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ABSTRACT

This study explores the integration of culturally responsive mathematical modeling (CRMM) in primary grades to promote racial justice and critical consciousness. The research focuses on how CRMM can support problem-based learning (PBL) by engaging students in analyzing real-world issues through mathematics. CRMM leverages students' cultural knowledge, identities, and experiences to critically examine and address social justice issues, such as racial representation in classroom libraries. The study utilizes a case study approach to investigate the supports and challenges faced by two primary grade teachers in designing and implementing a CRMM task, the Library Diversity Project. This project aimed to evaluate and improve the fairness of classroom library collections by analyzing the representation of different racial and cultural groups in children's books. The research highlights the importance of collegial support, critical reflection, and the use of specific pedagogical tools, such as decision matrices and graphical representations, in facilitating CRMM. Key findings indicate that CRMM tasks can cultivate critical awareness and empathy among students, enabling them to take action to address inequities. The study also unveils the complexity of implementing CRMM, particularly in navigating political, contextual, and mathematical challenges. Teachers' critical reflexivity and collaboration were essential in addressing these challenges and

fostering a learning environment that allowed students to use mathematics to analyze an issue and take action. The research contributes to the literature on equity-focused mathematics education by providing empirical evidence of the potential of CRMM to promote social justice in the primary grades. It emphasizes the need for ongoing professional development and support for teachers to effectively integrate CRMM into their practice. The study concludes with recommendations for future research and practice in culturally responsive mathematics teaching.

Keywords: *culturally responsive math modeling, critical consciousness, problem-based learning*

This paper details how a culturally responsive approach to mathematical modeling can support problem-based learning in primary grades that cultivates critical awareness and advocates for justice. Mathematical modeling (MM) is a cyclical process that uses mathematics to make sense of and analyze relevant, real-world situations by posing problems; making assumptions and defining variables; and creating, interpreting, revising, and sharing models (COMAP & SIAM, 2016). Problem-based learning (PBL), like mathematical modeling, centers inquiry, fosters curiosity, and engages students in analyzing complex real-life phenomena (Barell, 2007). While both MM and PBL have the potential to engage students in investigating complex phenomena related to justice issues (e.g. clean water access, political representation, traffic stops, food deserts), to do so explicitly necessitates a culturally responsive approach that includes cultivating critical consciousness, analyzing power dynamics and taking action (Aguirre, et al., 2019; Anhalt et al., 2018; Cirillo et al., 2016; Gutstein 2006; Ladson-Billings, 1995; Zavala & Aguirre, 2024;).

Culturally responsive math modeling (CRMM) uses a culturally responsive lens to (a) critically analyze real-world situations by connecting to students' knowledge, identities, and experiences; (b) support and sustain student engagement with high cognitive demand tasks; and (c) broaden student participation by distributing intellectual authority, minimizing status issues, and exercising collective critical mathematical agency (e.g. taking action by using mathematics in personally and socially meaningful ways) when the phenomenon under investigation warrants it (Turner, 2003; Zavala & Aguirre, 2024). We think of CRMM as an equity lever for a more humanizing, just, and liberatory mathematics education. We use the following equity definition to ground our work in CRMM (Aguirre, Mayfield-Ingram & Martin, 2024):

> All students in light of their humanity–their personal experiences, histories, identities, and physical and emotional well-being–must have the opportunity and support to learn rich mathematics that fosters meaning making, empowers decision-making, and critiques, challenges and transforms inequities and injustices. (p.10)

Thus, justice is inextricably linked to equity in mathematics as a tool for change.

One of the critical steps to designing justice-oriented CRMM tasks is working with teachers to consider local community problems relevant to students. As teachers learn about local contexts, they are more equipped to build students' critical awareness about contextual factors that have transformative potential (Anhalt et al., 2018; Jemal, 2017).

Few studies focus on CRMM in the elementary classroom (Albarracín, 2020; English, 2009; Turner et al., 2023;), particularly in the primary grades (kindergarten-grade 2). This special issue on PBL for equity and social justice echoes the need to highlight projects that engage diverse learners in meaningful real-world problems, and that reflect multicultural perspectives (Caires-Hurley et al., 2020). To address this gap in the literature, we utilize a case study analysis to identify the supports and challenges two primary grade teachers encountered when designing and enacting a CRMM task focused on racial justice–Library Diversity Project (Tate et al., 2022).

Relevant Literature

Culturally Responsive Mathematics Teaching

Culturally responsive mathematics teaching (CRMT) is a holistic instructional practice that includes explicit attention to cultural/community funds of knowledge and lived experiences in the teaching and learning of mathematics (Caswell et al., 2011; González et al., 2005; Hunter & Miller, 2022). Zavala & Aguirre (2024) define CRMT as,

> a set of specific pedagogical knowledges, dispositions, and practices that privilege mathematics, mathematical thinking, cultural and linguistic funds of knowledge, and issues of power and social justice in mathematics education. CRMT interrogates and innovates mathematics instruction to be a transformative and humanizing experience for everyone. (p. 6)

In addition, culturally responsive mathematics teachers hold high expectations for all students, foster critical analysis, and build authentic partnerships with students, families, communities, and sovereign Indigenous nations to support learning mathematics (Averill et al., 2009; Nicol et al., 2013). This is significant work for teachers of mathematics, and particularly challenging for teachers that may not share cultural identities with their students (Nicol et al., 2013).

Developing critical consciousness and self-reflection around racial, cultural, and sociopolitical contexts is a hallmark of culturally responsive teaching and learning (Gay, 2000; Ladson-Billings, 1995; Muhammad, 2023). Ladson-Billings (1995) argued that this pedagogy allows students to "critique the cultural norms, values, mores and institutions that produce and maintain social inequities"—in other words, to engage the world and others critically (p.162). Similarly, Muhammad (2023) included *criticality*, or increasing understanding of social issues and social constructions such as privilege, oppression, marginalization, and social justice as one of the key pursuits in learning. We concur with Muhammad that the elementary level is an ideal time to "build social political consciousness so students are not passive learners, but rather empathetic, critical thinkers working to see, name and root out discrimination and oppression in all forms" (2023, p. 75). With a focus on mathematics, CRMT offers a criticality space for children to engage complex situations and mathematize them to build critical consciousness and take action. For example, Sylvester's (1994) research on a justice-oriented economics unit found that a third-grade class grappled with authentic social and economic issues including homelessness, entrepreneurship, economic competition, and unemployment. Tate (1995) described an activity where middle school students investigated the ratio of liquor stores to schools in a given neighborhood and students wrote letters to city officials to enforce new zoning laws. Recent studies have documented young students using mathematics to explore racial, social, and economic injustice including economic boycotts, food insecurity, and clean water access (Aguirre et al., 2019, 2022; Chao & Jones, 2016; Koestler et al., 2022; Suh et al., 2018, 2021, 2023a; Tate et al., 2022, 2024; Turner et al., 2023a, 2023b). Many of these projects foster equity, critical consciousness, civic empathy, and community action (Mirra, 2018; Zavala & Aguirre, 2024).

Problem-Based Learning and Mathematical Modeling

Problem-based learning (PBL) and mathematical modeling (MM) are complementary approaches to ambitious, student-centered pedagogy in which students learn through solving open-ended problems (Suh & Seshaiyer, 2019). PBL is defined as "an inquiry process that resolves questions, curiosities, doubts, and uncertainties about complex phenomena in life" (Barell, 2007, p. 3). The role of the teacher in a PBL environment using MM is quite different from a traditional direct instruction

environment because the teacher becomes the expert facilitator. MM is a cyclical, iterative, and inquiry-based process that uses mathematics to make sense of and analyze relevant, real-world situations (CCSSM, 2010; Kaiser et al., 2011; Kaiser, 2017). Teaching MM involves supporting the development of modeling competencies (Maaß, 2006), including posing problems; making assumptions and defining variables; and creating, interpreting, validating, and sharing models (COMAP & SIAM, 2016). Emerging research demonstrates that elementary children engage with MM as an iterative cycle (Suh et al., 2021). So, while PBL is an instructional approach to teaching mathematics, MM is a mathematical domain with specific competencies that can be learned and assessed, even at the elementary level (COMAP, 2016).

