Diagnosis, Remediation, and Error Correction for Mathematics: How to Teach Pre-service Teachers

AUTHORS
Scott A. Dueker
Jill D. Grande

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ABSTRACT

Mathematics is a core academic subject, regardless of grade level or setting. Everyone uses mathematics in their everyday life, so being competent in basic mathematics is critical to independent living. One thing teachers can do to ensure learners are learning the mathematics concepts being taught is to diagnose and remediate the errors they are seeing. This skill involves digging deeper into the work of the learners and looking for error patterns. Unfortunately, this skill is not a focus of mathematics instruction courses that pre-service teachers (PSTs) take in their program. This article is aimed at mathematics instruction faculty and describes three main error types as well as what to do when they are identified.

KEYWORDS

Diagnosis, errors, mathematics, pre-service teachers, remediation

Numbers are everywhere: phone numbers and addresses, prices at the store, recipes for cooking, and sizes for clothing, to name just a few. Interpreting these numbers and manipulating them through operations and analysis to provide meaning are important life skills. However, teaching mathematics, particularly to learners in special education, can be a difficult process. What, at face value, seem to be simple concepts can be problematic for some learners. The ability to complete basic operational computations can set up a learner for vocational opportunities and independent living (Newman et al., 2009). For learners in special education environments, these skills often do not come easy (Browder & Spooner, 2006, 2011). According to the National Assessment of Educational Progress (NEAP) testing, scores for mathematics for learners with disabilities have fallen each administration since peaking in 2007 for third-grade learners and 2011 for eighth-grade learners (U.S. Department of Education, 2022). Teachers need to carefully plan instruction with the idea that all learners have the ability to learn as long as the most appropriate methods are chosen. Learning the most effective ways to provide that instruction comes during their pre-service teaching programs.

Fluency with basic operations (addition, subtraction, multiplication, and division) is a critical skill learners need to be able to complete more advanced mathematical tasks. Codding et al. (2017) suggested that fluency with basic fact retrieval can be positively related to math performance through high school. They also suggested that learners without that fact fluency struggle to perform the computational tasks required for things like word problems and data analysis. Operations can pose problems for learners who are not fluent in basic facts. However, as a classroom teacher, there are two things you can do to redress the situation. Diagnosis and remediation of learner errors can allow instruction to be individualized to the learner or groups of learners with similar error patterns. Riccomini (2005) found that teachers did not look for error patterns or adjust instruction based on error analysis for subtraction problems, instead focusing on reteaching basic facts. For this reason, discussion of diagnosis and remediation strategies should occur during the pre-service teacher (PST) preparation program as part of mathematics teaching instruction. In mathematics courses for PSTs, diagnosis and remediation need to be considered as important as the pedagogy itself.

Diagnosis refers to identifying the error type the learner is making. An initial grading based on problems completed incorrectly can be utilized, but the additional step of identifying the errors will allow teachers to specifically target problems for the learners (Kubina & Yurick, 2012). If a learner makes many errors on a worksheet, the teacher may not need to examine each incorrect response to identify that error type. These errors will generally follow a predictable pattern, with some slight variance. However, one error type will generally be dominant for a learner and guide the planning for remediation. A minimum of three data points are required to identify a trend (Collins, 2012). However, Browder and Spooner (2011) suggested six data points since using the intersection method for drawing a trend line requires six points, and the trend is an important indicator of learning. If the error types are consistent across the first few problems, there is a high likelihood that the issue has been identified, and the teacher can plan for remediation without assessing the remaining missed problems.

Remediation is the process of applying an intervention that is aimed at correcting errors (Merriam-Webster, 2022). There are three main types of errors that learners make (Hudson & Miller, 2006; Kubina & Yurick, 2012; Stein et al., 2018). Although the names of these error types vary slightly, they have common descriptions. Hudson and Miller (2006) refer to them as factual, procedural, and conceptual errors. Kubina and Yurick (2012) and Stein et al. (2018) call them fact, component-skill, or strategy errors. These three error types can provide the teacher with information about where the learner falls in relation to the current lesson. Because these error types increase in complexity, deciphering the error type for the learner will give the teacher a starting point for remediation of the skill, reducing the amount of time the learner will practice errors and increasing the fluency with which the learner completes the skill. As previously mentioned, these error types will present differently and be consistent as errors on worksheets.

