THE INTERDISCIPLINARY JOURNAL OF PROBLEM-BASED LEARNING

# Reimagining Mathematical Understandings Through an Equity Lens: How Students Demonstrate Robust Understandings in Project-Based Classrooms

**Siqi Huang (School of Education, University of California, Berkeley)**
**Mozzie DosAlmas (XQ Institute)**

IJPBL is Published in Open Access Format through the Generous Support of the College of Information at University of North Texas, the College of Education and Human Development at University of North Dakota, and the Friday Institute for Educational Innovation at North Carolina State University.

# The Interdisciplinary Journal of Problem-based Learning

2024 SPECIAL ISSUE

# Reimagining Mathematical Understandings Through an Equity Lens: How Students Demonstrate Robust Understandings in Project-Based Classrooms

Siqi Huang[1] and Mozzie DosAlmas[2]

[1]School of Education, University of California, Berkeley

[2]XQ Institute

## ABSTRACT

This study reports on the design and implementation of an equity-focused, project-based algebra curriculum in two advanced eighth-grade classrooms. We document with substantial detail a promising approach to capturing how students demonstrate and build robust mathematical understandings in ways that strongly connect to equity issues. We illuminate important avenues through which the proposed framework, design principles, and the Teaching for Robust Understanding framework (Schoenfeld, 2020) can effectively support the design, implementation, and assessment of equity-focused PjBL curricula. The paper concludes by discussing implications for researchers, teachers, curriculum designers, and the broader project-based learning (PjBL) community.

Keywords: *project-based learning, mathematics education, diversity and equity*

Numerous studies endorse project-based learning (PjBL) as an alternative to traditional pedagogies, providing fertile ground for expansive learning (Bell, 2010; Kokotsaki et al., 2016; Lucas Education Research, 2021). In mathematics, PjBL not only offers increased opportunities for students to enhance their disciplinary understandings but also promotes collaboration, motivation, problem solving, critical thinking, and social-emotional skills (Boaler, 2002; Holmes & Hwang, 2016; Rehman et al., 2023). However, researchers have yet to reach a consensus on how to precisely define PjBL

(Condliffe et al., 2017; Thomas, 2000). In this study, we adopt the definition provided by the Buck Institute for Education (n.d.) as “a teaching method in which students gain knowledge and skills by working for an extended period of time to investigate and respond to an authentic, engaging, and complex question, problem, or challenge” (para. 1). Our open-source, modeling-focused, project-based algebra curriculum (to be elaborated in the Method section of this paper) was designed in accordance with the Gold Standard PjBL model, which involves (a) a challenging problem or question, (b) sustained inquiry, (c) authenticity, (d) student voice and choice, (e) reflection, (f) critique and revision, and (g) public product (Larmer et al., 2015).

In recent years, advocacy has increased (Becton-Consuegra, 2020; Hypolite & Rogers, 2023; Miller et al., 2021; Tierney et al., 2023) for the adoption of PjBL as a strategy to promote equity and alleviate the entrenched “educational debt” (Ladson-Billings, 2006, p. 3) that has accumulated over time in American society. Moving away from viewing equity as merely closing the achievement gap, we follow R. Gutiérrez (2009) in conceptualizing equity as consisting of four dimensions: access, achievement, identity, and power. In this paper, we (a) examine how students demonstrate robust mathematical understandings (achievement), (b) analyze who contributes to classroom discourses and in what ways (access and power), and (c) explore what cultural-historical resources students bring to bear and how they leverage mathematics to address problems of personal significance (identity).

With exceptions such as Gutstein (2016) and Cross et al. (2012), a paucity of research details the descriptions and implementations of equity-focused PjBL mathematics curricula. Additionally, to the best of our knowledge, we are not aware of any studies that reconceptualize mathematical understandings in relation to equity in the PjBL context. This study addresses these gaps by (a) describing the design and implementation of an equity-focused project-based algebra curriculum in two eighth-grade classrooms and (b) proposing a framework that characterizes how students demonstrate robust mathematical understandings in ways that critically intersect with equity issues. Specifically, we ask, “How did students demonstrate mathematical understandings in equity-focused project-based algebra classrooms? In what ways did these behaviors intersect with issues of equity?”

# Literature Review

## Learning and Understandings

This study is grounded in Wenger’s (1998) learning theory, which frames learning in terms of participation in communities of practice. Moreover, we recognize that learning is fundamentally shaped by cultural, racial, and political factors that have systematically empowered some while marginalizing others (Martin, 2007; Nasir et al., 2020).

Our conceptualization of mathematical understandings combines the mainstream notion of building connections (Hiebert & Carpenter, 1992; Sidney & Alibali, 2015) with Perkins and Blythe’s (1994) performance-based perspective. According to Perkins and Blythe, students demonstrate mathematical understandings through various performances, such as correctly following procedures; explaining concepts; generalizing ideas; applying knowledge to new situations; and deriving consequences.

Building on this perspective, Huang (2022) proposes an observation system to identify behaviors related to understanding within problem solving, including problematizing ongoing work, suggesting multiple solution approaches, and examining problem statements and proposed ideas. In a subsequent work, Huang (2023) introduced a refined list of behavioral indicators of robust mathematical understandings. These indicators encompass spending time understanding complex ideas, even after reaching a solution; posing “what if” questions to problematize ongoing work; proactively suggesting multiple solution approaches; and examining conflicting ideas (Huang, 2023). Notably, existing behavioral indicators of robust mathematical understandings do not account for equity issues, such as how cultural-historical resources might play a role in students' generation of mathematical knowledge or who has access to demonstrate understandings in the first place. In this study, we will expand upon these referenced behavioral indicators with an eye towards equity within the context of PjBL mathematics classrooms.

## Designing for and Analyzing Productive, Equitable Mathematics Learning

Our work can be understood as a social design experiment (K. D. Gutiérrez & Jurow, 2016; K. D. Gutiérrez et al., 2020) rooted in Style's (1996, p. 21) idea of providing "windows and mirrors" for historically marginalized students. Social design experiments combine design-based research traditions (Design-Based Research Collective, 2003) with equity goals to (a) develop theoretically grounded and practical interventions, (b) iteratively refine local theories for consequential learning, and (c) ameliorate and redress historical injustices (K. D. Gutiérrez & Jurow, 2016). Importantly, researchers collaborate with various stakeholders to discern the pressing needs of marginalized communities and reorganize learning ecologies to challenge existing inequities and nurture individuals to become "historical actors" and designers of their own futures (K. D. Gutiérrez et al., 2019, p. 291).

Our design of mathematically rigorous PjBL activities aimed at encouraging active knowledge generation was guided by the productive disciplinary engagement (PDE) framework (Engle & Conant, 2002; Engle, 2012). The emphasis of the framework is that PDE entails active participation in the ways of thinking, reasoning, and communicating, which define the discipline. Three principles of the PDE framework particularly resonate with our intention to foster robust mathematical understandings: (a) problematizing, or encouraging students to tackle intellectual problems; (b) authority, or positioning students as authors and producers of knowledge; and (c) accountability, or holding students' intellectual work accountable to each other and to disciplinary norms (Engle & Conant, 2002, pp. 400–401).

Agarwal and Sengupta-Irving (2019) extend the PDE framework into a connective and productive disciplinary engagement (CPDE) framework that foregrounds issues of history, power, and culture in students' learning. This extension introduces two additional principles necessary for fostering connective learning: *epistemic diversity* and *historicity and identity*. Epistemic diversity centers on "heterogeneity in knowing and doing a discipline—e.g., perspectives, meanings, practices, values—that are historically and culturally constituted" (Agarwal & Sengupta-Irving, 2019, p. 351). Guided by this principle, we explicitly leveraged non-dominant ways of knowing and doing as legitimate bases for understanding mathematics and broader society. The principle of historicity and identity explicitly recognizes students as both individuals in the present moment and individuals shaped by socio-cultural-political history (Agarwal & Sengupta-Irving, 2019). Accordingly, we wove together the immediate academic contexts, students' histories of experiences, and broader socio-cultural-historical practices when designing for and examining students' learning.

