Written Calculation Ability and Numerical sense in Grades 6 and 7 among the Arab Sector in Israel

Wafiq Hibi

https://orcid.org/0000-0003-2897-0175

Sakhnin Academic College for Teacher Education

Sakhnin College, 805, Sakhnin, 3081000 Israel

Contact e-mail: wafiq.hibi@gmail.com

Wafiq.hibi@sakhnin.ac.il

Abstract: This investigation delves into the numerical comprehension among elementary and middle school students within Israel's Arab sector, challenging the conventional metrics of mathematical ability. Despite the apparent adeptness in standard mathematical tests, a significant discrepancy exists between calculative prowess and genuine numerical understanding among students. This discrepancy prompted an exploration into the nuanced domain of numerical sense, specifically in the context of fractions, among sixth and seventh graders. Engaging 244 students across these grades, the study employed a dual-assessment approach: a traditional written calculation test juxtaposed with a test emphasizing numerical sense. Results unveiled a parallel achievement pattern across both grades, with a pronounced superiority in calculation-based tests over those assessing numerical sense. Such outcomes underscore the critical distinction between mere calculation skills and the application of numerical sense. Further depth was added through interviews with 18 students, representing a spectrum of performance levels in both tests, to glean insights into the underlying competencies. This research underscores the imperative of re-evaluating educational strategies to foster a more holistic mathematical understanding, transcending beyond rote calculations to nurture a profound numerical sense.

Keywords: numerical sense, various math tests, benchmarks, fractions, magnitude

INTRODUCTION

The genesis of this research idea can be traced to a profound experience encountered during my tenure as a math instructor in Israel's Arab sector. In close engagement with a diverse student cohort, a notable pattern emerged: a distinct ability among students in executing mathematical algorithms and procedural steps contrasted sharply with their difficulties in conceptual comprehension and the practical application of mathematical principles, especially evident in their understanding of fractions—a crucial component for advanced mathematical thought and reasoning.

This observed discrepancy between computational skill and conceptual grasp prompted a reevaluation of the efficacy of prevailing educational methodologies in fostering a comprehensive, deep understanding of mathematics. It became apparent that the educational framework, while effective in instilling algorithmic abilities, might neglect the essential development of a numerical sense critical for engaging meaningfully with mathematics in real-world scenarios. Driven by a commitment to address this educational shortfall and enhance the quality of mathematical education, I initiated this research to investigate and advocate for pedagogical strategies that promote a holistic mathematical competency, encompassing both computational ability and a deep understanding of numerical concepts.

Numerical sense

Sowder (2020) defined numerical sense to be the emotion that refers to the general understanding of the student which includes the ability and tendency to use numbers and quantitative methods as means of communication. It is also the ability to use the processing and interpretation of information, and to use flexible ways to reach mathematical judgments and to develop useful strategies for managing numbers and actions.

The numerical sense is emphasized in the new reforms of mathematics education because it characterizes the study of mathematics as an activity with logical meaning, and because one of its objectives is that the student will understand that the numbers are useful and that there is a certain legality in mathematics. Hence, the authors of mathematics curricula should add it to the programs as early as preschool age (Sood & Mackey, 2015; Hornung *et al*., 2014; Dyson et al., 2013). Besides, Castronovo and Göbel (2012) and Sood and Mackey (2007) reported that experiencing numerical wisdom from an early age predicts future success and perseverance in mathematics in general and in algebra in particular.

In-field teachers, curriculum writers and researchers in mathematics education and psychology always discuss the essential components of numerical sense, the description of the students who have it, the students who lack it, and theoretically analyze in depth the numerical sense from the psychological aspect (Mock et al., 2016). Despite all the many discussions, the conventions on the main characteristics of numerical sense, according to (Sowder, 2020), include:

- The use of many representations of the number.
- Recognition of the number magnitude proportionally and absolutely.
- Selection and use of reference points, i.e. benchmark.
- Disassembly and assembly of numbers.

- Understanding the relative effects of operations on numbers.
- Flexibility in performing mental calculations and estimates.

In this work, the analysis of the results will discuss two characteristics: Recognition of the number magnitude proportionally and benchmark.

In many countries around the world, great emphasis has been placed on numerical sense over the past decade (Sood & Mackey, 2015), as in Israel, of which the Arab sector is a part. This is reflected in the curricula and is referred to among mathematical educators and their publications. However, a review of worksheets prepared by teachers and observations in classrooms shows that in the Arab sector in Israel there is a greater emphasis and appreciation on algorithmic actions and accurate results. Although this emphasis may lead to good results when an accurate calculation is required. The extent to which this process will lead to the transfer to numerical sense is unknown, and even imperceptible.

Jordan et al. (2010) declares that children with good numerical sense fully understand the meaning of numbers, have many interpretations and representations of numbers, are able to recognize the proportionality and absoluteness of the number magnitude evaluate the effect of operations on the numbers and develop a system of numerical reference points. On the other hand, Fischer et al. (2020) states that it is not clear whether students who are skilled in paper and pencil calculations and always try to apply an algorithm will often pave their success in mathematics if they cannot develop numerical sense.

The great attention nowadays paid to numerical sense is the response of the overemphasis given so far to computational methods that are usually algorithmic and the neglect of numerical sense. For example, if you ask a student who has been educated on algorithmic emphasis if a given calculation is logical, he usually tends to answer by recalculating in most cases in the same way, and does not try to reflect the outcome in the context of the situation and the numbers involved in it (Yang et al., 2008). Many studies have indicated that important perceptions of numbers and actions develop over time and this development is best cultivated when focused upon, or when it occurs frequently and in every math class (Kroesbergen et al., 2012; Shumway et al., 2019; Chard et al., 2005).

The continuous development of numerical sense was also discussed by Boaler et al. (2015), who stated that numerical sense is not an absolute entity that the student has or does not have, but a process that develops with experience and knowledge.

The researcher is aware of the fact that the study was carried out at middle schools in the Israeli Arab community, which is a minority population, but he thinks it is potential for a new target audience, and that the focus is on 'skills', which is interesting.

Fractions

Fractions emerge as complex yet essential elements within the realm of mathematics, playing a crucial role in various measurements and computations (Van de Wall et al., 2013), while also laying the foundation for the development of algebraic thinking and proportional reasoning

(Chinnappan, 2005; Clarke et al., 2008; Wijaya, 2017). The teaching and learning of fractions continue to attract the attention of scholars and educators worldwide, despite the persistent challenges faced in the instruction and mastery of fractions within the elementary mathematics curriculum (Charalambous & Pitta-Pantazi, 2007; Wijaya, 2017). Siegler et al. (2013) identify a link between poor understanding of fractions in early education and later underachievement in general mathematics and algebra at the secondary level. In a similar vein, Lazića et al. (2017) argue that difficulties in mastering fraction arithmetic can hinder advancement in more complex mathematical topics, negatively impacting career opportunities.

Bruce et al. (2013) highlight how a basic understanding of fractions can limit access to higher-level mathematical discussions. The challenge with fractions persists from elementary through to higher education, with a significant number of students failing to achieve ability in fraction arithmetic (Gabriel et al., 2013; Bentley & Bossé, 2018).

Bentley and Bossé (2018) support Gabriel et al.'s findings, revealing that college students make errors in fraction operations, reflecting the misconceptions observed among younger students. However, researchers like Behr et al. (1993) offer a different perspective, suggesting that the difficulties encountered in learning fractions pose considerable obstacles to students' mathematical growth.

Despite enduring challenges inherent in educational methodologies, research on the didactics of fractions uncovers a widespread difficulty with fraction concepts among students. Fazio and Siegler (2011) note that students across various nations often fail to develop a conceptual understanding of fractions. Gabriel et al. (2013) identify fractions as one of the most challenging mathematical subjects in primary education, especially highlighting difficulties in operations with fractions where students utilize approaches devoid of deep comprehension. Additionally, Bentley and Bossé (2018) shed light on common misconceptions among learners, encompassing issues such as fraction equivalence, the use of common denominators, and reliance on procedural algorithms without full understanding.

