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Research Article

Prospective mathematics teachers' mental actions related to debugging in technology supported mathematical modeling

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The study aims to investigate prospective middle school mathematics teachers' mental actions related to debugging, which is one of the computational thinking skills in the modeling process. The study was conducted with a single-case embedded model. The collaborative working group consisted of three prospective mathematics teachers selected by criterion sampling. The data were collected from the video analysis, screen excerpts, and GeoGebra files explaining the solution process of three prospective mathematics teachers for the designed two mathematical modeling problems (experimental and theoretical). According to the results obtained from the data through content analysis based on the theoretical framework, it was identified that the prospective teachers conducted sub-activities such as recognizing/detecting the error, extracting the error, and correcting the error, which is one of the dimensions of computational thinking in technology-supported mathematical modeling. These skills as the basic steps of interpretation, verification, and revision were developed in the process of technology-supported mathematical modeling. GeoGebra was involved as an important mental trigger in the debugging process. In further studies, computational thinking studies describing all the components in the process of technology-supported mathematical modeling can be conducted, and computational thinking skills can be revealed in the process of mathematical modeling in non-computerized environments.

Keywords: Mathematical modeling; Computational thinking; Debugging; Prospective mathematics teacher

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1. Introduction

In the 21st-century world, which is rapidly changing with the impact of technological developments, the dominant paradigms also show rapid development. One of the main reasons for these rapid changes in education is the rapid change in the knowledge and skills that people will need in the future. Future generations need to be able to make sense of information and process it, distinguish important or unimportant information, and relate this information to daily life. It is stated that skills such as critical thinking, reasoning, verification, analytical thinking and solving complex problems will be more important for individuals in the world of the 21st century (English & Gainsburg, 2016; Gray, 2016; National Council of Teachers of Mathematics [NCTM], 2000; National Research Council [NRC], 2011; Partnership for 21st Century Skills [P21], 2010).

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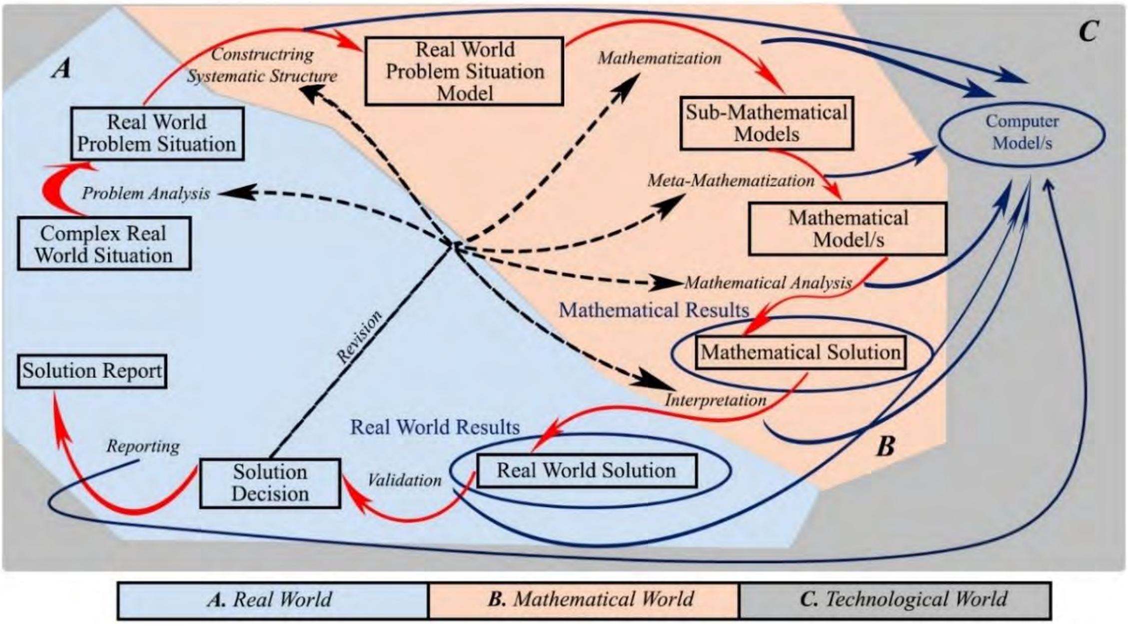
These changes, which have taken place with the significant impact of technology, have affected the works in mathematics education at all levels (Baykul, 2012). Through the skills needed by individuals in the 21st century and a greater and more effective integration of technology into the learning process, it can be pointed out that technology-supported mathematical modeling can play an important role in learning. Mathematical modeling which aimed at achieving a solution by mathematizing non-routine real-life problems is an open-ended problem-solving process (Berry & Houston, 1995; Borromeo Ferri, 2007; Hıdıroğlu, 2012; Peter-Koop, 2004). Recently, studies integrating mathematical modeling and technology stand out in the literature regarding the impact of technology on education (Ang, 2020; Greefrath & Siller, 2017; Greefrath et al., 2018; Hıdıroğlu, 2015; 2021; Wiedemann et al., 2020). Computational thinking is an important skill for the 21st century (Wing, 2006). Moreover, strong relationship between computational thinking and mathematical modeling is emphasized in the literature (Gadanidis et al., 2017; Sunendar et al., 2020; Voskoglou, 2012).

With the development of technology, the solutions to problems involving routine and simple algorithms can be done by computers (Autor et al., 2003). Therefore, it is important to educate individuals with the ability to solve non-routine and complex problems (Hoyles et al., 2010; Kaput et al., 2008). In mathematics education today, mathematical modeling can be an important tool when students develop real-life problem-solving skills that are complex and non-routine. There are many mathematics software (GeoGebra, The Geometer's Sketchpad, TinkerPlots, MatLAB, Cabri, Derive, Code.org, CODAP vb.) that we encounter in technology integration in mathematics education. There are studies showing that such software can be used effectively in the process of mathematical modeling (Flehantov & Ovsiienko, 2019; Hıdıroğlu, 2015; Wiley & Lingefjärd, 2017). However, regardless of mathematical modeling, 21st century students should have another important skill which is computational thinking (Angeli et al., 2016; Wing, 2006). Technology integration in mathematical modeling expands the range of mathematics that students can interact with, creates environments in which they can put forward their ideas for solutions involving higher-order thinking processes, and supports the deep abstraction that makes static situations dynamic (Buteau et al., 2017; Hıdıroğlu, 2015; Ndlovu et al., 2011). The solvers' computational thinking skills in the process have begun to be seen as an important research topic in the study of technology integration during mathematical modeling (Ang, 2020; Barcelos & Silveira, 2012; Barr & Stephenson, 2011; Costa et al., 2017; Gadanidis et al., 2017). Mathematical modeling is seen as an important tool for the development of 21st-century skills, which supports students relating daily life and mathematics, understanding the world better, and learning mathematical concepts (Blum & Borromeo Ferri, 2009; English & Gainsburg, 2016; NCTM, 2000). It is emphasized in the literature that these two fields are intertwined related areas and that integrated studies involving the two components should be carried out (Ang, 2020; English, 2018; Hickmott et al., 2018; Ndlovu et al., 2011).