While our intentional leverage of students' cultural-historical resources and identities was informed by CPDE, we acknowledge that this strategy aligns with core practices of culturally relevant pedagogy (Ladson-Billings, 1995), culturally responsive pedagogy (Gay, 2000), and culturally sustaining pedagogy (Paris, 2012). We drew further inspiration from justice-centered mathematics education (Berry et al., 2020; Gutstein, 2006, 2016) and critical race curriculum (Yosso, 2002) to foster sociopolitical consciousness, position mathematics as a tool for understanding and challenging inequitable systems, and support students in developing plans and taking actions for change.

Our examination of classroom interactions and the orchestration of PjBL activities by our teacher partners were guided by the Teaching for Robust Understanding (TRU) framework (Schoenfeld, 2014, 2017, 2020). TRU offers a theoretically robust and empirically tested approach that provides actionable insights for crafting powerful learning environments (Schoenfeld et al., 2023). It delineates five key dimensions of instruction: (a) the mathematics, (b) cognitive demand, (c) equitable access, (d) agency, ownership, and identity, and (e) formative assessment, positing that classrooms scoring high on these dimensions cultivate knowledgeable, flexible, and resourceful disciplinary thinkers (Schoenfeld, 2020). Focusing on the *mathematics* and *cognitive demand* in our context entails designing PjBL activities around core mathematical ideas and practices, echoing PDE's problematizing principle that engages students in disciplinarily rigorous and intellectually challenging problems. Our understanding of equity was deepened by TRU's elaboration of *equitable access*. This concept not only informed our design of providing opportunities for students to define core mathematical concepts (as opposed to imposing standard definitions on them) but also focused our analysis on who participated and in what ways. It also revealed how students leveraged non-dominant resources to justify their ideas that might otherwise be considered invalid. We incorporated TRU's dimension of *agency, ownership, and identity* with PDE's authority principle to design opportunities for students to generate, propose, and make consequential decisions in their learning process, supporting both mathematical identities highlighted by TRU and cultural-historical identities emphasized by CPDE. While *formative assessment* was not a focus of this study, we note that the open-ended activities we designed into the curriculum elicited students' ideas, allowing them to build on or challenge each other's perspectives, and enabling our teacher partners to adjust subsequent learning activities accordingly. Applying TRU to PjBL contexts is novel (A. H. Schoenfeld, personal communication, June 7, 2024), and we hope that our work can illuminate some promising ways in which TRU can support the design and implementation of equitable PjBL practices.

# Method

## Designing Equity-Focused Project-Based Algebra Curriculum

As researcher-educators, we recognize the influence of our identities and experiences on this work. The first author, Huang, was a PhD candidate in Science and Mathematics Education concurrently pursuing a Master's degree in Mathematics at the University of California, Berkeley during this research. As a Chinese woman in mathematics, she has often been frustrated by societal assumptions that attribute her mathematical success to her Asian background while linking her female peers' struggles to their gender. These experiences have deepened her commitment to creating inclusive spaces where students from all backgrounds can engage meaningfully with rigorous mathematical thinking and be recognized as capable mathematical thinkers. The second author, DosAlmas, brings a liminal Black, multiracial, nonbinary, queer perspective shaped by navigating boundaries within an intersectional identity matrix. Rather than fitting neatly within predefined categories, DosAlmas's liminality informs their worldview and drives their insistence on transgressing binaries. This perspective fuels their dedication to addressing the teaching and learning needs of marginalized students and ensuring that STEM, particularly mathematics, is broadly accessible. Together, these backgrounds inspire a curriculum design approach that actively involves diverse stakeholders and challenges the binary constructs often taken for granted in society.

Our equity-focused project-based algebra curriculum was developed through extensive collaboration among researchers, teachers, coaches, and a student advisory committee, reflecting our commitment to involving various stakeholders in the design process. We started by incorporating insights from the student advisory committee on youth cultural experiences (e.g., gossip in social media) and systemic inequities (e.g., school-to-prison pipeline) to identify meaningful problems. Modeling was an intentional focus of our curriculum, rooted in our team's dedication to engaging students in core mathematical practices (e.g., defining; connecting; representing; abstracting;

justifying; problem solving) and fostering 21st-century skills[1] such as innovation, critical thinking, and social responsibility. The open-source nature of our curriculum (available at https://xqsuperschool.org/) helped reduce barriers to educational resources beyond the students we could physically reach.

The full curriculum included eight modules: one module on identity- and community-building and seven modules on mathematical content. Due to space constraints, this paper will focus on two mathematical content modules: "Change the Stories," which focuses on understanding and transforming linear models, and "Going Viral," which leverages exponential models to represent and challenge social issues. In what follows we describe three deceptively simple design principles, illustrate their connections to existing works, and provide examples[2] of how these principles came to life in our PjBL classrooms.

## Design Principle 1: Engage Students with Socio-Historically Meaningful Problems While Maintaining Mathematical Rigor

We intentionally incorporated socio-historically meaningful problems to reflect the historicity and identity (Agarwal & Sengupta-Irving, 2019) of students, foster authenticity (Larmer et al., 2015), and promote socio-political consciousness (Gutstein, 2006; Yosso, 2002). To avoid the pitfall of backgrounding students' disciplinary learning in equity-focused pedagogies, we upheld mathematical rigor by engaging students in core mathematical practices with intellectually challenging problems and high expectations. This approach aligns with the challenge element of the Gold Standard PjBL model (Larmer et al., 2015), the problematizing and accountability principles of PDE (Engle & Conant, 2002), and the mathematics and cognitive demand dimensions of TRU (Schoenfeld, 2020).

For example, the "Change the Stories" module began with a recent news article (Desjardins, 2021) highlighting the disproportionate arrest and detention of Black children in Rutherford County, Tennessee. Students were prompted to analyze a statement from the Rutherford County judge, "It's not my job to know statistics," alongside the fact that 48% of children passing through her court were incarcerated, a sharp contrast to the state average of 5%. A subsequent activity involved brainstorming powerful statistical questions about the situation—questions whose answers could motivate and empower advocates for change in Rutherford County. One common question raised by students related the annual number of unjustly detained children and the annual cost of incarcerating them. This question prompted the development of a linear model incorporating variables such as total annual cost, annual cost per person of juvenile detention, average number of incarcerated children, and annual cost to maintain a jail even without incarceration.

Given the multitude of variables students had to meaningfully incorporate and the complex data awaiting their research in reports and news sources, the intellectual challenge of transforming real-world problems into mathematically valid models was substantial. Students were held accountable for justifying their proposed models and explaining how different representations (equations, tables, graphs) connected to one another. In their presentations, students were also expected to "change the stories" by proposing alternative scenarios, such as Rutherford County not incarcerating any children and redirecting funds toward quality education.

1 Our dedication to fostering 21st-century skills aligns with the goals of the Gold Standard PjBL model (Larmer et al., 2015).

2 We note that these examples were taken from observed classroom interactions to illustrate the design principles. They could be viewed as results of our curriculum implementation, but they are not considered results of this paper as they do not directly address our research questions.

## Design Principle 2: Leverage and Nurture Students' Sense of Agency and Authority in the Learning Process

We explicitly leveraged and nurtured students' sense of power (R. Gutiérrez, 2009), agency (Gutstein, 2006; Schoenfeld, 2020), and authority (Engle & Conant, 2002) in the learning of mathematics. We achieved this plan in two complementary ways. First, we supported students in pursuing their own driving questions, recognizing the diversity in their histories, aspirations, and problems that they genuinely wanted to address (K. D. Gutiérrez & Jurow, 2016; Agarwal & Sengupta-Irving, 2019). Second, we infused the PjBL process with opportunities for defining; connecting; creating; revising; explaining; reflecting; and presenting. This approach promoted students' voice and ownership over mathematics while engaging them in core mathematical practices (Engle & Conant, 2002; Larmer et al., 2015; Schoenfeld, 2020). Notably, the first strategy incorporated the first four elements of the Gold Standard PjBL model (Larmer et al., 2015), while the second strategy incorporated the last four.

