ELECTRONIC JOURNAL FOR RESEARCH
IN SCIENCE & MATHEMATICS EDUCATION
VOL. 28, NO. 3, 1-15

Effects of Using Patty Paper Practices During Geometric Constructions on Attitude, Self-Efficacy, and Achievement in Geometry

Burçak Boz-Yaman
Mugla Sitki Koçman Üniversitesi

ABSTRACT

This study aims to examine changes in students' achievement in geometry, attitude towards geometry, and their self-efficacy perception of geometric knowledge while working on paper folding activities through an experimental research methodology. The sample for this experimental study was 108 ninth-grade high school students. The study's data were gathered through attitude, self-efficacy, and achievement scales which were employed both as pretest and posttest measurements for experimental and control groups. The findings revealed a significant difference between the experimental group's pretest and posttest scores regarding the variables of achievement and self-efficacy. However, no significant difference was observed for attitude towards geometry for the experimental group. The study then addresses these results in light of the existing literature.

Keywords: geometry teaching, patty paper, paper folding, attitude, self-efficacy, geometry achievement

Introduction

Geometry plays a significant role in daily life along with mathematics. It is frequently used in architecture, art, and engineering, as well as several sub-branches of mathematics itself. Euclid's 13-volume geometry work, "The Elements," which has maintained considerable impact since 300 BCE, laid the foundation for contemporary geometry. Geometric constructions that date back to the time of Euclid are still relevant today for learning geometry, and also for mathematicians studying geometry (Pandiscio, 2002). Around 300 BCE, geometric constructions were generally created by means of a compass and a straightedge. However, the use of geometric constructions not only provides a different perspective to classic geometry teaching methods, but also reinforces the meaningful learning of geometric concepts through the promotion of mathematical thinking (Pandiscio, 2002; Serra, 2003). Geometric construction applications are considered particularly important in geometry teaching since they relate closely to the primary teaching goals of geometry, such as discovering the main axiomatic concept of geometry, proving, developing estimation capabilities, meaningful learning and problem solving, and the improvement of geometric thinking levels (Coad, 2006; Erduran & Yesildere, 2010; Leung, 2011).

The importance of geometric constructions has been emphasized in international and national standards and curricula (Australian Mathematical Sciences Institute, 2011; Common Core State Standards, 2010; Milli Eğitim Bakanlığı [Turkish Ministry of National Education], 2010a, 2010b; National Council of Teachers of Mathematics, 2000). For example, the National Council of Teachers of Mathematics Standards (2000) indicates that the use of construction tasks can help encourage students to "draw and construct representations of two- and three-dimensional geometric objects using a variety of tools" (p. 308) and "to recognize and connect mathematical ideas as a way to develop robust understandings of problems" (p. 354). Similarly, the Common Core State Standards (2010) highlighted geometric construction particularly in the Congruence section, where it advised to "make formal geometric constructions with a variety of tools and methods (e.g., compass and straightedge, string, reflective devices, paper folding, dynamic geometric software)" (p. 76). In Türkiye, the Ministry of National Education (Milli Eğitim Bakanlığı, 2010a, 2010b) drew attention to geometric construction with the "efficient use of a compass, ruler, protractor, and setsquare" (p. 18) under psychomotor skills and called for the formation of geometric constructions by promoting compass and ruler usage in model applications of mathematics curricula for several types of achievement.

Teaching environments for geometric construction should be equipped with various tools and learning strategies, with several studies in the literature having indicated that the use of different tools (e.g., dynamic geometry software, MIRA, protractor, setsquare, paper folding, compass-ruler) can help students to learn geometry more effectively (Clements & Battista, 1992). The literature includes studies on the use of paper folding in geometry teaching (Arıcı & Aslan-Tutak, 2015; Boakes, 2008; Coad, 2006; Pope, 2002), demonstrating that paper folding is a powerful yet simple teaching tool that can be used to perform geometric constructions. For example, third-degree equations can be solved through paper folding, whereas traditional tools such as the ruler and compass can only resolve equations of the second order (Geretschläger, 1995).

A study by Sanchez and Glassmeyer (2017) used patty paper to examine parabolas. The researchers utilized this material because when patty paper is folded, a white crease mark becomes visible. This makes the material useful for the examination of parabolas (Scher, 1996), ellipses, and hyperbolas (Smith, 2003) since sharp visible creases are retained after folding. Similarly, Spanik (2009) explained that a favorite lesson used patty paper to construct a unit circle and emphasized that students meticulously engaged with lessons where patty paper was used. Spanik (2009) noted that the material offered ample opportunity to clarify mathematical expressions and notations through systematic investigation to determine the exact values of sine, cosine, and tangent of 30°, 45°, and 60° angles. Draper (2007) was another mathematics teacher-researcher who stated that patty paper was a most effective tool and afforded students the opportunity to trace and fold actions in geometry and mathematics activities. King (2016) used patty paper to enhance students' understanding of piecewise functions through the use of the paper's transparency for tracing. In classes, King (2016) had students place a sheet of patty paper on the graphs of different portions of a line and a parabola within different intervals, then traced the axes onto the patty paper in order to construct piecewise functions. Additionally, Empson and Turner (2006) examined primary students' multiplicative thinking and reasoning using patty paper folding and revealed that it led to a potentially powerful development of students' thinking about fractions and multiplicative reasoning. In these studies, patty paper was notably preferred due to it being easy to fold, that the fold lines are easily distinguishable, and the paper can also be marked by students if needed. Therefore, the current study opted to utilize patty paper in place of either regular or origami paper. In recent years, the use of paper folding has been observed in geometry lessons in addition to the use of rulers, compasses, and concrete tools. Nevertheless, with patty paper often now utilized in geometric construction problems in addition to the traditional compass and straightedge method, there is a clear need to scientifically study the effectiveness of the patty paper method in both the cognitive and affective domains of geometry learning.