Evidence-Based Practices and High-Leverage Practices

Education law requires the use of evidence-based practices (EBPs) in the classroom. According to the Every Student Succeeds Act, teachers should look for practices that meet one of the two highest levels of evidence (Every Student Succeeds Act, 2015). In addition, research has identified certain practices, called high-leverage practices (HLPs), as particularly important in the delivery of quality instruction (McLeskey et al., 2017). Twelve of 22 HLPs relate to instruction. The more of these practices that are taught to PSTs who bring them to the classroom, the higher the quality of the instruction learners will receive. Instructors of PSTs should include these in their course design.

PST Preparation

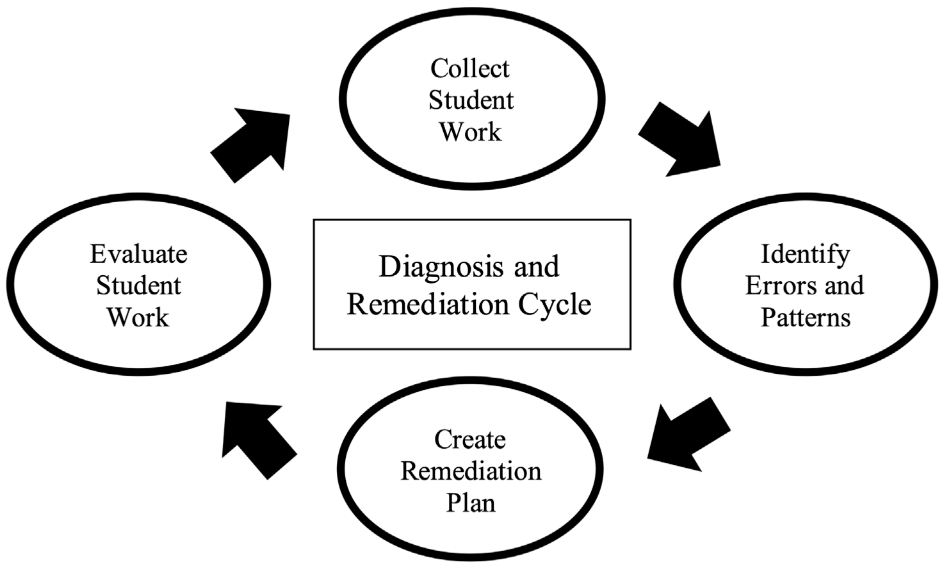
During teacher preparation programs, PSTs are required to take different types of mathematics courses to help them develop a basic understanding of the material they will need when they transition to the classroom. Not all PSTs enter their teaching programs with the same level of teaching self-efficacy. Many PSTs have had poor experiences with mathematics in their academic careers, leading to negative attitudes toward mathematics and the potential for modeling anxiety in their instruction (Olson & Stoehr, 2019). Mathematics anxiety can lead to PSTs opting for teaching grades with easier mathematics or focusing on areas without mathematics included. However, according to Aksu and Kul (2019), a survey of over 400 PSTs found that those with higher levels of pedagogical content knowledge were less likely to experience anxiety and felt higher levels of teaching efficacy. This suggests that providing PSTs with coursework aimed at addressing their self-efficacy and anxiety can yield better instruction when they enter the classroom. This improved self-efficacy can have an important impact on learners down the road.

In their longitudinal study of 113 children, Vukovic et al. (2013) suggested that effective instruction for learners may need to include aspects of explicit instruction, review and practice, and connection to provide relevance. The relevance provided by teaching the material in a way that connects with the learner's life connects the concepts with familiar things and allows for greater understanding (Herron et al., 2009). Instruction designed to build skills rather than be regurgitated on a test will be more beneficial for the learner. This is important for inclusion courses preparing PSTs for mathematics instruction.