### *Examples of Strategy 1*

After exploring the Rutherford County example, students were prompted to reflect on societal issues they genuinely cared about and choose their own counter-stories for retelling in their projects. Students were scaffolded with brainstorming questions such as the following:

1. Why is this story important to you?
2. Who is impacted?

At a later stage, we offered a storyboard template to guide students' storytelling:

1. What problem is your story addressing? How did it impact the involved community?
2. Who played a role in making a change? What powerful statistics did they uncover?
3. What was the process of changing the powerful statistics?
4. How has the situation evolved, especially for the most affected community?
5. What work remains to be done, and what are the next steps?

These prompts served as a catalyst for students to direct their attention not only towards identifying inequities within their communities or the broader society but also towards exploring proactive measures that could be implemented to address and resolve these disparities.

### *Examples of Strategy 2*

We supported student agency, authority, and robust understandings of exponential functions by encouraging them to (a) precisely *define* their understandings of "going viral," (b) *connect* their definitions with prior knowledge and real-world phenomena, (c) *create* examples to problematize initial definitions proposed by themselves or their peers, (d) *revise* their definitions based on new insights, and (e) *explain* the rationales underlying their revised definitions. Students were regularly prompted to *reflect* in their journals by answering questions such as the following:

1. What did you accomplish today?
2. How did you collaborate with your classmates?
3. What would you like to work on next time we work on projects?
4. What was challenging about today's work?
5. How can your teacher help you?

Additionally, students had ample opportunities to *present* ideas in both small-group and whole-class discussions, as well as to out-of-classroom audiences.

## Design Principle 3: Support PjBL with Cohesive Instructional Stages and Progress-Focused Assessment

This design principle combined a structured approach to PjBL module development with an assessment strategy that prioritized student progress over identifying shortcomings. Each of our modules consisted of six stages. The *hook* stage sparked students' curiosity using intriguing, relevant, and socio-historically meaningful problems similar to those in Gutstein (2006) and Yosso (2002). The

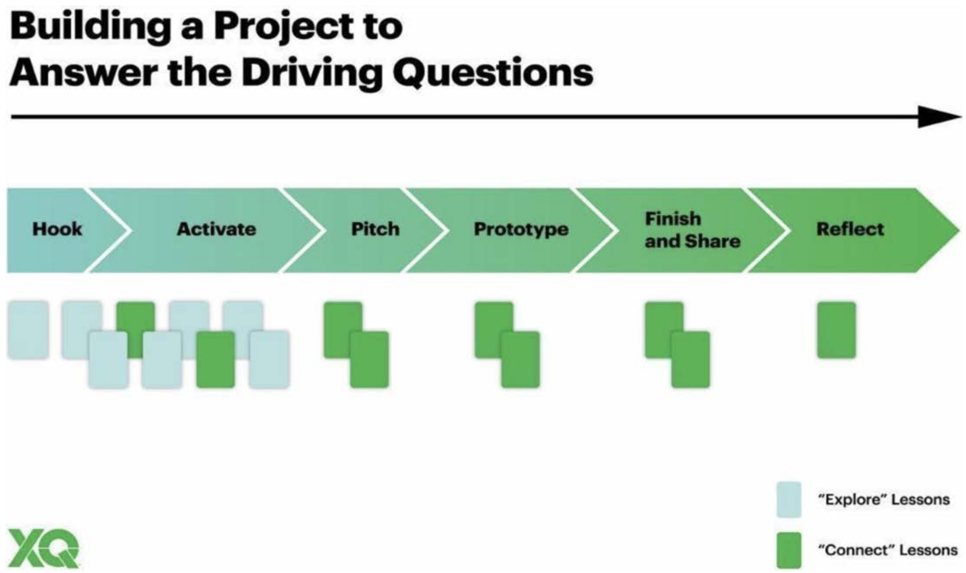


Figure 1. Overview of PjBL Instructional Stages

*activate* stage involved several inquiry-based lessons, with some focusing primarily on exploring new mathematical ideas and others on connecting with prior knowledge and experiences. The pitch stage supported students in developing, presenting, and refining their project ideas through active knowledge generation, peer sharing, and feedback exchange processes.[3] During the prototype stage, students brought their project ideas to life while upholding disciplinary norms and standards.[4] The finish and share stage supported students in revising their projects and presenting them to broader audiences. In the final reflect stage, students self-assessed their completed projects and the entire learning journeys, appreciating skills developed and identifying areas for further improvement. Figure 1 provides an overview of the instructional stages.

We followed Agarwal and Sengupta-Irving (2019) in emphasizing students' strengths and progress while avoiding a focus on assumed deficiencies. Our teacher partners' assessment of student learning was guided by three principles: (a) grading based on what students know rather than what they do not know; (b) evaluating the learning process, not just the end product; and (c) recognizing exemplary work, fostering a culture where students learn from mistakes without being penalized for errors. We ensured the accessibility of assessment rubrics for students and tasked them with providing examples for each rubric item that best represented their work. Specifically, students were encouraged to build portfolios (Figure 2) showcasing success with relevant topics, progress made in understanding them, and ongoing attempts towards mastery.

3 Both the activate and pitch stages emphasized student voice and choice (Larmer et al., 2015) while foregrounding mathematics, cognitive demand (Schoenfeld, 2020), and problematization (Engle & Conant, 2002).

4 Guided by Engle and Conant (2002) and Schoenfeld (2020), we ensured students were held accountable to the field of mathematics throughout the PBL process. Reflection, critique, and revision (Larmer et al., 2015) were integrated throughout the PBL process but were more salient in the prototype, finish and share, and reflect stages.

| | | | |
|---|---|---|---|
| **I can** write equations that accurately model a real-world situation, using variables and parameters in their correct relationships. | | | |
| Link to work that you think shows your learning: | Work 1 | Work 2 | Work 3 |
| How does this work show your learning? What are you still working on? | | | |
| **I can** calculate a reasonable estimate of a powerful statistics. | | | |
| Link to work that you think shows your learning: | Work 1 | Work 2 | Work 3 |
| How does this work show your learning? What are you still working on? | | | |
| **I can** change variables and parameters in an equation and describe how changes to those variables or parameters change the output of the equation, using a graph. | | | |
| Link to work that you think shows your learning: | Work 1 | Work 2 | Work 3 |
| How does this work show your learning? What are you still working on? | | | |
| **I can** construct arguments about how policies impact communities and advocates for how policies might better meet the needs of those communities. | | | |
| Link to work that you think shows your learning: | Work 1 | Work 2 | Work 3 |
| How does this work show your learning? What are you still working on? | | | |
| **I can** find personal relevance in my learning and use it to build curiosity that helps me to learn. | | | |
| Link to work that you think shows your learning: | Work 1 | Work 2 | Work 3 |
| How does this work show your learning? What are you still working on? | | | |

Figure 2. Portfolio Prompt (Change the Stories Module, Condensed Version)

## Data Collection & Analysis

We implemented our curriculum in two advanced eighth-grade algebra classrooms. The first classroom, with 21 students, was located in the Southeastern United States, serving primarily Black (61.9%) and White (33.3%) students. The second classroom, consisting of 15 students, was located in the Midwestern United States, where eight students (53.3%) were White, three (30%) were Black, two (13.3%) were Latiné, and two (13.3%) were of mixed race. We collected a total of 2,150 minutes of classroom videos; documented 35 ethnographic fieldnotes; conducted 860 minutes of semi-structured, audio-recorded interviews individually with each student and teacher partner; and gathered all project artifacts. Each student interview lasted approximately 8–13 minutes and each teacher interview lasted approximately 30–40 minutes, with initial interviews conducted before and final interviews conducted after the PjBL classes.

We adapted grounded coding (Charmaz, 2006; Vollstedt & Rezat, 2019) to examine how students demonstrated mathematical understandings in relation to equity in PjBL classrooms. Given the insufficient connection between mathematical understandings and equity in existing studies, grounded coding facilitated an iterative, fluid process where empirical data played a pivotal role in generating new concepts and categories (Vollstedt & Rezat, 2019). In the first phase, we developed an overview of the data by reviewing all recordings and partitioned the dataset into smaller segments. During the second phase, we created analytic memos for each recording, which documented observed behaviors, corresponding timestamps, and our initial hypotheses regarding their connections to mathematical understandings and equity. Concurrently, we identified emerging themes.[5]

The third phase involved refining our analysis through an extensive exploration of relevant literature, examining how existing studies operationalized behavioral indicators of mathematical understandings and how they might shed light on equity issues. We integrated these insights to refine our emerging themes and streamline codes to avoid redundancy. In the fourth phase, we developed highly detailed transcripts capturing students' utterances and behaviors, utilizing frequent codes from the previous phase to pinpoint, categorize, and structure the extensive data.[6] In the fifth phase, we conducted a holistic review of all collected data to evaluate whether the emerging categories provided a comprehensive narrative. This review was compared with our initial impression of the data, guiding revisions to the categories to ensure accuracy and thoroughness.

