Pattern Recognition in Computational Thinking: Semiotic Perspective

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Abstract: Pattern recognition is an aspect of computational thinking and a vital competency for solving mathematical problems. By recognizing patterns, one can develop possible solutions. This paper aims to describe the pattern recognition of prospective mathematics teachers in solving mathematical problems from a semiotic perspective. This study based on a qualitative case study approach. The research subjects were prospective mathematics teachers from a private university in Indonesia with impulsive and reflective cognitive styles. The data was collected using mathematical problem-solving tasks and interviews. Research findings indicate that individuals with impulsive and reflective cognitive styles employ a five-sign trichotomous process for pattern recognition. In the trichotomous system of signs, the final object is the pattern, like a determine the triangle height. The representamen consists of mathematical equations, such as equations with variables height. The interpretant is to determine patterns from the solved mathematical equations. In the case of the given problem-solving task, both students determine the height of a triangle. Further research is recommended for problem-solving activities conducted in groups so that the object, representamen, and interpretant from various student perspectives can be understood.

Keywords: semiotic perspective, pattern recognition, prospective mathematics teacher

INTRODUCTION

Semiotics plays a fundamental role in human communication systems, functioning as a theory that examines signs and their meanings (Chandler, 2007). Presmeg et al. (2016) state that in mathematics education, semiotics serves as an analytical lens for exploring how learners understand concepts and solve problems. Ernest (2006) also stated that semiotics is suitable for mathematics education to see how concepts are used in understanding material or solving mathematical problems. Through semiotic analysis, educators can identify the underlying cognitive activities that individuals engage in during problem-solving, thus revealing their thinking processes. Problem-solving is an integral part of mathematics learning (Wicaksono et al., 2019), and successful problem-solving requires careful consideration of each step and the logical order needed to arrive at a solution (Yanti et al., 2018).

Recently, computational thinking has gained prominence as a critical component of school curricula (Fraillon et al., 2020). It is an essential skill set for students to navigate and solve problems in the digital era (Bacelo & Gómez-Chacón, 2023). Computational thinking involves organizing and structuring information into logical sequences, using conditions for decision-making, and implementing loops for repetition until certain conditions are met (Van-Borkulo et al., 2021). These elements are fundamental for pattern recognition, which helps individuals identify similarities or recurring patterns to simplify and solve complex problems effectively (Rosali & Suryadi, 2021). While Yasin & Nusantara (2023) have examined the characteristics of pattern recognition in computational thinking, there remains a gap in understanding how students apply pattern recognition during problem-solving.

Pattern recognition in problem-solving correlates strongly with speed and accuracy. Speed refers to the time taken to solve a problem, while accuracy refers to the correctness of the solution. These two factors align with cognitive styles that influence how students approach problem-solving. According to Kagan (1966), individuals can be categorized as having either impulsive or reflective cognitive styles. Students with an impulsive cognitive style tend to act quickly, often without extensive deliberation, leading to more errors but greater spontaneity. On the other hand, students with a reflective cognitive style are more deliberate, emphasizing accuracy, which often means taking longer to complete tasks (Nietfeld & Bosma, 2003). These cognitive styles impact how students structure their problem-solving processes and their decision-making during mathematical problem-solving.

Semiotics can provide insights into how students utilize computational thinking during problem-solving by analyzing their use of signs. Peirce’s semiotic theory is particularly relevant, focusing on the sign trichotomy involving the representamen, object, and interpretant (Presmeg et al., 2016; Atkin, 2005; Buchler, 2012). This trichotomy enables educators to study how students’ semiotic activities reflect their pattern recognition capabilities in computational thinking. Recognizing these patterns can facilitate deeper conceptual understanding, supporting students in developing strategies for more effective problem-solving.

This research seeks to explore and describe how prospective mathematics teachers demonstrate pattern recognition during problem-solving, using semiotic activities as an analytical framework. By focusing on students with impulsive and reflective cognitive styles, the study aims to uncover how these styles influence their problem-solving approaches. The ultimate goal is to provide valuable insights for mathematics educators, helping them support students in grasping mathematical concepts and enhancing their problem-solving skills.

LITERATURE REVIEW

Computational Thinking

Computational thinking refers to an individual's ability to identify real-world problems suitable for computational formulation and develop algorithmic solutions to address and operationalize these problems (Fraillon et al., 2020). Furber (2012) defines computational thinking as recognizing computational aspects in the real world and applying tools or techniques from computer science to understand both natural and artificial systems and processes. This process allows for the analysis of complex problems, the development of understanding, and the creation of appropriate solutions. Essentially, computational thinking involves identifying problems and determining algorithms to find solutions.

Several aspects define computational thinking. Selby (2018), outlines aspects include abstraction, decomposition, pattern recognition, generalization, and automation. Similarly, Huang et al. (2021), highlight decomposition, pattern recognition, abstraction, and algorithmic thinking as key components. Dong et al. (2019) refers to these aspects with the acronym PRADA (Pattern Recognition, Abstraction, Decomposition, Algorithmic Thinking). These components are crucial when solving mathematical problems, with this research emphasizing the pattern recognition aspect. Pattern recognition is essential for problem solvers to understand the structure of problems and achieve their goals. It is a vital skill in computational thinking and an indispensable competency for effective problem-solving (Çoban & Korkmaz, 2021; Barron-Estrada et al., 2022), particularly in mathematics, which demands various strategic approaches.

Researchers	Pattern recognition description
Dong et al. (2019)	Emphasizes observing and identifying patterns, trends, and regularities in data, processes, or problems.
Selby (2018)	Emphasizes a particular generalization process. Generalization is a powerful component of the problem-solving process that can help define computational thinking. This aspect describes the ability to express problem solutions in general terms, which can be applied to different problems with some characteristics in common with the original.
Huang et al. (2021)	Emphasizes identifying similarities or common elements between two or more items. This process is closely related to abstraction or generalization.