Concept of Fraction

Kieren (1993), a luminary in the sphere of mathematical understanding, heralded the inaugural proposition that the essence of fractions is not anchored in a singular construct but rather emerges from a constellation of interwoven sub-constructs. This pioneering perspective catalyzed further scholarly endeavors, notably by Behr et al. (1993), and Lamon (1999), who, building upon Kieren's foundational insight into fractions, elucidated a quintuple sub-construct theoretical framework for fractions. These sub-constructs: part–whole, ratio, quotient, operator, and measure, imply that a specific fraction may embody divergent significances contingent upon the contextual backdrop. For example, the fraction 3/5 might be interpreted as a segment of a totality (three out of five equivalent segments), as a quotient (three divided by five), an operator (three-fifths of a specified quantity), a ratio (three to five), and, ultimately, as a measure (representing a point on a numerical continuum) (Pantziara & Philippou, 2012).

Sense in fractions reveals that one of the cardinal complexities in assimilating fractions arises from this intricate notion that encapsulates five interconnected sub-constructs (namely, part-whole, ratio, operator, quotient, and measure). Kilpatrick et al. (2001) have underscored that a significant portion of the befuddlement associated with fractions can be attributed to disparate interpretations (constructs), representations (models), and symbolic conventions. The discourse on the catalysts of learners' challenges with understanding fractions is ongoing. Wittmann (2013) posits that a majority of the errors encountered in operations involving fractions stem from discrepancies in pedagogical strategies. In this vein, Lazić et al. (2017) scrutinized the methodological paradigm of introducing fractions through propaedeutic learning and investigated its impact on pupils' mathematical achievement within the elementary educational spectrum. Their findings suggest that students exposed to the propaedeutic learning modality exhibit markedly superior outcomes compared to their counterparts who were not privy to this methodological approach, thereby affirming the value of "Sense in Fractions" in enhancing educational practices.

Fraction Sense

McIntosh et al. (1992) defined number sense as an individual's comprehensive understanding of numbers and operations, alongside the ability and inclination to apply this understanding flexibly for making mathematical judgments and devising effective strategies for handling numbers and operations. Fractions, as numerical constructs, and akin to common sense, embody an abstract concept of fraction sense. This concept is described through various interpretations. McNamara and Shaughnessy (2015) posited that fraction sense is crucial for learners' mastery over fraction operations, describing it as a deep and versatile understanding of fractions, not confined to any single context or type of problem. Students with a well-developed fraction sense can thoughtfully engage with fractions without resorting to the rote application of rules and procedures or providing irrational responses to fraction-related problems. Carpenter et al. (1993) suggested that an individual with fraction sense is proficient in recognizing both the numerator and denominator, as well as the individual and combined quantities they represent, thus viewing them as an integrated whole.

Furthermore, Way (2011) recognized fraction sense as a synthesis of key concepts related to fractions, including various forms of representation. She identified three dimensions of fraction sense: (1) flexibility in the visual representation of fractions, (2) comprehension of the magnitudes represented by fractions, and (3) consideration of numerical relationships. Therefore, fraction sense can be viewed as an extensive framework that incorporates fraction-related concepts and cognitive skills such as visualization and logical reasoning.

Reflecting on McIntosh et al.'s (1992) delineation of number sense and subsequent definitions of fraction sense, this study suggests that fraction sense encompasses four fundamental components: (1) concepts - acknowledging that within a fraction, both the numerator and denominator represent a cohesive entity; (2) visualization - demonstrating flexibility in fraction representations; (3) strategy - applying practical methods in solving fraction operations; and (4) rationalization - the capability to estimate and reason logically about fractions.

Fractions magnitude

According to McMullen and Van Hoof (2020), Xu et al. (2022), and Sowder (2020), In developing strategies for understanding magnitude fractions, students must acquire a foundational set of skills:

According to McMullen and Van Hoof (2020), Xu et al. (2022), and Sowder (2020), students need to develop a core set of skills to effectively understand magnitude fractions. One key skill is the ability to transform mixed numbers into improper fractions and to convert between simple fractions, including those greater than one and mixed numbers, and their decimal equivalents. This conversion skill is essential for deepening students' understanding of fraction representations. Another crucial aspect is the ability to compare fractions, whether they are in the same format or presented differently, such as comparing a simple fraction to a decimal. This comparison skill enables students to classify and arrange fractions by their magnitude. Additionally, students should be able to estimate the magnitude of a fraction by determining whether it is larger or smaller than a given value, and estimate the result of operations like addition and subtraction between fractional values

A final fundamental concept for students to grasp is the density of rational numbers, meaning that between any two rational numbers, an infinite quantity of numbers exists. Various strategies can be employed to find a fraction situated between two given fractions. For instance, one can generate a new fraction between any two positive fractions by adding the numerators of the given fractions to form the new numerator, and similarly adding the denominators to form the new denominator. This newly created fraction's value lies between the values of the two original fractions. In other words, for two fractions a/b and c/d (where a, b, c, and d, are natural numbers) with $c/d > a/b$, the fraction $(a+c)/(b+d)$ is less than c/d and greater than a/b.

Benchmark in Fractions

I can define, a benchmark fraction is a fraction that is used as a reference point. It is a common fracture that is used for comparing other fractions. I think that I can explain Benchmark and convince my students, for example, with a statement like this "Fractions are everywhere, understanding fractions is important for many careers and everyday life activities, architecture and construction, interior design, and engineering are some fields of work that use fractions. Chefs also depend on fractions to accurately measure ingredients, some tools like wrenches come in sizes that are benchmark fractions, all of these activities utilize benchmark fractions to make measurements".

Benchmark fractions make ordering and comparing fractions easier. By remembering just, the benchmark fractions, estimations of other fractions can be made. Unknown fractions can also be changed to equivalent fractions with the same denominator of a benchmark fraction for comparison (Liu & Jacobson 2022). Wiest and Amankonah, (2019), explained that Benchmark fractions can be used in several ways, comparisons, estimations, ordering, simplifying, and finding equivalent fractions are all ways to use benchmark fractions.

In the book of Petit et al. (2015), they see that Benchmark fraction is a reference or guide for identifying other fractions. Common fractions that are more familiar are used as benchmarks to help find the less familiar fractions. Important benchmark fractions include 1/8, 1/4, 1/3, 3/8, 1/2, 5/8, 2/3, 3/4, and 7/8. These benchmark fractions can be used to compare the sizes of other fractions, locate where they are on the number line, and find equivalent fractions.

Fitousi and Noyman (2024), sum up correctly that in order for students to be able to use Benchmark they must be proficient in comparing fractions with the same denominators, comparing fractions with the same numerators, comparing fractions with reference to their proximity to integers and comparing fractions with reference to their proximity to other fractions. When referring to proximity to integer number, there is reference to fractions that are smaller than integer number and fractions that are larger integer number and of course a common denominator (expansion and reduction).

The academic achievement of Arab students in Israel

Israel has about 2 million Arab residents, who are geographically spread out in three regions: The Galilee, the Mosholash and the Negev. The schools in the Arab sector are identical in resources and the teachers are required to teach according to the national program. The development of the students' computational ability is completed at the end of the sixth grade. In elementary school, students learn the four operations of arithmetic, simple and decimal fractions by way of a standard written algorithm, in traditional teaching. In junior high school, they experiment more complex procedures in the context of algebra, including continued support for computational skills.

According to the results of the PISA tests conducted in 2022 and published in December 2023, Israeli students ranked 38 in mathematics and 37 in science. These positions indicate that Israeli students' achievements are significantly lower than the average of the countries participating in the international test (OECD, 2023a). Further analysis reveals that the achievements of Israeli students in both mathematics and science fall below the average scores of 625,000 students across 81 countries that participated in the study (OECD, 2023b). Additionally, a report by the National Authority for Measurement and Evaluation in Education in Israel (2023) highlighted that over 60% of Arab-Israeli students tested were classified as having difficulties in each of the content areas.

