The development of knowledge of content and teaching task instruments for pre-service mathematics teacher

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ABSTRACT

The knowledge of the materials to be taught to the students is the basic knowledge that preservice mathematics teachers should possess, as they need to prepare themselves for teaching. In order to research preservice teachers' understanding of the subject matter and teaching skils, valid and reliable test instruments are required. Knowledge of content and teaching (KCT) is one of the tools that can be used. This research was conducted using the Plomp model. Based on the research results, it was found that the KCT task instruments are valid, reliable, and legible. The instruments were utilized in this study with several revisions to describe the content and teaching knowledge of the subjects. The results of this investigation strongly support the use of KCT instruments. This tool is crucial to implement in understanding the capability of preservice mathematics educators in planning and successfully executing math lessons. Additionally, future research should be conducted on content understanding and classroom instruction on mathematics education. Such research will be able to reveal information on the expertise and experience of prospective mathematics teachers.

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1. INTRODUCTION

Assessment of mathematical knowledge for teaching (MKT) [1] requires the teacher to evaluate students' understanding of mathematical concepts, analyse their non-biased comments, understand unconventional algorithms, and prevent typical students' arguments in mathematics classes. To increase student learning, teachers need to posses a solid understanding of mathematics and the necessary skills to deliver that knowledge using effective teaching methods [2]. A teacher's knowledge can be divided into different domains, such as knowledge about individual differences, organizational and management methods, and knowledge about the subject matter for learning [3]. More specifically, Shulman discovered that teacher knowledge is divided into three categories: knowledge of the subject matter, knowledge of pedagogy, and knowledge of the curriculum. MKT is the knowledge of content used in analysing, comprehending, and solving mathematical problems encountered while studying [4], [5]. In addition, they defined MKT as a practice-based theory summarising the mathematical knowledge needed to carry out repetitive mathematics teaching tasks. It was noted that they had adopted a flexible conception of the "needs" that allows for the habit of mind perspective and sensitivities necessary for effective content teaching [4].

Two fundamental knowledge bases, i.e., subject matter knowledge and pedagogical content (PCK), comprise one domain [6]. Moreover, every knowledge base is divided into three parts. Common content knowledge (CCK), defined as the understanding of and proficiency in mathematics used in academic contexts or by other people in related professions, is at the core of the curriculum. Specialized content knowledge (SCK) is a valuable mathematical understanding for teaching. It includes knowledge and experience that are typically not required for tasks outside the classroom, such as understanding how a mistake is made. In contrast, what is referred to as "horizon content knowledge" connects all the mathematics the curriculum covers. An explanation of the framework of MKT can be seen in Figure 1.

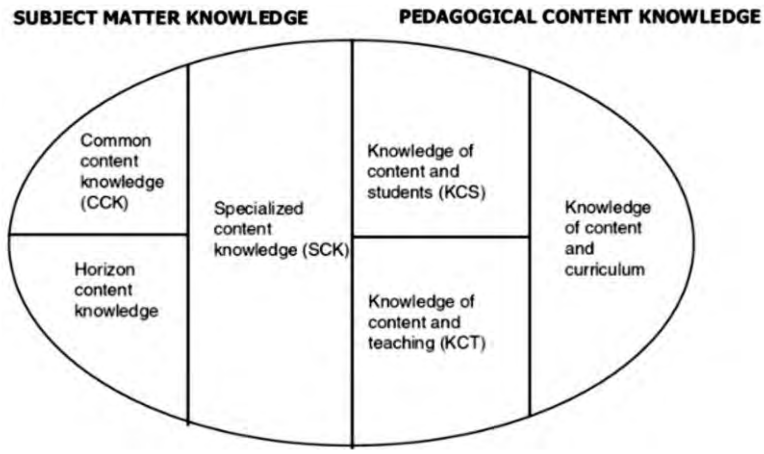

Figure 1. The framework of MKT [4]

Knowledge of content and students (KC)S combines knowledge of students and mathematics. Knowledge of content and teaching (KCT) emphasizes teacher knowledge of pedagogy and mathematics. KCC refers to the method teachers teach mathematics in their classes which can allow students to gradually develop the necessary mathematical understanding [5]. The context used in several MKT assessment tasks identifies the type of knowledge teachers need and details how they use it. Given the critical tenets of MKT, any attempt to establish MKT will include a discussion of context's role in MKT design [7].