## Reporting Results

The analysis process described above enabled us to categorize over 200 examples of students demonstrating mathematical understandings into five categories. Considering the extensive dataset, we purposefully selected examples for the next section using the following criteria: (a) capturing the essence of the corresponding categories, (b) spotlighting different activities to illustrate our curriculum from various angles, (c) featuring as many students as possible, (d) balancing the number of examples from each classroom, and (e) balancing examples that hinted at within-classroom equity issues with those transcending classroom walls. Notably, despite the differences in school locations and student demographics between the two classrooms, we identified at least 12 examples of each category from each classroom.

We chose not to follow the common practice of describing students solely by their race or gender, intending to emphasize that students, even within the same racial or gender group, are not homogeneous. Instead, we portrayed students' identities using descriptions primarily derived from their self-reflections in interviews, complemented by teachers' reports and our own impressions. All names in this paper are pseudonyms chosen by the students themselves.

# Results & Discussion

To answer the first research question, we developed a five-category framework (Figure 3) that categorized how students demonstrated robust mathematical understandings in the equity-focused PjBL classrooms. *Connecting mathematical ideas* foregrounded students' ability to discern and establish meaningful connections, resonating with the importance of interconnectedness in robust understandings (Hiebert & Carpenter, 1992). *Innovating problem solving* spotlighted students' flexibility and creativity in proposing alternative approaches to thinking about, representing, or solving problems, underscoring their epistemic diversity (Agarwal & Sengupta-Irving, 2019) while fostering deeper understandings by exposing students to diverse perspectives (Schoenfeld et al., 2023).

*Concretizing abstract objects* involved students drawing on prior knowledge and real-world experiences to generate tangible examples, reflecting research that underscores the integration of concrete and abstract reasoning (Purwadi et al., 2019). *Justifying with relevance* highlighted how students leveraged their personal identities and sociocultural resources to justify ideas, merging the fundamental practice of justification (Thurston, 1994) with the practical application of mathematics as a tool to reflect on and address real-world problems (Gutstein, 2006). *Problematizing underlying*

5 The first two phases correspond to initial coding in Charmaz (2006) and open coding in Vollstedt and Rezat (2019).

6 The fourth phase aligns with focused coding in Charmaz (2006) and axial coding in Vollstedt and Rezat (2019).

*assumptions* showcased students actively scrutinizing and grappling with the complexities inherent in their mathematical and social endeavors, aligning with the problematizing principle (Engle & Conant, 2002) and the behavioral indicator of problematizing ongoing work (Huang, 2022, 2023).

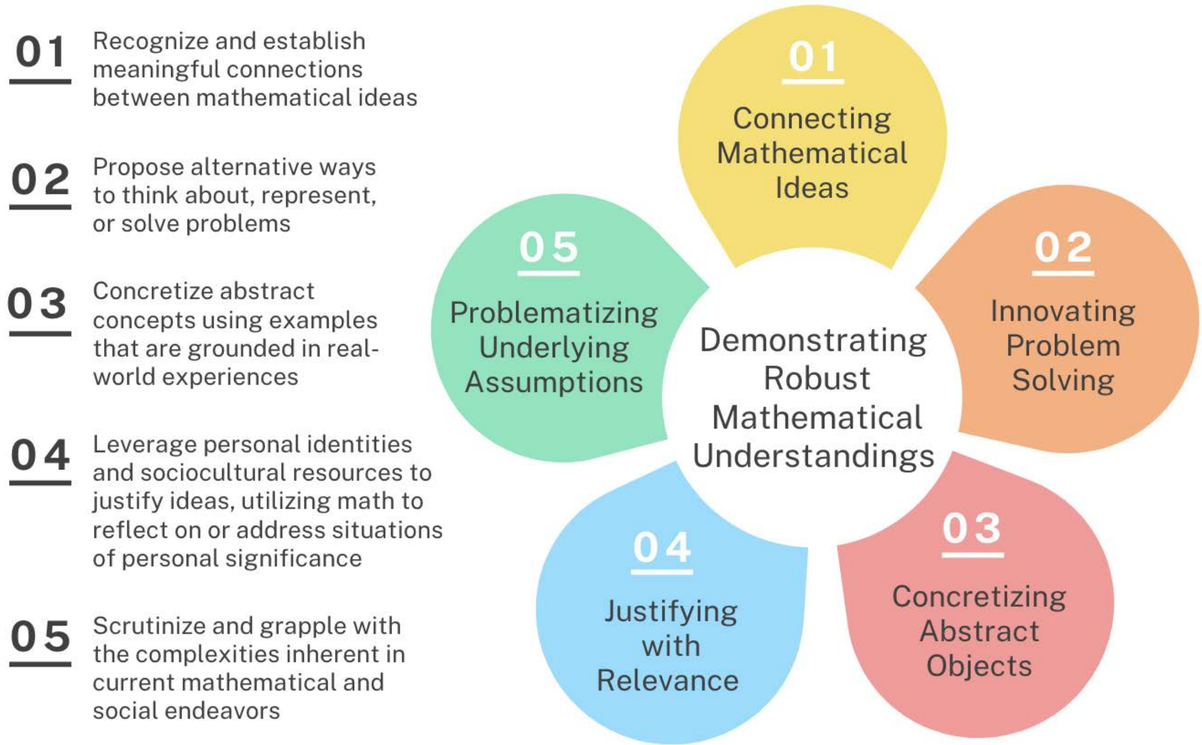


Figure 3. Demonstrating Robust Mathematical Understandings in Equity-Focused PjBL Classrooms

In the remainder of this section, we will provide examples for each category and discuss how they intersect with issues of equity.

## Connecting Mathematical Ideas

After extensive exploration of real-world data from Rutherford County, three students demonstrated robust mathematical understandings by collaboratively establishing connections among various variables:

Jordan: So, the number of kids who were locked up for nothing over ten years, is the percent of kids in Rutherford County who got locked up, multiplied by the number of kids who got sent to court over ten years, whether or not they got locked up, subtracted—

Roman: By the number of kids who were locked up for something over ten years.

Zendaya: By the percent of kids locked up fairly multiplied by the total number of kids sent to court over ten years.

Jordan: Yeah, yeah. They are the same.

In this conversation, Jordan (Black, charismatic, cool, kind) clarified how specific variables[7] contributed to the modeled variable of the number of children unjustly incarcerated over ten years. Notice that multiplying the variables Jordan specified resulted in the total number of incarcerated children in the past ten years. Roman (Black, energetic, funny, optimistic) seamlessly extended Jordan's thought, noting that the modeled variable equaled the result of Jordan's ideas subtracted by Roman's variable.[8] This collaborative sensemaking produced a correct model of the underlying mathematical relationships. Concurrently, Zendaya (Black, giving, smart, social) proposed an alternative connection, which was also correct, by breaking down Roman's variable into a product of two components.[9] Jordan's affirmation that Roman's and Zendaya's ideas were essentially equivalent highlighted the trio's ability to synthesize various components and connect them meaningfully, demonstrating robust understandings of both the mathematical relationships at play and the inequities being modeled.

To answer the second research question, we underscore the Black students' active positioning of themselves as central contributors to mathematical discourse in the above excerpt. This suggested productive authority (Engle & Conant, 2002) and ownership over mathematical content (Schoenfeld, 2020), sharply contrasting with the observation that White students often dominated classroom discussions in non-PjBL courses with this same class of students (teacher interview, December 13, 2022). By meaningfully connecting mathematical ideas, these students not only developed robust mathematical understandings that supported their subsequent advocations for change but also challenged existing power dynamics within the classroom. This example further substantiates findings from Gutstein (2016) that designing mathematical activities around relevant societal problems can broaden historically underrepresented students' access to and participation in mathematical discourse.