ERROR ANALYSIS

In determining why errors are occurring, Hudson and Miller (2006) suggested looking first at whether the learner is making errors due to carelessness or not knowing the procedures. This suggests that teachers should pay particular attention to the presentation of the lesson and how learners are engaged with the material. Carelessness can come from poor-quality instruction. Kenny (1980) identified several components of instruction common to a high-quality instructional format, including pacing, choral responding, and corrective feedback. For some learners, slower pacing can lead to inattention. Hudson and Miller posited this inattention can lead to a lack of understanding during instruction or carelessness in completing the work. After ruling out simple carelessness, teachers can then focus on issues related

FIGURE 1: Workflow Diagram of Diagnosis and Remediation Process



Note: Created based on Witzel, B.S., Riccomini, P.J., & Herlong, M. L. (2012). *Building number sense through the common core.* Corwin Press.

to procedural knowledge.

As mentioned previously, diagnosis and remediation are important steps in the learning process. Witzel et al. (2013) described four distinct steps a teacher should follow to provide the best outcomes for their learners, as described in Figure 1. The first step is to collect some sort of permanent product from the learner. This most often looks like worksheets completed independently. As with any type of data analysis, examining only one instance is not enough to determine patterns. Several instances of the learner's work need to be compared. Teaching PSTs the importance of data-based decision-making during class can reinforce the point. Modeling, an EBP, can be helpful in this regard. As an instructor of PSTs, you will be collecting some permanent product from your learners based on the assignments in your class. You can model data-based decision-making by describing how grades are calculated and how the errors are used to drive future content.

< Insert Figure 1 here >

The second step is to identify errors in the work (Witzel et al., 2013). Identifying error patterns is important to help guide the instruction to reach learners who may be struggling. When evaluating learner work, teachers should first determine whether the learner has identified the correct response. The number of problems answered correctly is not the only metric used to determine learning. Because incorrect procedures can still result in correct responses, counting the number of incorrectly answered problems on a worksheet can tell the teacher only how successfully the learner completed the work. To evaluate the level of understanding of those mathematical concepts, the teacher needs to examine each incorrect response to identify the error type the learner is exhibiting (Forbringer & Fuchs, 2014). Howell et al. (1993) suggested that having the learner demonstrate and explain their work can be a helpful additional step in identifying the cause of errors. This can guide the additional instruction the learner might need moving forward. For an already overworked teacher, this sounds like an additional burden. However, if done correctly, it can reduce the overall teaching load by focusing instruction where needed to improve correct responding and concept acquisition.

When working with PSTs in the classroom, there are a few ways to practice this step. First, the PSTs can use existing permanent products. These can be obtained from local school classrooms if access is permitted. They can also be generated from the computer. This exercise could be treated like a lesson, with some practice before a graded quiz. Another method, using technology, could be creating a simple Kahoot! (https://getkahoot.com/) for the PSTs to complete in class (Wang & Tahir, 2020). Kahoot! (2023) is a game-based application that requires participants to use a handheld device or computer to respond to questions. The instructor can create problems with different error types and give the PSTs time to review each problem and decide on the error type, choosing an option provided. PST responses are anonymous individually, but Kahoot! does show correct and incorrect responses. This allows the instructor to provide immediate corrective feedback without calling attention to anyone specifically (Plump & LaRosa, 2017).

However, these recommendations do not allow for Howell et al.'s (1993) suggestion to include the learner in the analysis process. They also recommend looking for exceptions to the common error patterns. These exceptions can be things like getting the correct response despite not following the typical algorithm for solving the problem. If the teacher only looks at the answer, they will miss the fact the learner did not completely understand how to complete the problem.