Table 1. Aspects of Pattern Recognition in Computational Thinking

Table 1 summarizes the aspects of pattern recognition in computational thinking from multiple studies, analyzing their similarities and differences. Pattern recognition involves identifying patterns within sub-problems of a mathematical task. Once identified, these sub-problems can be broken down, revealing connections, similarities, or repetitions. These patterns provide potential solutions that aid in addressing the broader mathematical problem.

Semiotics

A semiotic perspective on mathematical activities offers an alternative framework for conceptualizing mathematics (Ernest, 2006). Semiotics is the study not only of signs but of anything that stands for something else (Noth, 2021; Arzarello et al., 2009; Chandler, 2007). A sign holds meaning by referencing an object and producing an interpretive effect or interpretant. According to Peirce (Sáenz-Ludlow & Kadunz, 2016) the sign trichotomy comprises three components:

a. Representamen

The outcome of the representation process, functioning as a sign or depiction of something. It acts as a substitute for the object and may take on multiple forms. According to Sáenz-Ludlow & Kadunz (2016), a representamen itself cannot signify a real entity in isolation, particularly when dealing with complex objects.

b. Object

The entity indicated by the representamen, which can be a mental construct or a physical object. In Sáenz-Ludlow & Kadunz (2016), objects are categorized as real, dynamic, or direct. Real objects remain consistent during encoding or representation, whereas dynamic objects evolve subjectively in the interpreter's mind. Direct objects, parts of real objects, are defined by their initial representations. A final object may emerge during this conceptualization process, contributing to or comprising a real object.

c. Interpretant

The meaning or effect produced by the representamen, resulting from the interpretation process.

Peirce’s semiotic component indicators used in this study are presented in Table 2.

Peirce's Semiotic Components	Descriptor
Representamen	Representing something as something else
Object	Something referred to by the representamen
Interpretant	The effect resulting from representamen

Table 2. Peirce’s Semiotic Indicators

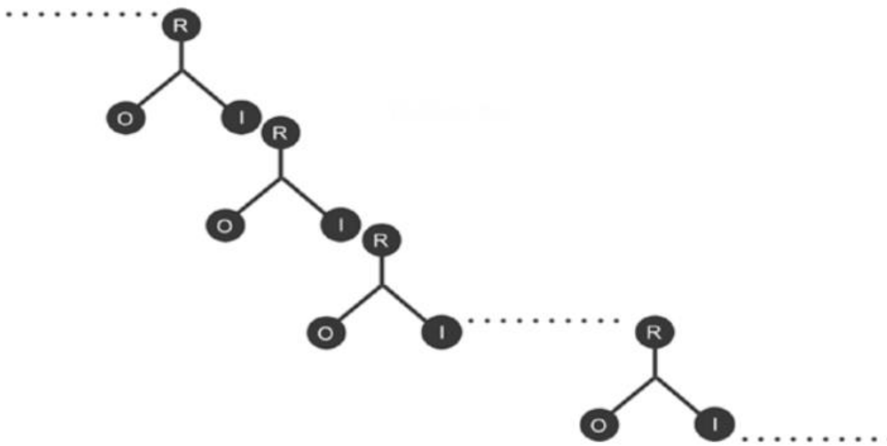

Figure 1. Trichotomy process of Peirce's semiotics

This process forms a sign trichotomy. In mathematics and computational thinking, this trichotomy process can recur multiple times during problem-solving or pattern recognition, depending on how thoroughly patterns are identified. Peirce views the interpretation process as inherently continuous and dynamic (Sáenz-Ludlow, 2007). Thus, an interpretant may become the representamen in the next trichotomy cycle, continuing until the sought pattern is fully recognized. Figure 1 illustrates Peirce's trichotomy model, with O representing the object, R the representamen, and I the interpretant (Sáenz-Ludlow & Kadunz, 2016).

Cognitive Style

Differences in thought processes or cognitive processes are known as cognitive styles. Cognitive style, learning style, and conceptual style are related terms that refer to individual characteristics and a consistent approach to organizing and processing information (Tennant, 2007). Cognitive style was first described by Allport (1937), as the consistent and characteristic tendencies individuals display when understanding, remembering, organizing, processing, thinking, and solving problems. It can be detected through language and nonverbal behavior patterns (Martin, 1998).

Rozencwajg and Corroyer (2005) identify types of cognitive styles, including impulsive, reflective, fast-accurate, and slow-inaccurate. This research focuses on impulsive and reflective cognitive styles because these two groups comprise the majority of individuals (approximately 70%), as stated in their findings. Additionally, Kagan (1966), initial hypothesis supports this focus, asserting that individuals who respond too quickly (impulsive) tend to make more errors. The impulsive cognitive style is characterized by students who provide rapid answers with less accuracy, while the reflective cognitive style is characterized by students who take longer to respond but provide more accurate answers. Reflective individuals are said to apply analytical processes and demonstrate cognitive maturity, whereas impulsive individuals tend to use holistic processes and show less cognitive maturity.

To better understand how individuals process information for problem-solving, detection through nonverbal behavior can be utilized (Allport, 1937; Martin, 1998). This can be achieved using measurement tools or instruments developed by Kagan (1966), Rozencwajg and Corroyer (2005) and Herianto (2013). Such approaches also address Rozencwajg and Corroyer (2005) observation that 70% of individuals exhibit impulsive cognitive styles and provide insight into Kagan's hypothesis about differences in processing time and accuracy between impulsive and reflective cognitive styles.

METHODS

Research Background

This research adopts a case study method with a qualitative approach. A case study provides a detailed example in an actual context, allowing readers to gain a clearer understanding of ideas than through abstract theories or principles alone (Cohen et al., 2018). The case study method is chosen for its ability to explore in depth how semiotics manifests in pattern recognition during computational thinking by individual students.