It is well known that the academic success of individuals is dependent upon not only on the cognitive, but also the affective dimension, with attitude and self-efficacy both key components of these dimensions. Among studies that employed various teaching methods and tools (e.g., technology, teaching through drama, concrete materials), several research studies have demonstrated that students' attitude towards geometry was more positive than where traditional teaching and learning methods were used (Duatepe, 2004; Funkhouser, 2003). In a meta-analysis study, Ma and Kishor (1997) determined that attitude and achievement were strongly related, and that students' attitude towards mathematics had negatively changed after starting school. Therefore, the examination of students' attitudes toward geometry is considered essential to understanding student performance in geometry learning.

Another affective aspect considered to influence student achievement in both mathematics (Hannula et al., 2014) and geometry (Erkek & Işıksal, 2012) relates to the concept of self-efficacy. Also referred to as self-efficacy perception or self-efficacy belief, this aspect is defined as an individual's thoughts regarding their capabilities and skill to organize their own activities (Bandura, 1986). Self-efficacy does not relate to how capable an individual is at a certain subject but is the concept of their belief in themselves. The higher an individual's belief, the more likely they are to leave failure behind them, and accordingly display ambition to attain their desired goals and achievement (Bandura, 1997). Individuals who are self-confident and feel at ease are likely to be more productive than those who are not, and in the school environment, this type of confidence may also assist learners in obtaining higher test scores. On the contrary, individuals who lack self-confidence may have low social skills and a lower chance of achievement in academic settings (Pajares, 1996). Bandura (1997) remarked on there being four basic sources of self-efficacy belief; "mastery experiences, vicarious experiences acquired through social models, verbal persuasion, and an individual's own physiological-emotional state" (p. 79). According to Bandura (1997), the most important of these four factors is "mastery experience" or life experience based on students' interpretation of their own personal performance. In research studies that addressed self-efficacy perception in relation to geometry (Karakuş, 2014; Pintrich & De Groot, 1990), it has been argued that in order to improve students' self-efficacy perception, which is generally determined as being low or average, different teaching models and tools should be employed in lessons in addition to enhanced student participation. Students' life experiences should be considered in order to support their development of self-efficacy beliefs in geometry, with appropriate environments provided in which they can encounter positive experiences. In the current study, positive student experiences were aimed to be provided through the use of patty paper folding activities in geometric construction.

Geometry is often regarded as a challenging subject that consists of many rules and thereby considered rote-based and boring by many students (Adolphus, 2011). It was also evident in studies by Mason (1998) and Mullis et al. (2012) that student achievement in geometry is lower compared to other branches of mathematics. In the current study, patty paper practices were used to teach geometric construction, and these practices were examined based on students' self-efficacy perceptions, attitudes, and achievement in geometry. Accordingly, the current study's aim was to examine the effects of patty paper practices for geometric construction on ninth-grade students' self-efficacy perceptions, attitudes, and achievement in relation to geometry. Among other methods and strategies employed for learning geometry, patty paper practices require further investigation in order to be better understood. In this respect, the following research questions form the basis of the current study:

(1) Is there a significant difference in attitudes towards geometry between ninth-grade students who participated in patty paper folding activities (over a six-week duration) for geometric construction in an "Auxiliary Elements of Triangles" unit and students who were instructed according to traditional methods?
(2) Is there a significant difference in self-efficacy perception between ninth-grade students who participated in patty paper folding activities (over a six-week duration) for geometric construction in an "Auxiliary Elements of Triangles" unit and students who were instructed according to traditional methods?

(3) Is there a significant difference in geometry achievement between ninth-grade students who participated in patty paper folding activities (over a six-week duration) for geometric construction in an "Auxiliary Elements of Triangles" unit and students who were instructed according to traditional methods?

Methodology

Research Design

Using a static group pretest, posttest design (Fraenkel & Wallen, 2006), this study compared students' pretest and posttests on measures of geometry-related attitude, self-efficacy, and achievement in order to determine whether or not a statistically significant difference was evident between the two scores. Since randomization of the students was not possible, two intact classes, which had already been formed at the beginning of the 2015-2016 school year, were used as the experimental and control groups of the study.

The design of the research is as presented in Table 1, where "O" represents the measurement tools conducted prior to and following the application. The application was divided into two, with patty paper practices conducted for the experimental group and traditional teaching for the control group.

Table 1

Pretest/posttest Design with Control Group

Group	Pretest	Application	Posttest
Experimental	O1, O2, O3	Patty paper practices	O1, O2, O3
Control	O1, O2, O3	Traditional teaching	O1, O2, O3

Note. O1: Geometry Achievement Questionnaire, O2: Attitude Towards Geometry Scale, O3: Self-Efficacy Regarding Geometry Scale.

The use of patty paper folding methods in the mathematics classroom was the treatment applied (to the experimental group) and was therefore considered as the independent variable of the study. During instruction of the "Auxiliary Elements of Triangle" unit, the experimental group performed patty paper practices, while the control group received standard traditional instruction from their teacher. The dependent variable was the students' posttest scores for the Geometry Achievement Questionnaire that had been prepared by the researcher, the Attitude Towards Geometry Scale (Bulut et al., 2002), and the Self-Efficacy Regarding Geometry Scale (Cantürk-Günhan & Başer, 2007).