Suh & Seshaiyer (2019) detailed the characteristics of a PBL learning environment (Marra, et al., 2014) that integrates MM, sharing that both are: a) problem-focused, authentic, ill-structured, and knowledge building; b) student-centered, where instructors serve in a supportive role; c) self-directed, where students individually and collaboratively assume responsibility; d) self-reflective, where learners monitor their understanding and learn to adjust strategies for learning; e) facilitative, where instructors support reasoning processes, group processes and interpersonal dynamics. MM begins with authentic, ill-structured problems connected to students' lived experience and context. These open-ended problems tend to be messy, require application of multiple math concepts, include multiple strategies and equally valid solutions, and involve a cyclical process of analysis and revision. MM involves building mathematical models that simplify the complexity of real-world situations while at the same time offering solutions that best fit the reality of the situation being analyzed.

Both PBL and MM have been studied as effective pedagogical approaches for student learning (Carlson et al., 2018; English, 2009; Krajcik et al., 2023; Lee & Galindo, 2021; Suh et al., 2017, 2021), yet these approaches are often more accessible to privileged students in special programs (i.e., Gifted and Talented), denying many historically marginalized students the opportunity for rich learning experiences (NCTM, 2020). It is important to situate these learning opportunity gaps in the broader context of systems, processes, structures, and policies that shape the racial and socioeconomic disparities that characterize U.S. schooling (Flores, 2007; Milner, 2012). These disparities include reduced expectations for students of lower SES and students of color (Peterson et al., 2016) and less access to high cognitive demand tasks that characterize PBL and MM (Boston & Wilhem, 2017). MM in the elementary grades has emphasized the importance of high cognitive demand tasks that examine complex real-world situations in schools, families, and communities so that students have opportunities to investigate and act upon situations that are meaningful to them (Anhalt et al., 2018; Suh et al., 2019, 2021; Turner et al., 2023). To better understand teaching MM in the PBL space with a justice-oriented task, research needs to attend to task design, as well as the supports and challenges that arise when tasks informed by culturally responsive approaches get enacted in elementary classrooms.

Theoretical Frameworks for Culturally Responsive Mathematical Modeling

This study on culturally responsive mathematics modeling (CRMM) draws upon two important frameworks including CRMT (Zavala & Aguirre, 2024) and MM (CCSSM, 2010). Building on previous work in CRMT (Aguirre et al., 2012; Aguirre & Zavala, 2013), Zavala & Aguirre (2024) provide a comprehensive framework made up of three interwoven strands each with multiple dimensions (See Figure 1). The *knowledges and identities* strand centers children's cultural/community funds of knowledge and lived experiences as resources for mathematics learning (Civil, 2007); rehumanizes mathematics by broadening what counts as math and affirming student math identity (Gutiérrez, 2018; Martin, 2000); and honors diverse mathematical thinking and ideas (Carpenter et al., 2014). The *rigor and support* strand attends to sustaining cognitive demand that emphasizes mathematical analysis and problem-solving (Smith & Stein, 1998), builds in social and analytic scaffolds to maximize access and sustained engagement in high cognitive demand activities (Anhalt,

2014), and affirms multilingualism by uplifting the translanguaging and multi-modal repertoires to communicate ideas (Chval et al., 2022; Maldonado Rodriguez et al., 2020). The *power and participation* strand focuses on broadening intellectual authority that maximizes student agency and authorship of mathematical ideas rather than the knowledge residing with the teacher, textbook or a few students (Dunleavy, 2015; Langer-Osuna, 2018); disrupting status and power by minimizing status issues that impact collaboration and dismantling racialized and gendered stereotypes about who can do mathematics inherent in classrooms (Featherstone et al., 2012; Zavala & Hand, 2019); and providing opportunities to critically analyze complex real-life situations and take action (Gutstein, 2006; Zavala & Simic-Muller, 2022). The analogy of strands (versus pillars) is intentional as it reflects a flexible interconnectedness across each strand's dimensions that can contribute to a positive, productive, and culturally sustaining mathematics learning environment. The design and strength of this *tapestry* is dependent on depth and breadth of the dimensions included.

Knowledges & Identities

Centering Cultural & Community Funds of Knowledge
Helping students connect mathematics with relevant/authentic issues or situations in their lives

(Re) Humanizing Mathematics
Supporting creativity, broadening what counts as mathematical knowledge, and affirming positive math identities for all students

Honoring Student Thinking and Ideas
Creating opportunities to elicit, express, and build on student mathematical thinking in multiple ways (e.g., through gesture, pictures, words, symbols)

Rigor & Support

Sustaining Cognitive Demand
Enabling students to closely explore and analyze math concepts(s), procedure(s), and problem-solving/reasoning strategies

Scaffolding Up
Maintaining high rigor with high support for all students

Affirming Multilingualism
Making space for multilingual learners (MLL) to be central participants in mathematics activities

Power & Participation

Distributing Intellectual Authority
Distributing mathematics authority and make space for multiple forms of knowledge and communication

Disrupting Status and Power
Disrupt status differences, entrenched stereotypes and inequitable power relationships present in all mathematics classrooms

Analyzing and Taking Action
Supporting student use of mathematics to analyze, critique, and address power relationships and injustice in their lives

Figure 1. Culturally Responsive Mathematics Teaching (CRMT) Zavala & Aguirre, 2024

The second framework informing this study focuses on mathematical modeling (MM). There are many versions of the MM cycle (CCSSM, 2010; Cirillo et al., 2016; COMAP, 2016; Kaiser, 2017). All involve some kind of iterative cyclical process that starts with making sense of a situation; identifying important quantities and assumptions; building and operating on a model; analyzing, interpreting, and refining the model; and ends with validating and reporting out the results of the model in the context of the situation (See Figure 2). Arrows show advancement as well as revisiting earlier phases to signal how modelers often revise their assumptions and refine models. Thick arrows signal entry and exit points to *build critical awareness* while making sense of justice-oriented situations and to *take action* at the conclusion (Anhalt et al., 2018; Cirillo et al., 2016; Zavala & Aguirre, 2024).
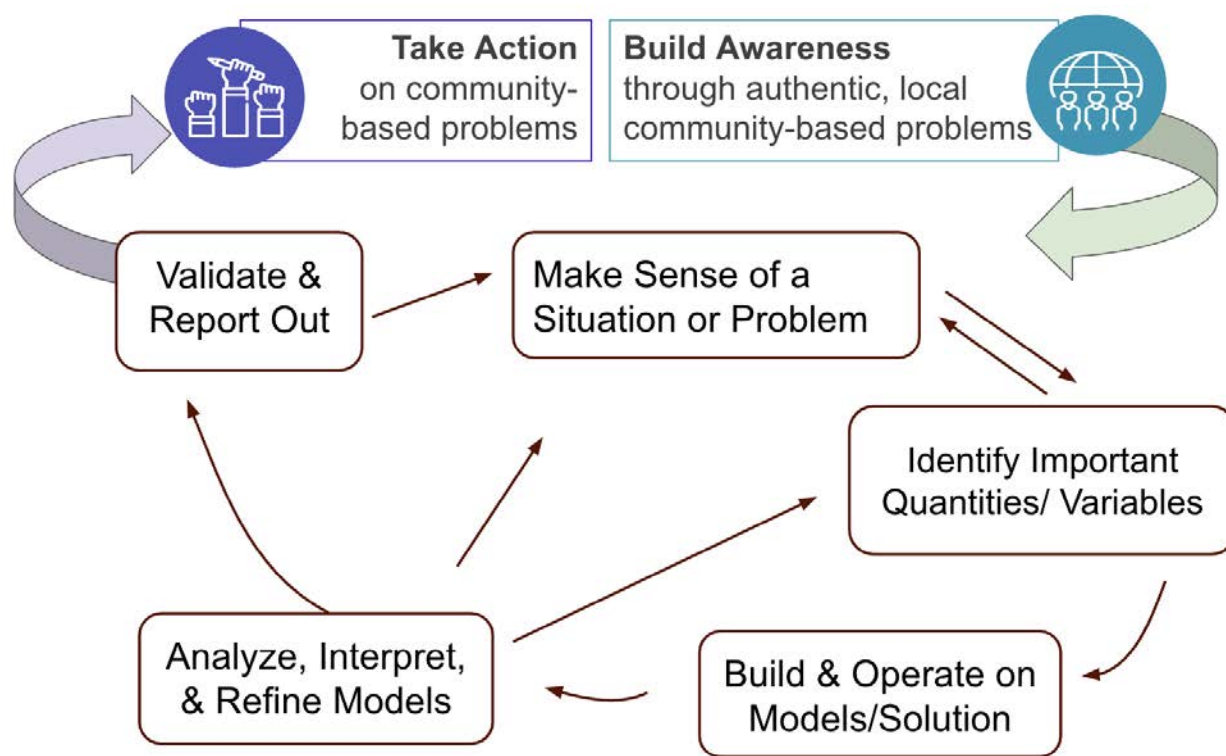
Figure 2. Culturally Responsive Mathematical Modeling Cycle