Mathematical content knowledge is frequently cited as a crucial component of teacher knowledge in the literature [8]. It is proposed as a didactic-mathematical knowledge model to characterize and enhance teacher knowledge [9], and developed the theory based on the practice of mathematical knowledge for learning (MKT), which is the knowledge of math that teachers use in schools to deliver instruction and oversee student progress [3], [4], [10]. Previous research has indicated that teachers' MKT and subject matter knowledge (SMK) [11] are crucial for students learning [10]. Shulman stated that unique knowledge for teachers is a mixture of special content knowledge, subject content, professional content, and curriculum knowledge (knowledge of educational materials, teaching procedures, and student objectives) [3]. Three types of essential PCK encompass CCK, KCS, and KCT [4].

PCK in mathematics is associated with high levels of aptitude for learning, rigour in learning, positive student attitudes towards mathematics, and proficiency in learning [12]. As a result of the teachers' PCK persistently low-quality instruction, it is anticipated that studies on preservice teachers' PCK will proceed [13]. PCK is closely related to proficiency in teaching advanced mathematics and calculus [14]. Hammack and Ivey discovered that proper instruction strongly connects to PCK [15]. In the literature, knowledge of PCK is widely defined as knowledge of effective teaching that includes, but is not limited to, in-depth and conceptual knowledge of teachers about a wide range of mathematical topics that are broad and specific to the class level taught; knowledge of curricula that allows teachers to use and choose curriculum materials effectively; and knowledge of instructional teaching and strategies [16], [17].

According to Blömeke et al, the definition of teacher's skill may be integrated into a single process-oriented model which is conceptualized as a horizontal continuum composed of many dimensions [18]. The primary focus is on the teacher's disposition, which is cognitive, affective, and motivating abilities. The objectives lie on the values of education and its philosophical and historical foundations [3]. Ball *et al*. [4]

explained that content and teaching (KCT) knowledge is necessary for instructional design. It includes the sequence of teaching and the selection of examples and representations to introduce the materials. In addition, the teacher must understand what actions must be taken to improve students' understanding, such as requiring them to use a thick angle to assist them in completing homework or to use angle to complete homework (KCT) [19]. KCT is required for instructional design, classroom instruction, and various other tasks, such as introducing materials, deepening student's understanding, and many more [20]. Thus, KCT, including all that a teacher chooses to do to improve student learning [20].

KCT explains the knowledge of mathematical planning and teaching that requires the selection of appropriate tasks, representations, and materials, ordering mathematics content and activities, and investigating the right questions [10], [21]. It teaches the principle of reality in a growing learning unit by locating the relevant content for instructions [10], learning about many popular learning models, evaluating the teaching advantages and disadvantages of the representation or model used to help teach the idea. It also identifies a language that can help or hinder students in learning particular notions. It focuses on concepts and activities with a robust educational system using the benefits and instructional costs of the representation or model used to teach the subject matter.

KCT includes instruction knowledge, often limited to specific topics [22]. Daily work in teaching requires specialized mathematical and pedagogical knowledge. It includes organizing and designing tasks and examples, choosing ways to demonstrate concepts, and using contextual situations. Wright [23] uses this instruction text to identify tactical scenarios to illustrate the principle [23], by considering the strengths and weaknesses of current representations and procedures in advancing discussion, choosing a model, term, and design for the growth of students' understanding from the appropriate use of language, analogy, and metaphor [24].