Sample and Data Collection Tools

Convenience sampling was used in the selection of one high school from a medium-sized provincial center in the Aegean Region of Türkiye. One mathematics teacher from the selected high school volunteered for the study during a preliminary interview, and two of the teacher's ninth-grade classes were selected at random to be included in the study. In total, 108 students participated in the study, with 56 students forming an experimental group and 52 students as a control group, and with both groups having been taught by the same mathematics teacher.

Two existing Likert-type scales were applied in the study as well as an open-ended questionnaire developed by the researcher. The instruments used were the Attitude Towards Geometry Scale, as developed by Bulut et al. (2002), the Self-Efficacy Regarding Geometry Scale developed by Cantürk-Günhan and Başer (2007), and the researcher-developed Geometry Achievement Questionnaire. The Attitude Towards Geometry Scale consisted of 17 items and three subdimensions and was applied to both the experimental and control groups as a pretest and following the application as a posttest. The Cronbach's alpha internal consistency score of the Attitude Towards Geometry Scale was α = .92. The Self-Efficacy Regarding Geometry Scale consisted of 25 items structured under three subdimensions. In the scale's reliability test, the Cronbach's alpha coefficient was calculated as α = .87. The Geometry Achievement Questionnaire was developed by the researcher and consisted of eight geometry questions covering the "Auxiliary Elements of Triangle" topic. Each question contained either the phrase of "please prove" or "please demonstrate" (see Appendix). During its development, the questionnaire was examined by three mathematics teachers for its suitability and validity for the target class level and subject topic, and the question wording was subsequently updated in accordance with their suggested corrections.

Data Collection Procedure

The research was implemented during the spring semester of the 2015-2016 academic year. The study was conducted over an eight-week period during which lessons from the "Auxiliary Elements of Triangle" sub-learning domain of the Secondary School (grades 9-11) Mathematics Curriculum for Geometry (Milli Eğitim Bakanlığı [Turkish Ministry of National Education], 2010a, b) were instructed to both the experimental and control groups of the study. For this sub-learning domain, the properties of angle bisector, altitudes, and median of a triangle were examined, and then each was discussed for each type of triangle.

In the first week of the study, preliminary application of the Geometry Achievement Questionnaire, Attitude Towards Geometry Scale, and the Self-Efficacy Regarding Geometry Scale were performed during two class hours, and these same instruments were also reapplied during the final week of the study following the application. The patty paper activities were led by the researcher for the experimental study group for a period of six weeks. During the activities certain geometry terms were emphasized, including angle bisector, interior angle bisector, exterior angle bisector, median, altitude, orthocenter, perpendicular bisector, centroid, inscribed circle, escribed circle, and circumscribed circle. These concepts were studied using a ruler, a compass, and patty paper folding activities. Views depicting the classroom practices of the experimental group are shown in Figure 1.

Figure 1

Students' Patty Paper Practices

During the patty paper activities, the instructor (researcher) distributed worksheets and afforded the students time to discuss the questions. The worksheets included questions about the related topics which the students were tasked with answering by constructing geometrical objects using patty paper. For example, in the second patty paper activity, the goal was to examine the bisector of an angle through paper folding. Instructions for folding a bisector of an angle and related questions were given in the worksheet as follows:

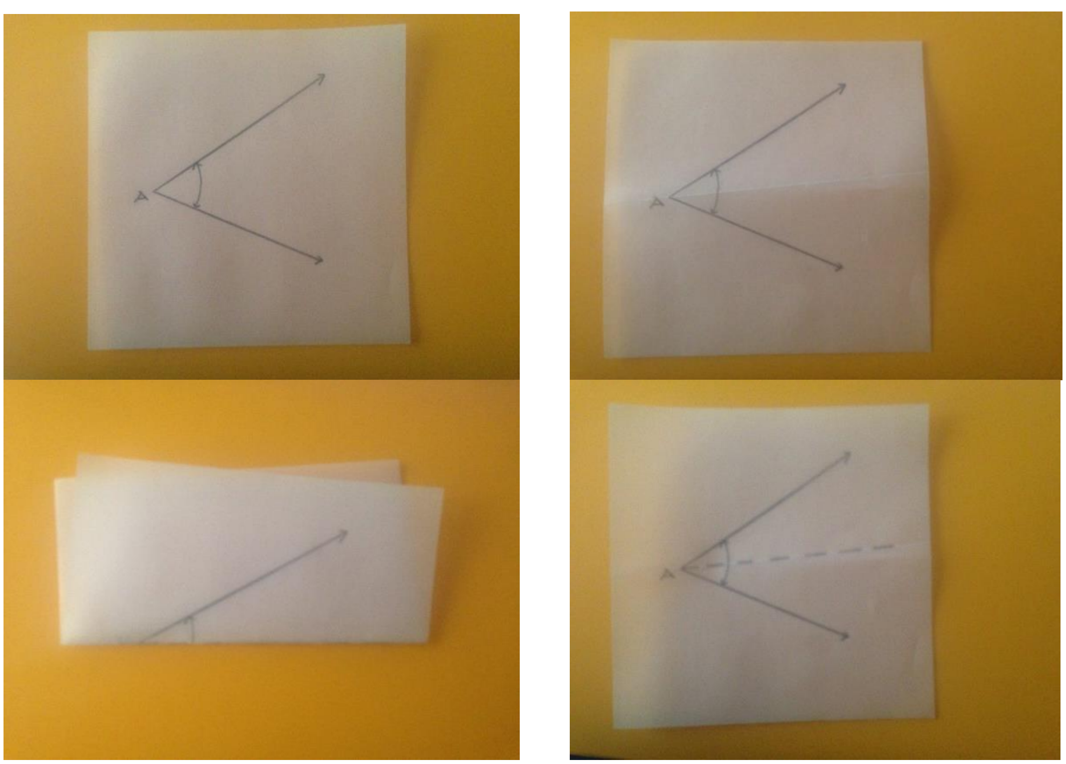
> Draw an angle on the patty paper. Fold one segment over the other ensuring that the fold passes through the vertex of the angle. Then, unfold the paper and draw along the fold line (crease). What do you think about the measurement of the angle? What changes when the angle is folded? Estimate the measurement of the two angles as they appear when the paper is unfolded.