Once error patterns have been identified, teachers need to create plans to target instruction for that learner, focusing on addressing the errors (Witzel et al., 2013). This third step in the process is critical because teachers will identify potential missing prerequisite skills that must be explicitly taught. When these skills are identified, and instruction is focused on improving them, the learner

FIGURE 2: Potential error points in two mathematical operations

$$\begin{array}{rrr} & 1 & \\ & 4 & 2 \\ + & 3 & 9 \\ \hline & 8 & 1 \end{array}$$

1. Add the two numerals in the ones column
2. Place the ones digit from the result below the line under the ones column
3. Place the tens digit from the result above the top numeral in the tens column
4. Add two numerals in the tens column together
5. Add the additional numeral above the tens column to the result
6. Place the result below the line under the tens column

$$\begin{array}{rrr} & 1 & \\ & 2 & 5 \\ \text{x} & 1 & 3 \\ \hline {}_{1} & 7 & 5 \\ 2 & 5 & 0 \\ \hline 3 & 2 & 5 \end{array}$$

1. Multiply the two numerals in the ones column
2. Place the ones digit from the result below the line under the ones column
3. Place the tens digit from the result above the top numeral in the tens column
4. Multiply top numeral in the tens column and the lower numeral in the ones column
5. Add the numeral above the tens column to the result (if applicable)
6. Place the new result below the line with the ones digit in the result under the tens column and the tens digit in the result next to it (if applicable)
7. Add a placeholder 0 under the ones column.
8. Multiply the lower numeral in the tens column and the upper numeral in the ones column
9. Place the ones digit from the result under the tens digit column
10. Place the tens digit from the result above the tens column above the line
11. Multiply the lower numeral in the tens column and the upper numeral in the tens column
12. Add the numeral above the tens column to the result (if applicable)
13. Place the new result
14. Add the numerals in the ones column between the lines
15. Place the ones digit from the result under the lower line
16. Place any tens digit from the result above the top numeral in the tens column between the lines
17. Add all numerals in the tens column
18. Place the ones digit from the result under the tens column
19. Place any tens digit from the result above the top numeral in the hundreds column between the lines
20. Add the numerals in the hundreds column
21. Place the result under the hundreds column below the lower line

is more likely to be successful with that process in the future. This step is critical for moving the learners forward. Assignments in the PST mathematics class may focus on lesson planning. Referring to previous lessons to reteach missing skills or provide additional practice to weaker skills, the PSTs may design more comprehensive lesson plans moving forward.

Finally, Witzel et al. (2013) recommended that teachers continue to monitor learner performance to ensure the additional instruction has the intended effect. This creates a circular workflow (see Figure 1) that guides the teacher through a continual process of evaluating the teaching process used for that lesson. Although the key step in the process is identifying the errors the learner is making, it is just as important for the teacher to create an appropriate way to teach the skill that may be leading to the error. In a PST program, teaching this cycle should be included with each topic taught so the PSTs gain experience completing the cycle with each type of mathematical concept.

A formal task analysis, an EBP, can be useful in teaching and be presented to learners in forms like graphic organizers. Task analysis is also a HLP that is similar to systematically designing instruction toward a specific learning goal or scaffolded supports (McLeskey et al., 2017). It can also be just as helpful for teachers in identifying the errors learners make on their formative assessments. Task analysis is simply breaking a multi-step job into its most simple components, the sequential combination of which completes the task (Cooper et al., 2020). Math textbooks may provide suggested task analyses, or they can be situation-specific. By examining the individual steps required to complete the problem, teachers may be able to find the patterns in learners' responses that lead to errors. Additionally, it can provide guidance on what specific skills would need to be taught to remediate the errors. Figure 2 shows an example of the potential number of responses a learner must make to complete two different mathematical operations. Each step is a point at which the learner could possibly make an error.

The idea of diagnosis and remediation through task analysis stems from error prediction theory in consumer ergonomics (Stanton & Baber, 2005). Knowing the points at which errors may be made can allow for more effective, directed teaching. In the addition problem, there are seven possible points at which the learner can make an error. The task requires several prerequisite skills, including vertical addition, addition with carrying, and knowledge of place value. Errors in any one of the steps can result in an incorrect final calculation. For the multiplication problem, the learner can make an error in any one of 21 different spots. Because the operation requires knowledge and execution of both multiplication and addition, there is a greater likelihood of an error during initial teaching. Again, several prerequisite skills are required to successfully complete the problem. Learners must be able to distinguish between multiplication and addition parts of the algorithm to complete the problem. Additionally, they need to know the same general skills required for addition problems. It is important for teachers to figure out why the errors are occurring.