The gaps in math achievement are particularly large and evident between Jewish and Arab education, with these gaps widening as the proportion of struggling students (students whose grades were at levels 1-2 out of 6 levels) in Arab education rose to 69% compared to 31% in Hebrew education (National Authority for Measurement and Evaluation in Education in Israel, 2023). It is noted that the gaps between the achievements of students in Arab education compared to Jewish education are significantly larger than the gap between Israel and the average of the organization's countries (National Authority for Measurement and Evaluation in Education in Israel, 2023).

Tzuriel et al. (2023) identify critical issues within the Arab education sector, including outdated teaching methods, an emphasis on assessment over pedagogy, and lower teacher status and pay, which hinder recruiting quality educators. Moreover, a per-student budget remains low by international standards. David (2023) and Sabra and Alshwaikh (2023) suggest that achievement gaps between Arab and Jewish students also stem from socio-economic disparities. Despite challenges such as insufficient funding and non-merit-based appointments, efforts are underway to bridge these gaps through better resource management and improved educational practices, aiming to enhance the quality of education for all students.

Another factor contributing to the modest performance of Israeli Arabs in international mathematics assessments is the predominant reliance on algorithmic mathematical practices within their educational framework. This approach has become deeply ingrained in their mathematical consciousness and identity, leading to a familiarity with procedural tasks without a profound comprehension or the development of advanced mathematical thinking. Notably, Sharkia and Kohen (2021) undertook a research project examining high-achieving high school students within the Israeli Arab community. Their study, titled "Class flipping among minorities in the context of learning mathematics: The Israeli case," highlights the prevalence of rote algorithmic techniques, which are not complemented by an in-depth understanding.

Acceptance of the findings from this research presents a valid justification for integrating numerical methods into the mathematics teaching paradigms employed within the Arab sector in Israel. Such an integration aims at not only addressing current educational disparities but also at bolstering students' mathematical competencies and analytical reasoning, contributing to a more equitable and effective educational framework. And this is what we are attempting to prove in our research.

METHOD

This investigation delves into the understanding of numbers among elementary and middle school students within the Arab sector in Israel, focusing on identifying potential disparities between mere calculation skills and a deeper numerical comprehension, with a particular emphasis on fractions among 6th and 7th graders. Given the distinct regional divisions within the Arab sector—namely, the Negev, the Mosholash, and the Galilee—this study encompasses a diverse sample, including 6 schools: 3 elementary and 3 junior high schools, all public. From each geographical area, two schools were selected randomly, one elementary and one junior high, resulting in a participant pool of 244 students, comprising 121 sixth graders and 123 seventh graders from three elementary and three junior high classes.

To assess the students' mathematical capabilities, the study adopted a dual assessment methodology: a conventional written calculation test paired with an assessment aimed at gauging numerical sense. This approach intends to highlight discrepancies between rote procedural ability and genuine understanding of numerical concepts.

Further enriching the study's depth, interviews with 18 students—chosen to represent a broad range of achievement levels from both assessments—were conducted. These interviews aimed to shed light on the participants' foundational skills and perceptions regarding their mathematical abilities and challenges. Through this comprehensive methodology, the research seeks to uncover nuanced insights into the numerical understanding of students in the Arab sector, thereby contributing valuable perspectives to the discourse on educational strategies and interventions tailored to this demographic.

Study question

Does a disparity exist between written calculation ability and numerical sense regarding fractions among 6th and 7th grade students in the Arab sector of Israel?

Instruments

The questions were developed by a research committee that included the researcher, the teachers instructing the students, the principals of the participating schools, the mathematics teachers at those schools, and the general inspector of mathematics in the Arab sector. All items were tailored to fit the curriculum. It is important to note that any item not meeting the consensus of the committee members was modified or replaced with one that was unanimously agreed upon.

The first component of the assessment, the Written Computation Test, consisted of 14 items and was designed to be identical for both fifth and seventh graders. The second component, the Numerical Sense Test, included 17 items. Both the Written Computation Test and the Numerical Sense Test maintained consistency across sixth and seventh grades.

Fourteen items in both test types were parallel, meaning they featured the same numbers but varied in question format. The items in both test types encompassed multiple-choice questions, open-ended questions, and a mix of multiple-choice with short answers.

Upon finalizing the questions, their quantity in each test type, and determining which questions would have identical numbers (though not necessarily the same wording) and appear in both tests, the committee decided to administer the Written Computation Test first and the Numerical Sense Test about a month later. This scheduling was to ensure that students would not remember the questions, although the risk was minimal since the question wording differed.

The committee also resolved to award one point for each correct answer that included a correct interpretation, and half a point for each correct answer with an incorrect interpretation. Consequently, the maximum score a student could achieve was 14 points in the Written Computation Test and 17 points in the Numerical Sense Test.

All tests were evaluated by the educators who taught the students, overseen by both the researcher and the mathematics supervisor. It is noteworthy that the reliability and validity of both test types were assessed and confirmed across three sixth and seventh grades before the data reported in this study were collected. Refer to Tables 3 and 4.

Interviews

Eighteen (18) students took part in the study. They were divided into 9 students from the sixth grade, of which 5 are from the higher level and 4 from the medium level, and 9 students from 7th grade, of which 5 are from the higher level and 4 from the medium level. They were randomly summoned to an interview aimed at discovering a number of characteristics of numerical sense, which include the understanding of the number magnitude, the benchmarks, the relative effects of arithmetic operations, decomposition and assembly, and the application of the knowledge of numbers and operations to computational situations.

RESULTS

Test Results

Table 1 presents the outcomes of assessments conducted to evaluate the numerical sense and writing abilities of 6th and 7th grade students, along with the corresponding grade levels. It provides a detailed breakdown of mean percentages and standard deviations, illustrating the students' performance in each area. Additionally, it includes the range of percent correct answers, offering insights into the variability of student achievement within these domains. The data are segmented by grade level. This structured overview aids in understanding the nuanced performance patterns across different mathematical competencies among the students evaluated.

	Grade	Test in Writing	Test in Numerical Sense
Mean Percent (Standard Deviations)	6 (*n* = 121)	68 (13.1)	46 (26.5)
	7 (*n* = 123)	74 (10.9)	61 (23.5)
Range of percent correct	6	25-95	20-74
	7	17-100	26-97

Table 1. Summary of the percentage of correct responses in the written calculation test and in the numerical sense test for grades 6 and 7

The results presented in Table 1 compare student performance on a written calculation test and a numerical sense test, specifically in the context of fractions, for 6th and 7th graders, within the Arab sector of Israel. The table provides a detailed breakdown of the mean percentage of correct responses alongside standard deviations, as well as the range of percent correct for both tests across the two grades. The mean percentage of correct responses indicates that students performed better on the written calculation test than on the numerical sense test.

The standard deviations and ranges indicate variability in student performance on both tests. However, the broader range in scores for the numerical sense test, especially in grade 7, suggests a wider dispersion of abilities in numerical sense compared to written calculation skills, indicating a stronger ability in written calculation skills, Specifically, 6th graders scored a mean of 46 with a standard deviation of 26.5 on the numerical sense test and 68 with a standard deviation of 13.1 on the written calculation test. 7th graders showed a similar pattern, with a mean score of 61 (SD = 23.5) on the numerical sense test and 74 (SD = 10.9) on the written calculation test. The range of percent correct responses further highlights this discrepancy, showing a wider distribution in numerical sense scores compared to written calculation scores for both grades. The results showed that the achievements in the written test were higher than those in the tests of numerical sense.

The correlation found between the written calculation test and the numerical sense test was 0.53 and 0.69 for grades 6 and 7, respectively. Conversations with teachers and students confirmed the fact that the students knew the items of the test in writing while the items of the numerical sense were a new experience and they thought it was challenging.