When MM is taught with an explicit justice-oriented culturally responsive approach, specific CRMT strands and dimensions are foregrounded (Cirillo et al., 2016). For example, to make sense of a situation that is grounded in a community-context, students make connections to their cultural funds of knowledge and lived experiences. This builds critical awareness by tapping into their knowledges and identities. As students collaborate to build and operate on models, they consider different mathematical ideas and reasoning. This foregrounds rigor and support dimensions such as scaffolding up and sustaining high cognitive demand. As students validate and report out their model's findings, intellectual authority is distributed as they critically analyze the situation including power dynamics, and identify ways to take action to transform injustice.

In this study, we draw on these two frameworks to investigate CRMM in a PBL space that facilitates critical consciousness and taking action – two important features of culturally responsive mathematics teaching (Anhalt et al., 2018; Turner et al., 2023). Using a case study approach, we examine how two primary grade teachers designed and enacted a specific modeling task that focused on racial representation and fairness of children's books (Tate et al., 2022). We address the following research questions:

Q1: What are the supports and challenges for teachers in designing and enacting a primary-grade justice-oriented CRMM task focused on representation and fairness?

Q2: How do these supports and challenges connect to the CRMT framework?

Method

Context of our Study

This study is part of a broader research project focused on CRMM in the elementary grades across four different regions in the United States. Via a year-long professional development program (2021-2022), teachers were introduced to frameworks for CRMT and MM, explored modeling tasks and routines, collaboratively planned activities, and reflected on classroom enactments. They also had access to a digital library with modeling tasks and planning materials. The Library Diversity Project was one of several justice-oriented modeling tasks in the digital library (Tate et al., 2022). Teachers self-selected which task they wanted to work on or co-designed with researchers their own modeling task.

Participants

This study focuses on two primary grade teachers from our project. Both teachers identified as white women who taught in schools with racially, culturally, and linguistically diverse students from underserved communities. The demographics matched the broader school demographics in both cases. Kelly is a grade 1 teacher at Willard elementary in the Mid-Atlantic region working in a state that does not support conversations related to race in schools. At the time of the study, she had nine years of teaching experience. Kelly's school has a diverse student population: Latinx 63%; Asian 19% , White 11%; and Black 3% , with 4% identifying themselves as "other". The majority of students (69%) are designated low-income and 53% receive English language support services.

Isabella (grade 2) is a first-year teacher at Hillside Elementary, in the Northwest region of the country. Hillside has an active ethnic studies committee that leads school-wide projects around race/racism, identity, community, and service. Hillside also serves a diverse student community: Asian 30%; Black/African American 18%; Hispanic/Latino 26%; Native Hawaiian/Pacific Islander 6%; Two or More races 8%; White 12%. In addition, 77% of students are designated low-income and 41% receive English language support services.

Our Positionality and Engagement in Co-designing Process

To understand our role as co-designers, it is helpful to share our positionality in this work.

Jennifer Suh:

I am an Asian American mathematics teacher educator and have a passion and commitment to teaching mathematics through social justice and to more intentionality in bringing criticality to the work. CRMM has been a highly authentic way to engage teachers and students by working together in community to develop critical consciousness and dismantle inequities. As a Korean American born in the United States and having attended school in both Korea and the U.S., I have navigated the complexities of privilege alongside the burden of racial stereotypes. This dual experience often left me feeling like a perpetual foreigner, caught between two cultures without fully belonging to either. My research is informed by a commitment to equity and culturally sustaining pedagogy in mathematics education and acknowledging and honoring students' intersecting identities. I work mostly with schools that are racially, culturally, and linguistically diverse and receive Title 1 funding. My research focuses on the use of CRMM to connect teaching mathematics for social justice interrogating power, privilege, and inequity.

Julia Aguirre:

I am a bi-racial third generation US born Chicana scholar and mathematics teacher educator who specializes in critical equity mathematics education that seeks to end systemic oppression and the role mathematics plays in it: racism, sexism, classism, and ableism. I have taught mathematics in formal K-12 school settings and out-of-school settings. My research is centered on equity-focused mathematics teacher education, culturally responsive mathematics pedagogy, social justice mathematics, and mathematical modeling. CRMM supports my efforts to uplift children's mathematical thinking, agency, lived experiences, and family/community connections as resources for mathematical learning and teaching. I am grateful to work with K-12 schools serving historically segregated and underserved communities of color and those impacted by poverty.

Erin Turner:

I am a white, cisgender woman, and mathematics teacher education scholar and teacher educator whose work addresses issues of equity and social justice in elementary mathematics teaching and learning, with a specific focus on mathematics classrooms that serve students from diverse cultural, racial, and linguistic backgrounds, including multilingual students from Mexican-origin and Mexican immigrant families in the Southwest. In my former work as a Spanish/English bilingual classroom teacher, and my current work supporting prospective and practicing teachers of multilingual students, I have had the privilege of building relationships with teachers, students and families, and learning from our collective efforts to build empowering, culturally responsive mathematics learning spaces. An ongoing goal in my work is to recognize my privilege as a white woman and academic scholar as I work to build collaborative relationships with schools that challenge power differentials and center the experiences of teachers and children. My current research interests include culturally responsive mathematical modeling in elementary classrooms and advancing equity in mathematics classrooms.

Mary Alice Carlson:

I come to this work as a white woman and mathematics teacher educator who focuses on recognizing the strengths in all communities with which I am privileged to work. Informed by my past and ongoing experiences facing my racial privilege, and by my position in a predominantly white institution, city, and state, I aim to build trusting relationships with teachers and walk alongside them as they develop the courage, compassion, and critical consciousness needed to confront racial injustice. As a researcher, my goal is to always keep the shared humanity of teachers, mathematics education researchers, and especially students at the forefront of my work.

Holly Tate:

I situate myself in this work as a white woman and an elementary mathematics coach. My positioning within justice-oriented teaching and learning is shaped by how I have been forced to face my racial privilege in new ways as a parent of multi-racial, Black children. I enter the field of mathematics education as a co-conspirator (Love, 2019), hoping to leverage my privilege in ways that elevate teacher and student action and empathy. I also believe in the need for (and power of) democratic learning, and I strive to learn and grow alongside communities to leverage collective and individual expertise. My research interests include teaching mathematics for social justice, the development of critical consciousness in teachers, and the role of participatory action research in creating humanized mathematical spaces for children.

Elzena McVicar:

I am a Black woman, mathematics teacher educator, mamma, and former elementary school teacher. My scholarly work focuses on how liberatory pedagogies inform approaches to teaching mathematics to young students, particularly Black and Brown students, by empowering them as mathematical doers and thinkers. I approach CRMM as an opportunity for mathematics teachers to foster positive mathematics identity development for students by using children's mathematical thinking and cultural and community funds to fuel mathematics learning. Ultimately, my aim is to cultivate mathematics learning environments that are joyous, promote creativity, and pursue criticality that moves towards justice in our world.