The instructor also used patty paper during the activities. While the students were reading and folding the required geometrical concept, the instructor also folded the same geometrical concept which was then presented to the class as an example of how the task should be achieved. For students who had not understood how to construct the fold, the instructor then showed them the example and then assisted them where necessary to complete the task themselves. After the patty paper had been folded and the required concept constructed, the students answered the questions prescribed in the worksheets through peer discussion. The instructor provided assistance when the students experienced any problems reaching the correct answers or had difficulties during the patty paper task construction. The instructor helped to direct the students' discussions and led them to construct the tasks appropriately and to answer the worksheet questions.

In the following bisector construction example, the students could see the area between the two closed half lines were divided into two equivalent areas. Therefore, the closed half line represented by a dashed line in Figure 2 is referred to as an angle bisector.

Figure 2

Constructing an Angle Bisector



While the experimental group received instruction on patty paper folding with student-centered activities, the control group received traditional teacher-led instruction. In the control group, since there were no patty paper tasks, the topic of "bisector of an angle" was only shown on the whiteboard where the teacher first drew an angle and then the bisector. The teacher gave the properties of the bisector of an angle and demonstrated some bisector exercises. The control group students were knowledge receivers only and took no part in the lessons as either performers or constructors.

In addition, the researcher taught the control group origami and paper folding activities (e.g., folding crane) for one hour per week over and above their normal mathematics class time of six hours, whilst the experimental group were taught by the instructor for two hours (out of six) each week (i.e., no additional teaching hour). However, it should be noted that the one-hour weekly origami lesson offered to the control group was not associated with the content of the geometry lesson and was designed purely as an extracurricular activity. This approach was opted for in order to ensure that the researcher did not teach only one specific class and also to minimize possible effects caused by interaction between the students of either group. The two hours per week of paper folding activities conducted for the experimental group were led by the researcher (as instructor), whereas the experimental group's remaining four hours of weekly mathematics classes were taught by their usual assigned mathematics teacher. The activities were each designed in accordance with the current mathematics curriculum in order that the class teaching schedule was not unnecessarily interrupted, meaning that the teaching schedule continued in a way that was able to encompass the activities. The practice was completed in a total of 12 hours over a period of six weeks, split equally as two hours per week. Table 2 details the content of the activities applied to the experimental group.

Table 2

Activity Content for Experimental Group

Week	Activity	Activity content
1	Introduction of folding axioms	Seven folding axioms were introduced, with each demonstrated using patty paper plus worksheet questions.
2	Drawing a bisector	Bisectors folded to a line segment from points on/outside.
3	Forming a circumscribed circle	Perpendicular bisectors found in various triangle types, circles drawn taking their intersection as center, introduced as circumscribed circle of triangles.
4	Drawing an inscribed circle	Angle bisectors found in various triangle types by folding, circles drawn taking their intersection as center, introduced as inscribed circle of triangles.
5	Forming and studying medians in triangles	Medians of sides determined in various triangle types and bisectors folded, relation between intersection of medians and median length studied.
6	Studying orthocenters in triangles	Orthocenters found in various triangle types and their intersection studied.

These two-hour sessions involved patty paper folding and examination of the folding process through worksheet questions which helped students to critique the patty paper folding process based on geometrical thinking. The paper folding and accompanying worksheet questions were aimed at helping the experimental group students to construct valid geometry knowledge.

Data analysis

Two of the three measurement tools applied in the study, the Attitude Towards Geometry Scale (Bulut et al., 2002) and the Self-Efficacy Regarding Geometry Scale (Cantürk-Günhan & Başer, 2007), are both formed as five-point, Likert-type scales, with higher scores indicating a higher level of attitude towards geometry or self-efficacy regarding geometry. In the data analysis of the scales' scores, paired sample *t*-test statistical examination was used in order to test whether or not any statistically significant differences existed between and within the pretest and posttests scores.

Findings

The findings of the study are presented in three separate foci in accordance with the study's three research questions: attitude towards geometry, self-efficacy regarding geometry, and geometry achievement.

Attitude Towards Geometry

According to the pretest test scores of the Attitudes Towards Geometry Scale, there was no statistically significant difference found between the experimental and control groups (t = -0.186; p = .822). Therefore, it can be deduced that prior to the application, the two groups of students were similar in their attitude towards geometry. However, after having conducted both a pretest and posttest of the Attitudes Towards Geometry Scale, the students in the experimental group showed no statistically significant difference in their attitude towards geometry after having received six weeks of patty paper folding instruction.

Similarly, no significant difference was found from examination of the control group's pretest and posttest scores. Although there was a decrease in the control group's mean attitude score after six weeks, this difference was not found to be statistically significant. Furthermore, upon analysis of the posttest scores of both the experimental and control groups, no significant difference was observed regarding the students' attitudes towards geometry. See Table 3 for this information.