Error Types

Solving mathematical problems is best done by following the prescribed steps in a problem-solving strategy specific to the operation. Learners may commit errors at any point in the process. As mentioned previously, there are three main error types learners can make in mathematics operations (Hudson & Miller, 2006; Kubina & Yurick, 2012; Stein et al., 2018). Each has unique characteristics, but they can often be combined to cause errors. This can mean that diagnosing the errors becomes more important because the teacher will need to develop an appropriate remediation plan for that learner.

Basic Fact Errors

Basic fact errors are just that. Learners making fact errors have not mastered basic math facts related to the four operations: addition, subtraction, multiplication, and division. The earlier learners become fluent with these facts, the more successful they will be with more complex mathematical concepts in the future (Gersten & Chard, 1999). *Fluency* is the ability to recall information quickly and correctly (Baroody, 2011). In addition, fluency exercises can be added to Tier 2 and Tier 3 interventions in a RtI/MTSS framework. Teachers often post basic math facts as tables in their classrooms. These tables can be helpful prompts for learners as they work to acquire fluency with the facts.

Errors with basic facts are generally obvious and follow predictable patterns. Fact errors normally require only additional practice on those math facts; however, this may need to be specifically targeted toward the error patterns discovered (Hudson & Miller, 2006; Stein et al., 2018). As mentioned, teaching PSTs to identify fact errors can be accomplished by embedding sample learner worksheets into lessons.

Teachers can provide additional practice with basic facts with explicit instruction and timed fluency exercises. Many different commercial packages are available for teachers to use; however, this can be accomplished with simple worksheets and a timer. Teachers can use any worksheet, including ones they might already be using for general instruction. Practice can include the entire class or may be targeted to individual learners that require additional time and attention. College instructors can expose PSTs to these types of interventions during class by setting aside time for the PSTs to act as learners and practice using the materials. PSTs should be taught to change worksheets daily to avoid having the learners try to memorize the answers in a specific pattern. Problems may repeat but not in the same order on the sheets. For example, five to 10 worksheets may be rotated so that the same worksheet is not provided on consecutive days.

Practice across the facts should progress in a focused manner. Generally, easier facts should be introduced first, followed by related facts, and then reverse facts (Stein et al., 2018). Because of the inverse relation between addition and subtraction as well as multiplication and division, concurrently introducing both can create some problems for learners. By teaching basic math facts in this particular order, the learner practices sets that are different enough to avoid confusion. Sets with similar responses are introduced later once the learner is more fluent in the initial sets. Discrimination practice can be achieved by including mastered problems with those in acquisition.

Manipulatives are another tool that learners can use to learn basic facts. Both concrete and virtual manipulatives are EBPs that provide a connection to the material. PSTs should be exposed to using manipulatives in their instruction course. Providing them with access to the materials and time to use them in mock lessons can help their understanding of the best ways to implement them while creating engagement for the learners in their class. Objects such as Unifix Cubes or Lego® bricks can demonstrate addition in a tangible, visual way. By dividing the class of PSTs into three groups, the instructor can simulate a classroom experience of stations. One group can work on fact problems using one type of concrete manipulative. Another group can practice fluency using fact family diagrams. A third group can use virtual manipulatives or games for their practice on a Smart™ Board. By combining explicit instruction and real-world applications, teachers can effectively address fact errors in mathematical operations and support students in developing fluency and accuracy in arithmetic.

Component-Skill Errors

The second type of error, component-skill errors, directly reflects how well the learner is absorbing the lesson content. With component-skill errors, the learner attempts to use the strategies taught to complete the problems but misses some elements or performs the steps out of order (Hudson & Miller, 2006; Stein et al., 2018). Although the learner may also commit some fact errors, the error patterns show some confusion about applying the skills accurately. Component-skill errors are not solitary. Learners often make multiple types of errors, but they typically follow a predictable pattern.