## Innovating Problem Solving

In the next example, students demonstrated robust mathematical understandings by generating various methods for understanding and simplifying the expression $2 \times 2 \times 2 \times 2^4$.

Chris: $2^7$ because if we break up the exponent, $2^4$ is really $2 \times 2 \times 2 \times 2$. Combining the first three 2's, we have seven 2's multiplied together. So, that's $2^7$.

Megan: I also got $2^7$ because there are three 2's on the left—that makes it $2^7$. And then since they have the same base, you can add the exponent. And $3 + 4 = 7$.

Robyn: My approach is to look at $2^4$ first and then counting the single 2's from the right to the left. So $2^5$, $2^6$,, and $2^7$

In this exchange, Chris (White, confident, smart, athletic) applied the mathematical definition of exponents as repeated multiplication, skillfully breaking down and reorganizing numbers to reach a correct solution. Megan (White, athletic, creative, hesitant) employed a different strategy, recognizing the common base and accurately employing exponent rules to arrive at the same result. Robyn (Black

---

7 The two variables, in Jordan's words, were "the percent of kids in Rutherford County who got locked up" and "the number of kids who got sent to court over ten years, whether or not they got locked up."

8 Roman's variable was, in his own words, "the number of kids who were locked up for something over ten years."

9 The two components, in Zendaya's words, were "the percent of kids locked up fairly" and "the total number of kids sent to court over ten years."

mixed with Latiné, trustworthy, humble, quiet) took a sequential approach, starting with $2^4$ and incrementing the exponent for each additional factor, presenting yet another valid method. These distinct approaches not only underscored the students' mathematical proficiency and flexibility in addressing mathematical expressions but also created rich opportunities for meaningful connections among diverse perspectives.

In response to the second research question, we observed that embracing alternative approaches in the classroom provided opportunities for students who were not among the fastest problem-solvers, such as Megan and Robyn, to voice their ideas and be publicly acknowledged. Importantly, we do not view meaningful mathematical engagement as a zero-sum game where providing more opportunities for underprivileged students means that traditionally privileged students will receive less. This example illustrated that encouraging students to innovate problem solving opened the door for all students, including both traditionally privileged (e.g., Chris) and underprivileged (e.g., Robyn), to equitably shape their classroom conversations. Therefore, innovating problem solving could help address issues of access (R. Gutiérrez, 2009; Schoenfeld, 2020) while supporting students' authority (Engle & Conant, 2002), agency, ownership, and mathematical identities (Schoenfeld, 2020).

The second example of innovating problem solving featured Jake (Black, energetic, playful, imaginative) and Ben (Black, logical, careful, deep thinker), who demonstrated robust mathematical understandings by presenting a novel, culturally resonant representation of "going viral" (Figure 4). This occurred in the context of intuitively graphing the spread of gossip in the movie "Crazy Rich Asians,"[10] which disseminated through social media to the protagonist's mother before he had a chance to talk to her personally. Jake and Ben drew inspiration from the culturally embedded concept of a "family tree" (Bouquet, 1996, p. 43), thoughtfully integrating cultural-historical resources into their mathematical innovation while accurately capturing key figures and the direction of information flow. With respect to the second research question, this example illustrated that encouraging students to innovate problem solving provided opportunities to honor student's epistemic diversity (Agarwal & Sengupta-Irving, 2019); support meaningful connections between cultural-historical resources and disciplinary content (Ladson-Billings, 1995); and develop their sense of authority (Engle & Conant, 2002) and cultural-disciplinary identities (Agarwal & Sengupta-Irving, 2019; Schoenfeld, 2020).

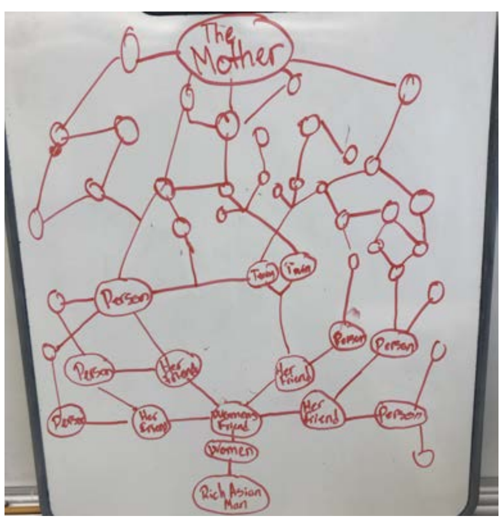


Figure 4. Innovative Problem Solving: Alternative (Family Tree) Representation

10 The activity was designed to support students' engagement with different representations, serving as a pivotal reflection point that scaffolded students' subsequent refinement of their intuitive criteria for determining whether something "went viral."

## Concretizing Abstract Objects

In the following example, students demonstrated robust mathematical understandings by creating concrete examples of three abstract graphs (see Figure 5):

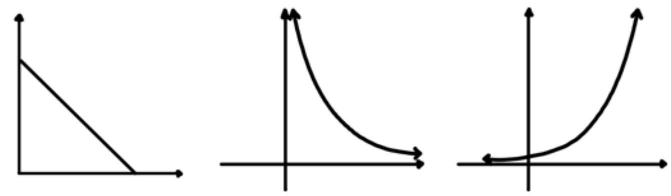

Figure 5. Illustrating Abstract (Linear and Exponential) Graphs

| | |
|---|---|
| Teacher: | What is a scenario that can be represented by one of these graphs? |
| Thomas: | For the left one, just a guy having 75 apples, and then he eats like five every day. So, it goes down the [inaudible text segment] and ends when there's no more apples. |
| Teacher: | He's eating five apples a day, that's a constant rate of change. Good. Who has a scenario to represent the graph on its right? |
| Demetrius: | I got it. There's a surprise party happening for Tyrese, three of his friends are planning it. The whole school knows about it, from his three friends. And then every day, three kids tell three other kids that they shouldn't go. They're full. And then the next day, all nine, well, nine plus the beginning three, each tell three other kids. And so on. So less and less kids come. |
| Teacher: | Okay, so the number of people who are going to attend the party would be decreasing exponentially. |
| Demetrius: | Yes. |
| Teacher: | Okay. That works. That works. Does somebody else have a scenario? |
| Ryan: | Serial murder? |
| Teacher: | Can you elaborate on how a serial murder death is exponential? |
| Ryan: | For every person they kill, they kill two more. |
| John: | Yeah, once they start killing, they can't stop. |
| Alexis: | What about crimes in general? |
| John: | Once you commit one crime and get away with it— |
| Alexis: | You won't stop. |
| John: | You're most, yeah, you're most likely going to keep doing it. |
| Alexis: | That's what my uncle did that got him to prison. |
| Josh: | There might be something with addiction. |

In the above discussion, Thomas (White, smart, kind, lazy)[11] linked his linear graph scenario to the familiar context of apple consumption, enhancing its relatability and ensuring a robust application of linear models. Demetrius (Black, funny, athletic, creative) incorporated a shared enthusiasm for surprises among his peers into his accurate concretization of the exponential decay graph. Ryan (White, loud, funny, chill) broached a topic of serial murder and presented it in a manner that withstood mathematical scrutiny. John (Latiné, enthusiastic, sociable, diligent) built on his out-of-school understanding to elaborate further, emphasizing how engagement in activities such as murder can lead to perpetuated and escalated actions. Alexis (Black, outgoing, caring, smart) expanded the scope of the dialogue by reflecting on the nature of crimes in general, while Josh (White, quiet, smart, annoying) added a socio-psychological layer by discussing addiction.

This rich and open-ended discussion, influenced by popular culture narratives surrounding crime, showcased the diverse ways in which students leveraged their lived experiences and socio-cultural-historical knowledge to contribute meaningfully to mathematical discussions. The active roles taken by students of various racial backgrounds[12] in generating this sophisticated dialogue transcended the traditional approach of discussing canned textbook problems. Examples that might be swiftly dismissed in a standard mathematics classroom not only became integral to the ongoing discourse but were recognized as significant contributors to the overall mathematical discussion.