Research Subject

The subjects in this study comprised 46 semester VI prospective mathematics teacher students from a private university in Indonesia. Two participants were selected using purposive sampling to explore how semiotics influences pattern recognition in computational thinking. One participant had an impulsive cognitive style, while the other had a reflective cognitive style. These subjects were chosen due to their varied backgrounds in information processing when understanding concepts in mathematical problem-solving, as observed in their coursework. This research aims to contribute to cognitive theory by providing empirical data that enriches our understanding of brain function in the context of pattern recognition. The selected subjects also had comparable mathematics ability test scores and demonstrated good communication skills, both orally and in writing.

Instruments and Procedures

This study utilises three research instruments for data collection. The first instrument is the Matching Familiar Figure Test (MFFT), which measures the cognitive style of prospective mathematics teacher students. The researcher used an MFFT instrument adapted from Herianto (2013) and Warli (2013). This instrument presents a standard image alongside several identical images, and the subject is required to select the one that matches the standard image. This adaptation aligns with the concepts and images from Kagan (1966), original instrument for assessing impulsive and reflective cognitive styles. Additionally, the MFFT instrument has been validated and empirically tested on subjects in a mathematics education class.

The second instrument is a mathematics ability test, administered to all participants who completed the MFFT. This test assesses students' mathematical abilities and selects participants with similar levels of capability. It consists of five descriptive questions focusing on Algebra, Numbers, and Geometry to evaluate students' problem-solving skills. The mathematics ability test was subjected to validity testing by experts before being used.

The third instrument is the Mathematical Problem-Solving Task (MPST), which was provided after the selection of participants who met the research objectives. This task evaluates how students engage in computational thinking from a semiotic perspective, according to their cognitive style.

In addition, interviews were conducted after the students completed the mathematical problem-solving tasks to further investigate their semiotic processes in computational thinking according to their cognitive style.

Data Analysis

The data from the cognitive style test were analysed by examining the time taken and the frequency of errors. Cognitive styles were classified based on the median response time and the median frequency of correct answers (Rozencwajg & Corroyer, 2005). The median served as the threshold for distinguishing between impulsive and reflective cognitive styles. The analysis of the mathematics ability test involved scoring student responses according to criteria set by the researchers. The study found a median response frequency of 4 and a median response time of 19:45 minutes. Analysis showed that 5 students did not participate in the mathematics ability test, resulting in 18 impulsive students, 6 fast-accurate students, 5 slow-inaccurate students, and 17 reflective students. Figure 2 illustrates the analysis of subject selection based on cognitive style and mathematical ability, where the vertical axis represents the mathematics ability test scores and the horizontal axis denotes cognitive style categories (impulsive, reflective, fast-accurate, and slow-inaccurate).

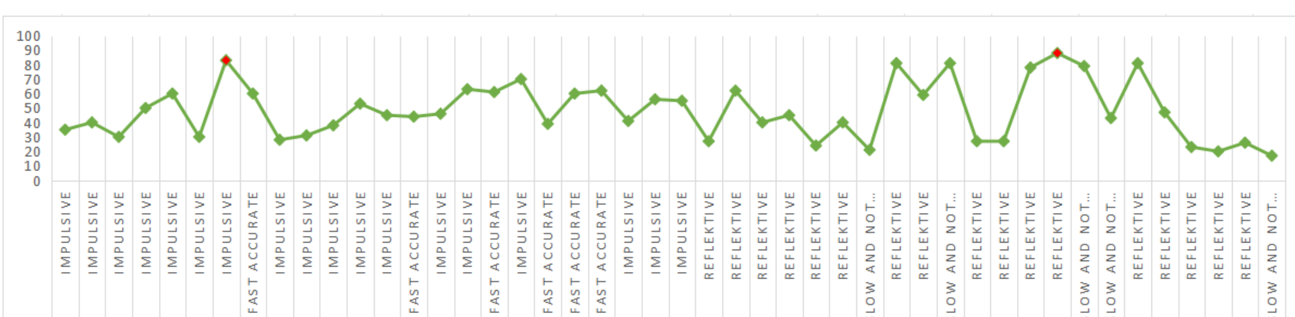

Figure 2. Analysis of research subject selection data

The data from the mathematical problem-solving tasks were analysed using Yin (2016) five-stage approach: compiling, disassembling, reassembling (arraying), interpreting, and concluding. The compiling stage involved gathering research data, such as students' work documents and interview recordings, which were then transcribed. The disassembling stage involved breaking down the data into smaller components, including coding of the students' work and interview transcripts. Table 3 and Table 4 are the coding used in data analysis activities.

Information	Code
Researcher	P
Impulsive Students	SI
Reflective Students	SR
Pattern recognition	CT

Table 3. Coding of Researcher, Subject and Pattern Recognition

Peirce's semiotic components	Code
Representamen	z
Object	o
Interpretant	i

Table 4. Coding of semiotic components

The reassembling (arraying) stage involved rearranging and combining the students' work documents, field notes, and interview transcripts. These data were then integrated and presented using Peirce's semiotic framework. The interpreting stage analysed the patterns identified during the reassembling stage, leading to a deeper understanding of the data. Finally, at the concluding stage, conclusions were drawn about the computational thinking of prospective mathematics teacher students from a semiotic perspective, considering their impulsive or reflective cognitive styles.

RESULTS

Semiotic Perspective on Pattern Recognition in Computational Thinking of Prospective Mathematics Teacher Students with a Cognitive Impulsive Style

The SI introduced the pattern recognition stage by finding the heights of triangles CDF and FCB. Figure 3 shows the SI's work on pattern recognition in the MPST.

The SI recognised patterns by calculating the area and height of triangles. In Figure 3, the SI represents the height of the triangle FEB as x. After identifying triangle FEB and and the parallelogram's height, the SI represents the height of triangle FCD as $30 - x$. Using the equation $\mathrm{L}\ \Delta\mathrm{FEB} + \mathrm{L}\ \Delta\mathrm{FCD} = 132$ and substituting the known elements in finding the area of the triangle, the SI found the value x. By obtaining this value $x = 12$, the students get the height of the two triangles. This allowed the SI to determine the heights and areas of ΔFEB dan ΔFCD. The interview excerpts confirm these findings.