1.1. Theoretical Framework

There are many process models that explain the mathematical modeling process (Galbraith et al., 2007; Hıdıroğlu, 2015). Among these process models, Hıdıroğlu's (2015) technology-supported mathematical modeling process stands out, which allows technology to be used as a tool, not an end in the process; a tutee and a tool, not a tutor, and which also offers a detailed analysis to examine the mental acts in the process. For this reason, in this study, the process model of Hıdıroğlu's (2015) technology-supported mathematical modeling was used in order to explain their mental actions related to debugging, which is one of the computational thinking skills in the technology-supported mathematical modeling process of prospective middle school mathematics teachers. The process model of Hıdıroğlu's (2015) technology-supported mathematical modeling is composed of the real world, the mathematical world, and the technological world. In addition, there are nine key components and nine basic steps that explain this process. It has 55 cognitive sub-steps describing the basic steps, and 22 metacognitive sub-steps (see Figure 1).

Figure 1
Technology-Supported Mathematical Modeling Process of Hıdıroğlu (2015)



According to this model, (1) in the problem analysis, the problem is read and simplified by explaining it in simple terms, the strategic factors in the problem are considered, the data in the problem are analyzed, and simple assumptions are made by interpreting the content. (2) In constructing the systematic structure, the basic solution strategy is designed, strategic factors and information necessary/unnecessary for the solution are eliminated, strategic factors are grouped, higher-order reasoned assumptions are encountered, simple transitions between real world, technology and the mathematical world are initiated by using experiences. (3) Mathematization involves determining dependent/independent variables/constants/parameters, expressing strategic factors in mathematical symbols, making preliminary estimates of sub-mathematical models [SMMs], using numerical estimates of strategic factors for which data are not available in the problem situation, deriving algebraic/graphical representations of the SMM, and making transitions between technological and mathematical representations. (4) In the meta-mathematization; dependent/independent variables/constants/parameters and SMMs belonging to master mathematical model [MMM] are determined, algebraic/graphic representations of SMMs are used, a technological system is established by revealing the relationships between SMMs, data required for BMM are obtained from SMMs, strategic factors are interpreted and preliminary predictions regarding BMM are made and algebraic/graphic representations of BMM are obtained. (5) In mathematical analysis; graphical or algebraic representations of SMM/MMMs are utilized, calculations are made to reach mathematical solution and results, a technological system that gives a mathematical solution and results is established, and mathematical results are obtained regarding critical points of SMM/MMMs. (6) In the interpretation, the real world equivalents of the mathematical solution/results are determined, the relationship between the real world situation and the mental model and the real world equivalents of the critical points of the MMM is revealed, the real world solution and results are analyzed in terms of the problem situation and the assumptions are examined in line with the real world solution/results. (7) In verification, unexpected situations in real world results are analyzed, real world results are compared with predictions/measurements based on experience, data in the problem, videos and photographs, processes and thoughts are checked by making a decision on the adequacy of real world solution/results. (8) In revising, the source of errors in the solution is identified, the procedures and ideas are reviewed and improved, alternative solution strategies are identified if necessary, and changes are made in higher-order assumptions. (9) In reporting, the important ideas to be written in the report are emphasized, the solution is supported by detailed mathematical expressions and the things to be written in the report are listed. Although the mathematical modeling cycle is gradual, there are often irregular transitions between the steps. The solver may jump backward or forward a few steps from the current step. Technology does not change the basic steps in the process and enriches the sub-steps and makes the basic steps more obvious.

Common Core State Standards for Mathematics (2011) emphasize that mathematical modeling and technology are important strategies for doing mathematics and should be included in all levels of education (K12). Students engage in many mental (cognitive or metacognitive) actions while solving mathematical modeling problems (Maaβ, 2006) and especially at this stage, technology enriches these mental actions and increases the quality of thoughts (Ang, 2020; Hıdıroğlu & Özkan Hıdıroğlu, 2016). Nowadays, technology appears as an important guide and supporter of the human mind in all areas of life. Considering that mathematical modeling aims to explain real world contexts mathematically, the learning environments that emerge with the cooperation of technology and mathematical modeling will be important in developing the competencies that future individuals should have. Corlu et al. (2014) clearly emphasize the impact and importance of these two skills in the learning process by considering mathematical modeling in the mathematics discipline and computational thinking in the technology discipline as basic skills in integrated STEM. In this sense, the combination of computational thinking and mathematical modeling is an important strategy for 'transdisciplinary' understanding, which is the highest dimension of Bybee's (2012) curriculum understanding. Students need to develop computational thinking as a part of their mathematical literacy (mainly mathematical modeling) in order to make deeper evaluations of mathematics in a rapidly changing world with new technologies and trends (PISA-2022 Turkey Report). In PISA mathematical literacy proficiency levels, mathematical modeling and computational thinking skills stand out at level 3 and above. Therefore, in order to be successful in international examinations such as PISA, it is necessary to educate students with high mathematical modeling and computational thinking skills. Considering that 39% of students in Turkey (OECD average is 31%) did not reach level 2 in mathematics in PISA 2022, it is imperative that all countries, especially Turkey, adopt a learning approach based on computational thinking and mathematical modeling in mathematics curricula.

There are many studies explaining computational thinking with different dimensions in the literature (Angeli et al., 2016; Barr & Stephenson, 2011; Maharani et al., 2019). Computational thinking skills are handled in a number of ways by different researchers (Özkan Hıdıroğlu & Hıdıroğlu, 2021). In this study, the theoretical framework of Maharani et al. (2019) was taken into consideration, and the mental actions involved in debugging, one of the dimensions of this approach are described due to the fact that it is simple and understandable, its dimensions are more distinguishable, and it is handled from the perspective of mathematics education. Maharani et al. (2019) explain computational thinking with five sub-skills (see Table 1).