In many cases, the learner will attempt to follow the specific instructions provided by the teacher but will often show the same error across multiple problems on a worksheet. This indicates that the learner understood the basics of the lesson but simply implemented it incorrectly. Different remediation methods will depend on the specific error. Typically, remediation involves reteaching a part of the basic algorithm. For example, when a learner makes an error in renaming, a teacher can provide additional practice with place value and placing digits in the correct location, critical skills for multi-digit addition. Because all operations are related, remediating this skill can have long-term implications for the learner. Using prompts like graph paper to assist with demonstrating place value, the teacher can help the learner orient the problems vertically and remediate this problem. Because addition and subtraction are inversely related operations, remediation looks similar for subtraction problems. Practice with renaming during addition instruction can potentially reduce errors with subtraction.

As the problems become more complex, the opportunity to commit component-skill errors increases. With multi-digit multiplication and division problems, learners must also use addition and subtraction and may confuse the skills required to perform those operations. Like addition, remediating multiplication can involve practice with skip counting, which can remediate this error. A multi-digit problem error involving renaming could be the result of the learner placing digits in the wrong column. This error is similar to the type seen in multi-digit addition problems. Practice on place value can correct this error since the learner knows the process for completing the problem.

Division problems present differently than the other three operations because the problems look substantially different. The division sign (÷) is replaced with the more general checkmark-looking sign. This does not mean that learners make different errors. Identifying these errors allows the teacher to efficiently target the errors for remediation while keeping the learner engaged with the current lesson. For example, learners can still make renaming or place value errors. These can be addressed by the same methods used for addition or subtraction. By reviewing each learner's work, a teacher can determine remediation needed for both specific learners as well as the entire class. Multiple learners are likely making the same errors.

Instruction for PSTs on component-skill errors can include practice in task analysis of the operations. Providing a few problems in each of the operations and discussing the results as a class can facilitate dialogue on how to teach the steps. In addition, by introducing manipulatives, PSTs can demonstrate how to complete the steps. This can become part of their lesson plans as an assignment.

Strategy Errors

The final error type that learners could make is strategy errors. Teachers provide strategies, or algorithms, to the learners, giving them a method for solving the problems. These typically consist of a set of steps (task analysis) the learner should follow. With strategy errors, learners show they have not learned the concepts being taught. Strategy errors differ from component-skill errors because the learner does not demonstrate the skills required to complete the strategy. With component-skill errors, the learner can follow the strategy and complete some steps correctly but lacks skills with some of the components. Strategy errors are some of the easiest errors to identify but require the most effort to correct.

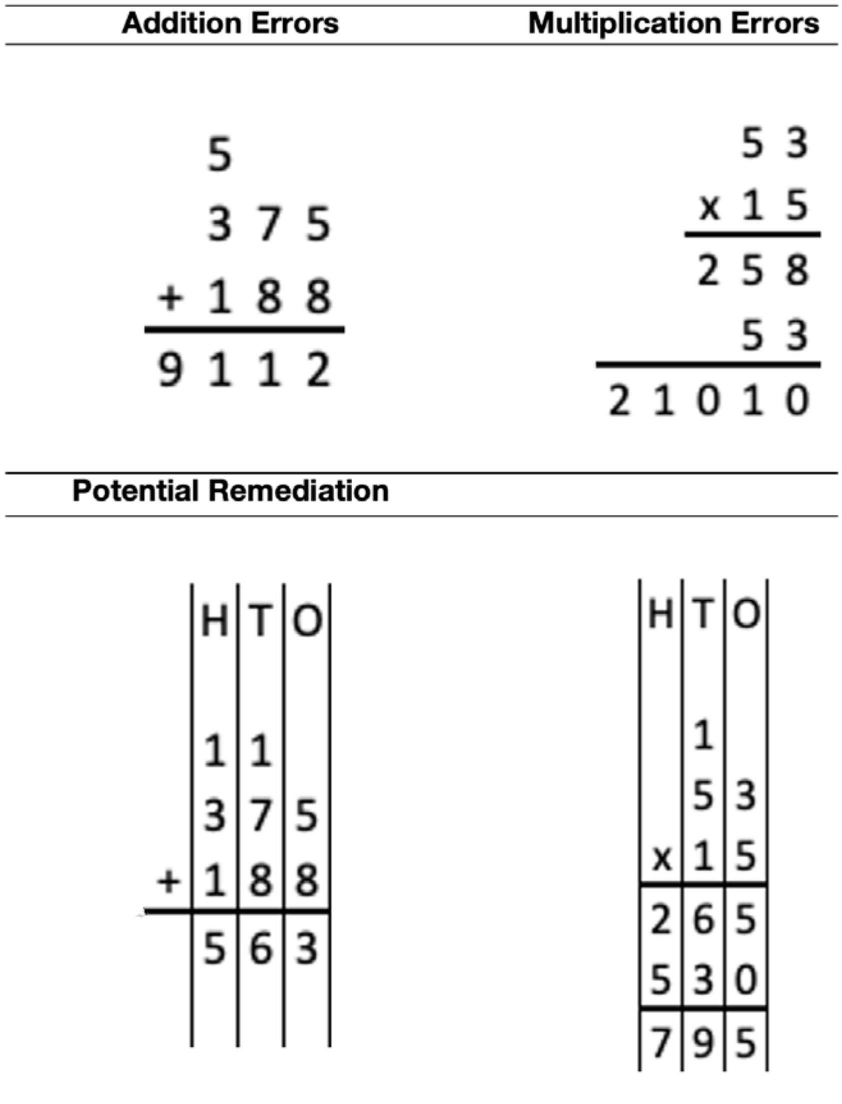
Remediation for strategy errors involves reteaching the concepts from the beginning. It will also likely involve identifying and teaching missing preskills as well. One of the more common strategy errors would be the learner using an addition algorithm to complete subtraction or multiplication problems, which could also be related to a deficiency in fact knowledge. Facilitating this knowledge with prompts, the teacher can label each part of the problem and its place on the fact family diagram for the learner to include either on the worksheet or in a graphic organizer.