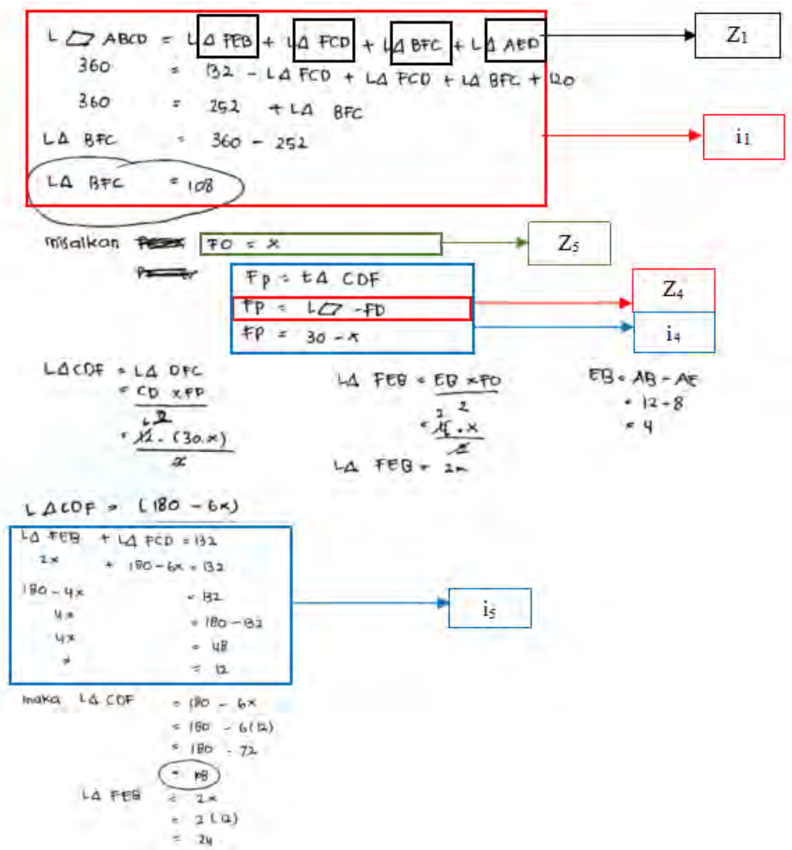

Figure 3. Pattern Recognition of SI

P-CT : *Here are the stages, which means you must break them down. Then, from these stages, do you think you found a pattern or characteristic from the problem, or the method used?*

SI-CT : *This is because everything is divided into triangles, so find the area of the triangle together.*

P-CT : *Anything else?*

SI-CT : *It is finding the height of the triangle. It can be obtained from the height of the parallelogram.*

P-CT : *Which one?*

SI-CT : *That is triangle* DCF *and* EBF *because I understand this as an arbitrary triangle so that the height can be from this parallelogram. Here, I assume that the height of the* EBF *triangle is* FO, *that is,* x, *then the height of the* DCF *triangle is* FP, *that is* $30-x$*, and then the height of the parallelogram is* x *reduced by the height of the* EBF *triangle.*

P-CT : *Where is it located?*

SI-CT : *This is* FO *equals* x *then* FP *equals* $30-x$

The trichotomy sign system in Peirce's semiotics can be observed in the students' work and interview results. This system comprises objects, representamen, and interpretants, which are interconnected. In the initial conversation, the SI stated "This is because everything is divided from triangle-triangle, so equally find the area of the triangle". The interview excerpts show that the SI identifies triangles as the object $(\boldsymbol{o_1})$ in recognising patterns in problems within MPST. The representamen $(\boldsymbol{z_1})$ is depicted in Figure 3, where the SI divides a triangle into four parts: ΔFEB, ΔFCD, ΔBFC, and ΔAED. In this representamen, the SI stated that the next step involved finding the area of each triangle. Thus, the interpretant $(\boldsymbol{i_1})$ is the process of finding the area of the triangle that make up the parallelogram ABCD.

For further pattern recognition, the SI continues to use triangles as objects $(\boldsymbol{o_2})$. The representamen $(\boldsymbol{z_2})$ is evident from the conversation, specifically the triangle's height. The SI mentioned that the height of a triangle can be obtained from the height of a parallelogram, a statement that also functions as an interpretant $(\boldsymbol{i_2})$.

The researcher then posed follow-up questions to confirm the SI's explanation of pattern recognition. The SI explained, "Those are triangles DCF and EBF. Because of this, I understand them as arbitrary triangles". From this conversation, the SI still considers triangles as objects $(\boldsymbol{o_3})$ for recognizing patterns in MPST. The representamen $(\boldsymbol{z_3})$ is shown by the mentioning ΔFEB and ΔFCD as arbitrary triangles, leading to the interpretant $(\boldsymbol{i_3})$.

It does not end there; the SI states that the triangle's height can be derived from the parallelogram. Thus, the object $(\boldsymbol{o_4})$ is the triangle's height, while the representamen $(\boldsymbol{z_4})$ is evident from the result of the SI's work, expressed as $\mathrm{FP} = t_{\square} - \mathrm{FO}$. Therefore, the interpretant $(\boldsymbol{i_4})$ in this process is the triangle's height, which can be derived from the height of the parallelogram. In the subsequent process, the object $(\boldsymbol{o_5})$, the height of ΔFEB represented by x, and the height of ΔCDF is represented by $30-x$. The representamen $(\boldsymbol{z_5})$ is illustrated through the SI's work, showing $\mathrm{t}\ \Delta\mathrm{FEB} = \mathrm{FO} = x$ and $\mathrm{t}\ \Delta\mathrm{CDF} = \mathrm{FP} = 30-x$.