With KCT, teachers could gain the necessary understanding to plan their teaching to minimize misunderstandings. This planning combines attention to the sequence of instructions to overcome misconceptions, and shows practical examples to highlight misconceptions. This is also necessary for designing a series of instructions that provides a task track built with complexity and at a rate that proffers adequate consolidation of understanding [25]. Teachers need KCT knowledge to plan, teach, and choose suitable activities, exercises, and representations for various topics. A critical aspect of this knowledge for teachers is understanding situations in which teachers must deviate from their initial planning, such as when a student makes a mathematical invention [26]. Instruments are needed for researching geometry teaching at high school and teacher education and development study in mathematics (TEDS-M) can be used for learning and knowledge of mathematical content [27]. However, this research aims to develop an understanding of content and teaching (KCT) of task instruments on the material about system of linear equations of two variables.

2. METHOD

Research and development (R&D) were the research method employed in this study. The Plomp model, which consists of three phases (preliminary investigation, prototyping, and assessment), was the development paradigm used in this work [28]. This study was aimed at increasing the understanding of KCT that was valid and reliable. Purposive sampling was used in this study's sample strategies to choose participants from pre-service mathematics teachers. The data collection techniques were tests, instrument validation sheets, and lifts. The instruments used in this research consisted of KCT tasks, instrument validation sheets, and user response lifts. The test instrument was by the indicator of KCT, i.e., designing the sequence of teaching materials, identifying examples that match the material to demonstrate concepts, and using specific representations and procedures to support student understanding. The distribution of indicators of KCT is presented in Table 1.

Table 1. Indicators of KCT

Indicators	Item number
Design of linear equation system of two variables	1 and 2
Identify examples that match the material to demonstrate the concept of a system of linear equations of two variables	3 and 4
Use specific representations and procedures to support student understanding	5

2.1. Preliminary research phase

The objective of the preliminary research stage was to examine the main issues underlying the importance of making knowledge content and learning task instruments (KCT). In addition, this stage also aimed to create a conceptual framework that would be utilized as a basis for further studies. Two analyses were carried out in the early stages of the research: needs analysis and literature review. The analysis of needs and content began with a review of the mathematics education curriculum, teaching materials, and student knowledge. Furthermore, the literature was analyzed to find references related to potential mathematics teachers' content knowledge and teaching knowledge. The mathematics education curriculum analysis aimed to know the inside of the curricula used and the ability of the content knowledge and knowledge teaching students of potential mathematical teachers.

2.2. Prototyping phase

This stage aimed to create a KCT task tool designed based on preliminary research. At this stage, the prototype consisted of understanding the KCT task sheet, response sheet, and criteria. This study aimed to create an essay-shaped question tailored to the indicators of KCT and based on preliminary research results.

2.3. Assessment phase

The assessment process consisted of three parts. First, the question was validated by a chosen expert with deep understanding on KCT. The trial's second part was limited to subjects that experts had reviewed. After this stage, the data was tested for reliability, level of difficulty, and operating power. After a limited trial, further analyses were carried out if the criteria were met.

2.4. Data analysis

The data validation procedure was conducted through the expert's validation of the KCT task instrument, as well as the readability test of the instrument which used a rating scale. Expert confirmation ensureed that the instrument's measurements covered a wide range and accurately represented the constructs to be measured. The expert assessed the appropriateness of the questions, indicators, or items in reflecting the essential aspects of the variables studied. Expert validation helped ensure the instrument was relevant to the context and the studied population. Analysis of the results of specialist validation and legibility testing of KCT assignments used the categories in Table 2.

Analysis of the reliability and difficulty level of KCT tasks used the Alpha formula and difficulty level test. The reliability test in this study was used to see the accuracy of the KCT tasks. The category used for the reliability test was the high or very high category. Each reliability category is explained in Table 3.

As for the difficulty level categories, good assignments should have varying levels of item difficulty; there are questions with complex types to motivate students to study and moderate questions to see students with a certain level of ability work on the questions. Questions with easy categories can also be selected to distribute questions with various types. Each category of task difficulty level is explained in Table 4.