Teaching strategy errors in mathematical operations involves identifying common miscalculations and providing targeted instruction to address these errors. A common theme across all error types is the use of explicit instruction on problem-solving strategies. By breaking down complex operations into systematic steps and demonstrating problem-solving techniques, students can develop a deeper understanding of the underlying concepts and learn how to approach mathematical problems strategically. Teachers may also identify missing prerequisite skills that can be expressly taught to assist in strengthening the learner's use of the algorithm.

Utilizing visual aids and both concrete and virtual manipulatives can also help students grasp abstract concepts and visualize problem-solving strategies. For example, graphic organizers, number lines, base-ten blocks, or geometric shapes can provide concrete representations of mathematical operations and aid in comprehension. Additionally, interactive activities and games that simulate real-world scenarios can engage students and encourage them to apply problem-solving strategies in context. Culturally relevant examples can also assist in making connections (Cook et al., 2023; Dueker & Chitiyo, 2023).

Encouraging students to explain their problem-solving strategies to their peers or the teacher and justify their solutions can deepen their understanding and help them identify and correct errors. By combining explicit instruction, visual aids, interactive activities, and collaborative learning opportunities, teachers can effectively teach strategy errors in mathematical operations and support students in becoming more proficient problem solvers. Teachers should think carefully about how to adjust the lesson to better present the concepts.

FIGURE 3: Addition and Multiplication Errors with Potential Remediation



Remediation Examples

The problems in Figure 3 are examples of addition and multiplication problems with common errors learners might make. The addition problem has both fact and component-skill errors. In this example, the learner has correctly tried the addition but erred in the factual computation. Because this error might be due to simple inattention, the teacher should first try to identify a consistent pattern of similar fact errors across multiple problems and worksheets. If that is shown to be the case, the teacher would want to employ a remediation strategy of providing additional practice on single-digit addition fact problems.

In addition, the learner has made errors related to renaming. This caused the written sum to be significantly higher than the correct answer. These could be either component-skill or strategy errors. Since the learner would place the entire answer under the line in a typical single-digit addition problem, the learner demonstrates he understands how to complete that part of the problem when solving the ones column. However, renaming is required for multi-digit addition problems. Also, when adding the tens column, the learner tried renaming but placed the tens value under the tens column and placed the ones value above the hundreds column. The first of these errors might suggest that the learner had not learned the basic algorithm for multi-digit addition. The fact that the learner tried but was unsuccessful at renaming in the tens column shows that the learner did know to rename but failed to do so in the ones column. Remediating this would likely require discussions of place value and its relation to the columns in an addition problem. The second remaining error shows the learner tried to place the numerals in the columns but mixed them up by placing the tens value under the line and the ones value above the hundreds column. This resulted in the incorrect addition of the hundreds column and a vastly larger incorrect sum.