Table 3

Experimental and Control Group t-test Results: Attitude Towards Geometry

Scale	Groups		*N*	$\bar{x}$	*SD*	*df*	*t*	*p*
Attitude Towards Geometry Scale	E.G.	Pretest	56	3.32	0.671	55	-1.480	.145
		Posttest	56	3.46	0.786			
	C.G.	Pretest	52	3.35	0.713	51	0.468	.642
		Posttest	52	3.32	0.773			
	E.G.	Pretest	56	3.32	0.671	106	-0.186	.822
	C.G.		52	3.35	0.713			
	E.G.	Posttest	56	3.46	0.786	106	0.955	.913
	C.G.		52	3.32	0.773			

Note. Experimental Group: E.G., Control Group: C.G., $p < .05$

From Table 3, the experimental group had increased scores ($\bar{x}_{pretest} = 3.32$, $\bar{x}_{posttest} = 3.46$) after the intervention, whereas the control group exhibited a decline ($\bar{x}_{pretest} = 3.35$, $\bar{x}_{posttest} = 3.32$). Upon comparing the two groups based on their initial attitude scores, no statistically significant difference emerged, indicating a comparable disposition towards geometry. However, whilst the mean score of the experimental group ($\bar{x}$ = 3.46) following the intervention surpassed that of the control group ($\bar{x}$ = 3.32), no statistical significance was found between them.

Self-efficacy Regarding Geometry

In order to answer the second research question of the study, the examination focused on the results from the Self-Efficacy Regarding Geometry Scale. As illustrated in Table 4, there was no statistically significant difference observed in the pretest mean scores between the experimental ($\bar{x}$ = 3.01) and control groups ($\bar{x}$ = 3.06), suggesting that students in both groups exhibited similar levels of self-efficacy at the onset of the study. However, following the six-week treatment period, notable differentiation emerged between the pretest and posttest mean scores of the experimental group, where the paper folding approach of instruction had been employed (see Table 4, $\bar{x}$ = 3.41, *SD* = 0.587, $t(106) = 2.238$, $p < .005$).

Table 4

Experimental and Control Group t-test Results: Self-Efficacy Regarding Geometry Scale

Scale	Groups		*N*	$\bar{x}$	*SD*	*df*	*t*	*p*
Self-Efficacy Regarding Geometry Scale	E.G.	Pretest	56	3.01	0.518	55	-5.405	.000*
		Posttest	56	3.41	0.587			
	C.G.	Pretest	52	3.06	0.577	51	-1.140	.259
		Posttest	52	3.14	0.662			
	E.G.	Pretest	56	3.01	0.518	106	-0.462	.343
	C.G.		52	3.06	0.577			
	E.G.	Posttest	56	3.41	0.587	106	2.238	.027*
	C.G.		52	3.14	0.662			

Note. Experimental Group: E.G., Control Group: C.G., * $p < .05$

In contrast, there was no significant difference detected between the pretest ($\bar{x}$ = 3.06) and posttest mean scores ($\bar{x}$ = 3.14) of the control group's students. This indicates that the self-efficacy scores of the control group remained largely unchanged after having received traditional geometry instruction in their lessons. Conversely, comparison of the posttest mean scores ($\bar{x}_{E.G.}$ = 3.41, $\bar{x}_{C.G.}$ = 3.14) indicated that there is a significant effect of the patty paper folding approach on the ninth grade students' self-efficacy in geometry (see Table 4, $\bar{x}$ = 3.41, *SD* = 0.587, $t(106) = 2.238$, $p < .005$).

Geometry Achievement

Scores taken from the Geometry Achievement Questionnaire were used to test the third research question and the third quantitative assessment of student performance applied in the study. Initially, there was a noticeable difference in performance on the Geometry Achievement Questionnaire pretest. The control group statistically performed better than the experimental group ($\bar{x}$ = 9.48, SD = 7.41, $t(106) = -3.44$, $p < .005$). This finding suggests that the control group was comprised of students with a higher level of geometry knowledge when compared to the experimental group.

However, as shown in Table 5, a significant difference was observed between the pretest and posttest mean scores of the Geometry Achievement Questionnaire for the experimental group, after having been engaged in patty paper folding practices ($\bar{x}$ = 8.41, *SD* = 6.60, *t*(55) = -5.062, *p* < .005). The difference in scores between the pretest and posttest indicated that the use of paper folding practices had a positive effect on the experimental group's geometry achievement level.

Table 5

Experimental and control group t-test results: Geometry Achievement Questionnaire

Scale	Groups		*N*	$\bar{x}$	*SD*	*df*	*t*	*p*
Geometry Achievement Questionnaire	E.G.	Pretest	56	5.34	4.95	55	-5.062	.00*
		Posttest	56	8.41	6.60			
	C.G.	Pretest	52	9.48	7.41	51	-1.235	.223
		Posttest	52	10.04	7.56			
	E.G.	Pretest	56	5.34	4.95	106	-3.44	.00*
	C.G.		52	9.48	7.41			
	E.G.	Posttest	56	8.41	6.60	106	-1.195	.235
	C.G.		52	10.04	7.56			

Note. Experimental Group: E.G., Control Group: C.G.,*p < .05

No significant contrast was observed between the mean pretest and posttest scores of the Geometry Achievement Questionnaire among students in the control group. However, despite the lack of statistical significance, it is notable that there was an overall improvement in their achievement scores. Similarly, when looking at the posttest scores for geometry achievement between the experimental and control groups, no substantial difference was found. This could be explained by the control group having had higher achievement mean scores even before the treatment began when compared to the experimental group, indicating that the control group consisted of students who were generally more academically successful. However, upon closer examination of the mean pretest and posttest scores for the experimental group, there was a noticeable increase in the students' geometry achievement levels (see Table 5).