Conversations with teachers and students highlight a crucial insight: students were familiar with the format and content of the written calculation test but found the numerical sense test to be a novel and challenging experience. This suggests that traditional teaching methods may emphasize procedural calculation at the expense of developing a deeper, intuitive understanding of numbers.

Although the seventh graders presented better knowledge than the sixth graders, the nature of the performance within each age group was consistent. Thus, for example, there were no significant gender differences ($p > 0.05$) in both types of tests, and there was also better success in the test of written calculations than in the items of numerical sense all along.

Table 2 reports the results of the shared items in both tests, as well as the T-test results for students in grades 6-7. The written calculation scores were significantly higher (p=.02) than the numerical sense scores in both class levels. This finding indicates that students who succeed in performing a written calculation are not necessarily able to apply a sense of numbers in making similar decisions that are not driven by computation.

Grade	Form	*n*	Mean (Std Dev)	*df*	*t*-ratio
6	Numerical sense test	121	50 (22.49)	114	2.96 (p = .02)
	Written computation test	121	67 (13.08)		
7	Numerical sense test	123	6 (18.95)	118	3.03 (p = .02)
	Written computation test	123	73 (10.89)		

Table 2. Means of Percent Correct (with Standard Deviations) and t-test Results

The results presented in Table 2 illustrate a persistent gap across both 6th and 7th grades between students' ability in executing written computations and their understanding of numerical concepts, specifically in the context of fractions. This discrepancy highlights the imperative for educational strategies within the Arab sector of Israel to not merely enhance students' computational abilities but also to cultivate a more profound conceptual grasp of numerical principles. The statistically significant differences, evidenced by p-values of .02 for both grade levels, substantiate the conclusion that these observations are not likely to be the result of random variation.

Performance in the written computational test items (shown in Table 3 of the Appendix) was generally higher than in the numerical sense test items (shown in Table 4 of the Appendix), this phenomenon is found in every class for items reported. In Table 4, we see that while the test of written calculation tested a student's skill in finding an exact solution of an exercise when he solved in writing, the test of sense tested the student's ability to apply different aspects of the numerical sense in the same calculation. These findings indicate that success in calculation does not necessarily indicate the ability to apply numerical sense. This phenomenon was repeated at every grade level. For example, in a written calculation test (Table 3), students were required to calculate the result of 0.532·231.5. In the calculation test 58% of the sixth graders succeeded. In the corresponding test of numerical sense (Table 4), students are told that the 123158 digits are a correct result of the above exercise, but the decimal point is missing. Their task was to indicate the location of the point; the thinking in the test of the numerical sense should have been that 231.5 multiplied by a number a little greater than half would give a result of approximately between 100 and 150. 59% of sixth graders said that the answer was 12.315 that is, they seemed to apply the rule of positioning the decimal point. The results in seventh grade were similar.

Even in the $\frac{8}{7}+\frac{11}{13}$ exercise (Table 3), more than 60% of the seventh grade students were able to solve in a regular calculation, while in the estimate only 37% of them succeeded (Table 4). The students did not realize that the value of each fraction is actually close to 1, so the amount is close to 2. Although most of the seventh graders knew how to use the standard calculation method, many of them did not perceive the value of the numbers they calculated.

Analysis of the Interviews

18 students took part in the study. evenly divided between the sixth and seventh grades, with each grade comprising a mix of higher and middle-level achievers. These students were selected for interviews designed to explore various dimensions of numerical sense, focusing specifically on their grasp of number magnitude and the concept of benchmarks, among other attributes. The interviews aimed to uncover insights into students' understanding and application of these critical numerical concepts. Herein, we present the findings from these interviews, highlighting the participants' competencies in understanding number magnitude and benchmarks, which are essential for a robust numerical sense.

Number magnitude

In an interview that examined the understanding of the number magnitude, the students were given 3 items which focused on different aspect ratios of numbers, comparison, arrangement, and understanding of the density of rational numbers. Table 5 in the Appendix summarizes the results.

It can be seen that there was a large difference in the levels of numerical sense acquired between the high-level and medium-level students in each grade level. To the question of how many fractions between 2.51 and 2.52, all the high-level students in each grade class knew how to answer that there is no end. They also supported their answer with at least one explanation. One student, for example, explained: "There are infinite numbers exemplifying: 2.51, 2.511, 2.5111...;" another student wrote: "You can add an infinite number after the 5 of 2.51".

In contrast, 6 out of 8 middle-level students claimed that there were only 9 numbers or 10 numbers. They identified numbers like: 2.51, 2.512, 2.513...,2.519 as the only options for numbers between 2.51 and 2.65. For example, one of the students answered that the answer was exactly 10 numbers. Each of these students gave the same examples, and even when they continued to ask, they did not find any other fractions. In a discussion with them, they showed that they did not see the relationship between 2.51 and 2.510. Two students from the middle level thought that there was no decimal number between 2.51 and 2.52. One student claimed that the consecutive number of 2.51 was 2.52 and therefore there was no decimal number between them.

To the question of which fractions there are between $\frac{3}{5}$ and $\frac{4}{5}$, the students of the high level of the seventh grade knew how to answer correctly. All the explanations focused on the ability to change the denominators of the fractions, for example: $\frac{4}{5}=\frac{40}{50}=\frac{400}{500}$ and $\frac{3}{5}=\frac{30}{50}=\frac{300}{500}$. On the other hand, no medium-level student answered correctly. They thought that the two fractions $\frac{3}{5}$ and $\frac{4}{5}$ were consecutive. One of the students said, "The difference between 4 and 3 is 1. So the next fraction after $\frac{3}{5}$ is $\frac{4}{5}$".

Two students turned the fractions into decimal fractions 0.8 and 0.6 and then said that 0.7 lies between them. But they could not use it to find a simple fraction between them. All the students at the highest level of seventh grade knew how to correctly answer the instruction "Sequentially arrange: $0.595, 61\%, 0.3562, \frac{5}{8}, \frac{3}{8}$" . Their explanation included converting simple fractions to decimal ones. One of the students explained that after conversion, he compares the tenth and hundredth digits. Most of the students at both levels knew how to sequence decimal numbers. They were less successful in the arrangement of simple fractions. Most of the mid-level students treated simple and decimal fractions as two separate entities and found no connection between them. One student said that he did not know how to compare decimal and simple even though he studied in class, because he did not understand it.

Benchmarks

The effective use of benchmarks is related to the estimation capability and numerical sense (Hulse et al.,2019). One-half (1/2) and 1 are the most common benchmark fraction. An example of how to use this benchmark fraction is something like this: Is 3/8 more or less than one half? Since 4/8 is 1/2 and 3 are less than 4, 3/8 is slightly less than 1/2. In the interview that examined the use of the dot, the students received 4 items that focused on the use of the Benchmark of 1 or 0.5 using multiplication and division indices.

The first item in Table 6 (see Appendix) allows to use benchmarks in multiplication. All the high-level students and half of the mid-level students knew how to use the benchmarks $\frac{1}{2}$ or 0.5 to multiply with 110 and decide that it was less than 55. Although this was the easiest question, one sixth grader replied that duplication always increases the result.

The second item in Table 6 encouraged students to use the benchmark of 1. 7 out of 10 higher-level students knew how to act correctly, and only 1 in 8 mid-level students knew how to use that benchmark. It seems that the emphasis placed on algorithmic calculation in math studies within the classrooms seems to have an impact on students' thinking and attitudes.

Even the high-level students tended to initially solve the $\frac{4}{9} \cdot \frac{21}{22}$ by way of $\frac{4}{9} \div \frac{22}{21}$ and then get the result, and at this point they knew how to say that it was multiplication by a number greater than 1 and, therefore, the answer is greater than $\frac{4}{9}$. Only when asked for an alternative strategy, did the high-level students say that the benchmark could also be used in division, in addition to multiplication.