Table 1
Computational Thinking Skills (Maharani et al., 2019)

Sub-skills	*Students' activities*
Abstraction	Students can decide whether to use or reject an object, important information can be interpreted to separate it from unused information.
Generalization	Ability to formulate a solution in general form that can be applied to different problems can be interpreted as the use of variables in solving problems.
Decomposition	Ability to break down complex problems into more understandable and easily solvable problems
Algorithmic Thinking	Ability to design a process/action that describes/explains step by step how problems are solved
Debugging	Ability to identify, dispose of, and correct errors.

Debugging, one of the computational thinking skills, is defined as the ability to identify, dispose of and correct errors (Maharani et al., 2019). Identifying errors is identifying situations that exist in problem-solving and expressing what they are. Error correction is the correction of these identified errors in order to obtain a more qualified result as a result of the solution. Error elimination is the removal of the things that led to the error from the solution, and a different solution is pursued instead of trying to correct the situation that created the error. While debugging involves identifying and correcting errors in the solution process, verification involves analyzing the effectiveness of the solution (Kurtuluş & Öztürk, 2017). When the studies in the literature on computational thinking skills are examined, it is seen that debugging skill is one of the most common skills (Shute et al., 2017). Debugging skills are as important in mathematics education (Schoenfeld, 1992) as they are in computer science (Bers et al., 2014). In the literature, it is emphasized that problem solvers show weak mental actions in the mathematical modeling process, especially in the verification step (Hıdıroğlu & Bukova Güzel, 2013; Tekin Dede & Yılmaz, 20143). In addition, it is also stated that it is more difficult to correct errors in a program than to detect them and that it becomes easier or harder for students to detect errors according to their experiences (Fitzgerald et. al, 2008; Lewis, 2012; Murphy et al., 2008). According to another study (Gugerty & Olson, 1986), students with no experience have relative difficulty in detecting errors.

Accordingly, the aim of this study is to explain the mental actions of prospective middle school mathematics teachers regarding debugging, one of the computational thinking skills, in the process of technology-supported mathematical modeling. The problem statement for this purpose is "What are the mental actions of prospective middle school mathematics teachers regarding debugging, which is one of the computational thinking skills in the process of technology-supported mathematical modeling?"

2. Methods

2.1. Research Design

This study was conducted with a case study, one of the qualitative research methods. A case study is a research method that aims to bring detailed and in-depth information about one or more existing situations and processes (Fraenkel et al., 2011). This study focuses on one (debugging skill) of the multiple sub-units (five dimensions of computational thinking) that explain a concept (computational thinking) within a process (technology-supported mathematical modeling) (Yin, 2003). This occurs in nested case studies. In addition, in this study, since the solution processes of different prospective middle school mathematics teachers for two different mathematical modeling problems were addressed, multiple cases were encountered. Considering Yin's (2003) classification of case studies, the model of this study is defined as a single-case embedded model. Single-case embedded models aim to explain more than one sub-unit within a single case and perform in-depth analysis with more than one data source (Yin, 2003). The unit of analysis of this case study is the mental actions of prospective middle school mathematics teachers regarding debugging, which is one of the computational thinking skills in the process of technology-supported mathematical modeling.

2.2. Participants

This study was conducted with a case study, one of the qualitative research methods. A case study is a research method that aims to bring detailed and in-depth information about one or more existing situations and processes (Fraenkel et al., 2011). This study focuses on one (debugging skill) of the multiple sub-units (five dimensions of computational thinking) that explain a concept (computational thinking) within a process (technology-supported mathematical modeling) (Yin, 2003). This occurs in nested case studies. In addition, in this study, since the solution processes of different prospective middle school mathematics teachers for two different mathematical modeling problems were addressed, multiple cases were encountered. Considering Yin's (2003) classification of case studies, the model of this study is defined as a single-case embedded model. The unit of analysis of this case study is the mental actions of prospective middle school mathematics teachers regarding debugging, which is one of the computational thinking skills in the process of technology-supported mathematical modeling.

Table 2
Demographic Characteristics of the Participants

Pseudo-names	*Algorithm and Programming Success Grade*	*Grade Point Average*	*Age*	*Gender*
Ayşe	A1	3.73	19	Female
Adile	A1	3.52	19	Female
Ali	A1	3.47	19	Male

2.3. Data Collection Tools

The data in the study consisted of video transcripts, written response papers, GeoGebra files, and the researcher's observation notes about three prospective middle school mathematics teachers as to how they found technology-supported solutions to two mathematical modeling problems. The data collection techniques used in the study were interview, observation, and document analysis. Two mathematical modeling problems designed by the researcher were used as data collection tools in the study (Aktaş, 2022). In order to obtain a rich data set, especially in the solution process, Berry and Houston's (1995) classification of mathematical modeling problems was taken into consideration in the designed problems, and care was taken to ensure that the problems met the characteristics of theoretical (Ferris Wheel Problem) and experimental (200 Meter Run Records Problem) mathematical modeling problems (see Appendix 1). We took extra care to ensure that the mathematical modeling problems were appropriate for prospective middle school mathematics teachers' prior learning about mathematics and technology, that they were comprehensible and interesting, that they supported the use of technology, and that they were in accordance with the characteristics of mathematical modeling problems found in the literature (Baki, 2002; Berry & Houston, 1995; Blum, 2002; Borromeo Ferri, 2007; English, 2003; Schoenfeld, 1994). For these two mathematical modeling problems, expert opinions were obtained from six researchers who had studies on mathematical modeling in mathematics education, and later on, the problems were revised. Afterwards, the pilot study of the problems was conducted online with a prospective mathematics teacher, and necessary revisions were made for both the data collection process and the effectiveness of the problems.