In this inclusive environment, students engaging in discussions about crime—whether speaking humorously, invoking sociocultural narratives, or referencing family members' encounters with the law—found themselves centrally involved rather than on the periphery of mathematical conversations. It challenged conventional norms of what was relevant or appropriate in mathematics classrooms, increasing opportunities for all students—particularly those often marginalized in typical mathematics conversations—to contribute and see themselves as significant contributors to the advancement of collective disciplinary discourse. In the language of TRU (Schoenfeld, 2020), this open discussion provided equitable access to core mathematical ideas while supporting students' sense of agency, ownership, and identities as powerful mathematical learners and practitioners.

## Justifying with Relevance

In the next example, students explored potential stakeholders in disseminating Beyoncé's private performance, leveraging their sociocultural identities and real-world experiences to justify their ideas:

Carri: I'm trying to find someone who does research on the spread of things.

Megan: Oh, like a scientist? Who else would care?

Zoey: I'd be caring. I'd be the one sitting on social media waiting for the video.

Megan: Maybe Beyoncé and her family care about how the video is getting out there too. I mean she has a team that does research and figures out what's going on. They probably don't want the performance to be leaked.

Carri: Yeah, every time something popped up it was gone.

Embedded within the broader "stan culture," where dedicated fans, known as stans,

11 We wish to underscore our commitment to preserving the authenticity of students' language in our descriptions. It is important to note that, for us, terms such as "lazy" are not uniformly negative but rather reflective of the student's own expressions and experiences.

12 This excerpt was taken from the predominantly White classroom in the Midwestern United States.

passionately idolize celebrities (Bermudez et al., 2020), the girls explored a dynamic online phenomenon with which they genuinely identified. Carri (Black, spontaneous, social, compassionate) connected professional researchers with their own mathematical and socio-scientific exploration. Megan (White, athletic, creative, hesitant) expanded the perspective, suggesting that Beyoncé and her team would likely be concerned about the video's dissemination, possibly working to control its release. Leveraging her identity as a devoted fan of Beyoncé, Zoey (Black, thoughtful, quiet, goal-oriented) contributed by articulating the heightened interest of fans in actively monitoring social media for the leaked video. Carri, drawing from personal experience, further substantiated the claim that unauthorized content would quickly disappear from online platforms.

This collaborative example illustrated the thoughtful sensemaking embedded in students' socio-mathematical explorations, showcasing that mathematics served not as a confined formula but as a tool for deeper engagement with phenomena that held personal and sociocultural significance to students. To address the second research question, justifying with relevance provided opportunities for students to engage in justification—a core mathematical practice (Thurston, 1994)—in ways that leveraged their socio-cultural-historical identities (Agarwal & Sengupta-Irving, 2019). This approach bolstered productive disciplinary learning while also enriching traditional perspectives on the nature of mathematical engagement and integrating socio-historical and identity dimensions to enhance students' access to meaningful mathematics.

## Problematizing Underlying Assumptions

In this final example, students demonstrated robust mathematical understandings by scrutinizing and addressing the limitations of their emerging exponential model. Their model represented the spread of news surrounding George Floyd, an African American man whose demise in police custody triggered global protests and discussions on racial injustice and police brutality.

| | |
|---|---|
| Cabinet: | But the trend won't go forever, right? |
| DeMarcus: | Yeah the map will. Maybe we should say it stops. |
| Carrington: | Well … it won't stop spreading, but it will calm down. |
| DeMarcus: | It won't be as popular. |
| Cabinet: | Ah, so it has a restricted domain. |

In the above exchange, Cabinet (Black, positive, Christian, athletic) asked, "But the trend won't go forever, right?" This student leveraged real-world intuitions to invite collaborative sensemaking regarding the constraints inherent in their emerging model. DeMarcus (Black, funny, smart, risk-taker) proposed a potential plateauing of the dissemination ("Maybe we should say it stops"), elaborating on the idea that public interest in the news would naturally diminish over time. Carrington (Black, energetic, sensitive, clingy) contributed by suggesting that the spread might not entirely cease but rather gradually subside, possibly involving a smaller exponential growth factor or warranting a transition to a different, slower-growing model. Cabinet synthesized these insights, proposing that a resolution to this nuanced challenge lay in restricting the domain of their exponential model.

To answer the second research question, we note that the students' active scrutiny and problematization of their model showcased vigorous agency (Schoenfeld, 2020) in holding themselves accountable to the underlying mathematical principles (Engle & Conant, 2002) while addressing nuanced real-world constraints. This provided opportunities for students to develop authority (Engle & Conant, 2002); ownership over mathematical content and productive mathematical identities (Schoenfeld, 2020); and socio-political consciousness (Gutstein, 2006). By meaningfully connecting mathematics with societal problems of profound personal, racial, historical, and social significance and problematizing assumptions underlying emerging models, the students developed deeper understandings of both the mathematical and the societal problems being modeled, enabling them to be better equipped to "read" (i.e., understand) and "write" (i.e. change) the world with mathematics (Gutstein, 2006, p. 23).

# Conclusion

We have presented a promising approach to capturing how students demonstrated robust mathematical understandings in PjBL classrooms and meaningfully connected these insights to issues of equity. It is worth noting that although the framework was developed with a primary focus on how students demonstrate understandings, subsequent analysis revealed that engaging in the behaviors highlighted by our categories also provided opportunities for students to build deeper understandings. Given the preliminary nature of the proposed framework, further research is essential to test and refine it across various PjBL contexts in order to enhance its effectiveness as an analytic and teaching tool.

Analytically, each category in our framework offers valuable insights into students' mathematical learning while shedding light on issues of access, power, and identity that R. Gutiérrez (2009) identifies as important dimensions of equity. *Connecting mathematical ideas* underscores the interconnectedness in robust understandings (Hiebert & Carpenter, 1992), calling attention to who has access to publicly make connections and position themselves as central contributors to classroom discourse (Schoenfeld et al., 2023). *Innovating problem solving* spotlights students' flexibility and creativity in proposing alternative approaches, explicitly honoring students' epistemic diversity (Agarwal & Sengupta-Irving, 2019) and opening the door for all students—including those not among the fastest problem solvers or otherwise underprivileged—to voice their ideas, develop authority (Engle & Conant, 2002), and equitably shape classroom discussions. *Concretizing abstract objects* foregrounds how students' build on their real-world experiences to generate tangible examples of abstract mathematics, inviting collaborative sensemaking around issues students find intriguing (e.g., crime) and broadening traditional notions of what counts as appropriate in mathematics classrooms. *Justifying with relevance* highlights how students leverage their personal identities and sociocultural resources to justify ideas, merging the core mathematical practice of justification (Thurston, 1994) with the practical application of mathematics as a tool to reflect on and address societal problems they genuinely care about. Finally, *problematizing underlying assumptions* centers on students actively scrutinizing and addressing complexities inherent in their mathematical and social endeavors, providing opportunities for them to develop agency, ownership, productive mathematical identities (Schoenfeld, 2020), and socio-political consciousness (Gutstein, 2006).

As a teaching tool, our framework illuminates crucial learning objectives and design heuristics that curriculum designers and teachers can leverage to tailor instructional strategies. For instance, a focus on connecting mathematical ideas could help teachers encourage students to connect different concepts, while guiding curriculum designers to create learning activities that integrate ideas across different modules. *Innovating problem solving* suggests recognizing and nurturing students' strengths and perspectives, embracing diverse forms of intellectual expressions, and vigilantly guarding against mislabeling or overlooking students' non-normative ideas. We facilitated *concretizing abstract objects* by encouraging students to create stories and represent graphs, recognizing this as one example of using open-ended tasks to stimulate rich discussions (Wu, 1994) and support students in bridging concrete and abstract reasoning (Purwadi et al., 2019). *Justifying with relevance* emphasizes engaging students in core mathematical practices (Schoenfeld, 2020) while supporting them to integrate personal identities and sociocultural resources into mathematical conversations. Finally, *problematizing underlying assumptions* can encourage teachers to model critical scrutiny of nuanced complexities in complex problems while supporting students to develop agency, confidence, and power in challenging inequitable systems with mathematics.