The third and fourth items allowed students to use benchmarks in fractions. Students at the intermediate level used the rules learned, more than benchmarks. For example: in the fourth item, all the students at the intermediate level first calculated the exact answer, and only then compared the result to 1. Only after the encouragement of the researcher was a different strategy proposed. The high-level students also initially tended to the algorithmic path, but when asked for a different path, they were helped by benchmarks.

An example of an interview with a high-level student in 7th grade:

R = researcher, S = student

R: Without calculating an exact answer, do you think that the sum of $\frac{7}{13} + \frac{3}{7}$ is bigger than $\frac{1}{2}$ or less than $\frac{1}{2}$?

S: I think the sum of $\frac{7}{13} + \frac{3}{7}$ is bigger than $\frac{1}{2}$.

R: Explain!

S: because $\frac{7}{13}+\frac{3}{7}$ is equal to $\frac{7}{13}+\frac{3}{7}=\frac{88}{91}$ and that is bigger than $\frac{1}{2}$.

R: Do you have another way without using a common denominator to answer this question?

S: (After he thinks); I think the two numbers are slightly less than half, so if you put them together, we get a result greater than half.

Each interview helped to reveal the students' tendency to depend on certain approaches. In general, all students initially tended to apply algorithmic calculation. No students have been observed reflecting on the problem and using the numerical sense, but after the researcher has asked for another way, they showed a different conduct.

Although there were differences between the interviewed students, what they all had in common was that they initially relied on algorithmic calculation. So it can be clearly concluded that the written algorithms highlighted in the mathematics curriculum have a huge impact on students' thinking and attitudes.

DISCUSSION

The study in question aimed to determine whether there is a disparity between written calculation ability and numerical sense regarding fractions among 6th and 7th grade students in the Arab sector of Israel. The findings can be summarized as follows; Data indicated that students generally performed better on tests assessing written calculation abilities compared to those evaluating numerical sense in the context of fractions. This was evidenced by higher mean percentages of correct responses in written calculation tests across both grades. The study found a consistent gap between students' abilities to perform written calculations and their understanding of numerical concepts related to fractions. This gap was statistically significant for both 6th and 7th graders, suggesting a need for educational strategies that not only improve computational skills but also deepen students' conceptual understanding of numbers. Students generally knew how to sequence decimal numbers but struggled more with arranging simple fractions. There was a noticeable disconnect in how mid-level students perceived simple and decimal fractions, treating them as unrelated entities. This indicates a conceptual misunderstanding or a gap in learning about the relationship between different types of fractions. High-level students demonstrated a tendency to solve problems involving fractions using a direct, algorithmic approach before considering alternative strategies, such as using benchmarks for comparison. Intermediate students also relied on learned rules rather than benchmarks, only considering alternative strategies upon prompting.

This suggests a reliance on rote learned methods over conceptual understanding or flexible thinking in problem-solving among students of varying ability levels.

Overall, the study highlights a significant disparity between students' written calculation ability and their numerical sense regarding fractions, pointing to a broader issue in the educational approach to teaching mathematics within the Arab sector of Israel. There's a clear need for pedagogical strategies that bridge the gap between computational skills and deep conceptual understanding, encouraging students to grasp the underlying principles of fractions and numerical relationships more thoroughly.

The study meticulously investigates the mathematical abilities of 6th and 7th grade students in the Arab sector of Israel, focusing on the interplay between written calculation ability and numerical sense with an emphasis on fractions. The research uncovers a significant disparity: students demonstrate notable ability in executing written calculations but encounter substantial challenges when tasked with exercises that require a deep-seated numerical sense. This divergence highlights a critical educational challenge: the need to balance computational skills with a profound understanding of numerical concepts.

Corroborating Tsao's (2004) assertion, the findings indicate that adeptness in routine calculations does not necessarily translate to a solid numerical sense. This insight is vital, challenging the traditional pedagogical emphasis on computational ability as the sole hallmark of mathematical success. Supporting the arguments of Chard et al. (2005) and Sasanguie et al. (2013), the study reinforces the idea that correct answers are not the sole indicators of understanding; the cognitive processes and conceptual underpinnings are equally, if not more, important.

Conducted among 288 students, the study provides a robust dataset that enhances the reliability of its conclusions. These findings significantly advocate for an educational shift towards fostering a more integrated approach to understanding fractions, both simple and decimal. This recommendation aligns with Ni and Zhou (2005) and Sembiring et al. (2008), who emphasize the critical yet often overlooked connection between these mathematical representations.

The research further delves into students' problem-solving approaches, revealing a prevalent reliance on memorized calculation methods. This reliance, while effective for certain tasks, may inhibit the development of adaptable problem-solving skills and a deeper numerical understanding. Notably, the study observes that higher-level students occasionally diverge from these rote techniques, particularly when encouraged to explore alternative problem-solving strategies. This observation underscores the potential benefits of promoting exploratory learning and conceptual understanding in mathematics education.

A pivotal aspect of the study is its focus on the use of benchmarks and understanding number magnitude as critical components of numerical sense. The findings suggest that students' difficulties in these areas contribute significantly to the observed disparity between calculation ability and numerical understanding. The ability to use benchmarks effectively and to grasp the magnitude of numbers is fundamental to developing a robust numerical sense. Therefore, educational strategies that emphasize these aspects can play a crucial role in bridging the gap identified in this research.

In essence, this study serves as a clarion call for a pedagogical paradigm shift within the Arab sector of Israel, urging an approach that equally values computational fluency and deep numerical understanding. Such a balanced approach promises not only to enhance students' mathematical capabilities but also to equip them with the critical thinking and problem-solving skills necessary in an increasingly complex world.

In conclusion, by reevaluating educational priorities to foster both computational skills and numerical sense, this research advocates for a holistic approach to mathematics education. Such an approach, by emphasizing benchmarks and the significance of number magnitude, aims to prepare students not just as proficient calculators but as insightful thinkers capable of applying their mathematical knowledge creatively and effectively.

RECOMMENDATIONS

Reflecting on the study's findings, which highlight a clear difference between students' ability in written calculations and their numerical sense, particularly concerning fractions, it is evident that educational strategies need realignment. The recommendations provided herein are fortified by insights gained from the study, aiming to bridge the gap and enrich students' mathematical competencies comprehensively. Given the study's revelation that students possess a significantly greater ability in written calculations than in applying numerical sense, there's a pressing need for educators and curriculum developers to prioritize numerical understanding. This initiative should permeate the entirety of primary and preparatory education, ensuring that students develop a robust and intuitive grasp of numbers from early on. The observed disparity underscores the importance of offering teachers specialized training that not only highlights innovative teaching methods but also delves into how to effectively integrate numerical sense into daily math lessons. Workshops and professional development courses should include practical examples and strategies proven to enhance numerical understanding, as observed in the study. The study’s findings suggest the benefit of incorporating numerical sense into classroom activities, emphasizing real-world applications and practical problem-solving. Curriculum designers should craft lessons that encourage students to engage with mathematical concepts in a hands-on manner, thereby solidifying their numerical intuition. The need for textbooks and teaching materials to systematically include numerical strategies is evident. These resources should provide a gradual buildup of skills in numerical sense, ensuring that students' progress from basic concepts to more complex applications, mirroring the ability levels observed in the study.

Future research should aim at dissecting the components of numerical sense further, exploring strategies for effective number decomposition, operational impacts, and the cultivation of mental calculation skills. Such research will augment our understanding and inform more targeted educational practices. Expanding the research to include a wider range of demographics will shed light on how numerical sense develops across different cultural and educational contexts. This broader perspective is essential for creating inclusive and effective teaching strategies that cater to diverse learning needs. Activities designed to enhance creativity in mathematical problem-solving, particularly those that offer multiple avenues for solution, can stimulate deeper numerical understanding. The study's insights into students' struggles with conceptualizing fractions underscore the need for tasks that encourage thinking beyond standard calculation methods. The benefits of collaborative learning, as highlighted by the study, suggest that peer interactions can play a crucial role in developing numerical sense. Group activities should be structured to encourage discussion, debate, and the sharing of strategies, facilitating a communal learning experience that enhances individual understanding. Implementing these strengthened recommendations will not only address the disparities identified but also foster an educational environment where numerical sense is valued and nurtured alongside computational skills. This balanced approach promises to equip students with a more comprehensive mathematical foundation, readying them for the challenges of higher education and beyond.