2.4. Data Collection Process

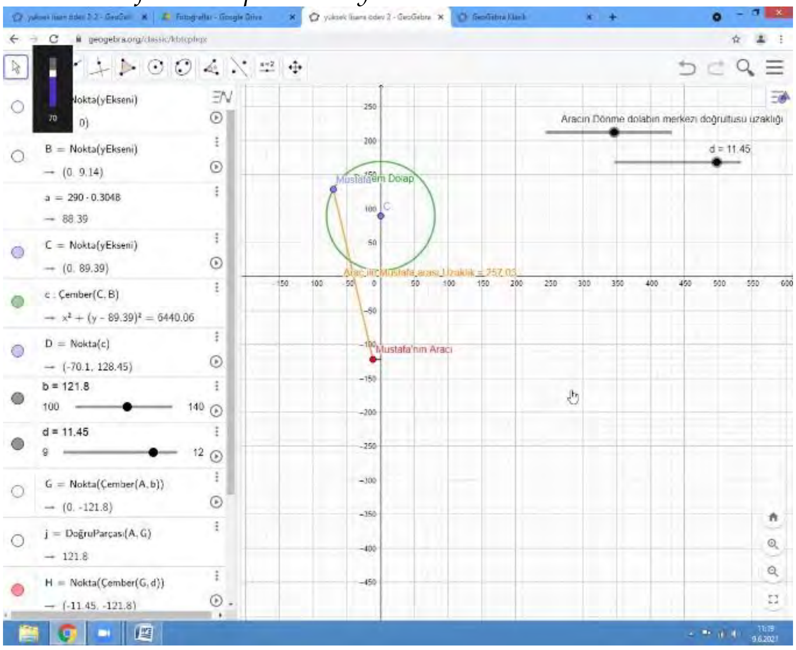
The preparation and data collection process of the study proceeded in the following order:

1) Within the scope of the Algorithm and Programming course, which is a compulsory course in Pamukkale University, Elementary Mathematics Teaching undergraduate program in the 2020-2021 academic year, students were given education on the mathematical modeling process, technology-supported mathematical modeling problems and solutions. Within the scope of the course, students worked with eight mathematical modeling problems in the technology classroom.
2) As well as taking into consideration the opinion of the instructor of the course, three students who had passed the Algorithm and Programming course with grade A1 (the highest grade) and who were willing to participate in the study were included in the collaborative working group.
3) While creating the problems, mathematical modeling problems in the literature were examined, and it was ensured that the mathematical modeling problems designed were suitable for the basic features of mathematical modeling problems.
4) When two mathematical modeling problems were designed, expert opinions were taken, and necessary corrections were made. As a result of the corrections, it was decided to use these two

mathematical modeling problems as the data collection tool after the expert opinion was taken again and the approval was obtained (see Appendix 1).

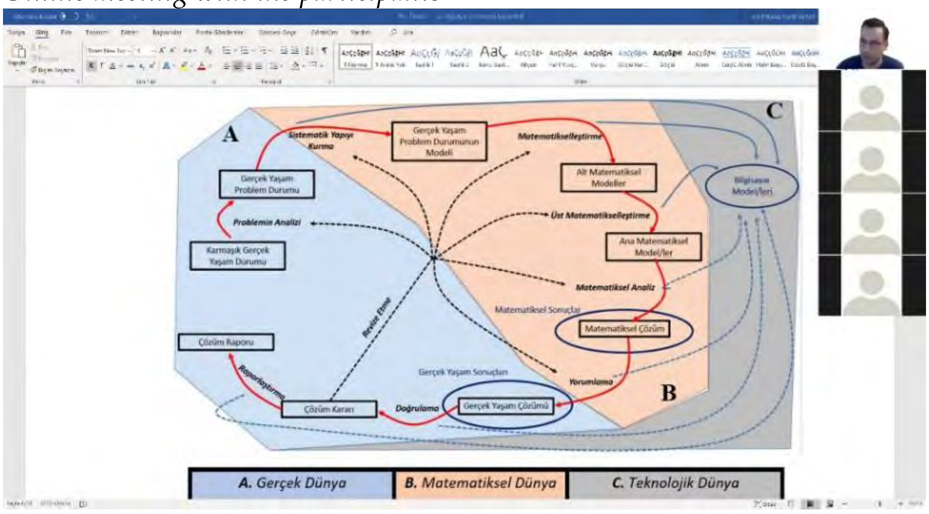
5) The pilot study of the data collection tools was conducted with a prospective teacher. After the pilot study, it was seen that there was no need for any correction, and it was decided to proceed with the actual study (see Figure 2).

Figure 2
Screenshot from the pilot study



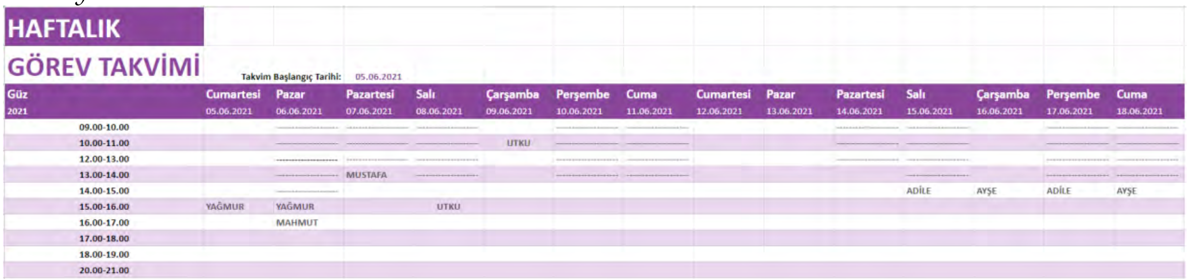
6) An online meeting was held with the collaborative working group, and they were informed about the purpose, scope, and process of the study (see Figure 3).

Figure 3
Online meeting with the participants



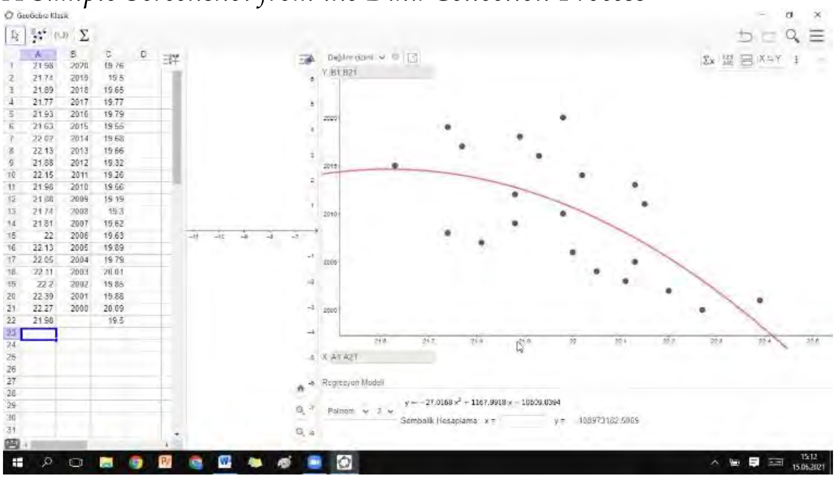
7) Different days and times were set for individual interviews with the collaborative working group (see Figure 4)