From the processes carried out by the SI, it can be concluded that the interpretant $(\boldsymbol{i_5})$ is the determination of the heights of triangles CDF and EBF. Peirce considers the interpretative process to be continuous and dynamic, as a general process. Sáenz-Ludlow (2007), in his research, also states that the concept of trichotomy persists until a conclusion or result is obtained from a problem-solving phase. In this context, the interpretant becomes the representamen in the next trichotomous cycle. Figure 4 illustrates a trichotomous sign system that the SI performs in the aspect of pattern recognition.

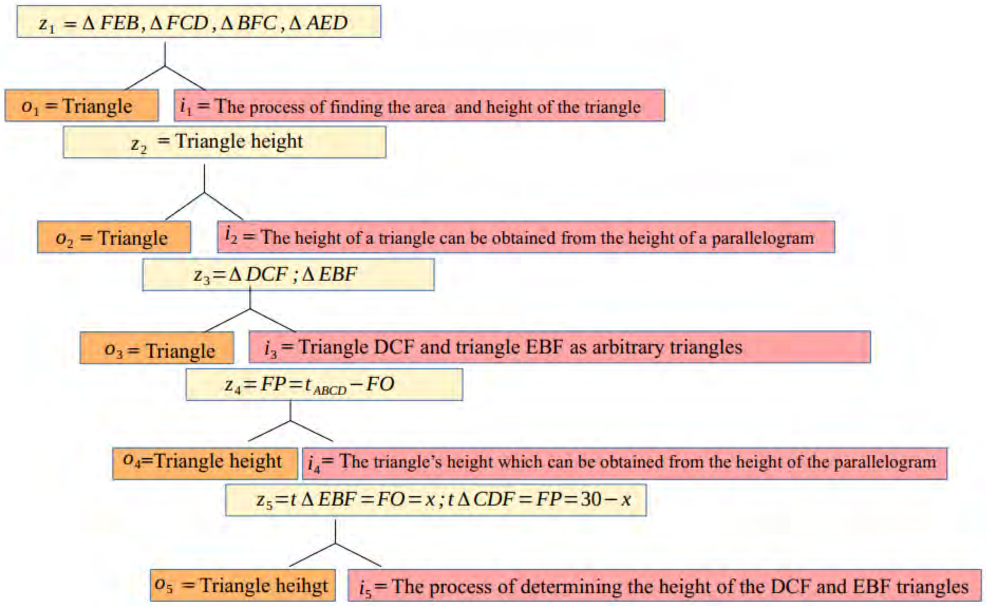

Figure 4. Trichotomy Signs Pattern Recognition of SI

Semiotic Perspective on Pattern Recognition in Computational Thinking Students of Mathematics Teacher Candidates with Reflective Cognitive Style

Pattern recognition identified by SR in this MPST involves determining the height of triangles. This pattern is then applied to find the height variable of triangles FCB and FCD. Below are excerpts from interviews conducted by the researchers.

P-CT : *Based on the formula, not the image. After that, what characteristics or patterns were found from the information in the problem?*

SR-CT : *(Thing long) As for the specific pattern, if the questions are different, you cannot get the pattern, ma'am. Questions number one and two are almost the same. They both asked about the area of triangles. Let us look for the variable of height first. After looking for a new height variable, we can find the area of one of the unknown triangles and then reduce it to the area of the entire parallelogram.*

The interview excerpts illustrate the trichotomy of signs: objects, representamens, and interpretants. Based on the excerpts, the object $(\boldsymbol{o_0})$ in pattern recognition, recognized by SR, is the total area of the triangle within the MPST. The representamen $(\boldsymbol{z_0})$ is evident from SR's work, as shown in Figure 5.

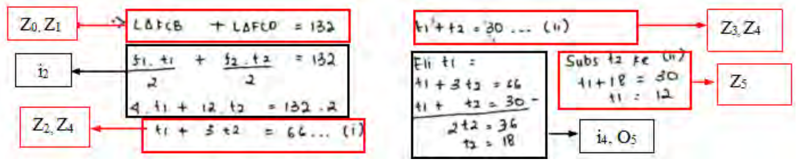

Figure 5. Pattern Recognition of SR

Figure 5 demonstrates how SR found the heights of triangles FCB and FCD using substitution and elimination methods on equations (i) and (ii), derived from the representamen of each triangle's height with variables $t_1$and t_2. The calculated heights are $t_1 = 12$ and $t_2 = 18$. The interpretant (i_0) is reflected in the statement from the interview: "Let us find the height variable first. After that, we can find the area of one of the triangles we do not know yet. Then we can reduce it with the area of the entire parallelogram." The trichotomy of signs recognized by SR in pattern recognition shown in Figure 6.

$z_0 = L\ \Delta\text{FCB} + L\ \Delta\text{FCD} = 132$

O_0 = sum of two areas triangle; triangle

i_0 =find the height variable value

Figure 6. Trichotomy Signs Pattern Recognition

SR provided a detailed explanation of the pattern recognition process in the excerpt below.

P-CT : *Okay, that means we do not randomly write down the height of t_1 and t_2. Then, from that stage, write down the area of the FEB triangle plus the area of the FCD triangle equal to* 132 *until you get this first equation. What is the basis of this, and what do these stages mean?*
SR-CT : *Write the area of the triangle based on the problem. We know the sum of the triangles FEB and FCD areas, which is equal to 132. The triangle area formula is base times height divided by two. The base is represented with $f_1$and f_2, and the height is represented with t_1 and t_2, divided into two. We do it like algebra.*
P-CT : *Okay, here we get t_1 plus three times t_2 which equals 66 dots of this one. What does this mean?*
SR-CT : *Oh, that one*
P-CT : *What do you mean by that?*
SR-CT : *It is to interpret it as an equation. There is the first equation and second equation for eliminations and substitutions.*
P-CT : *OK, how about $t_1 + 3t_2$?*

SR-CT : *We have $f_1 \times t_1$ plus $f_2 \times t_2$ equals 132. Because it is divided by 2, we moved it to the right side, and now it becomes $f_1 \times t_1$ plus $f_2 \times t_2$ equals 132 times 2. We know that f_1 is 4 and f_2 is 12, so we get 4 times t_1 plus 12 times t_2 equals 132 times 2. After that, we divided it by 4. So, the result is t_1 plus 3 times t_2 equals 66.*