Three design principles formed the core of our curriculum: 1) engaging students with socio-historically meaningful problems while maintaining mathematical rigor, 2) leveraging and nurturing students' sense of agency and authority in the learning process, and 3) supporting PjBL with cohesive instructional stages and progress-focused assessment. Regarding our equity goals, the first principle enabled us to systematically connect mathematics with problems that students identified with, facilitating meaningful integration of students' resources, identities, and mathematical learning[13]. We were guided by the second and third principles to broaden access to meaningful mathematical engagement[14] and to develop more agentic, resourceful advocates for change[15]. We hope that these principles can spark deeper insights into designing equity-focused mathematics curricula.

We would like to emphasize that an equity-focused PjBL curriculum alone may not be sufficient to replicate the richness of equitable learning and robust mathematical understandings documented in this paper. The synergy of (a) an open learning environment where diverse perspectives were respected and all students were positioned as valid contributors, and (b) adept teachers with expertise in facilitating discussions and advancing pertinent ideas mathematically in collaboration with students, appeared to have significantly contributed to the results. Therefore, we urge readers to not lose sight of how classroom cultures and teachers' moment-to-moment decisions might play a role in advancing or constraining students' experiences with equity-focused PjBL curricula.

Given the limited exposure of TRU (Schoenfeld, 2020) in PjBL communities, we would like to highlight two significant ways in which TRU advanced our equity goals. First, the equitable access dimension prompted us to scrutinize how opportunities for participation were and could be allocated among students. Second, we anchored our design principles in the agency, ownership, and identity dimension, using it to stimulate discussions about opportunities for students to generate ideas and be recognized as valuable contributors to classroom discourse. We believe that TRU offers additional avenues for supporting broader equity-focused initiatives in PjBL, and we encourage readers to explore these further.

Advancing equity-focused PjBL initiatives necessitates new tools, and we are confident that our proposed framework, design principles, and TRU hold great potential in supporting more equitable, ambitious PjBL practices.

## Limitations

Our study's focus on two 8th grade algebra classrooms may limit the generalizability of findings, as students in these classrooms had higher prior mathematical achievements than typical 8th grade students in the United States, which is consistent with recent research findings (Remillard et al., 2017). Moreover, we recognize that our framework does not account for the subtle differences afforded and constrained by various sub-areas of mathematics. Therefore, future research is needed to test and refine our framework in other sub-areas such as geometry and probability. Additionally, replicating our approach, which involves incorporating a student advisory committee to customize PjBL curriculum, may pose significant challenges for those constrained by time, resources, or administrative support.

---

13 The student advisory committee played a key role in identifying meaningful problems. The Rutherford County example and the gossip example were designed to connect mathematics with issues students identified with. The last four examples in the Results & Discussion section illustrate how students built on their identities, real-world experiences, and sociocultural resources to advance their mathematical learning.

14 The first four examples in the Results & Discussion section demonstrate broadened access to mathematics.

15 A heightened sense of agency, resourcefulness, and socio-political consciousness was evident in the students' portfolios and final interviews.

# Acknowledgments

We extend our sincere gratitude to Teacher D, Teacher M, and their students for graciously welcoming us into their classrooms and allowing us to learn with them. We are also deeply appreciative of the collaborative efforts of the Algebra 1 PjBL Design Collaborative for their invaluable contributions to the design and implementation of the PjBL curriculum. Finally, we are grateful for the insightful feedback from Alan Schoenfeld, Lloyd Goldwasser, and our anonymous reviewers, whose input has significantly enriched this manuscript.

Correspondence concerning this article should be addressed to Siqi Huang, University of California, Berkeley, 2121 Berkeley Way, Berkeley, CA 94704, United States. Email: shuang123@berkeley.edu

# References

Agarwal, P., & Sengupta-Irving, T. (2019). Integrating power to advance the study of connective and productive disciplinary engagement in mathematics and science. *Cognition and Instruction, 37*(3), 349–366. https://doi.org/10.1080/07370008.2019.1624544

Becton-Consuegra, D. (2020, January 1). Getting clearer: HQPBL as an equity imperative. *Getting Smart*. https://www.gettingsmart.com/2020/01/01/getting-clearer-hqpbl-as-an-equity-imperative/

Bell, S. (2010). Project-based learning for the 21st century: Skills for the future. *The Clearing House: A Journal of Educational Strategies, Issues and Ideas*, *83*(2), 39–43. https://doi.org/10.1080/00098650903505415

Bermudez, R., Cham, K., Galido, L., Tagacay, K., & Clamor, W. L. (2020). The Filipino "Stan" phenomenon and Henry Jenkins' participatory culture: The case of generations X and Z. *Asia Pacific Journal of Education, Arts and Sciences*, *7*(3), 1–7.

Berry, R., Conway, B., Lawler, B., & Staley, J. (2020) High school mathematics lessons to explore, understand, and respond to social injustice. Corwin Press.

Boaler, J. (2002). Learning from teaching: Exploring the relationship between reform curriculum and equity. *Journal for Research in Mathematics Education, 33*(4), 239–258. https://doi.org/10.2307/749740

Bouquet, M. (1996). Family trees and their affinities: The visual imperative of the genealogical diagram. *Journal of the Royal Anthropological Institute, 2*(1), 43–66. https://doi.org/10.2307/3034632

Buck Institute for Education. (n.d.). What is PBL? *PBLWorks*. https://www.pblworks.org/what-is-pbl

Charmaz, K. (2006). Constructing grounded theory: A practical guide through qualitative analysis. SAGE Publications.

Condliffe, B., Quint, J., Visher, M. G., Bangser, M. R., Drohojowska, S., Saco, L., & Nelson, E. (2017). *Project-based learning: A literature review* (Working Paper). MDRC. https://www.mdrc.org/sites/default/files/Project-Based_Learning-LitRev_Final.pdf

Cross, D. I., Hudson, R. A., Adefope, O., Lee, M. Y., Rapacki, L., & Perez, A. (2012). Success made probable: Creating equitable mathematical experiences through project-based learning. *Journal of Urban Mathematics Education*, *5*(2), 55–86. https://doi.org/10.21423/jume-v5i2a129

Design-Based Research Collective. (2003). Design-based research: An emerging paradigm for educational inquiry. Educational Researcher, 32(1), 5–8. https://doi.org/10.3102/0013189X032001005

Desjardins, L. (2021, October 12). *Tennessee judge jailed minors on bogus charges following playground fights, cursing*. PBS News. https://www.pbs.org/newshour/show/tennessee-judge-jailed-minors-on-bogus-charges-following-playground-fights-cursing

Engle, R. A. (2012). The productive disciplinary engagement framework: Origins, key concepts, and