Figure 4
Weekly task schedule



8) Individual online interviews were conducted with prospective teachers at designated times. The online interviews were conducted because the prospective teachers were not in the province where the university was located during the pandemic. The audio recording of the interviews, the screen recording, the photograph of the paper used in the solution process, and the GeoGebra file in which the model was created were collected by the researcher at the end of the interview (see Figure 5). During the interviews, while the prospective teachers were solving the problem, the researcher did not communicate with the prospective teachers except in certain situations. These situations were repeating the sentences that were not heard due to the weak internet connection or interference in the microphone and answering the questions of the prospective teachers about whether they could use some data from the internet source. During the problem-solving process, all the operations performed by the prospective teachers on the screen were instantly observed through screen sharing at the online meeting. The actions of the prospective teachers on paper could not be examined during the process, but only at the end of the process by taking their photographs. However, in order to better examine what the prospective teachers did in the process, they were not allowed to use erasers during the operations on paper.

Figure 5
A Sample Screenshot from the Data Collection Process



9) The audio recordings and screen-sharing records were transcribed verbatim. While transcribing, not only the voices but also the prospective teachers' operations on the screen were transcribed. A total of 870 minutes of video and audio recordings were obtained during the interviews. The time allocated by each prospective teacher to each problem is given in Table 3.

Table 3
Time allocated by the collaborative working group to solve the problems

Collaborative Working Group	*Problems*	*Time Allocated (minute)*
Ayşe	Ferris Wheel	106
	200 m Run Records	96
Adile	Ferris Wheel	67
	200 m Run Records	76
Ali	Ferris Wheel	73
	200 m Run Records	42

During the data collection process, the environment was described in detail to all prospective teachers in the meeting held before the individual interviews. The features that this environment should provide are listed below:

- A quiet, comfortable room for problem-solving, where they can study alone at a table.
- Personal computer with GeoGebra and Zoom applications installed.
- Internet connection for uninterrupted Zoom interview.
- Built-in or external microphone for use during Zoom calls.
- Sufficient paper, pencil, and eraser that they can use during problem-solving if needed.

The limitations of the study: we could not collect the data online due to the global pandemic during the data collection process, we were not able to instantly observe what the prospective teachers had done with paper and pencil during this process, and we were not able to reveal their in-group behaviors since the individual solution process was handled.

2.5. Data Analysis

The data analysis was conducted by the content analysis method based on the theoretical framework put forward by Strauss and Corbin (1990). Content analysis is a method in which some words and word groups in the text are analyzed according to codes identified within the framework of predetermined rules and summarized according to categories (Büyüköztürk et al., 2012). The interviews were transcribed and combined with screen recordings, written response sheets, GeoGebra files and observation notes to ensure synchronization among the data types.

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Table 4
Indicators of computational thinking skills

Computational thinking skills (Themes)	*Indicators of computational thinking skills (Codes)*
Debugging	Identifying/detecting errors
	Disposing of errors
	Correcting errors

While creating the codes and themes in the technology-supported mathematical modeling process, Hıdıroğlu's (2015) theoretical framework of mathematical modeling was taken into consideration. The nine basic steps in this theoretical framework were considered as themes and the sub-steps as sub-themes, and the indicators (codes) belonging to these themes are presented in Table 5 below. In the presentation of the themes, the themes and sub-themes that emerged in the study are presented.

Table 1
Indicators Related to Mathematical Modeling

Mathematical Modeling Steps (Themes) and Mathematical Modeling Sub-steps (Sub-themes)	*Indicators of Mathematical Modeling Steps (Codes)*
Interpretation	
Analyzing assumptions in line with real world solutions and results	Expressing the effects of the basic assumptions taken at the beginning on real world life solutions/results Expressing what happens if the basic assumptions taken at the beginning are changed
Validation	
Examining unexpected situations in the results of real world	Examining in which cases the formed master mathematical model [MMM] is inadequate Deciding whether the shortcomings of the formed MMM are sufficient for the solution. Comparison of critical points (such as defined/undefined points, outliers) of the master mathematical model [AMM] with values in real world situations.
Controlling processes, thoughts, and steps	Rethinking to check the thoughts addressed Rethinking to check the actions taken Rethinking to check the steps covered Checking and correcting problems between technological language and mathematical language
Revision	
Identify the source of the error/mistake in the solution	Revisiting the part of the solution where they think there is an error Reviewing what can be improved When you do not reach a satisfactory result
Revisiting the processes and reflections	Revisiting all thoughts and actions in the process when you are not satisfied with the solution Reviewing the entire process when the solution is found to be flawed
Identifying alternative solution strategies	Changing the overall solution strategy when errors are found in thinking and operations Using mathematical and technological knowledge to find MMM in a different way. Choosing an alternative path that can continue the solution instead of using the incorrectly generated best-fit line

As a result of the data analysis, 88% agreement was observed as a result of the inter-coder reliability test (Miles & Huberman, 1994). Discrepancies were discussed by the analysts, and a consensus was reached on a common point in the presentation of the findings.

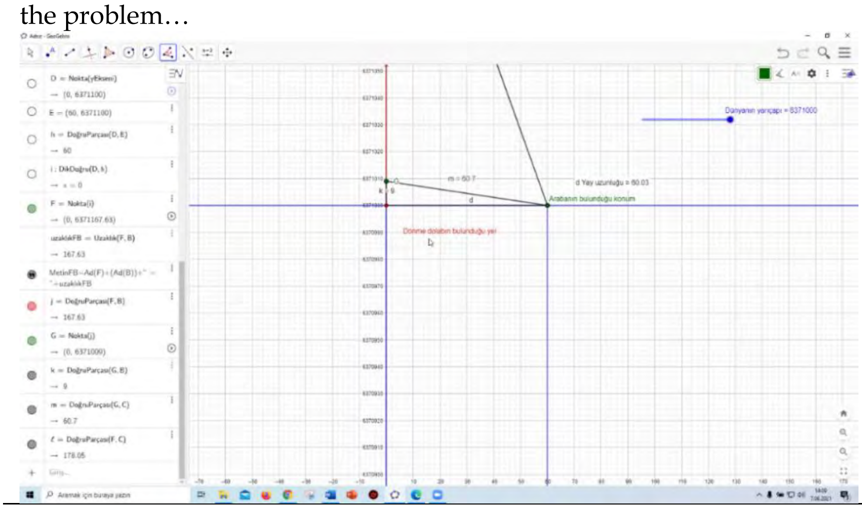
3. Results

This section presents prospective mathematics teachers' mental actions related to the debugging skill of computational thinking in the technology-supported mathematical modeling process.