Library Diversity Project

The Library Diversity Project is a justice-focused CRMM task that investigates cultural and racial representation of children's fiction books in the class library (Tate et al., 2022). Books shape how children learn about social norms and relationships via the representation of the characters (Adukia, 2023; Hill & Bartow Jacobs, 2020). It is well documented that children have limited access to fictional books written or illustrated by BIPOC (Black, Indigenous, and People of Color) creators or that have main characters who are BIPOC (Greenspan, 2023). In fact, Black youth activist, Marley Dias, created a hashtag #1000BlackGirlBooks to address the lack of children's literature that features Black girl main characters in her school (Anderson, 2016). In a recent economics study examining racial and gender representation of award-winning children's books, researchers found significant over-representation of books with white male characters and under-representation of Black and Latinx main characters (Adukia et al., 2023). Furthermore, from a demand standpoint, they show that people "consume books that center their own identities" in other words they read more books that showcase characters with similar social identities (e.g. race, gender, culture, etc.)(p.2226). However, they also found that suppliers like libraries cater more to the dominant group (e.g. white people). For example, an analysis of public library inventories in Seattle, WA found twice as many books stocked from mainstream collections than from diversity collections, and that the price point for books from the diversity collection was 22% higher. So, while demand to buy books is related to social identities of the consumer (e.g. families, children), access to books that reflect a broader more diverse population is limited and more expensive, raising equity and racial justice concerns. This matters because award winning books are often in demand by book sellers and libraries, including school librarians and teachers, who select books for their classroom libraries and readalouds.

Given this background, the Library Diversity Project was created as a justice-focused PBL activity that fosters mathematical inquiry into complex phenomena relevant to children's lives. Children were asked to examine the fairness of their classroom library collection using mathematics to analyze social identity representations (e.g. race, culture) and generate math models that could help make the library collection *more fair*. During the planning, teachers mapped out potential mathematical pathways that aligned to the mathematics standards (see Figure 3).
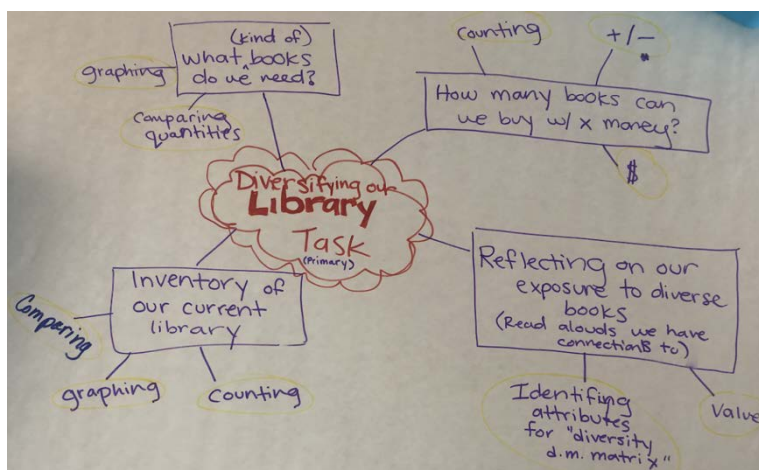
Figure 3. Concept Map of the Potential Mathematical Modeling Pathways

This project resonated with both Kelly and Isabella because both were trying to diversify their library collections with read-aloud books featuring main characters of color. Kelly had already started this process the year prior. Isabella had inherited a classroom library from a retired teacher and thrown out books that were outdated or offensive. There were several cornerstone activities that anchored the Library Diversity Project including the following (See Table 1): A read aloud book focused on race and diversity (Madison et al., 2021); a routine called "Mathematizing the World", where teachers used an image (Figure 4) to do a notice and wonder activity with students (Tate et al., 2022) and a book sort activity to generate data of their library books. The primary math focus was analyzing the data and making decisions to purchase books that would make the library *more fair*. Kelly introduced a graphic organizing tool called a decision matrix to help students rank and rate books they would like to purchase. Isabella introduced bar graphs to represent the book data generated. Both Kelly and Isabella provided students with a budget constraint to further help make decisions. Students presented their findings to critical stakeholders such as the principal, librarian, and potential donors to take action purchasing books that would make their libraries more fair.
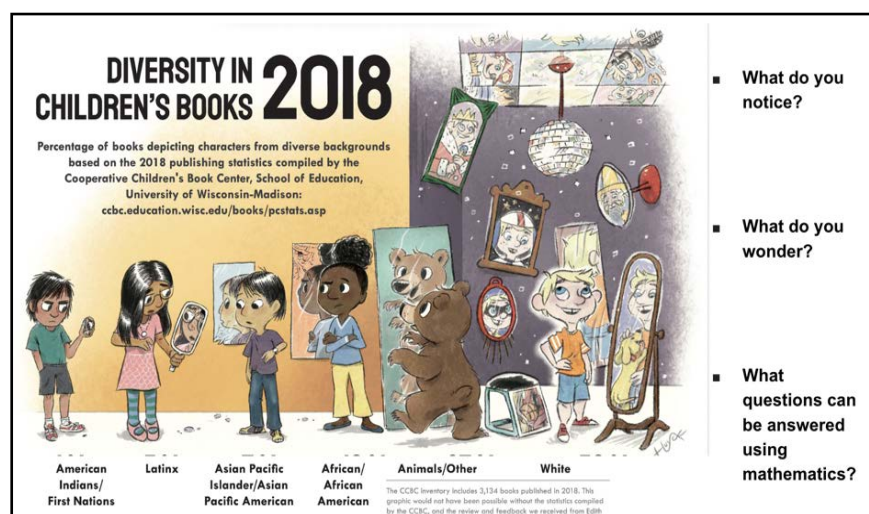

Figure 4. Numberless Graph Used to Launch the Library Diversity Project

Purpose	Activity	Description
I. Building background knowledge	Read-Aloud: Our Skin Children's book on race	Used the read-aloud to remind students about race and racism. Sparked connections with some students' lived experiences and family member's experiences.
	What is culture?	Class engaged in discussions about race and culture
	Find a book (like me/not like me)	Students selected a book that was like them in some way.
II. Evaluating the library for fairness	Mathematizing the World Routine with the numberless graph	Notices and Wonderings with the numberless graph and then provided a bar graph representation of the same data.
	Diverse representations within racial groups	Students showed different images of people who identify as a specific racial/cultural group.
	Book sort	Sorted books from classroom libraries based on specific race/culture categories. Counted books in each category and recorded their findings

III. Using math to make decisions

(1st grade, Site A)	(2nd grade, Site B)
A decision matrix was introduced with criteria to assign points to evaluate books. The decision matrix had categories like "the book is written by a person of color", "character is African American, Asian American, Native American" and other criteria for diversifying the library.	To further analyze the fairness of the library, the teacher introduced a bar graph of the school demographic data. The students compared bar graph data with school demographic data. The students revised the book sort graph to make each category "more fair" by adding additional books.

IV. Taking action

Purchasing new books	Students used a given budget to purchase books to make their library more fair. Students shared their findings with other teachers and students, as well as administrators, librarians, and school boards. Books were purchased.

Table 1. Design of the Library Diversity Project with Timeline of Lesson Activities Across Two Sites

Data Sources & Analysis

Our data sources included transcripts from teacher planning sessions and interviews. The primary data source was individual teacher interviews following the lesson enactments. These debrief

interviews focused on reflection of the planning and enactment stages, critical incidents, supports and challenges with student learning and teaching, and how they integrated CRMT practices during their lessons. Additionally, teachers participated in hour-long individual interviews at the end of the year that focused on perceptions of MM and connections to the CRMT framework, experiences implementing modeling lessons, student learning, and support and challenges for teaching modeling.