Table 2. Expert validation criteria and question legibility

Grade (%)	Category	Description
35-54	Not good/not suitable	Not suitable to use
55-69	Not good/not suitable	Suitable to use with major revisions
70-84	Good/appropriate	Suitable to use with minor revisions
85-100	Very good/very suitable	Suitable to use

Table 3. Reliability category

Grade	Description
0 .800-1.000	Very high
0.600-0.799	Tall
0.400-0.599	Enough
0.200-0.399	Low
0.000-.0199	Very low

Table 4. Difficulty level categories

Grade	Description
0-0.30	Hard
0.31-0.70	Medium
0.71-1.0	Easy

3. RESULTS AND DISCUSSION

3.1. Instrument validation

The development of the KCT task instruments began with preliminary research and continues with the preparation of prototypes. After producing a prototype of the KCT task, the next phase was expert validation. Instrument validation was carried out with validation sheets and KCT assignments and answered questions. The expert validators who understand the ability of the analysis were asked to validate the instrument. The validation stage of this instrument aimed to assess language aspects, material aspects, and the instrument's suitability that had been prepared in the form of prototype 1. Furthermore, all validators stated that the prototype can be used with several revisions. Based on the results of the expert validation conducted, it was found that five assessment criteria fell into the excellent or appropriate category, and four criteria fell into the very good/outstanding category. A complete explanation is demonstrated in Table 5. The results of the expert validation analysis showed that the instrument was valid and feasible to use with minor revisions to the sentences.

Table 5. Expert validation test results

Criteria	Expert validation (grade in percentage)			The average of each criterion	Category
	1	2	3		
The tasks for KCT have been delivered systematically and organised.	85	83	88	85	Very good/very suitable
Task items measuring content and teaching knowledge (KCT) are markers of technical proficiency.	87	81	85	84	Good/appropriate
The KCT task covers all aspects of the information that must be revealed.	88	85	83	85	Very good/very suitable
Instructions are developed for content and teaching (KCT) knowledge.	85	86	81	84	Good/appropriate
Instructions are written in a way that prevents various interpretations.	88	84	85	85	Very good/very suitable
Clear presentation of the material on the question	84	83	81	82	Good/appropriate
The language used adheres to the formal style of the Indonesian language.	87	87	86	86	Very good/very suitable
The preservice teachers can easily understand the words utilised.	85	86	81	84	Good/appropriate
There are no conflicting or ambiguous interpretations of the sentences.	84	85	81	83	Good/appropriate

3.2. Limited trial

Students were asked to complete the attached readability test form in order to provide feedback on the questions after finishing the assignment. Following improvements to the prototype, a readability test was administered by the preservice mathematics teachers. Seven students were given the instrument. Following that, a readability questionnaire was given to preservice mathematics teachers. The survey's findings indicate that three criteria fell into the very good/excellent criteria, while two fell into the suitable category. Table 6 provides a detailed description of how the readability questionnaire was calculated.

Table 6. Readability test results

Criteria	Student (grade in percentage)			The average of each criterion	Category
	1	2	3		
The KCT task structure is organised to be easily understood.	88	85	87	86	Very good/very suitable
The KCT assignments use proper spacing, type, and font sizes that are simple to read.	85	83	84	84	Good/appropriate
Assignments for KCT typically employ simple language.	87	87	86	86	Very good/very suitable
I found the KCT task instructions to be simple and straightforward to follow.	81	88	84	84	Good/appropriate
It is possible to complete the KCT assignment.	86	85	84	85	Very good/very suitable

3.3. Advanced trial

The validated and revised instrument was tested on 30 mathematics education program students. The Alpha formula calculation received a reliability score of 0.897 in the high category based on the test results. These findings suggest that when applied to the same group under comparable circumstances, KCT activities are reliable and consistent in delivering comparably similar. The results of the difficulty levels revealed that questions 1, 2, and 4 belonged in the moderate category, question 3 in the difficult category, and the last item was categorized as easy. Table 7 explains and analyses the questions' difficulty levels.