In the solution of the Ferris Wheel problem, Ali first made the assumption that the Ferris wheel was on the Earth and that the horizontal distance should be neglected in the solution. Towards the end of his solution, he wanted to improve the technological system he created by asking himself the question "What would change if there was the same Ferris wheel on a different planet?" (see Table 6). Ali considered the effect of the assumptions he considered at the beginning on the solution and thought about where his solution would develop if different assumptions were considered. In this case, in the process of technology-supported mathematical modeling, it was observed that Ali exhibited mental actions related to the basic step of interpretation of the sub-step of analyzing the assumptions in line with real world solutions and results.

Table 6
A part of Ali's solution process for the Ferris Wheel problem

Ali: It varies between 60 meters and 178 meters, this is how he sees the distance to Mustafa's car. In this way, it varies between 60 meters and 178 meters. Here I have neglected the horizontal distance of the Ferris wheel, I have taken it vertically. I found the maximum and minimum values, 60 meters and 178 meters. The reason why I handled the radius of the world with a slider was that the values would change when it was done in a place other than this world, so that's why I wanted to handle it with a slider. Because when the radius changes, the distances change, and students can see it like this. It should be handled in this way in solving the problem…



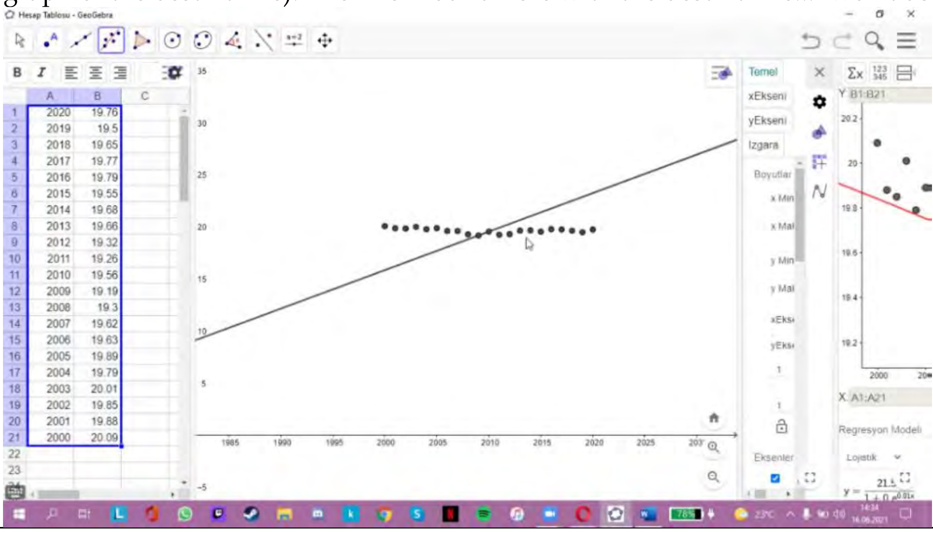
Ali interpreted his mathematical model with its assumptions and limitations and related his mathematical model to real life results. These mental actions have been seen as an indication that he detected an error in this section.

In the solution of the 200 Meter Run problem, Ayşe examined the data in the problem situation and predicted that the solution would be made with the best-fit line. She continued her solution by selecting the "Best Fit Line" tool in the GeoGebra window. However, while constructing the best-fit line, he selected only two points instead of selecting the list of points. She thought that the line she obtained in GeoGebra, and the point list were not compatible (see Table 7). This situation caused Ayşe to think that the mathematical model she created was incomplete or inadequate. The fact that Ayşe realized the problem in the solution showed that she exhibited mental actions related to the sub-step of comparing real world results with problem data in the basic step of verification during the technology-supported mathematical modeling process.

Table 7

A part of Ayşe's solution process for the 200 Meter Run problem

Ayşe: It looks linear at first, but... here, maybe I can use the best fit line for data like this before, but I couldn't. (It selects only two points from the point set and creates a best fit line.) I will do it again (Selects only two points from the point set and creates a best fit line). Why couldn't I do that? (According to Ayşe, the best fit line doesn't look as it should.) Shouldn't I have chosen two points? When you think about it, it also makes sense that it should be around a line. Looking at the dots, I mean. Huh, it doesn't make sense when I look here, either (She compares the linear regression model with the best fit line). I will also do something. I also want to compare the equation I found here (the equation of the regression line) with the thing (the graph of the best fit line). The line I found here with the best fit line...I won't use the best fit line...



The fact that Ayşe thought that the GeoGebra-based mathematical model she created was not compatible with the model in her mind and the regression lines showed that she made an error detection here. As a result of her error detection, Ayşe stated that she would not use the best-fit line in the solution instead of correcting it. In this case, Ayşe exhibited mental action to eliminate the error.

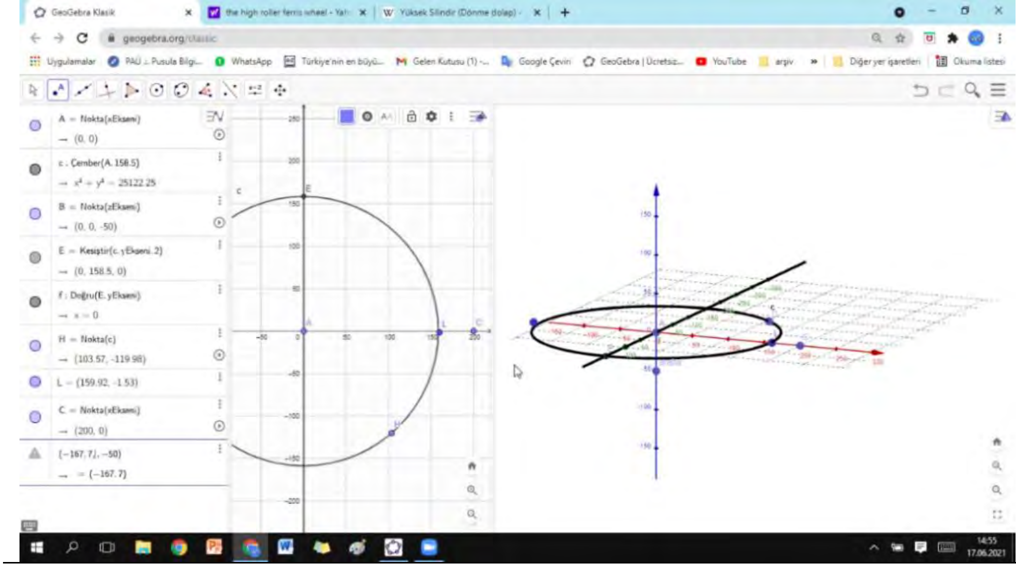
In the solution of the Ferris Wheel problem, Adile defined a three-dimensional point from the algebra input section in GeoGebra to construct a moving point on the circle she created. However, she realized that the point she defined was not on the circle from the 3D graphical window of GeoGebra and made corrections in her solution by checking her operations (see Table 8). Thus, in this section, Adile exhibited mental actions related to the sub-step of controlling the operations, thoughts, and steps in the basic step of verification in the technology-supported mathematical modeling process.