developments. In D. Dai (Ed.), *Design research on learning and thinking in educational settings* (pp. 170–209). Routledge.
Engle, R. A., & Conant, F. R. (2002). Guiding principles for fostering productive disciplinary engagement: Explaining an emergent argument in a community of learners classroom. *Cognition and instruction, 20*(4), 399–483. https://doi.org/10.1207/S1532690XCI2004_1
Gay, G. (2000). Culturally responsive teaching: Theory, research, and practice. Teachers College Press.
Gutiérrez, R. (2009). Framing equity: Helping students "play the game" and "change the game." *Teaching for Excellence and Equity in Mathematics, 1*(1), 4–8.
Gutiérrez, K. D., Becker, B., Espinoza, M., Cortes, K., Cortez, A., Lizárraga, J. R., Rivero, E., & Yin, P. (2019). Youth as historical actors in the production of possible futures. *Mind, Culture, and Activity*, *26*(4), 291–308. https://doi.org/10.1080/10749039.2019.1652327
Gutiérrez, K. D. & Jurow, A. S. (2016). Social design experiments: Toward equity by design. *Journal of Learning Sciences, 25*(4), 565–598. https://doi.org/10.1080/10508406.2016.1204548
Gutiérrez, K. D., Jurow, A. S., & Vakil, S. (2020). Social design-based experiments: A utopian methodology for understanding new possibilities for learning. In N. I. S. Nasir, C. D. Lee, R. Pea, & M. McKinney de Royston (Eds.), *Handbook of the cultural foundations of learning* (1st ed., pp. 330–347). Routledge. https://doi.org/10.4324/9780203774977-23
Gutstein, E. (2006). Reading and writing the world with mathematics: Toward a pedagogy for social justice. Taylor & Francis.
Gutstein, E. (2016). "Our issues, our people—Math as our weapon": Critical mathematics in a Chicago neighborhood high school. *Journal for Research in Mathematics Education*, *47*(5), 454–504. https://doi.org/10.5951/jresematheduc.47.5.0454
Hiebert, J., & Carpenter, T. P. (1992). Learning and teaching with understanding. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 65–97). Macmillan.
Holmes, V. L., & Hwang, Y. (2016). Exploring the effects of project-based learning in secondary mathematics education. *The Journal of Educational Research*, *109*(5), 449–463. https://doi.org/10.1080/00220671.2014.979911
Huang, S. (2022). Mathematical problem solving with understanding. In A. E. Lischka, E. B. Dyer, R. S. Jones, J. Lovett, J. F. Strayer, & S. Drown (Eds.), *Proceedings of the forty-fourth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 808–812). Middle Tennessee State University.
Huang, S. (2023). Developing a tool for measuring student orientations with respect to understanding in mathematical learning. In T. Lamberg & D. Moss (Eds.), *Proceedings of the forty-fifth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 49–58). University of Nevada, Reno.
Hypolite, L. I., & Rogers, K. D., Jr. (2023). Closing STEM opportunity gaps through critical approaches to teaching and learning for Black youth. *Theory Into Practice*, *62*(4), 431–447. https://doi.org/10.1080/00405841.2023.2278972
Kokotsaki, D., Menzies, V., & Wiggins, A. (2016). Project-based learning: A review of the literature. *Improving schools*, *19*(3), 267–277. https://doi.org/10.1177/1365480216659733
Ladson-Billings, G. (1995). But that's just good teaching! The case for culturally relevant pedagogy. *Theory into practice*, *34*(3), 159–165. https://doi.org/10.1080/00405849509543675
Ladson-Billings, G. (2006). From the achievement gap to the education debt: Understanding achievement in US schools. *Educational researcher*, *35*(7), 3–12. https://doi.org/10.3102/0013189X035007003
Larmer, J., Mergendoller, J., & Boss, S. (2015). *Setting the standard for project based learning*. ASCD.
Lucas Education Research (2021). *The evidence is clear: Rigorous project-based learning is an effective lever for student success.* https://www.lucasedresearch.org/research/research-briefs/
Martin, D. B. (2007). Mathematics learning and participation as racialized forms of experience: African American parents speak on the struggle for mathematics literacy. In M. Allexsaht-Snider (Ed.), *Urban parents perspectives children's math. Mtl V8# 3* (1st ed., pp. 197–229). Routledge.

https://doi.org/10.4324/9780203764152-2

Miller, E. C., Reigh, E., Berland, L., & Krajcik, J. (2021). Supporting equity in virtual science instruction through project-based learning: Opportunities and challenges in the era of COVID-19. *Journal of Science Teacher Education*, *32*(6), 642–663. https://doi.org/10.1080/1046560X.2021.1873549

Nasir, N. I. S., de Royston, M. M., Barron, B., Bell, P., Pea, R., Stevens, R., & Goldman, S. (2020). Learning pathways: How learning is culturally organized. In N. I. S. Nasir, C. D. Lee, R. Pea, & M. McKinney de Royston (Eds.), *Handbook of the cultural foundations of learning* (1st ed., pp. 195–211). Routledge. https://doi.org/10.4324/9780203774977-14

Paris, D. (2012). Culturally sustaining pedagogy: A needed change in stance, terminology, and practice. *Educational Researcher, 41*(3), 93–97. https://doi.org/10.3102/0013189X12441244

Perkins, D., & Blythe, T. (1994). Putting Understanding Up Front. *Educational Leadership*, *51*(5), 4-7.

Purwadi, I., Sudiarta, I., & Suparta, I. N. (2019). The effect of concrete-pictorial-abstract strategy toward students' mathematical conceptual understanding and mathematical representation on fractions. *International Journal of Instruction*, *12*(1), 1113–1126. https://doi.org/10.29333/iji.2019.12171a

Rehman, N., Zhang, W., Mahmood, A., Fareed, M. Z., & Batool, S. (2023). Fostering twenty-first century skills among primary school students through math project-based learning. *Humanities and Social Sciences Communications*, *10*(1), 1–12. https://doi.org/10.1057/s41599-023-01914-5

Remillard, J. T., Baker, J. Y., Steele, M. D., Hoe, N. D., & Traynor, A. (2017). Universal Algebra I policy, access, and inequality: Findings from a national survey. *Education Policy Analysis Archives, 25*(101). http://dx.doi.org/10.14507/epaa.25.2970

Schoenfeld, A. H. (2014). What makes for powerful classrooms, and how can we support teachers in creating them? A story of research and practice, productively intertwined. *Educational researcher*, *43*(8), 404–412. https://doi.org/10.3102/0013189X14554450

Schoenfeld, A. H. (2017). Teaching for Robust understanding of essential mathematics. In T. McDougal, (Ed.), *Essential mathematics for the next generation: What and how students should learn* (pp. 104–129). Tokyo Gagukei University Press.

Schoenfeld, A. H. (2020). Reframing teacher knowledge: A research and development agenda. *ZDM*, *52*(2), 359–376. https://doi.org/10.1007/s11858-019-01057-5

Schoenfeld, A., Fink, H., Zuñiga-Ruiz, S., Huang, S., Wei, X, Chirinda, B. (2023). *Helping students become powerful mathematical thinkers: Case studies of Teaching for Robust Understanding*. Routledge. https://doi.org/10.4324/9781003375197

Sidney, P. G., & Alibali, M. W. (2015). Making connections in math: Activating a prior knowledge analogue matters for learning. *Journal of Cognition and Development*, *16*(1), 160–185. https://doi.org/10.1080/15248372.2013.792091

Style, E. (1996). Curriculum as window and mirror. *Social Science Record*, *33*(2), 21–28.

Thomas, J. W. (2000). A review of research on project-based learning. Autodesk Foundation.

Thurston, W. P. (1994). On proof and progress in mathematics. *Bulletin of the American Mathematical Society*, *30*(2), 161–177.

Tierney, G., Urban, R., & Olabuenaga, G. (2023). Designing for equity: Moving project-based learning from equity adjacent to equity infused. In J. D. Slotta & E. S. Charles (Eds.), *Proceedings of the 3rd Annual Meeting of the International Society of the Learning Sciences*. International Society of the Learning Sciences.

Vollstedt, M., & Rezat, S. (2019). An introduction to grounded theory with a special focus on axial coding and the coding paradigm. In G. Kaiser & N. Presmeg (Eds.), *Compendium for early career researchers in mathematics education* (pp. 81–100). Springer Nature. https://doi.org/10.1007/978-3-030-15636-7

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. Cambridge University Press. https://doi.org/10.1017/CBO9780511803932

Wu, H. (1994). The role of open-ended problems in mathematics education. *Journal of Mathematical Behavior*, *13*(1), 115–128. https://doi.org/10.1016/0732-3123(94)90044-2

Yosso, T. J. (2002). Toward a critical race curriculum. *Equity & Excellence in Education*, *35*(2), 93–107. https://doi.org/10.1080/713845283

# Author Bios

**Siqi Huang** https://orcid.org/0000-0003-2534-7708

Siqi Huang is a Ph.D. candidate in Science and Mathematics Education at the University of California, Berkeley. Her research focuses on the teaching and learning of mathematics, with particular emphasis on mathematical identities, equity, problem-solving, and the design of inclusive learning communities.

**Mozzie DosAlmas** https://orcid.org/0009-0004-0048-6044

Mozzie DosAlmas is a Senior Researcher and Curriculum Developer for XQ Institute, Co-Lead of the Research Justice at the Intersections program for the Mills Institute, and an adjunct professor. Mozz's work focuses on the teaching and learning needs of youth traditionally underserved by educational institutions and in STEM fields. They aim to broaden access to these fields by bringing a critical women of color feminisms and queer theory lens to curriculum development, educational research, and educational leadership.