beltrán-Pellicer, P., Giacomone, B., & Godino, J. D. (2018). Prospective mathematics teachers' knowledge and competence analysing proportionality tasks. *Educação e Pesquisa*, *44*, 1-22. https://doi.org/10.1590/S1678-4634201844182013

Burgos, M., Castillo, M. J., Beltrán-Pellicer, P., Giacomone, B., & Godino, J. D. (2020). Análisis didáctico de una lección sobre proporcionalidad en un libro de texto de primaria con herramientas del enfoque ontosemiótico [Didactical analysis of a lesson on proportionality of a primary school textbook using tools of the onto-semiotic approach]. *Bolema: Boletim de Educação Matemática, 34*(66), 40-68. https://doi.org/10.1590/1980-4415v34n66a03

Burgos, M., Giacomone, B., Godino, J. D., & Neto, T. (2019). Desarrollo de la competencia de análisis ontosemiótico de futuros profesores de matemáticas mediante tareas de proporcionalidad [Developing the Onto-Semiotic Analysis Competence of Prospective Mathematics Teachers Using Proportionality Tasks]. In E. Badillo, N. Climent, C. Fernández y M.T. González (Eds.), *Investigación sobre el profesor de matemáticas: formación, práctica de aula, conocimiento y competencia profesional* (pp. 241-261). Ediciones Universidad Salamanca.

Burgos, M., & Godino, J. D. (2021). Assessing the epistemic analysis competence of prospective primary school teachers on proportionality tasks. *International Journal of Science and Mathematics Education, 20*, 367–38. https://doi.org/10.1007/s10763-020-10143-0

Calle, E. C., Breda, A., & Font, V. (2021). Reflection on the complexity of mathematical items in initial teacher education. *Journal of Higher Education Theory and Practice, 21,* 197-214, 2021. https://doi.org/10.33423/jhetp.v21i13.4801

Calle, E., Breda, A., & Font, V. (2023). Significados parciales del teorema de Pitágoras usados por profesores en la creación de tareas en el marco de un programa de formación continua [Partial meanings of the Pythagorean theorem used by teachers in the creation of tasks within the framework of a continuing education program] *Uniciencia, 37*(1), 1-23. https://doi.org/10.15359/ru.37-1.1

Chapman, O. (2014). Overall commentary: understanding and changing mathematics teachers. In J.–J. Lo, K. R. Leatham & L. R. Van Zoest (Eds.), *Research Trends in Mathematics Teacher Education* (pp. 295-309). Springer International Publishing.

Cohen, L., Manion, L., & Morrison, K. (2011). *Research methods in education*. Routledge.

Contreras, J. M. (2011). *Evaluación de conocimientos y recursos didácticos en la formación de profesores sobre probabilidad condicional*. [Doctoral thesis, Universidad de Granada].

Cotrado, B., Burgos, M., & Beltrán-Pellicer, P. (2022). Análisis ontosemiótico de los contenidos de probabilidad en los documentos curriculares de Perú [Ontosemiotic Analysis of Peruvian Curriculum Documents about Probability]. *Educación Matemática, 34*(3), 97-131. https://doi.org/10.24844/EM3403.04

Estrada, A., & Batanero, C. (2007). Errores en el cálculo de probabilidades en tablas de doble entrada en profesores en formación. *Uno: Revista de didáctica de las matemáticas, 44*, 48-58.

Font, V., Godino, J. D., & Gallardo, J. (2013). The emergence of objects from mathematical practices. *Educational Studies in Mathematics, 82*, 97–124. https://doi.org/10.1007/s10649-012-9411-0

Font, V., Planas, N., & Godino, J. D. (2010). Modelo para el análisis didáctico en educación matemática [A model for the study of mathematicsteaching and learning processes]. *Infancia y aprendizaje, 33*(1), 89-105. https://doi.org/10.1174/021037010790317243

Giacomone, B., Godino, J. D., Wilhelmi, M. R., & Blanco, T. F. (2018). Desarrollo de la competencia de análisis ontosemiótico de futuros profesores de matemáticas [Developing the onto-semiotic analysis competence of prospective mathematics teachers]. *Revista Complutense de Educación, 29*(4), 1109-1131. http://dx.doi.org/10.5209/RCED.54880

Godino, J. D. (2024). *Enfoque ontosemiótico en educación matemática: Fundamentos, herramientas y aplicaciones*. Aula Magna. McGraw Hill.

Godino, J. D., Batanero, C., & Font, V. (2007). The onto-semiotic approach to research in mathematics education. *ZDM, 39*(1-2), 127-135. https://doi.org/10.1007/s11858-006-0004-1

Godino, J. D., Giacomone, B., Batanero, C., & Font, V. (2017). Enfoque ontosemiótico de los conocimientos y competencias del profesor de matemáticas [Onto-Semiotic Approach to Mathematics Teacher's Knowledge and Competences]. *Bolema: Boletim de Educação Matemática, 31*(57), 90-113. http://dx.doi.org/10.1590/1980-4415v31n57a05

Godino, J. D., Rivas, H., Arteaga, P., Lasa, A., & Wilhelmi, M. R. (2014). Ingeniería didáctica basada en el enfoque ontológico-semiótico del conocimiento y la instrucción matemáticos. *Recherches en Didactique des Mathématiques, 34*(2/3), 167-200.