Table 8

Some of Adile's solution process for the Ferris Wheel problem

Adile: This is the x (x-axis in the three-dimensional system). So, if I want a point here, I will write the coordinate of this point in the algebra window, and I already knew that -50. When I look at y, x is around 150, but it has a height, I will say plus 9,2. I just need to find x, z, y, and if I find y, I can draw this point. Now that y is here (pointing to the value axis on the three-dimensional system), huh okay. I think y is also related to x. Ah, y was here (notices the correct position of the y-axis).
(She is writing that (-167,7,-50) in the introduction of GeoGebra).
I want to draw the projection of that 50 at the lower point of that circle, that is, somewhere around there (she says she wants to draw a point in the direction parallel to the perpendicular depth to a point on the top of the circle.) I want to draw a point, but I'm not sure what it would be there. ... That's why I'm thinking. I think it's zero. (She is correcting the point she has already created in the algebra window.) I'll do it by saying zero. No good (She made a mistake in entering the coordinate. She changes the coordinate to (-167,7,.-50)). This is it, right? (She changes it again and makes the coordinate (-167,7,0,-50).) Yes, is that OK? It didn't work. I will delete this (She corrects again and makes the coordinate (-167,0,-50).). Huh, okay, now I've done it.

Table 8 continued



Although Adile used mathematical language correctly in the solution, she had problems in transforming her mathematical expression into the technological language in GeoGebra. Adile's identification and correction processes to look for the source of the error she encountered in her solution showed that she exhibited mental actions related to the debugging skill in computational thinking.

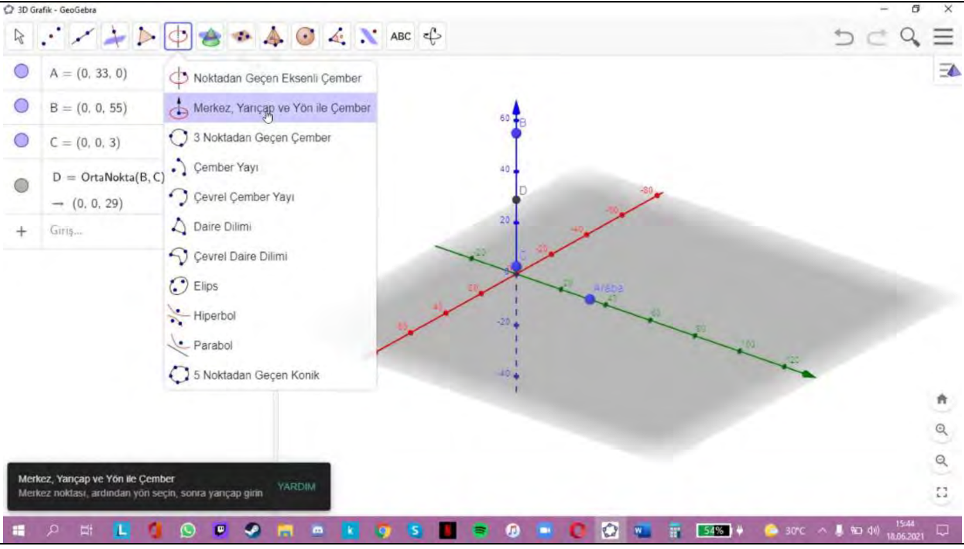
In the solution of the Ferris Wheel problem, Ayşe started to construct a circle in the three-dimensional view of GeoGebra by following the instructions of the software. However, since she chose the wrong axis while applying the instructions, the circle formed in a different plane than she expected. Then, she reviewed her operations, identified her mistake, and constructed the circle as she wanted (see Table 9). In this section, Ayşe's revisiting her operations, identifying her mistake and updating her strategy to obtain the MMM showed that she exhibited mental actions related to the sub-steps of identifying the source of the error/mistake in the solution, revisiting operations and thoughts, and identifying alternative solution strategies from the basic step of revision in the process of technology-supported mathematical modeling.

Table 9

A part of Ayşe's solution process for the Ferris Wheel problem

Ayşe: Between 2 points, then, you draw a circle like this. So, which were the circle tools? And what's the center of the circle? This will be the center of the Ferris wheel. Center, radius. I don't know the length of the radius right now. Which one do I use? I can't use the one that goes through 3 points, can I? I mean, I don't know a point here or there. I remember drawing a circle with the center and a point through it, but I don't know which one right now. What is this? (A circle parallel to my xy-plane has formed.) First choose the axis, then a point on the circle. Did I choose wrong? Why didn't it work? What did I do wrong? Was I supposed to click on B? No, not like that. I mean, that's why this circle is not around the axis right now, right? I don't know the center, the radius, the radius. ...measure it. Good, 26, okay. Center, why is this happening? Why is that? What exactly am I supposed to choose here? I'm going to choose a direction, right? I chose the center. How do I choose the direction? Didn't I choose the right thing? Uh, I mean, it's going to be towards here. How do I choose? Do I choose this? Should I choose the car? I'll try. I don't know. (Ayşe realized that the mistake she made in the rest of her solution was because she was not following the instructions of the GeoGebra tool correctly, and later, she followed the correct instructions and finally got the circle she wanted).