Acknowledgments

I would like to express my sincere appreciation to all of the teachers and the school staff who agreed to participate and worked well together to ensure that this study could be carried out smoothly. Additionally, I'd like to extend my sincere gratitude to each student who took part in the study making a significant contribution to future research by referencing the findings I found.

References

[1] Bentley, B., & Bossé, M. J. (2018). College students' understanding of fraction operations. *International Electronic Journal of Mathematics Education, 13*(3), 233-247. https://doi.org/10.12973/iejme/3881.

[2] Behr, M., Harel, G., Post, T., & Lesh, R. (1993). Rational Numbers: Toward a Semantic Analysis - Emphasis on the Operator Construct. In T. Carpenter, E. Fennema & T. Romberg (Eds.), *Rational Numbers: An Integration of Research* (pp. 13-47). Hillsdale, NJ: Lawrence Erlbaum Associates.

[3] Boaler, J., Williams, C., & Confer, A. (2015). Fluency without fear: Research evidence on the best ways to learn math facts. *Reflections*, *40*(2),7-12

https://www.cusd200.org/cms/lib/IL01001538/Centricity/Domain/1018/FluencyWithoutFear-2015.pdf

[4] Bruce, C., Chang, D., Flynn, T., & Yearley, S. (2013). *Foundations to learning and teaching fractions: Addition and subtraction*. Curriculum and Assessment Branch: Ontario Ministry of Education. http://www.edugains.ca/resourcesDP/Resources/PlanningSupports/FINALFoundationstoLearningand TeachingFractions.pdf.

[5] Carpenter, T. P., Fennema, E., & Romberg, T. A. (1993). Toward a unified discipline of scientific inquiry. In T. P. Carpenter, E. Fennema, & T. A. Romberg (Eds.), *Rational Numbers: An Integration of Research* (pp. 1-11). Lawrence Erlbaum Associates.

[6] Castronovo, J., & Göbel, S. M. (2012). Impact of high mathematics education on the number sense. *PloS one*, *7*(4), e33832. https://doi.org/10.1371/journal.pone.0033832.

[7] Charalambous, C. Y., & Pitta-Pantazi, D. (2007). Drawing on a theoretical model to study students' understandings of fractions. *Educational Studies in Mathematics, 64*, 293-316 https://doi.org/10.1007/s10649-006-9036-2.

[8] Chard, D. J., Clarke, B., Baker, S., Otterstedt, J., Braun, D., & Katz, R. (2005). Using measures of number sense to screen for difficulties in mathematics: Preliminary findings. *Assessment for Effective Intervention, 30*(2), 3-14. https://doi.org/10.1177/073724770503000202.

[9] Chinnappan, M. (2005). *Children's mappings of part-whole construct of fractions*. In P. Clarkson & A. Downtown (Eds.), Conference of the Mathematics Education Research Group of Australasia (pp. 241- 248). Sydney: MERGA.

[10] Clarke, D. M., Roche, A., & Mitchell, A. (2008). 10 Practical Tips for Making Fractions Come Alive and Make Sense. *Mathematics Teaching in the Middle School*, *13*(7), 372-380. Retrieved Oct 20, 2024, from https://doi.org/10.5951/MTMS.13.7.0372

[11] David, H. (2023). The failure of gifted education in Israel. *Journal of Gifted Education and Creativity*, *10*(3), 141-155. https://dergipark.org.tr/en/download/article-file/3059888.

[12] Dyson, N. I., Jordan, N. C., & Glutting, J. (2013). A number sense intervention for low-income kindergartners at risk for mathematics difficulties. *Journal of Learning Disabilities*, *46*(2), 166-181. https://journals.sagepub.com/doi/abs/10.1177/0022219411410233.

[13] Fazio, L., & Siegler, R. (2011). *Teaching fractions*. International Academy of Education (IAE). http://unesdoc.unesco.org/images/0021/ 002127/212781e.pdf.

[14] Fischer, U., Suggate, S. P., & Stoeger, H. (2020). The implicit contribution of fine motor skills to mathematical sense in early childhood. *Frontiers in Psychology*, *11*, 1143. https://doi.org/10.3389/fpsyg.2020.01143.

[15] Fitousi, D., & Noyman, R. (2024). Why fractions are difficult? Modeling optimal and sub-optimal integration strategies of numerators and denominators by educated adults. *Cognition*, *242*, 105656. https://doi.org/10.1016/j.cognition.2023.105656

[16] Gabriel, F., Coché, F., Szucs, D., Carette, V., Rey, B., & Content, A. (2013). A componential view of children's difficulties in learning fractions. *Frontiers in Psychology*, *4*, 715. https://doi.org/10.3389/fpsyg.2013. 00715.

[17] Hornung, C., Schiltz, C., Brunner, M., & Martin, R. (2014). Predicting first-grade mathematics achievement: The contributions of domain-general cognitive abilities, nonverbal number sense, and early number competence. *Frontiers in Psychology*, *5*, 272. https://doi.org/10.3389/fpsyg.2014.00272.

[18] Hulse, T., Daigle, M., Manzo, D., Braith, L., Harrison, A., & Ottmar, E. (2019). From here to there! Elementary: A game-based approach to developing number sense and early algebraic understanding. *Educational Technology Research and Development*, *67*(2), 423-441. https://doi.org/10.1007/s11423-019-09653-8.

[19] Jordan, N. C., Glutting, J., & Ramineni, C. (2010). The importance of number sense to mathematics achievement in first and third grades. *Learning and Individual Differences*, *20*(2), 82-88. https://doi.org/10.1016/j.lindif.2009.07.004.

[20] Kieren, T. E. (1993). Rational and fractional numbers: From quotient fields to recursive understanding. In T. P. Carpenter, E. Fennema & T. A. Romberg (Eds.), *Rational numbers: An integration of research* (pp. 49-84). Hillsdale, NJ: Lawrence Erlbaum.

[21] Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

[22] Kroesbergen, E. H., Van't Noordende, J. E., & Kolkman, M. E. (2012). Number sense in low-performing kindergarten children: Effects of a working memory and an early math training. *Reading, writing, mathematics and the developing brain: Listening to many voices*, 295-313. https://doi.org/10.1007/978-94-007-4086-0_16.

[23] Lamon, J. (1999). *Teaching fractions and ratios for understanding: Essential content and instructional strategies for children*. Nova Jersey: Lawrence Erlbaum.

[24] Lazića, B., Abramovichb, S., Mrđaa, M., & Romano, D. A. (2017). On the teaching and learning of fractions through a conceptual generalization approach. *International Electronic Journal of Mathematics Education, 12*(8), 749-767.

[25] Liu, J., & Jacobson, E. (2022). Examining US elementary students' strategies for comparing fractions after the adoption of the common core state standards for mathematics. *The Journal of Mathematical Behavior*, *67*, 100985.https://doi.org/10.1016/j.jmathb.2022.100985.

[26] McIntosh, A., Reys, B. J., & Reys, R. E. (1992). A proposed framework for examining basic number sense. *For the Learning of Mathematics, 12*, 2-8.

[27] McNamara, J., & Shaughnessy, M. M. (2015) (2nd ed.). *Beyond pizzas & pies, Grades 3-5: 10 Essential Strategies for Supporting Fraction Sense*. CA: Math Solutions.

[28] McMullen, J., & Van Hoof, J. (2020). The role of rational number density knowledge in mathematical development. *Learning and Instruction*, *65*, 101228. https://doi.org/10.1016/j.learninstruc.2019.101228.