Gómez, E. (2014). *Evaluación y desarrollo del conocimiento matemático para enseñar la probabilidad en futuros profesores de educación primaria*. [Doctoral thesis, Universidad de Granada].

Gómez, E., Contreras, J. M., & Batanero, C. (2015). Significados de la probabilidad en libros de texto para Educación Primaria en Andalucía. In C. Fernández, M. Molina y N. Planas (eds.), *Investigación en Educación Matemática XIX* (pp. 73-87). SEIEM.

Hoadley, U., & Galant, J. (2016). An analysis of the Grade 3 Department of Basic Education workbooks as curriculum tools. *South African Journal of Childhood Education, 6*(1), 1-12. https://doi.org/10.4102/sajce.v6i1.400

Lecoutre, M. P. (1992). Cognitive models and problem spaces in “purely random” situations. *Educational Studies in Mathematics, 23*(6), 557-568.

Lloyd, G. M., & Behm, S. L. (2005). Preservice elementary teachers’ analysis of mathematics instructional materials. *Action in Teacher Education, 26*(4), 48–62. https://doi.org/10.1080/01626620.2005.10463342

MINEDU (2019a). *Cuaderno de trabajo de Matemática: Resolvamos problemas secundaria 1*. Ministerio de Educación (Perú).

MINEDU (2019b). *Cuaderno de trabajo de Matemática: Resolvamos problemas secundaria 2*. Ministerio de Educación (Perú)

Mohamed, N. (2012). *Evaluación del conocimiento de los futuros profesores de educación primaria sobre probabilidad*. [Doctoral thesis, Universidad de Granada].

Parraguez, R., Gea, M. M., Díaz-Levicoy, D., & Batanero, C. (2017). ¿Conectan los futuros profesores las aproximaciones frecuencial y clásica de la probabilidad? *Revista Digital: Matemática, Educación e Internet, 17*(2). https://doi.org/10.18845/rdmei.v17i2.3077

Pepin, G., & Gueudet, G. (2018). Curriculum resources and textbooks in mathematics education. In: Lerman, S. (ed.). *Encyclopedia of mathematics education* (pp. 172-176). Springer. https://doi.org/10.1007/978-94-007-4978-8_40

Pino-Fan, L.R., Castro, W.F., & Font, V. (2023). A Macro Tool to Characterize and Develop Key Competencies for the Mathematics Teacher' Practice. *International Journal of Science and Mathematics Education, 21*, 1407–1432. https://doi.org/10.1007/s10763-022-10301-6

Pino-Fan, L., Godino, J. D., & Font, V. (2018). Assessing key epistemic features of didactic-mathematical knowledge of prospective teachers: the case of the derivative. *Journal of Mathematics Teacher Education, 21*(1), 63-94. https://doi.org/10.1007/s10857-016-9349-8

Pochulu, M., Font, V., & Rodríguez, M. (2016). Desarrollo de la competencia en análisis didáctico de formadores de futuros profesores de matemática a través del diseño de tareas [Development Of The Competence In Didactic Analysis Of Training Of Future Mathematics Teachers Through Task Design]. *Revista Latinoamericana de Investigación en Matemática Educativa, 19*(1), 71-98. https://doi.org/10.12802/relime.13.1913

Remillard, J. T., & Kim, O. K. (2020). *Elementary mathematics curriculum materials: Designs for student learning and teacher enactment*. Springer Nature. https://doi.org/10.1007/978-3-030-38588-0

Rondero, C., y Font, V. (2015). Articulación de la complejidad matemática de la media aritmética. *Enseñanza de las Ciencias, 33*(2), 29-49. https://doi.org/10.5565/rev/ensciencias.1386

Rubio, N. (2012). Competencia del profesorado en el análisis didáctico de prácticas, objetos y procesos matemático. [Teacher competence in the didactic analysis of mathematical practices, objects and processes] Tesis doctoral, Universitat de Barcelona, España. https://www.tdx.cat/handle/10803/294031#page=1

Shawer, S. F. (2017). Teacher-driven curriculum development at the classroom level: Implications for curriculum, pedagogy and teacher training. *Teaching and Teacher Education, 63*, 296–313. https://doi.org/10.1016/j.tate.2016.12.017

Thompson, D. (2014). Reasoning-and-proving in the written curriculum: Lessons and implications for teachers, curriculum designers, and researchers. *International Journal of Educational Research, 64*, 141–148. https://doi.org/10.1016/j.ijer.2013.09.013

Vásquez, C., & Alsina, A. (2015). El conocimiento del profesorado para enseñar probabilidad: un análisis global desde el modelo del conocimiento didáctico-matemático [Teachers' knowledge for teaching probability:A global analysis based on Didactic-Mathematical Knowledge model]. *Avances de Investigación en Educación Matemática 7*, 27-48. https://doi.org/10.35763/aiem.v1i7.104

Yang, K., & Liu, X. (2019). Exploratory study on Taiwanese secondary teachers' critiques of mathematics textbooks. *Eurasia Journal of Mathematics, Science and Technology Education, 15*(1), em1655. https://doi.org/10.29333/ejmste/99515