P-CT : *Okay, so next, where does it go? To t_1 plus t_2 or the equation below first?*

SR-CT : *To the side, ma'am.*

P-CT : *To the side, this means that image 2 equals 30 dots. What is your basis for writing this?*

SR-CT : *t_1 plus t_2 is based on what is known from the image analysis. There is an analysis of the t parallelogram equal to 30 equals the height of FEB plus the height of FCD, which has been represented with t_1 and t_2. We can directly write t_1 plus t_2 equals 30, as the second equation.*

P-CT : *What must we know if we want to get t_2 and t_1?*

SR-CT : *We must know the same variables. We have variables t_1 and t_2. We will analyze again which of the other variables are the same as t_1 and t_2. The height above is still unknown; the variables are the same as t_1 t_2, $t_1 + 3t_2$. So, that means we can eliminate or substitute to find out which variables we do not know from the problem. We eliminate t_1 because the coefficients are the same, and we get t_2.*

P-CT : *Next, do we substitute t_2 into the second equation?*

SR-CT : *Yes.*

P-CT : *Why must it be the second equation? Are we not allowed to substitute it with another equation?*

SR-CT : *We can substitute it with another equation. Just look at the coefficients. The first equation still has a coefficient of 3, which must be multiplied and divided again. So, we chose the second equation, which has a coefficient of 1, so we can immediately find the result.*

From the interview results, the SR can explain in detail the process of determining the heights of the two triangles. The semiotic components for the object $(\boldsymbol{o_1})$ in pattern recognition are illustrated in the interview, specifically regarding the sum of the areas of the two triangles. The student stated the representamen $(\boldsymbol{z_1})$ as L ΔFCB + L ΔFCD $= 132$. Based on the representamen, the interpretant $(\boldsymbol{i_1})$ is reflected in the interview results: "The formula to find the area of a triangle is base times the height divided by 2. The base is represented by f_1 and f_2, and the height is represented by t_1 and s, then divided by 2." From this statement, SR carried out the substitution process, for example, describing the area of a triangle as $\frac{1}{2} \times a \times t$ and substituting known base and height values into the established formula. This leads to the equation $t_1 + 3t_2 = 66$.

Subsequently, SR stated in the interview, "t_1 plus 3 t_2 is derived from the triangle formula." Therefore, the object $(\boldsymbol{o_2})$ in the pattern recognition stage is identified as a mathematical equation, with the representamen $(\boldsymbol{z_2})$ indicated by $t_1 + 3t_2 = 66$. Furthermore, the interpretant $(\boldsymbol{i_2})$ is evident in the interview statement: "t_1 plus $3t_2$ is from the triangle formula." The interview results show that the student recognizes $t_1 + 3t_2$ as the sum of the heights of the FEB triangle and the FCD triangle, which is known from the MPST and incorporated into the interpretant component.

The student also mentioned "t_1 plus t_2 based on which is known by image analysis", identifying the object $(\boldsymbol{o_3})$ in this stage as the height of the parallelogram. The student analyzed the images in the MPST and derived $t_1 + t_2 = 30$, which will serve as the representamen $(\boldsymbol{z_3})$. The interpretant $(\boldsymbol{i_3})$ is reflected in the interview results: "There is an image analysis t parallelogram equal to 30 equals to FEB height plus FCD height, represented by t_1 and t_2. So, we can directly write t_1 plus t_2 equals to 30. That is the second equation." Based on the interview excerpts, SR stated that $t_1 + t_2 = 30$ was obtained from the analysis of the MPST image. SR represented the height of the FEB triangle as t_1, the height of the FCD triangle as t_2 and concluded that the sum of the heights of both triangles equals the height of the parallelogram.

The next stage conducted by SR, as depicted in Figure 3, involves substitution and elimination from the first and second equations. The representamen $(\boldsymbol{z_4})$ is represented by $t_1 + 3t_2 = 66$ and $t_1 + t_2 = 30$. discerned from the results of SR's work in Figure 5, alongside the interview statements: "So we analyzed again, which of the other variables are same as t_1 and t_2. The height above is still unknown and is the same as first equation" and "$t_1$plus $t_2$equals 30 so that is the second equation". The object $(\boldsymbol{o_4})$ is identified as the height of the triangle, as articulated in the interview: "We must know the same variables. We still have t_1 and t_2". The interpretant $(\boldsymbol{i_4})$ is revealed through the elimination process from the first and second equations, as expressed in the interview: "So that means we can eliminate or substitute to find out the variables we do not know. We eliminate t_1 because the coefficients are the same and get t_2". SR then conducted the elimination and substitution process to obtain the value $t_2 = 18$.

SR carries out the substitution process from $t_2 = 18$ directly to the second equation. In this stage, the object $(\boldsymbol{o_5})$ is the height of the triangle, while the representamen $(\boldsymbol{z_5})$ is represented with $t_2 = 18$. When confirming the substitution, SR stated, "In the first equation there is still a coefficient of 3 which must be multiplied and divided again. So, we chose the second equation with a coefficient of 1 and found the value." The interpretant $(\boldsymbol{i_5})$ is the substitution of $t_2 = 18$ into an equation with a coefficient of 1. his indicates that the student understands the result of pattern recognition. An overview of the sign trichotomy performed by the student during pattern recognition is presented in Figure 7.