3.4. The knowledge of preservice mathematics teachers in making teaching sequences

The ability of preservice mathematics teachers to create instructional sequences needs to be improved and developed. Preservice mathematics instructors could get this lesson from lectures in disciplines that support instruction. Based on the findings of the experiments, pre-service mathematics teachers had a variety of learning design options available to them when teaching a system of two-variable linear equations in the context of the provided situation. Because the challenges had been identified so that they could organise their learning based on these contextual problems, 27% of preservice mathematics instructors adopted problem-based learning. Finding solutions to their challenges made it simpler for students to develop comprehension, which was another reason preservice instructors adopt problem-based learning. Along with problem-based learning, 17% selected project-based learning and realistic mathematics, 13% chose exploration and collaborative learning, and 7% employed cooperative-based learning. Figure 2 provides more clarification.

Table 7. Difficulty level calculation results

Item number	Difficulty level	Category
1	0.433333	Moderate
2	0.516667	Moderate
3	0.116667	Difficult
4	0.316667	Moderate
5	0.75	Easy

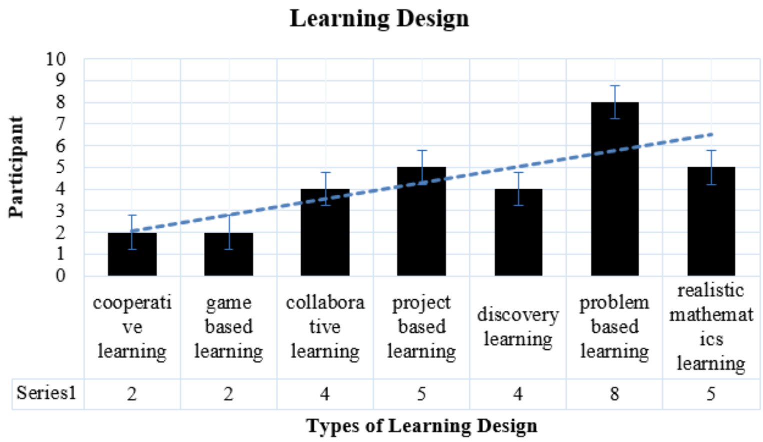

Figure 2. Learning design

3.5. The preservice mathematics teachers' knowledge in making sample questions

According to the findings of the experiment investigating the preservice mathematics teachers' understanding on how to create formal and contextual math questions that satisfy each of the criteria for higher-order thinking skills (HOTs), medium-order thinking skills (MOTs), and lower-order thinking skills (LOTs), some subjects still found it challenging to create formal math questions that satisfy the criteria for HOTs. In contrast, the prospective mathematics teachers already had the knowledge and skills necessary to create questions that address the requirements for MOTs and LOTs. They also already possessed the information and abilities necessary to create questions using all types of HOTs, MOTs, and LOTs. To help their students understand the topic, the prospective math teachers could differentiate each question's criteria.

3.6. The preservice mathematics teachers' knowledge in the representation used in learning

According to the findings of the follow-up trials by testing KCT assignments, 53% of preservice mathematics teachers chose to use both visual representations and mathematical representations or multiple representations in mathematics learning. Moreover, 27% of preservice mathematics teachers decided to use only mathematical models in conveying learning mathematics. Lastly, 20% of preservice mathematics teachers decided to use visual representations only. These findings suggest that visual representations and mathematical models can be crucial for learning mathematics. In addition, computer-based applications or media could support the usage of visual representations to maximize visual appeal and offer worthwhile learning activities for students to develop their knowledge, according to preservice mathematics instructors' explanations. The GeoGebra program is a piece of software that can assist pupils in comprehending two-variable linear equations. By entering straight-line equations in the available menu, the GeoGebra application may help pupils draw straight lines by displaying them on the screen. The example following is using

GeoGebra to create parallel, coincident, and intersecting straight lines. An explanation of the use of representation can be seen in Figure 3.