Based on the data provided in Table 5, it is evident that the utilization of the patty paper folding approach had a discernible impact on the geometry achievement of the experimental group's students. Conversely, the control group's students did not exhibit any significant variance in their geometry achievement scores. When comparing the pretest results in geometric achievement between the two groups, a notable finding emerges. A significant disparity in the average scores was shown, with the control group outperforming the experimental group. This difference can be explained by the fact that the students in the control group initially performed at higher levels. However, what is particularly noteworthy is the lack of any significant mean difference between the posttest scores of the experimental and control groups. This finding indicates that the introduction of paper folding activities enabled the experimental group to bridge the gap in achievement, bringing their scores closer to those of the higher-performing control group.

Discussion

The aim of this study was to evaluate the influence of paper folding activities on the geometry achievement, attitudes, and self-efficacy perceptions of ninth-grade students. A six-week treatment period was implemented, and inferential statistics were utilized in the analysis of the obtained data. This section presents the results of the study in accordance with the findings.

The results suggested that the experimental group who participated in paper folding activities experienced considerable gains in their geometry achievement. This result was similar to that revealed in several prior studies in the literature that established achievement as having increased through the application of different teaching methods (e.g., technology, compass-ruler use, origami) in geometry lessons (Arıcı & Aslan-Tutak, 2015; Güven, 2006; Napitupulu, 2001). The cause behind such increases may be interpreted such that these alternative practices help enable learners to actively form geometric constructions and to reach conceptual understanding and explanations more easily, unlike traditional methods. For instance, Güven (2006) found that geometric drawing instructed by means of paper folding and compass-ruler use for seventh and eighth grade students for a period of six weeks helped geometry success levels and saw a positive attitude change towards geometry. Similarly, Arıcı and Aslan-Tutak (2015) found that students who were taught geometry using origami for four weeks had significantly higher achievement compared to the control group. Unlike the findings of Güven (2006) and Arıcı and Aslan-Tutak (2015), the current study observed that the use of patty paper folding did not significantly affect the experimental group's attitude towards geometry. Similarly, Hull and Brovey (2004) determined that their experimental group was more successful with regards to geometry achievement, but that no significant difference was identified with regards to the students' attitudes. The main reason behind this finding may be that the time required to observe such a change is longer and that longer-term practices are required. For example, Hull and Brovey (2004) applied an implementation over a period of three weeks, whereas the current study was based on a treatment period that spanned six weeks. Thus, it would be beneficial for the process to be repeated in a longer-term study and the results subsequently comparatively analyzed. The questions used in the current study's Geometry Achievement Questionnaire were open-ended and required the students to provide explanation, which is another potential justification for the absence of any difference in the experimental group's geometry achievement. The participants of both the current study's control and experimental groups were unfamiliar with these kinds of open-ended questions, being more accustomed to answering multiple-choice questions requiring no additional explanation. Therefore, the participant students may have experienced difficulties in answering the open-ended questions they faced in the Geometry Achievement Questionnaire.

The results regarding another variable of the current study, self-efficacy perception, are considered to be of particular importance. Although no change in attitude was observed, the experimental group's self-efficacy perception exhibited a significant positive difference. The cause of this improvement may be explained by means of elements reported by Bandura (1997) as the sources for the development of self-efficacy, or the lack thereof. In the current study, the researcher continuously provided verbal confirmation and cues to the students, underlining that geometry may be an easier subject than they thought (verbal persuasion), which was reinforced by their having witnessed the achievements of their peers (indirect experience) and may be proposed as the cause for the observed change in self-efficacy. Furthermore, patty paper practices, which may be regarded as direct experience for the students, may also be considered a factor for positive change in self-efficacy as they largely formed abstract concepts by themselves and experienced geometry subjects as a phenomena that may be studied concretely. Similarly, Usher and Pajares (2008) remarked that the successful experiences of individuals in their lives could enhance self-efficacy beliefs.

Consistent with prior research by Lam and Pope (2016), one of the most significant results of the current study was that introducing paper folding into geometry lessons helped to enhance the students' motivation and self-confidence. For this reason, the use of paper folding activities is recommended to help improve class/activity involvement of students who generally experience anxiety or lack of self-confidence, particularly in relation to geometry lessons, to increase their motivation and thereby enhance their self-confidence, as well as their attention and interest during classes.

Based on an additional observation about psychomotor skills, the students notably experienced difficulties in the use of a compass, which may relate to there being no integrated compass application during classes as the reason behind this observation. It was noted that the mathematics teachers had not used any geometrical tools during the lessons, focusing instead on the solving of theoretical geometry questions, despite the mathematics curriculum clearly indicating that the "compass-ruler or their counterpart in dynamic geometry software should be used." The reasons underlying teachers' unwillingness to use such tools could be that "they were unable to perform geometric drawings" and that "they do not place the required importance on geometric drawings," as emphasized in the findings of Erduran and Yesildere (2010), leading to the students' lack of knowledge and skills required for the formation of geometric constructions. The result of a study by Napitupulu (2001) also produced findings whereby secondary education geometry students could not acquire the required skills for geometric constructions.

It may be argued that the difficulties experienced by the current study's students in folding patty paper was that they had not previously performed any paper folding activities. In earlier research, the researcher pointed out that paper folding should be performed a number of times in order to familiarize students with the general process of paper folding and to then study these processes with regards to mathematics (Boz, 2015). This approach would help students to develop the required fine motor skills and the ability to use multiple organs (e.g., eye, hand) simultaneously during paper folding activities. Having gained experience at paper folding, students would be likely to perform better in geometry-based paper folding activities.