[29] Mock, J., Huber, S., Klein, E., & Moeller, K. (2016). insight s into numerical cognition: Considering eye-fixations in number processing and arithmetic. *Psychological Research*, *80*(3), 334-359. https://doi.org/10.1007/s00426-015-0739-9.

[30] National Authority for Measurement and Evaluation in Education in Israel (2023). *PISA 2022 study, literacy in math, science and reading*, Israel Ministry of Education. https://meyda.education.gov.il/files/Rama/PISA-2022-report.pdf.

[31] Ni, Y., & Zhou, Y. D. (2005). Teaching and learning fraction and rational numbers: The origins and implications of whole number bias. *Educational Psychologist*, *40*(1), 27-52. https://doi.org/10.1207/s15326985ep4001_3.

[32] OECD (2023a). *PISA 2022 Results (Volume II)*. Learning During-and from-Disruption, PISA OECD Publishing, Paris. https://doi.org/10.1787/a97db61c-en.

[33] OECD (2023b). *PISA 2022 Results (Volume I)*. The State of Learning and Equity in Education, PISA OECD Publishing, Paris, https://doi.org/10.1787/53f23881-en.

[34] Pantziara, M., & Philippou, G. (2012). Levels of students' "conception" of fractions. *Educational Studies in Mathematics, 79*(1), 61-83. https://doi.org/10.1007/s10649-011-9338-x.

[35] Petit, M. M., Laird, R. E., Ebby, C. B., & Marsden, E. L. (2015). *A focus on fractions: Bringing research to the classroom*. Routledge. https://books.google.co.il/books/publisher/content?id=0zc-CgAAQBAJ&hl=iw&pg=PR6&img=1&zoom=3&sig=ACfU3U0MyqgZ7-a46asp8yWqsRQmnYTnmA&w=1280.

[36] Sabra, H., & Alshwaikh, J. (2023). The use of Arabic language by mathematics teachers as a resource to support teaching. *ZDM Mathematics Education*, 1-13. https://doi.org/10.1007/s11858-022-01462-3.

[37] Sasanguie, D., Göbel, S. M., Moll, K., Smets, K., & Reynvoet, B. (2013). Approximate number sense, symbolic number processing, or number–space mappings: What underlies mathematics achievement?. *Journal of Experimental Child Psychology*, *114*(3), 418-431. https://doi.org/10.1016/j.jecp.2012.10.012.

[38] Sembiring, R. K., Hadi, S., & Dolk, M. (2008). Reforming mathematics learning in Indonesian classrooms through RME. *ZDM Mathematics Education*, *40*(6), 927-939. https://doi.org/10.1007/s11858-008-0125-9

[39] Sharkia, H., & Kohen, Z. (2021). Flipped classroom among minorities in the context of mathematics learning: The Israeli case. *Mathematics*, *9*(13). https://doi.org/10.3390/math9131500

[40] Shumway, J. F., & Moyer-Packenham, P. S. (2019). A counting-focused instructional treatment to improve number sense: An exploratory classroom-based intervention study. *The Mathematics Enthusiast*, *16*(1), 289-314. http://doi.org/10.54870/1551-3440.1459

[41] Siegler, R. S., Fazio, L. K., Bailey, D. H., & Zhou, X. (2013). Fractions: The new frontier for theories of numerical development. *Trends in Cognitive Science*, 17, 13–19. https://doi.org/10.1016/j.tics.2012. 11.004

[42] Sood, S., & Jitendra, A. K. (2007). A comparative analysis of number sense instruction in reform-based and traditional mathematics textbooks. *The Journal of Special Education*, *41*(3), 145-157. https://doi.org/10.1177/00224669070410030101

[43] Sood, S., & Mackey, M. (2015). Examining the effects of number sense instruction on mathematics competence of kindergarten students. *International Journal of Humanities Social Sciences and Education*, *2*(2), 2349. https://citeseerx.ist.psu.edu/document?repid=rep1&type=pdf&doi=18ba17869cacb1ab1b99a511287b51e8c0873806

[44] Sowder, J. T. (2020). Making sense of numbers in school mathematics. In *Analysis of Arithmetic for Mathematics Teaching* (pp. 1-51). Routledge. https://www.taylorfrancis.com/chapters/edit/10.4324/9781315044606-1/making-sense-numbers-school-mathematics-judith-sowder

[45] Tsao, Y. L. (2004). Exploring the connections among number sense, mental computation performance, and the written computation performance of elementary preservice school teachers. *Journal of College Teaching & Learning*, *1*(12). https://doi.org/10.19030/tlc.v1i12.2022

[46] Tzuriel, D., Cohen, S., Feuerstein, R., Devisheim, H., Zaguri-Vittenberg, S., Goldenberg, R., ... & Cagan, A. (2023). Evaluation of the Feuerstein Instrumental Enrichment (FIE) program

among Israeli-Arab students. *International Journal of School & Educational Psychology*, *11*(1), 95-110. https://doi.org/10.1080/21683603.2021.1951409

[47] Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2013). *Elementary and middle school mathematics: Teaching Developmentally*. Upper Saddle River, NJ: Pearson

[48] Way, J. (2011). Developing number sense using learning objects. *In J., Way, & J., Bobis (Eds.), Fractions: Teaching for understanding* (pp.153–166). Adelaide: The Australian Association of Mathematics Teachers (AAMT) Inc.

[49] Wiest, L., & Amankonah, F. O. (2019). Conceptual versus Procedural Approaches to Ordering Fractions. *European Journal of Science and Mathematics Education*, *7*(1), 61-72. https://eric.ed.gov/?id=EJ1205761

[50] Wijaya, A. (2017). The Relationships between Indonesian Fourth Graders' Difficulties in Fractions and the Opportunity to Learn Fractions: A Snapshot of TIMSS Results. *International Journal of Instruction*, *10*(4), 221-236.

[51] Wittmann, G. (2013). The consistency of students' error patterns in solving computational problems with fractions. In B. Ubuz, C. Haser & M. A. Mariotti (Eds.), *Proceedings of the 8th Congress of the European Society for Research in Mathematics Education* (pp.393–402). Antalya

[52] Xu, C., Li, H., Burr, S. D. L., Si, J., LeFevre, J. A., & Huang, B. (2022). Divide and conquer: Relations among arithmetic operations and emerging knowledge of fraction notation for Chinese students in grade 4. *Journal of Experimental Child Psychology*, *217*, 105371. https://doi.org/10.1016/j.jecp.2021.105371

[53] Yang, D. C., Li, M. N., & Lin, C. I. (2008). A study of the performance of 5th graders in number sense and its relationship to achievement in mathematics. *International Journal of Science and Mathematics Education, 6*(4), 789-807. https://doi.org/10.1007/s10763-007-9100-0

APPENDIX

[1]The correct answer

Item	**Response Type**	**Class 6 in percent**	**Class 7 in percent**
Is $\frac{9}{17}$ or $\frac{4}{9}$ closer to 1/2? Why?	$\frac{9}{17}$ Correct Justification[1]	9	34
	$\frac{9}{17}$ Wrong justification	20	17
	$\frac{4}{9}$	44	26
	The same thing	3	28
	No answer	24	19
To which of the next two numbers would we have to add a bigger number, to get $2\frac{1}{2}$? $\frac{20}{11}$ or $\frac{15}{9}$. why?	$\frac{15}{9}$ Correct justification[1]	10	33
	$\frac{15}{9}$Wrong justification	18	0
	$\frac{20}{11}$	49	20
	The same thing	1	28
	No answer	22	19
Which is the number closer to the amount $1\frac{1}{5} + 2\frac{8}{18}$? a. 1 b. 2 c. 3 d. 4	a	11	6
	b	19	15
	c	33	33
	d[2]	37	46
Which is bigger? a. $1\frac{1}{2} + 2\frac{1}{4}$	a	31	19
	b	27	23
	Equal[1]	42	58

b. $\frac{1}{2} + 3\frac{1}{4}$			
Which sum is bigger than 1? $a. \frac{4}{6}+\frac{2}{7}$ $b. \frac{4}{6}+\frac{3}{7}$ $c. \frac{4}{6}+\frac{4}{7}$ $d. \frac{4}{6}+\frac{5}{7}$	a b c d^2	10 22 16 52	9 17 20 54
$5 \div 0.025$		58	61
$\frac{12}{5} - \frac{3}{10}$		55	73
$0.3 \cdot 0.5$		58	67
$2\frac{4}{8} \cdot 36$		55	73
$100 \div 0.25$		54	56
$\frac{6}{15} + \frac{4}{7}$		57	76
$\frac{8}{7} + \frac{11}{13}$		61	63
$\frac{44}{10} + \frac{61}{2}$		64	69
$0.532 \cdot 231.5$		58	66