All interviews were transcribed and coded using Atlas Ti. Using an inductive, open-coding process (Saldaña, 2021), we first identified quotations where teachers talked about challenges and supports during the planning and enactment phases of the Library Diversity Project. We also identified quotations that related to the CRMT dimensions such as honoring student thinking, centering cultural funds of knowledge, and disrupting power and status. We looked for code co-occurrences with codes including supports and challenges during design and enactment with the CRMT dimension codes. This examination of code occurrence supported our understanding of the research questions because we not only wanted to understand the supports and challenges, but also understand how CRMT dimensions emerged for the justice-oriented task focused on representation and fairness. The researchers reached consensus coding interviews using the CRMT framework as part of the coding book.

The secondary data sources included field notes of the planning sessions and classroom videos of the lessons where we identified clips that the teachers mentioned in the interviews. Using all these data sources, we created cases for each teacher that detailed the sequence of events with supports and challenges and specific critical incidents. Finally, we initiated a cross-case analysis identifying themes related to supports and challenges in the design and enactment of the Library Diversity Project and how these related to the CRMT dimensions.

Findings

Through our cross-case analysis, we elaborate key themes related to the supports and challenges of implementing a justice-oriented CRMM task using illustrative excerpts from interviews and professional development discussions. We connect these findings to salient strands of the CRMT framework as well as describe how these teachers worked across strands to design and enact the library diversity project.

Theme 1: Supports and Challenges to Raise Students' Critical Consciousness Around Racial Identity and Disrupting Stereotypes

In the Library Diversity Project, Kelly and Isabella described key *challenges* and *supports* related to raising students' critical consciousness around racial identity and stereotyping. Planning and enacting the book sort activity in ways that supported conversations about racial and cultural identity posed a challenge. In this section, we describe how collegial conversations among teachers and resource materials supported teachers as they navigated this challenge. We end with a discussion of how these supports and challenges connect to specific dimensions of the CRMT framework.

Being "Seen": Avoiding Stereotypes and Affirming Multiple Identities

Kelly was inspired by the Library Diversity Project introduced in the planning workshop. Collegial support pushed teachers to critically reflect on potential challenges. For example, when Kelly shared with her colleagues that she wanted students to sort her classroom books, her colleagues pressed her to think critically about the use of racial identity categories as a method of sorting.

Kelly: I think I will start with an inventory of our current library, getting the kids to help with

that. So that could involve graphing. We do count how many of these whatever categories and come up with comparing quantities or numbers. We've already read aloud and which ones they make their own connections to, and so getting them to identify different areas of diversity. So, food, language, activities, race, gender, but whatever they identify with. Then, the whole class, a diversity decision-making matrix based on which books do we want to buy, what we want to see more of in our library and the idea of value so then some other math modeling. Alright, our class was given this much money now how can we spend it?

Grace: How will students sort all those books? It would take me a while to read a book …and decide which category.

Kelly then explained to her colleagues that as part of critical literacy and social studies activities, she read aloud many books with diverse characters so that all students could see themselves and other culturally diverse families in the stories. Some books included in the sort had characters with racial background familiar to students, which would support their sorting. Kelly also had students map their identities with a culture map activity that she said connected to, "family celebrations, language, food, cultural activities, race, gender."

As Kelly shared these prior activities with her colleagues, she also posed questions about how she would distribute books for sorting.

Kelly: So, one of our social studies standards is like recognizing diversity. And so, at the beginning of the year, we did already do a lot of things like my family celebrations, our language and things like that…So I don't know if I want to mix it up and give each table a sample of each one to help like for them to go through and count just a smaller sample. And then we'd like to put it together as the whole class. There are so many that books have multi-races and just exactly how can they be quantified in different areas....

Missy: So there are a lot of assumptions…

Grace: Yes, the name Manuel doesn't necessarily mean that he is Latinx.

This excerpt showcases a challenge that Kelly encountered during the design phase (avoiding assumptions and stereotypes) and how her colleagues served as critical friends, anticipating the task with her and prompting her to think more deeply about tensions related to racial identities and stereotypes that might emerge as students sorted the books. The teachers were aware of intersectional identities and did not want to perpetuate stereotypes by assuming race, ethnicity and culture based on visible characteristics and names. This collegial conversation prompted the teachers to think more deeply about the task and how to make the activities more accessible for first graders.

A related challenge teachers encountered as they enacted the book sort was supporting students who struggled to find books that reflected their racial or cultural identities. For example, Kelly shared that she had been working on diversifying her read-aloud collection for some time, and that she was very invested in centering her students' identities in her classroom. She shared that she has a special connection with her students as she lived in the community and previously attended the school. The importance of affirming a child's racial/cultural identity was particularly salient during the *like me* book sort activity when one of her first-grade students, Chinho, could not find a book with an Asian boy character. In her debrief interview Kelly stated:

So, he was the one that had the hardest time. We did have a couple of books, and he did pick one that he got excited about because we have one about Chinese New Year which he celebrates. But he was the one that found it the hardest to find a character that looks like him.

Because we identified Asian boy characters as like a large hole in our library. Right? We have a decent amount of Asian characters, main characters, but far fewer boys than girls.

Kelly recognized how this deeply affected her student and took action to make sure that when students used the decision matrix to identify books from the publishing company circular, it would include books that this child could identify with. In this case, she made the circular more inclusive by adding a book with a Chinese boy character. This explicit action to center this child's knowledges and identities through books helped facilitate student empathy and critical consciousness as well. Kelly describes another meaningful moment involving Chinho advocating for a book that would help his Latinx classmate, Miguel, feel affirmed (See Figure 5).

So, I made sure that our circular would have books with Asian boy characters. Later he picked a fly guy book, because it was written in Spanish. And he knew that we had a student in our class who did not speak English at all that year. Miguel was a newcomer, brand new, spoke no English and Chinho said, 'but I know he loves it when Miss Kelly reads books in Spanish, so we need more books that are written in Spanish so that he can enjoy the read-alouds too.' So, he wasn't even shopping for himself. He was thinking about other kids that it would also be nice to have just like we weren't even thinking about, like not the animal characters. He thought it's written in Spanish.

In response to the challenge of students who struggled to find books that reflected their identities, Kelly prioritized the need for each and every one of her students to be "seen" by modifying the circular for book choices. This response was emulated by Chinho, when he also considered his classmate and purchased a book for Miguel to feel seen. Both Kelly and Chinho's actions show the power of empathy and affirmation of multiple identities (cultural, language) to support teachers as they navigate the book sort challenges.

Figure 5. Chinho's Selection of Books Gives Him Visibility as well as His Friend Miguel

Beyond Binary Sorting: Expanding and Reckoning with Racial Identity

Isabella also faced challenges supporting students who struggled to find books that reflected their racial and cultural identities. As a first-year teacher, Isabella had inherited a library from a retired teacher that had many outdated books with racial stereotypes. While she got rid of most of those books, she thought the Library Diversity Project would provide an opportunity to acquire more relevant and meaningful books for her classroom. To support students' critical consciousness about racial identities, she enacted many of the pre-work activities and lesson slides provided by Kelly and Holly. In particular, she found the slide deck that highlighted diverse faces that identified as a

particular racial group helpful in supporting further development of students' understandings about race. Isabella stated:

> We looked at the slides that one of the grades at Site B had used about like these are people who identify within certain racial group just because even though we've talked about it a lot in my class, I wasn't sure how familiar each of the kids were with, well, what could a character in a book possibly look like if they identify as Latinx. I liked these slides. I thought they were really nice in the way they showed a huge diversity of people within each racial group and identity. And then I liked when Lila stopped on the Black/African American/African slide. And she said, "Well, that guy looks white." I was like, "yeah, his skin is not as dark as the people around him, but can we always tell how someone identifies by looking at them?" Which I think is such an important part of the conversation. And that's also what I was kind of hoping one of them would do.