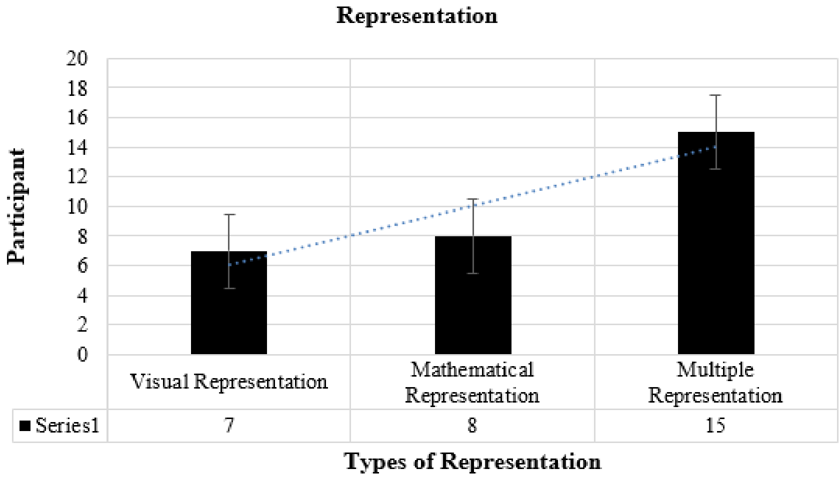

Figure 3. Representation

Based on the experts' validation, the KCT instrument was deemed feasible to discover the KCT of prospective mathematics teachers. Content and teaching knowledge can help preservice mathematics teachers in preparing them in the teaching-learning process. They can also create learning designs to prepare lessons and sample questions with various ability levels from low, medium, and high. Furthermore, preservice mathematics teachers can make formal mathematics and contextual mathematics questions. Finally, they can determine which representations will be used in learning. Furthermore, the prospective teachers know visual representations with the help of specific media or applications to assist teaching and mathematical models, making it easier for students to understand calculations in explaining concepts. Knowledge of content and education helps prepare preservice mathematics teachers to prepare for teaching more quickly. With this ability, teachers can plan and organize lessons well and use the right learning strategies [29]. In addition, they help teachers face students' challenges, prejudices, and misunderstandings [30]. The construction of competence is closely related to knowledge. When a person has knowledge-based abilities in a particular field and can apply them to real-life problems or situations, they are considered competent [18]. According to this explanation, competence does not just include knowledge. Competence is defined as a set of cognitive dispositions or specific motivations (including expertise) through situations-specific skills to observable performance [18]. Therefore, regarding teacher competence, knowledge is considered a component of competence.

Based on the test results, it was found that the KCT assignments could be read well with high-reliability criteria and overall difficulty levels of complex, medium, and accessible. The test results also showed that preservice mathematics teachers had several choices in determining the arranged learning design. Sample questions made it easier for students to understand concepts and choose different representations for use in learning. The prospective teachers decided to use multiple words in assisting teaching. KCT means knowing the most compelling example or sequence of instruction. In learning mathematics, it is essential to balance conceptual understanding, procedural smoothness, and the ability to communicate mathematical ideas [31]. One of the goals is to determine the quality of teachers as the main component that affects teaching and learning mathematics. Most of their efforts were spent finding the specialized knowledge teachers needed to teach mathematics well [29], [32]. When situations-specific skills such as perception, interpretation, and decision-making are used, these actions are mediated, allowing adequate performance in the context of teaching. Knowledge as a disposition is usually measured through questionnaires or other test tools [18].

4. CONCLUSION

This study aimed to create task instruments for measuring KCT. According to the study's findings, the KCT task instrument was valid and reliable to understand the prospective mathematics teachers' subject-matter expertise and classroom management skills; the KCT instrument for preservice mathematics teachers offers a variety of options for choosing the learning design, other representations to be used in learning, and examples of questions used to help students understand concepts. The prospective mathematics teachers were found to select multiple representations while teaching. Furthermore, the instrument is also crucial to understanding how capable prospective mathematics teachers are in planning and successfully executing math lessons. Additionally, more research should be done on content understanding and classroom instruction. Such research will be able to reveal information on the expertise and experience of prospective math teachers.

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