Suggestions

The formation of geometric constructions has been integral to both mathematics and mathematics teaching for centuries. However, these practices should be studied more with respect to geometry teaching and are deserving of greater attention in the teaching and learning of geometric concepts. It is contemplated that, based on the findings of the current study, the literature would benefit considerably if comparisons were made between classes where patty paper practices were performed and where other tools were used and to provide experimental findings. In addition, comparisons could be made between classes where dynamic geometry software and a compass-ruler were employed, and where paper folding activities were undertaken. Moreover, the current study also proposes that the literature would benefit from lesson planning that incorporates all of these processes; that is, paper folding activities followed by compass and ruler application, then supported by the use of dynamic geometry software, and finally testing the effectiveness of these approaches. The results of the current study suggest that geometry teachers could consider using paper folding activities in their lessons to develop students' geometry knowledge and geometrical thinking. Patty paper activities have the potential to improve students' achievement, positive attitude towards geometry, and self-efficacy perception with regards to geometry. However, it should be noted that students may need additional support during patty paper activities, hence teachers should develop appropriate strategies and pedagogical tools to scaffold students' geometry learning during such activities. These scaffolding types could also be examined in future studies.

This author received no financial support for the research, authorship, and/or publication of this manuscript.

Burçak Boz-Yaman (burcakboz@gmail.com) is an associate professor in mathematics education. Her interests include mathematics teacher's content knowledge and pedagogical content knowledge. In addition, she has studied geometry teaching, activity-based learning, and the role of technology in mathematics education.

References

Adolphus, T. (2011). Problems of teaching and learning of geometry in secondary schools in Rivers State, Nigeria. *International Journal of Emerging Sciences, 1*(2), 143-152.

Arıcı, S., & Aslan-Tutak, F. (2015). The effect of origami-based instruction on spatial visualization, geometry achievement, and geometric reasoning. *International Journal of Science and Mathematics Education, 13*, 179-200. https://doi.org/10.1007/s10763-013-9487-8

Australian Mathematical Sciences Institute. (2011). *The Improving Mathematics Education in Schools (TIMES) Project, Construction.* http://www.amsi.org.au/teacher_modules/pdfs/Construction.pdf

Boz, B. (2015). A Journey from two-dimensional papers to three-dimensional origamic cube. *Journal of Inquiry Based Activities,* 5(1), 20-33. https://www.ated.info.tr/index.php/ated/article/view/58/97

Bandura, A. (1986). *Social foundations of thought and action: A social cognitive theory*. Prentice Hall.

Bandura, A. (1997). *Self-efficacy: The exercise of control.* Freeman.

Boakes, N. (2008). Origami-mathematics lessons: Paper folding as a teaching tool. *Mathidues, 1*(1), Article 5. https://www.fau.edu/education/centersandprograms/mathitudes/documents/20080901bmathitudesoct08revisionfinalversionforpublicationoct242008.pdf

Bulut, S., Ekici, C., İşeri, A. İ., & Helvacı, E. (2002). A scale for attitudes towards geometry. *Education and Science, 27*(125), 3-7. https://egitimvebilim.ted.org.tr/index.php/EB/article/view/5146

Cantürk-Günhan, B., & Başer, N. (2007). The development of self-efficacy scale toward geometry. *Hacettepe University Journal of Education, 33*, 68-76. http://www.efdergi.hacettepe.edu.tr/shw_artcl-1006.html

Clements, D., & Battista, M. T. (1992*).* Geometry and spatial reasoning. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 420-460). Macmillan.

Coad, L. (2006). Paper folding in the middle school classroom and beyond. *Australian Mathematics Teacher, 62*(1), 6-13.

Common Core State Standards Initiative. (2010). *Common core state standards for mathematics*. https://learning.ccsso.org/wp-content/uploads/2022/11/Math_Standards1.pdf

Draper, C. (2007). Tracing geometry. *Connect, 20*(4), 22-25.

Duatepe, A. (2004). *The effects of drama based instruction on seventh grade students' geometry achievement, Van Hiele geometric thinking levels, attitude toward mathematics and geometry* [Doctoral dissertation, Middle East Technical University, Ankara, Turkey]. https://open.metu.edu.tr/handle/11511/14577

Empson, B. S., & Turner, E. (2006). The emergence of multiplicative thinking in children's solutions to paper folding tasks. *The Journal of Mathematical Behavior, 25*(1), 46-56. https://doi.org/10.1016/j.jmathb.2005.11.001

Erduran, A., & Yesildere, S. (2010). The use of a compass and straightedge to construct geometric structures. *Elementary Education Online, 9*(1), 331-345. https://ilkogretim-online.org/index.php/pub/article/view/687

Erkek, Ö., & Işıksal, M. (2012, October 17-21). *Exploring the relationships among 8th grade students' geometry achievement, geometry self-efficacy and spatial anxiety* [Conference presentation]. European Conference on Educational Research, Cadiz, Spain.

Fraenkel, J. R., & Wallen, N. E. (2006). *How to design and evaluate research in education* (6th ed.). McGraw-Hill.

Funkhouser, C. (2003). The effects of computer-augmented geometry instruction on student performance and attitudes. *Journal of Research on Technology in Education, 35*(2), 163-175. https://doi.org/10.1080/15391523.2002.10782377