However, during the book sort Isabella noticed that instead of noticing diverse racial identities in the books, students were sorting the books into only two categories, white characters and Black characters. Isabella shared that this binary sorting was a major challenge she faced in the project and that she consulted with another teacher at the school for assistance.

> And then as I was kind of going through the task and every day doing something different, I was able to come to her (Deborah, grade 1 teacher) and I was like, so I noticed this when we were book sorting that the kids were really struggling to get books with Latinx characters and Asian characters into categories that matched it. They kept putting them as white and she was like, okay, well let me look at some of my books and see if we're going to see that same issue in here. And then her suggestion was kind of like, well just do another day of it, just go back and revisit it. I did. I think that made a huge difference in the project. I think if I just tried to plow forward and be like, this is what we're doing next, it would not have worked as well. So, we went back and we spent another day talking about what were the demographics and how can we tell and what does that mean and what does that look like? And then I had them do another sort, can you find a book that looks like you?

When Isabella asked students to revisit their groupings of books, she focused their attention on specific books, and posed questions, inviting them to explain and justify their sorts.

> I was noticing with the other group that there were some people possibly getting miscategorized for the purposes of our graph. I decided, I'm just going to pull one of these and have them look at it because I don't want to take over and be like, let's look through every book. Is this right? No, I don't think this is right. But I just wanted to model for them like, "Hey, I have a question about this one. Let's look through it really intentionally, let's look for words, let's look at the back" with the hopes that they would then, with the following, books do that.…But I wanted the kids to really take ownership of this. And if this is how they are perceiving our library, that's also an indication to me that there's a lot more work for us to be doing.

For Isabella, an important part of supporting students' critical consciousness about race was allowing students to maintain ownership of their ideas and decisions. This meant that instead of correcting or redirecting students' book sorts, she provided additional opportunities, and guided students to use additional resources and careful observation to promote criticality in their sorts.

Isabella responded to this challenge by consulting her colleague, and by spending more instructional time helping students think critically about the demographics of the school. She engaged

students in a personal book sort to connect to their own identities, and completed additional read-alouds related to race to give students time to deepen their critical consciousness about racial identities of main characters in books. These supports broadened students' experiences and helped to disrupt narrow views of racial identities.

However, the Library Diversity Project also raised a significant challenge for Isabella as a white teacher implementing this project: She voiced internal tension and critical reflexivity (Le Bourdon, 2022) reflecting on how her identity and social position influenced her view of the world. While Isabella openly acknowledged her white racial identity while engaged in the project, there was a moment that made her question her positionality. Like Kelly, the data for her class had the most books with animals and white characters. The library was not fair based on the over representation of white character books versus books with BIPOC characters. Students were engaged in thinking about how this is unfair and how to take action to make it more fair. Isabella shared a story about how a student came to school one day in distress about the racial representation of teachers. This student's response caused Isabella to pause and critically reflect on the project and its impact on students and her role as their teacher:

> But there was a day where I was like, is this project stirring up too much for the kids? And I'm not the person to be leading it. They need someone else to be helping them go through it. There's been, kind of this weekend, uncertainty about what do I need to do ethically for these kids? And where do I need to go? Are there places I need to back off or are there places that I'm not giving them enough space and I need to give them more space? The dilemmas have been more on my end as I've reckoned with this one student's pretty extreme reaction to his feeling of a sense of safety and security in our classroom.

Isabella's reckoning points to her capacity to critically reflect on her contribution to this situation: How might she be harming and how might she be helping students navigate systemic racism as it plays out in school libraries and the teaching workforce? Isabella continued with the project, mindful of how this student felt and offering individual support if needed. In her final interview, Isabella stated:

> I see that it (disrupt status and power dimension) says disrupting entrenched stereotypes and I think the library task tackled that, not necessarily entrenched stereotypes in math, but stereotypes in books or stereotypes that are present in the classroom. And inequitable power relationships, definitely same sort of thing if we have, my classroom was shown to have a lot of books representing white students and giving this project to the students really helped them identify that and disrupt it as well as it was kind of hopefully intentional on my end where I was trying to do the same thing.

Through the first theme, we found connections to CRMT in the ways both Kelly and Isabella navigated through the challenge of supporting critical consciousness about race by attending to students' knowledges and identities, affirming multilingualism, disrupting status and power, while distributing intellectual authority. Through collegial discussion, Kelly and Isabella implemented PBL that attended to diverse learners' identities by taking on critical perspectives as they used mathematics to analyze inequities reflected in classroom libraries. Kelly *supported students' identities* when she adapted the book circular to include books with main characters from racial and gender backgrounds that were previously *invisible* to some of her students (e.g. books with Asian boy characters). She *affirmed multilingualism* by responding to her student's idea to add bilingual books to the options. Isabella employed the dimension of *disrupting status and power*, particularly in relation to disrupting stereotypes present in books and how students identified and categorized the racial/cultural backgrounds of main characters of books. As Isabella moved toward culturally responsive teaching,

this work required critical reflexivity, support from colleagues, time to revisit and focus students on specific details in selected books (e.g. pictures of main characters, language clues). Isabella also *distributed intellectual authority* by inviting students to frequently share their thinking and co-constructing their ideas of how to make the library more fair.

Theme #2: Supports and Challenges to Mathematize Fairness and Take Action

In the Library Diversity Project, Kelly and Isabella described key *challenges* and *supports* related to mathematizing fairness and developing agency to take action. Key supports for mathematizing fairness included a decision matrix, budget constraints which maintained a high cognitive demand, and honoring students' diverse mathematical ideas to help construct models that would lead to a *fairer* library. In addition, graphical representations provided both supports and challenges in how students analyzed fairness and took action to address inequities they identified in their library book collection. We end this theme with a discussion of how these supports and challenges connect to specific dimensions of the CRMT framework.

Fostering Critical Mathematics Through the Decision Matrix

For Kelly the decision-matrix (see Figure 6) was a key support that informed discussions about using mathematics to evaluate books, and mathematizing issues of representation and fairness. Kelly co-created categories with her students to evaluate potential books they would purchase for their class library. Students rated the books and awarded points (1-5) for each category. In her debrief interview, Kelly reflected:

> One of them (criteria) was does this [book] represent students at our school, our culture? [Our] heritage, I think, is how we had put it in the matrix. [They evaluated whether] the main character [was] a person of color. And, we had it broken down into two more specific groups: is the main character a person of color and is the author or illustrator, a person of color.

She felt that having these different criteria (i.e., racial/cultural background of author or illustrator, and of the book's main character) would support a critical discussion about how to evaluate the book's potential to enhance the diversity of their library. In addition, the decision matrix was a visual organizer that provided an important scaffold for students to record their sums, identify patterns, and justify book choices to make the library more fair. The final rating of a book involved adding five addends which surpassed grade 1 number and operation standards. Thus, helping to sustain high cognitive demand throughout the project.

The task also included a budget constraint for ordering books ($20). The budget constraint added realistic complexity to the situation sustaining the cognitive demand while supporting rich discussions about fairness and representation. Students added multiple addends to $20 and if they went over the budget, they had to trade out books and choose another that would satisfy their goal of making their library more fair. The $20 constraint also offered many opportunities for students to practice addition with multiple addends using numbers between 1-20 which was a first-grade standard. To illustrate, Kelly shared a critical moment during the enactment where two students working together had a novel solution to optimize their $20 budget.

> So, I had assigned some that each got $20. If they wanted to work together that was fine, but they had a budget of $20. Two students did realize they had spent $19, and so there were no books in the Scholastic flyers that were only a dollar. So at first I just asked like, 'So can you buy anymore?' And they said no, but one of them said, 'but if I share my dollar with my friend we'll have $2 and there were a couple books in there that only cost $2.' So that's when I let

them know. I said, 'Sure, right, like as long as you're within your budget'. One of them spent $19, and the other one spent $21, and they shared a book in their plan that cost $2. So that was really really cool, and yes, I had not even thought about them sharing their budget to like get more out of it.