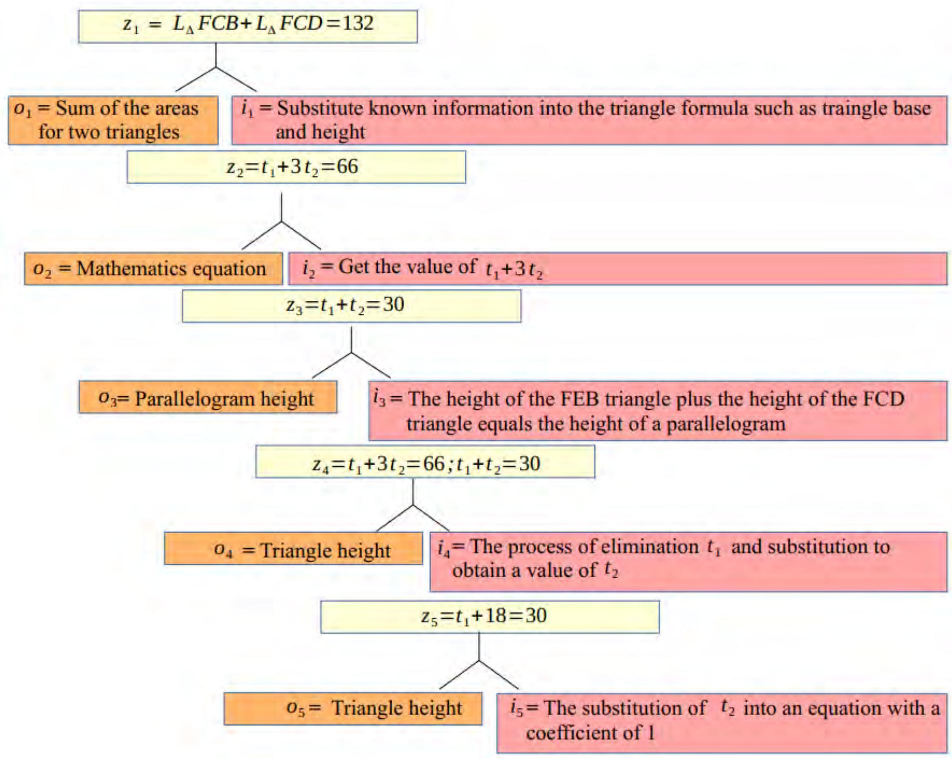

Figure 7. Trichotomy Signs Pattern Recognition of SR

DISCUSSION

Pattern recognition is an activity in which students identify a pattern or regularity in an object. It begins with examining the sub-problems within the given task. From these sub-problems, similarities or repetitions within the task are sought. Please note that this research is a case study of students selected based on specific criteria, namely impulsive and reflective students. Therefore, we do not attempt to generalize our findings regarding pattern recognition in these two students when solving mathematical problems from a semiotic perspective to all college students. In SR's activity of searching for similarities or repetitions, SR conveyed that it was known from the task that all flat shapes forming a parallelogram are triangles, and the base of each triangle is known, while some heights are unknown. Similarly, SI conveyed that all flat shapes forming a parallelogram are arbitrary triangles. Thus, in each triangle, both the base and height must be known to determine its area. This activity involves recognizing that there is some information in the task that is not yet known, which must be discovered first.

After identifying the similarities or repetitions in the task, the next step is to recognize the relationship between what is asked and what is known. From the interview excerpts, it can be understood that both students expressed that the height of one of the triangles in the task could be obtained from the height of the parallelogram. After both students recognized and recalled that necessary information in the task was unknown, their activities involved finding the value of this unknown information, which, in this case, is the height of the triangle. The height of the triangle is needed to determine its area, so its value must be found. Furthermore, the research results indicate that reflective students require more time to identify patterns. This was evident in the interview conversations, where the reflective student needed time to explain the questions posed by the researcher based on the given task. Reflective students explained their steps meticulously and elaborately.

Based on the explanation of pattern recognition conducted by SI, it is evident that they acknowledge patterns. During the MPST, SI recognizes patterns through the relationship between the height and area of a triangle. The student stated that the initial pattern recognition encountered was determining the area of each triangle, as it was evident in the problem. There is a parallelogram that is divided into four triangles. In addition, SI mentioned that the next step, after finding each triangle's area, is to find the height of the triangles, for example, the heights of triangles DCF and EBF. SI stated that these triangles are arbitrary, so their heights are the same as that of the parallelogram. By representing one of the heights of the triangle with a variable x, we obtain the height of the two triangles.

Object	Representamen	Interpretant
Triangle	$\Delta FEB, \Delta FCD, \Delta BFC, \Delta AED$	• Find the area of the triangle. • Arbitrary triangle
Mathematical equations	• $t\ \Delta FEB = FO = x$ • $t\ \Delta CDF = FP = 30 - x$ • $FP = t_{ABCD} - FO$ • $L\ \Delta FCB + L\ \Delta FCD = 132$ • $t_1 + 3t_2 = 66$ • $t_1 + t_2 = 30$	• Determines the height of the triangle. • Do the process of elimination to get the height of the triangle. • Do the substitution process to get the height of the triangle.
Variable	x, t_1, t_2	Representation of variable height of a triangle.
MPST Question	The sum of the areas of two triangles	Determine the sum of the areas of two triangles.
Parallelogram	ABCD parallelogram	• Determine the area of the parallelogram. • Determine the height of the parallelogram.

Table 5. Students Semiotic Analysis

A trichotomous sign performed by SR in pattern recognition identifies the triangle height as the object (o). The representamen (z) component can be seen from what the SI uses in solving the problem, as shown by writing the variable for the height of the triangle and the formula for the triangle's area that comprises the parallelogram. The next component is the interpretant (i), which can be expressed through the actions taken by SR to complete the stages, including determining the height and finding the area of the triangles that form a parallelogram — for example, determining the height of a triangle or a parallelogram. Additionally, it includes the substitution, elimination, and calculation processes to derive the height and area of the triangles that make up the parallelogram.

According to the description of the pattern recognition aspect carried out by SR, it was mentioned that SR identifies the height of the triangle. SR indicated that the initial step involves determining the value of the unknown height variable. It was stated that knowing the height of the triangle facilitates the calculation of the triangle's area as required in the MPST.