The multiplication problem in Figure 3 also shows multiple errors. When calculating the ones column, the learner added instead of multiplying. This is a basic strategy error left over from single-digit multiplication. Strategy errors typically require reteaching of the algorithm. From there, the learner correctly multiplied the digit in the tens column in the upper numeral with the digit in the ones column from the lower numeral, indicating knowledge of multi-digit multiplication properties. The learner then failed to add a placeholder under the digit in the ones column in the product. This could be a component-skill error if it is not a consistent pattern. If the learner makes this error for each problem, it would be a strategy error. The multiplication of the rest of the problem is correct. With a component-skill error, the teacher can provide practice with prompted worksheets using graph paper or column lines.

However, additional errors occurred during the addition of the products. This demonstrates the relationship between addition and multiplication and the importance of ensuring learners have a strong foundation in addition before beginning multiplication. First, the learner made a fact error in adding eight and three in the ones column and an additional error by not renaming. The learner committed the same renaming error in the tens column. Because these two renaming errors occur in the same problem, they may indicate a strategy error that relates back to addition. The final product, 21,010, is considerably higher than the correct answer of 795.

These two examples demonstrate that learner errors are not limited to one of the three types. Learners may make multiple errors in a single problem. If that error is consistently displayed, the teacher can create appropriate remediation strategies targeting specific deficits.

In the figure, the remediation is focused on the concept of place value. The problems are embedded in a series of columns corresponding to the different place values. Each column is labeled above with a single letter, which might be part of the problem. The vertical lines can guide the learner on the placement of the digits once they have identified the value. As with any prompt introduced to learners, these vertical lines and letters would need to be faded as the learner becomes fluent with the concept. An easy way would be to fade the letters and then the lines. However, consistent practice will be required until the learner achieves a level of understanding.

CONCLUSIONS AND IMPLICATIONS

Teachers in the field must understand the importance of diagnosis and remediation in mathematics. This should begin during PST training. During teacher preparation programs, mathematics instruction classes should focus on content creation and improving the basic understanding of mathematical concepts. However, by including examples of errors learners might make during PST instruction, where they fit into the various error types, and how to address the issues presented, teachers of mathematics instruction can set the stage for their understanding and more frequent use in future classrooms. Instructors of PSTs should embed discussions and practice with diagnosis and remediation into every mathematical concept taught in the class. By providing this additional instruction to PSTs, faculty may reduce mathematics anxiety, increase content knowledge, and provide a way for the PSTs to understand their learners' mathematical understanding (Olson & Stoehr, 2019). Diagnosis and remediation analysis can also be an important way to address struggling learners. Using the information from a formative assessment, teachers can easily target instruction to ameliorate misunderstandings. Ensuring PSTs understand the importance of that relationship is a critical part of their training and should be included in any mathematics instruction course they take.

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ABOUT THE AUTHORS

Scott Dueker, Ph.D.

Dr. Scott Dueker is an assistant professor of Special Education and Applied Behavior Analysis at Ball State University. He is also a Board-Certified Behavior Analyst-Doctoral Level. His interests include mathematics instruction and language development for learners with moderate to severe intellectual and developmental disabilities as well as using technology for instruction in both clinical and classroom settings.

Jill Grande

Jill D. Grande is a graduate student working toward a PhD in special education at Ball State University. She has a BA in psychology and sociology and an MA in Applied Behavior Analysis (ABA). Jill is a Board-Certified Behavior Analyst (BCBA) and holds certificates in autism and supervision. She currently works as a graduate assistant at Ball State University. Her main areas of interest include autism spectrum disorder (ASD), emotional and behavioral disorders (EBD), and behavioral consultation.