Figure 6. Decision matrix to evaluate books to diversify the class library

Kelly honored students' diverse mathematical ideas and encouraged them to pursue a novel mathematical strategy that not only optimized their spending but reflected a sophisticated compensation strategy they collectively generated. In reflecting on this incident, Kelly who lives in the community and is familiar with the families in the community shared that many of the Latinx families she works with prioritize collectivism where group responsibility is shared. Although this strategy was not anticipated and surprised her at the moment, Kelly reflected on how this emphasis on collectivism and pooling their funds reflected the way of thinking that was culturally rooted in the community.

Examining Equity and Equality Through Graphs

Isabella described how graphical representations, specifically bar graphs, supported and challenged students to make sense of their book sort data as they worked to mathematize fairness. Isabella felt the bar graph of their own classroom library data helped students evaluate fairness and make connections and comparisons to the graph of the national data. Students made observations about differences and tried to offer solutions to make their library more fair as they navigated these new graphical representations (see Figure 7). She noted in her debrief interview:

And it was really interesting because I honestly had expected more students to just draw their bars all the way up to our highest quantity, which we had, which was 92, and just be like, they

all have to be at 92. And only one student did that. And then some of the students didn't even put the bars up to be like, they all need to be at 50. Like each bar they were saying, 'Well I think we need this many because this one is the smallest, so we need to get it up to 50, but this one's not as small so we only need to get it up to 40.' [Students were] really trying to reason with what does it mean when this bar is so big and what does it mean compared to the others?....And so to me that was saying like, I'm seeing there's something that's unfair going on here and the way to make up is to really overcompensate in one group, but that doesn't necessarily fix things. So I've seen a lot of equity versus equality without having to name it. With them understanding it.
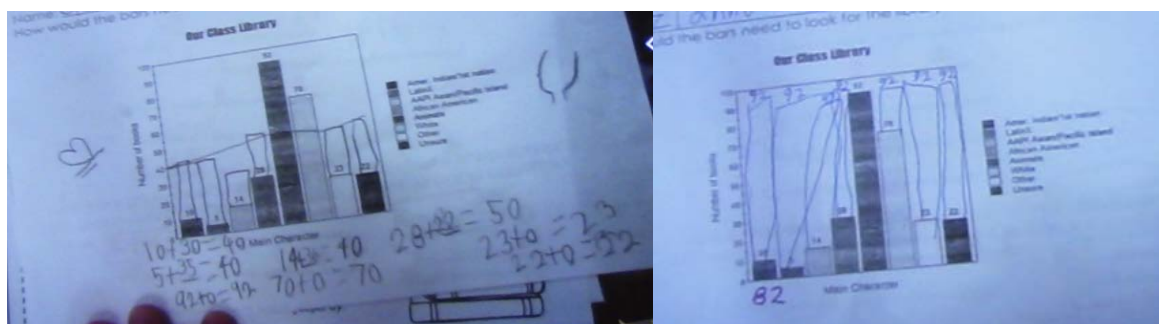

Figure 7. Different Ideas About Graphing Fairness

Isabella also recounted that some students were challenged to represent their diverse mathematical ideas particularly part-whole relationships and subtraction. Students could graphically represent "closing the gap", but had a hard time determining the missing part with symbolic notation. After noticing this challenge, Isabella paused the class and modeled an example using one student's idea and a number bond example. Then students returned to work. She stated:

When they were working on their graphs, there was a lot of trouble. I noticed the kids figuring out the missing part. [Students asking] 'So how do I get from 10 books to 50 books? And how do I use math to show that?' So they were able to draw it on their bar graph, but then when I was like, cool, how many more do we need? It was really hard. And we probably could have spent a full 90 minutes just doing that because some of them really just needed a lot more help and support there.

Isabella recognized the need to scaffold students across representations to fully interpret their solution and take action.

In addition, the Library Diversity Project pushed students to be change agents for their school using mathematics to develop systems for change and exercising a collective critical math agency. Isabella noted that her students have moved beyond mathematics to mathematical justification for building equity-based systems for checking out books.

…now they're starting to be like, 'ordering books isn't enough. We should also be making our own books. Why can't we make our own books? Why don't we have a system in place where you get out all the books with Asian characters and we hand them out equally. And that's how we share them. And everyone gets the same number.' … I think they've kind of moved past 'we need 10' to 'this is unfair' and that's how they're expressing it as opposed to 'we don't have enough of this one.'

Throughout the second theme, several supports and challenges to mathematize fairness and

take action connected to the CRMT framework. Kelly and Isabella found specific representations (e.g. decision matrix and bar graph) were supports that *honored diverse student mathematical thinking* and *sustained cognitive demand*. Students showed mathematical creativity and criticality through their bar graph representations (closing gap solutions) and budget models (novel optimal strategies to pool funds). The budget constraint of $20 also provided multifaceted support that promoted opportunities for students to draw on their *experiences and funds of knowledge*, showcase their *intellectual authority*, and *analyze and take action* through generating mathematical solutions collaboratively and, ultimately, advocating for new books that would make the library more fair. Teachers also reported mathematical challenges that occurred as students grappled with the complexity of the task. Isabella recognized some students struggling with subtraction and representing their models symbolically. She used a close analysis of one student's math idea to scaffold the connection between their bar graph representation and a symbolic representation to "close the gap", thus connecting to the CRMT dimension of *scaffolding up*.

Discussion

This study provides empirical evidence about the potential of CRMM to cultivate critical consciousness with mathematics and take action to address racial justice in the PBL space (Anhalt et al., 2018; Muhammad, 2023; Turner et al., 2023). Previous studies have shown that CRMT is complex with teachers navigating political, contextual, mathematical, and affective terrains (Aguirre et al., 2019; Aguirre & Zavala, 2013; Anhalt et al., 2018; Gutiérrez, 2013; Kokka, 2022; Leonard et al., 2010; Zavala & Aguirre, 2024). Our study confirms the complex terrains teachers navigate by analyzing the supports and challenges that arose as students participated in the Library Diversity Project.

We found the supports and challenges in enacting a justice-oriented modeling task traversed multiple CRMT dimensions across all three strands: Knowledges and identities; rigor and support; and power and participation. These connections demonstrate the potential for high impact and ambitious teaching with CRMM in the PBL space (Suh & Seshaiyer, 2019). Our findings highlight the importance of collegial support for co-designing tasks that meet mathematical and student engagement goals as well as critical reflection on teaching. Design supports include co-constructed lesson materials (slide decks, children's literature suggestions, decision-matrix); explicit time for building background knowledge about race, racism, and fairness prior through read-alouds and cultural mapping activities; and cross curricular connections to social studies and critical literacy units. Many of these supports connect to the CRMT framework. Specifically, criteria included in decision matrices *centered students' funds of knowledge and experiences* and acted as a catalyst to *rehumanize mathematics* by affirming students' math identity and their intersectional identities such as race, class, gender, and language. Both Kelly and Isabella identified collegial conversations as a support that helped them to navigate the political terrain of the task and to *disrupt stereotypes*, *power and status* in critical conversations about mathematizing race/cultural representation in children's books. Challenges such as raising students' critical consciousness, talking about race, and making sense of symbolic notation when mathematizing fairness connected to the dimensions of *honoring students' diverse mathematical thinking* and finding ways to *scaffold* rich discussions that *distributed intellectual authority* and *sustained high cognitive demand*.

In terms of enactment, the authenticity of the task motivated students to build critical awareness about racial representation and fairness in books and to take action to address inequities in their library collection. The critical thinking mathematics activities like the notice and wonder routine using the numberless graph that depicted the disparity of representation of racial diversity in books, and the book sorting activities, elicited deep conversations about race and culture as well as mathematics. Teachers also identified specific task supports, such as the decision matrix, budget constraints, and bar graphs, that helped students investigate this complex situation (*sustain the*

cognitive demand), as well as *honor and extend diverse mathematical thinking*. Kelly's students negotiated mathematical solutions involving multiple addends. Isabella's students discussed their book inventory graphs differentiating between equality and equity to purchase books and redress the racial injustices of their library collection. These supports leveraged student critical analysis and also fostered opportunities to be change agents to make their libraries more fair for themselves and future students (*analyzing and taking action*).