The description of the sign trichotomy in the pattern recognition aspect conducted by SR is as follows: the object $(o)$is the triangle height, the representamen (z) is indicated by the height variable written by the student in the mathematical equation $t_1 + 3t_2 = 66$ and $t_1 + t_2 = 30$, while the interpretant (i) is shown in the interview and the results of the SR work, which determine the height of a triangle, an essential piece of information necessary to find the area of the triangle asked in the MPST.

Table 5 summarizes the interpretations made by the students of the signs used in recognizing patterns in the problems in MPST, which are grouped according to the level of representamen, object, and interpretant.

Based on the description of the interpretations of the representamen presented in Table 6, it can be stated that in the process of pattern recognition carried out by the research students, the semiotic components of objects such as triangles, mathematical equations, variables, questions, and images contained in MPST are crucial. The object is the pattern. The object is the final item identified by the students. This final object is derived from the direct object when identifying pattern recognition. As the problem-solving process progresses, this object becomes dynamic. Eventually, the pattern is obtained as the final object, which is part of the actual object to be identified. The pattern can be something visible or something conceived (Ernest, 2006). A sign does not merely represent its object but refers to an idea sometimes called the basis of representamen (Otte, 2006). Presmeg (2016), in a book review, stated that this section implies that real object ontology is still, beyond human interpretation, something to aim for, at least as an approximation. For example, research conducted by Sabre (2015) states that the object can be a specification of the problem to be solved.

The semiotic component for the representamen presented in Table 5 can be said to encompass everything resulting from the student's representation. The representation can be anything written down, such as an equation in mathematics or variable examples used. Additionally, it can include anything conveyed by the students in representing the stages of the problem-solving process. Mudaly (2014) states that a representamen represents something else. An example given in his research is the quadratic function $y = x^2$.

The interpretant, the third semiotic component in Table 5, can be expressed as anything the students do in the problem-solving process. For example, finding the area of a triangle, performing substitution and elimination processes, determining the height of a flat shape, and representing the variables used in solving problems. Sabre (2015), in his research, explains that the argument is included in the interpretant. When the students mention the problems they will solve in the task, it can be categorized as an interpretant. Thus, the interpretant component can represent a problem-solving procedure used during pattern recognition. The interpretant reflects the students' interpretation of the representamen in problem-solving. Sáenz-Ludlow and Kadunz (2016) also stated that the interpretant is limited to the students' perspective on the representation used. Therefore, with the same object, there can be differences in perspective or the stages undertaken by each student.

Semiotics presents a conceptual approach to text and sign interpretation, creating meaning or effect (Mudaly, 2014). To understand how both students carry out pattern recognition, it is essential to conduct a detailed and careful analysis of each semiotic component of the students' work. The researcher acknowledges that this analysis process requires a literature review of semiotic components in problem-solving. This is necessary to discover how students' pattern recognition is supported by interview results. Consequently, it will be understood how semiotics plays a significant role in the pattern recognition conducted by the students. Particularly for individual student analysis, which parallels the individual semiotic analysis conducted by Mudaly (2014). Therefore, an in-depth literature review is conducted to examine how each component of semiotics plays a crucial role in pattern recognition. Thus, it can be said that the sign relationship can be described as a semiotic activity. It can be said that this supports Radford (2001) assertion that the relationship between representamen, object, and interpretant can help develop solutions to problems because it connects with someone's cognition. Sáenz-Ludlow and Kadunz (2016) emphasize that semiotic theory explains the mutual construction of knowledge and experience, where the representamen symbolizes an object and the object is interpreted to create meaning, such as a mathematical concept.

Based on the description of these results, the representamen plays a significant and fundamental role in forming and refining mathematical concepts. Peirce's semiotic components, which include the object, representamen, and interpretant, serve as valuable tools for solving complex problems, particularly for mathematics teachers. The object refers to the concept, phenomenon, or problem that is the focal point of attention. In the context of solving mathematical problems, the object could be a math problem, a specific mathematical concept, or a phenomenon that can be analyzed using mathematics. Teachers can help students recognize and understand the challenges they encounter. For instance, if the object is a complex math problem, the teacher will assist students in identifying its key elements.

In mathematics, the representamen may include mathematical notation, graphs, diagrams, or other visual forms used to illustrate a concept or problem. The representamen aids in visualizing the problem. For example, graphs or diagrams can help students understand patterns or relationships between elements in a mathematical scenario. The representamen can simplify complex concepts or problems. Visual or symbolic representations of a problem often make it easier to comprehend and resolve.

The interpretant is the meaning or understanding derived from the representamen. Teachers can help students develop a deeper grasp of mathematical concepts through the interpretation of these signs. For example, teachers can demonstrate how certain notations lead to the solution of an equation. The interpretant encourages students to reflect on their understanding. Teachers can ask students about their interpretation of specific signs, prompting them to think more critically and metacognitively about the problem-solving process. Through the interpretant, teachers can assess students' understanding and adjust their teaching strategies accordingly.

From this representamen, a solution will be derived based on the object, which serves as the foundation for problem-solving. Given that symbols and representations are fundamental to the teaching and learning of mathematics, it is crucial for all students to understand the use of symbols in the problem-solving process. These symbols should be grounded in the underlying object and their intended purpose.

CONCLUSION

In the given task, both impulsive and reflective students utilized the heights of triangles within the parallelogram to recognize the pattern. The identified object is this pattern, which serves as the final object recognized by the students. The representamen component is similarly manifested by both students, incorporating mathematical equations that display recognizable patterns, such as equations involving height variables. The semiotic component of the interpretant is evident in the interpretations and problem-solving procedures employed by the students, including determining the area of triangles, substitution, elimination, and arguments presented during the problem-solving process. Understanding the trichotomous system of signs—representamen, object, and interpretant—facilitates comprehension of mathematical concepts in problem-solving.

Furthermore, semiotics can be applied to understand the concepts of the material being taught. Therefore, as a development of this research, the researcher hopes to conduct further analysis from a semiotic perspective on the learning process as it is conducted in a traditional classroom setting, aiming for a variety of representations and interpretations.

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