Table 3. The items in the written test and the percentage of those who answered correct from both class levels

[1]The correct answer

Item	Response Type	Class 6 in percent	Class 7 in percent
Without calculating an exact answer, determine which of the next two numbers $\frac{9}{17}$ or $\frac{4}{9}$ closer to 1/2? Why?	$\frac{9}{17}$ Correct justification[1]	6	28
	$\frac{9}{17}$ Wrong justification	21	20
	$\frac{4}{9}$	45	23
	The same thing	4	2
	No answer	24	23
Without calculating an exact answer, determine which of the next two numbers would we need to add a bigger number to get $2\frac{1}{2}$? $\frac{20}{11}$ or $\frac{15}{9}$. why?	$\frac{15}{9}$ correct justification[1]	8	23
	$\frac{15}{9}$ wrong justification	19	25
	$\frac{20}{11}$	49	31
	The same thing	2	1
	No answer	22	20
Without calculating an exact answer, determine which is the number closer to the sum: $2\frac{8}{18}+1\frac{1}{5}$. a. 1 b. 2 c. 3	a	20	14
	b	17	22
	c	33	35
	d[1]	30	39

d. 4

Without calculating an exact answer, determine which is bigger: a. $1\frac{1}{2}+2\frac{1}{4}$ or b. $\frac{1}{2}+3\frac{1}{4}$ 1. a 2. b 3. equal	a	28	23
	b	33	27
	Equal[1]	39	50
Without calculating an exact answer, determine which amount is greater than 1? $a.\ \frac{4}{6}+\frac{2}{7}$ $b.\frac{4}{6}+\frac{3}{7}$ $c.\frac{4}{6}+\frac{4}{7}$ $d.\frac{4}{6}+\frac{5}{7}$	a	13	10
	b	23	22
	c	19	17
	d[1]	45	51
Without calculating an exact answer, enclose the best estimate to 5÷0.02. a. much less than 50. b. A little bit less than 50. c. A little bit more than 50. d. Much more than 50	a	11	15
	b	21	22
	c	16	16
	d[1]	52	57
Without calculating an exact answer, the best estimate to	a	23	21
	b[1]	49	68
	c	28	11

$\frac{12}{5} - \frac{3}{10}$ is:

a. less than 2

b. less than 3

c. More than 3.

Without calculating an exact answer, the best estimate to $0.3 \cdot 0.5$ is: a. Bigger than 2 b. Less than 1 c. Equal to 1.5	a b[1] c	25 49 26	21 60 19
Without calculating an exact answer, the best estimate to $2\frac{4}{8} \cdot 36$ is: a. Bigger than 100 b. Less than 50 c. Between 50 and 100	a b c[1]	24 25 51	14 21 65
Without calculating an exact answer, the best estimate to $100 \div 0.25$ is: a. Bigger than 500. b. Lager than 450. c. Less than 450.	a b c[1]	20 29 51	16 12 62
Without calculating an exact answer, the best estimate to $\frac{6}{15} + \frac{4}{7}$ is: a. bigger than 1.5 b. Bigger than 2	a b c[1]	31 26 44	21 18 61

c. Less than 1.5.

Without calculating an exact answer, the result of the connection of $\frac{8}{7}+\frac{11}{13}$ is: a. Approx. 1 b. Approx. 2 c. Approx. 3	a b[1] c	52 27 21	39 37 24
Without calculating an exact answer, determine which is bigger: $1.\frac{99}{100}$ or $2.\frac{999}{1000}$ a.1 b. 2 c. equal	a b[1] c	21 59 20	25 63 12
How many different decimal fractions are there between 0.53 and 0.54? Why? a. None. b. One. What is the number? c. Several fractures. Give two examples. d. A lot. Give three examples.	a b c d. Correct examples[1] Wrong examples No answers	29 12 9 29 7 14	9 10 6 58 11 6
How many different decimal fractions are there between 8.99 and 9?	a b c	36 10 5	5 5 3

a. None. Why? b. One. What is the number? c. Several fractures. Give two examples. d. A lot. Give three examples.	d. Correct examples[1]	29	71
	Wrong examples	3	7
	No answers	17	9
Without calculating an exact answer, determine the result of 0.532·231.5 is: a. 123.158 b. 12.3158 c. 1231.58	a[1]	23	30
	b	59	48
	c	18	22
Without calculating an exact answer, the best estimate to $\frac{44}{10}+\frac{61}{2}$ is: a. Bigger than 35 b Bigger than 40 c. less than 35	a[2]	22	12
	b	23	18
	c	55	60

Table 4. The items in the Numerical Sense the percentage of those who answered correct from both class levels

[1]The correct answer

The Interview Item	Response	High Level		Medium Level	
		Class 6 ($n = 5$)	Class 7 ($n = 5$)	Class 6 ($n = 4$)	Class 7 ($n = 4$)
How many decimal numbers are there between 2.51 and 2.52?	Correct with the use of the number magnitude	4	5	1	1
	Correct without using the number magnitude	1	0	0	1
	Wrong answers	0	0	3	2
How many fractions are there between: $\frac{3}{5}$ to $\frac{4}{5}$?	Correct with the use of the number magnitude	2	5	0	0
	Correct without using the number magnitude	1	0	1	2
	Wrong answers	2	0	3	2
Arrange in sequence: $0.595, 61\%, 0.3562, \frac{5}{8}, \frac{3}{8}$	Correct with the use of the number magnitude	3	5	0	1
	Correct without using the number magnitude	1	0	1	1
	Wrong answers	1	0	3	2

Table 5. The following are the answers to the items of the number magnitude among high and medium-level students from the sixth and seventh grades

[1]The correct answer

The Interview Item	Response	High Level		Medium Level	
		Class 6 (n = 5)	Class 7 (n = 5)	Class 6 (n = 4)	Class 7 (n = 4)
Without calculating an exact answer, do you think that the result of 0.47·110 is greater than 55 or less than 55?	Correct with the use of a benchmark	4	5	1	1
	Correct without the use of a benchmark	1	0	1	2
	Wrong answers	0	0	2	1
Without calculating an exact answer, do you think that the result of $\frac{4}{9} \div \frac{21}{22}$ is greater than $\frac{4}{9}$ or less than $\frac{4}{9}$?	Correct with the use of a benchmark	3	4	0	1
	Correct without the use of a benchmark	2	1	1	1
	Wrong answers	0	0	3	2
Without calculating an exact answer, do you think that the amount of $\frac{7}{13} + \frac{3}{7}$ is greater than $\frac{1}{2}$ or less than $\frac{1}{2}$?	Correct with the use of a benchmark	2	4	0	0
	Correct without the use of a benchmark	3	1	3	2
	Wrong answers	0	0	1	2

Without calculating an exact answer, do you think that the amount of $\frac{7}{13}+\frac{3}{7}$ is bigger than 1 or less than 1?	Correct with the use of a benchmark	2	4	0	0
	Correct without the use of a benchmark	3	1	2	1
	Wrong answers	0	0	2	3

Table 6. shows four items designed to encourage the use of benchmarks