Enactment challenges included navigating the tension that the teachers felt in honoring students' diverse ideas while developing students' critical consciousness about race. Challenges with mathematical concepts also arose as students constructed and interpreted mathematical models of fairness. Moving between graphical representation and symbolic notation involving subtraction to "close the gap" between books with main characters of animals or white children and books that included main characters of children of color necessitated explicit scaffolding and time. While this mathematical challenge with subtraction is not surprising in the primary grades (Carpenter et al., 2014), Isabella supported students by building on her students' work which honored their mathematical thinking (*scaffolding up and honoring diverse math thinking*).

The design of this justice-oriented CRMM task within PBL necessitates a multifaceted approach, incorporating collaboration, time, and the development of critical consciousness. Using a CRMT approach, these teachers created a safe environment for criticality, enabling students to examine social inequities reflected in their library book collection, foster dialogues centered on racial justice, and take action using mathematics to promote structural change in book collections that will benefit themselves and future students at their schools (Muhammad, 2023). This safe environment was also important for teacher participants who shared their personal reflections about their growing critical awareness around racial justice and how that is reflected in the books children have access to (Anjali et al., 2023; Greenspan, 2023).

In our ongoing efforts to co-design justice-oriented PBL initiatives using CRMM, we acknowledge the importance of recognizing both our own positionalities and those of the educators with whom we collaborate. This mutual understanding was instrumental as teachers, alongside their colleagues and professional development facilitators, embarked on the creation of PBL initiatives such as the Library Diversity Project. Our collaborative efforts with students not only fostered a profound critical consciousness but also cultivated a sense of collective critical mathematical agency aimed at transforming their classroom libraries in personal and socially meaningful ways (Turner, 2003). Moreover, a commitment to challenging the systemic conditions prevalent in educational contexts not only facilitated the development of teachers' critical consciousness but also demonstrated the remarkable capacity of primary grade students to grasp, empathize with, and connect to social justice situations (Chao & Jones, 2016; Koestler et al., 2022; Tate et al., 2022). This process underscores the potential of justice-oriented PBL that utilizes CRMM to serve as a powerful vehicle for both educators and students to engage in meaningful change.

Finally, the author team entitled this study "Exploring Racial Justice with culturally responsive mathematical modeling in the primary grades: Cultivating criticality in the problem-based learning space" to emphasize the Library Diversity Project's focus on racial and cultural representation in children's literature, directly engaging with issues of racial justice. By employing culturally responsive mathematical modeling (CRMM), the project allowed students to analyze and address the representation of different social identities in their library collection, making the learning experience relevant and meaningful. Focusing on primary grades underscored the importance of introducing these concepts early in children's education, fostering awareness, civic empathy, and critical thinking about social justice from a young age. The project encouraged students to think critically about the fairness and equity of their library collection, using mathematics to develop a deeper understanding of social justice issues. As a problem-based learning activity, it engaged students in solving real-world problems, helping them apply their mathematical skills to meaningful and relevant issues, thereby cultivating critical thinking and a deeper understanding of both

mathematics and social justice. Racial justice means working towards systemic change and solutions by targeting the root causes of inequities/injustices, dismantling them, and replacing them with systems that center equity, justice and liberation for everyone. By examining and addressing the disparities in the representation of different racial and cultural groups in their library, students actively participated in the pursuit of racial justice through bold actions like writing a letter to the publisher, presenting budget-based proposals to school principal and librarian, and advocating authorship of their own books to ensure future access to books that are more representative. This project not only highlighted the importance of equitable representation in educational resources but also empowered students to become agents of change in their communities.

Acknowledgments

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Jennifer Suh is a professor of mathematics education at George Mason University. Dr. Suh teaches mathematics methods courses in the Elementary Education Program and mathematics leadership courses for the Mathematics Specialist Masters and PhD Programs. As the principal investigator (PI) for numerous NSF-funded collaborative grants, including DRK12, STEM-C, ITEST, and RITEL, she has focused on immersing elementary teachers in mathematical modeling, culturally responsive mathematical modeling, and the equitable integration of technology in the mathematics classroom. Over the past 20 years, Dr. Suh has been deeply involved in Lesson Study, a teacher-led professional development model. This model elevates teachers as co-designers and co-researchers, centering equity in community-based mathematics modeling lessons. Currently she collaborates on a project with the coauthors of this article called EQSTEMM (www.eqstemm.org) focused on culturally responsive mathematical modeling in elementary grades. Her work has significantly impacted students in marginalized communities, empowering them to see mathematics as a powerful tool to address injustices and inspire action.

Julia Aguirre

Julia Maria Aguirre is a Professor of Education at the University of Washington Tacoma. Her work focuses on critical equity studies in mathematics education, teacher education, culturally responsive Suh et al. Exploring racial justice with culturally response mathematical modeling mathematics pedagogy, and mathematical modeling. In addition to multiple journal articles, Dr. Aguirre is co-author of four books: The Impact of Identity in K-12 Mathematics: Rethinking Equity-

Based Practices (2024); Cultivating Mathematical Hearts: Culturally Responsive Mathematics Teaching in Elementary Classrooms (2024); Transforming Mathematics Teacher Education: An Equity-based Approach (2019); and The Impact of Identity in K-8 Mathematics: Rethinking Equity-based Practices (2013). Current grant-funded project EQSTEMM focuses on culturally responsive math modeling. She has taught mathematics in formal K-12 school and out-of-school settings. She is committed to collaborating with educators to reimagine math pedagogy that leverages student authentic experiences; partner with families and communities; dismantle academic apartheid; and advocate for mathematics education that cultivates joy and justice. She welcomes all to join these commitments.

Erin Turner

Erin Turner is a Mathematics Education Professor in the department of Teaching, Learning and Sociocultural Studies at the University of Arizona. She is a former bilingual (Spanish/English) elementary teacher and leads courses and professional development focused on equity-oriented mathematics teaching in culturally and linguistically diverse settings. Dr. Turner's scholarship focuses on the critically important field of equity and social justice in mathematics education. Specifically, her work examines how mathematics instruction can draw upon children's multiple mathematical funds of knowledge in ways that support mathematical understanding and a sense of agency. Most recently, she has collaborated with elementary teachers and mathematics education researchers on a multi-site research and professional development project focused on teacher learning and practice related to teaching culturally responsive mathematical modeling across grade K-5. Dr. Turner has authored and co-authored numerous publications in these research areas, and her work has appeared in top mathematics and teacher education journals.

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Mary Alice Carlson is Associate Professor of Mathematics Education at Montana State University in Bozeman, MT. Her work focuses on teacher learning and professional development and on mathematical modeling in formal and informal settings. She is co-author of two books: Becoming a Teacher of Mathematical Modeling: Grades K-5 and Becoming a Teacher of Mathematical Modeling: Grades 7-12. A former elementary and middle school teacher, Dr. Carlson is especially interested in how teachers can learn in and from their practice, and on how teacher collaboration supports instructional change. Her current grant-funded projects focus on culturally responsive mathematical modeling and on creating opportunities for preservice and inservice teachers to learn together.

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Holly Tate is an Instructional Mathematics Coach in Virginia, now in her 10th year coaching

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Elzena McVicar is a Postdoctoral Scholar in mathematics education at the University of Washington Seattle. Her research focuses on creating anti-racist mathematics learning environments through liberatory pedagogies. Dr. McVicar's work includes learning from Black women teachers, creating joyous mathematics classrooms, and opening curriculum spaces for children's mathematical thinking and cultural/community funds of knowledge to fuel mathematics learning. Currently, she works with the Systems Leadership for Math Improvement project, supporting school leaders to engage teachers in taking up racially just ambitious mathematics teaching. She previously worked as a K-5 public school teacher and collaborated with EduDesign, a teacher-led professional development group with the aim of teaching for social justice.