Geretschläger, R. (1995). Euclidean constructions and the geometry of origami. *Mathematical Magazine, 68*(5), 357-371. https://doi.org/10.1080/0025570X.1995.11996356
Güven, Y. (2006). *The effect of different geometric drawing methods on students' achievements, attitudes and van Hiele geometry understanding levels* [Master's thesis, Karadeniz Technical University, Trabzon, Turkey]. https://tez.yok.gov.tr/UlusalTezMerkezi/tezDetay.jsp?id=hh_2s6zmMQVkGwYm3bh3Yg&no=ZSHTv2lazUiGWaFCglr9pA
Hannula, M. S., Bofah, E. A., Tuohilampi, L., & Metsämuuronen, J. (2014). A longitudinal analysis of the relationship between mathematics-related affect and achievement in Finland. In S. Oesterle, P. Liljedahl, C. Nicol, & D. Allan (Eds.), *Proceedings of the Joint Meeting of PME 38 and PME-NA 36* (pp. 249-256).
Hull, A. N., & Brovey, A. J. (2004). The impact of the use of dynamic geometry software on student achievement and attitudes towards mathematics. *Action Research Exchange, 3*(1), 24-37.
Karakuş, F. (2014). Pre-service elementary mathematics teachers' views about geometric construction. *Journal of Theoretical Educational Science, 7*(4), 408-435. https://dergipark.org.tr/en/pub/akukeg/issue/29355/314110
King, S. L. (2016). Piecing together piecewise function. *Mathematics Teacher, 90*(7), 550-552. https://doi.org/10.5951/MT.90.7.0550
Lam, T. K., & Pope, S. (2016). *Learning mathematics with origami*. Association of Teachers of Mathematics
Leung, H.-C. (2011). *Enhancing students' ability and interest in geometry learning through geometric constructions* [Master's thesis, The University of Hong Kong, Hong Kong]. http://hdl.handle.net/10722/177256
Ma, X., & Kishor, N. (1997). Assessing the relationship between attitude toward mathematics and achievement in mathematics: A meta-analysis. *Journal for Research in Mathematics Education, 28*(1), 27-47. https://doi.org/10.2307/749662
Mason, M. M. (1998). *The van Hiele levels of geometric understanding. The professional handbook for teachers: Geometry*. McDougal-Littell / Houghton-Mifflin.
Milli Eğitim Bakanlığı. (2010a). *Ortaöğretim Geometri Dersi 9-10. Sınıflar Öğretim Programı* [Secondary school 9-10th grade geometry curriculum]. http://ttkb.meb.gov.tr/www/ogretim-programlari/icerik/72.
Milli Eğitim Bakanlığı. (2010b). *Ortaöğretim Geometri Dersi 11. Sınıf Öğretim Programı* [Secondary school 11th grade geometry curriculum]. http://ttkb.meb.gov.tr/www/ogretim-programlari/icerik/72
Mullis, I. V. S., Martin, M. O., Foy, P., & Arora, A. (2012). *TIMSS 2011 international results in mathematics*. TIMSS & PIRLS International Study Center, Boston College.
Napitupulu, B. (2001) *An exploration of students' understanding and van Hiele levels of thinking on geometric constructions* [Master's thesis]. Simon Fraser University, British Columbia, Canada. https://library-archives.canada.ca/eng/services/services-libraries/theses/Pages/item.aspx?idNumber=56921023
National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Author.
Pajares, F. (1996). Self-efficacy beliefs in achievement settings. *Review of Educational Research, 66*(4), 543-578. https://doi.org/10.3102/00346543066004543
Pandiscio, E. A. (2002). Alternative geometric constructions: Promoting mathematical reasoning. *The Mathematics Teacher, 95*(1), 32-36. https://doi.org/10.5951/MT.95.1.0032
Pintrich, P. R., & De Groot, E. (1990). Motivational and self regulated learning components of classroom academic performance. *Journal of Educational Psychology, 82*(1), 33-40. https://psycnet.apa.org/doi/10.1037/0022-0663.82.1.33

Pope, S. (2002). The use of origami in the teaching of geometry. *Proceedings of the British Society for Research into Learning Mathematics, 22*(3), 67-73.

Sanchez, W. B., & Glassmeyer, D. M. (2017). Connecting parabolas and quadratic functions. *The Mathematics Teacher, 110*(5), 380-386. https://doi.org/10.5951/mathteacher.110.5.0380

Scher, D. P. (1996). Folded paper, dynamic geometry, and proof: A three-tier approach to the conics. *Mathematics Teacher, 89*(3), 188-193. https://doi.org/10.5951/MT.89.3.0188

Serra, M. (2003). *Discovering geometry: An investigative approach*. Key Curriculum Press.

Smith, S. G. (2003). Paper folding and conic sections. *Mathematics Teacher, 96*(3), 202-207. https://doi.org/10.5951/MT.96.3.0202

Spanik, A. (2009). Building the unit circle: A patty paper approach. *Mathematics Teacher, 103*(3), 232. https://doi.org/10.5951/MT.103.3.0232

Usher, E. L., & Pajares, F. (2008). Self-efficacy for self-regulated learning: A validation study. *Educational and Psychological Measurement, 68*(3), 443-463. https://doi.org/10.1177/0013164407308475

Appendix: Geometry Achievement Exam

(1) “Points that are at equal distance to two points, are a line that is perpendicular to the line segment that connects these two points.” Please demonstrate the expression by drawing and explaining why it is perpendicular.

(2) “In a triangle, interior angle bisectors intersect at a point.” Please demonstrate the expression by drawing and explaining why they intersect.

(3) “In a triangle, altitudes intersect at a point.” Please demonstrate the expression by drawing and explaining why they intersect.

(4) “In a triangle, bisectors intersect at a point.” Please demonstrate the expression by drawing and explaining why they intersect.

(5) “In an isosceles triangle, auxiliary elements of the equal sides are equal between themselves.” Please demonstrate the expression by drawing and explaining why they are equal.

(6) “In an equilateral triangle, all auxiliary elements are equal.” Please demonstrate the expression by drawing and explaining why they are equal.

(7) “If all corresponding sides of two triangles are equal, then the triangles are equal and the measurements of angles facing the equal sides are also equal.” Please demonstrate the expression by drawing and explaining why they are equal.

(8) “In a triangle, two external angle bisectors and a third internal angle bisector intersect at a point.” Please demonstrate the expression by drawing and explaining why they intersect.