Table 10 continued

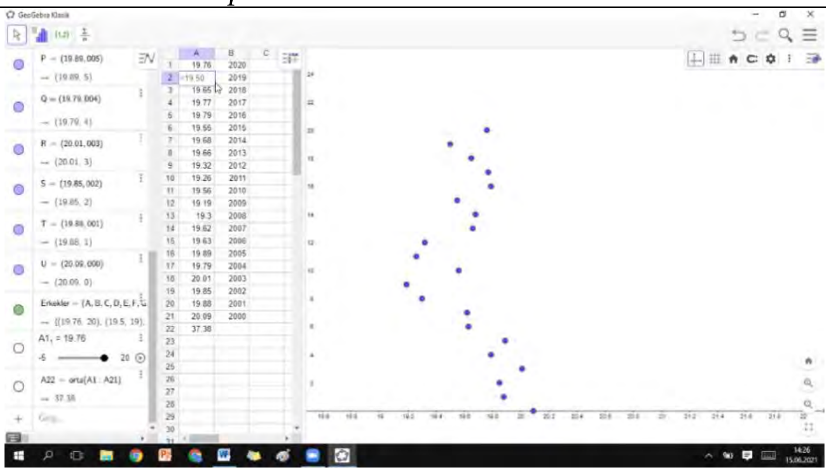


While Ayşe wanted to create a circle in the three-dimensional system in GeoGebra, she made mistakes while creating the circle due to her lack of command of the technological language. Afterwards, she looked at her solution again and tried to understand the cause of the error and corrected her solution. Here, it was seen that Ayşe exhibited mental actions related to the debugging skill in computational thinking.

In the solution of the 200 Meter Run problem, Adile calculated the arithmetic mean of the record times with the help of GeoGebra after entering the years and men's record times into the spreadsheet part of GeoGebra and obtained a numerical value that did not make sense to her. This value is 37.38. Since 37.38 is a much higher number than all the record times obtained, Adile thought that this value was not a reasonable value for the arithmetic mean. When Adile did not reach a satisfactory result in her solution, she checked her solution again, thinking that she had made a mistake, and while examining the data in the spreadsheet in GeoGebra, she realized that she had entered incorrect data in a row and corrected it (see Table 10). In this part of her solution, Adile exhibited mental actions related to the sub-steps of identifying the source of the error/mistake in the solution and reviewing the operations and thoughts in the revision basic step in the technology-supported mathematical modeling process.

Table 11

Part of the solution process of Adile's 200 Meter Run problem

Adile: Something happened here (pointing to block A2. The number value there is different than it should be. 391.76 when it should be 19.50.) 19.50 here has changed. (She corrected the changed A2 block with the correct one.) It looks like I am making a mistake while selecting, that's why this is happening. Let's delete this place (She deletes the average value she found before). Let's select it again and discard it. 19.65 (Average value found). I think the average I found is not enough. But it makes sense.	

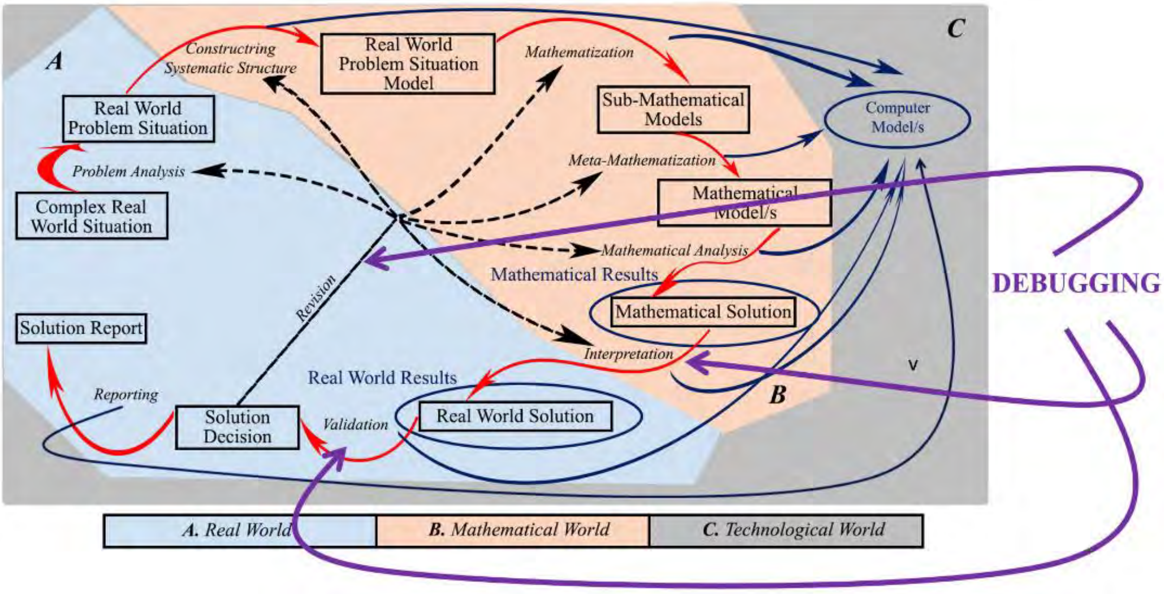
Adile identified the mistake she made in the solution process and corrected it. Thus, it was thought that she exhibited mental actions related to debugging skill in terms of computational thinking.

4. Conclusion, Discussion and Recommendation

In general, mental actions related to debugging skill, one of the computational thinking components of Maharani et al. (2019), were encountered in the basic steps of Hıdıroğlu's (2015) interpretation, verification, and revision of the technology-supported mathematical modeling process. There were no examples of mental actions related to debugging skill in the sub-steps of the basic steps of problem analysis, constructing systematic structure, mathematization, meta-mathematization, mathematical analysis and reporting of the technology-supported mathematical modeling process of middle school prospective mathematics teachers. Middle school prospective mathematics teachers exhibited mental actions for debugging in the interpretation, verification, and revision steps of the technology-supported mathematical modeling process (see Figure 6).

Figure 6

Basic steps in the technology-supported mathematical modeling process in which debugging skill is revealed as a mental action



In these steps, error detection, error correction, and error elimination sub-actions including looking for different ways in cases where they could not correct the error were encountered. It can be said that GeoGebra played an active role in helping students realize their mistakes and think about their thoughts during the mathematical modeling process. In summary, it can be stated that the emergence of computational thinking skills in the technology-supported mathematical modeling process was consistent with the views stated in the literature (Kallia et al., 2021; Selby & Woollard, 2013; Sunendar et al., 2020).

There were no examples of mental actions related to debugging skill in the sub-steps of the basic steps of debugging skill, which are problem analysis, constructing systematic structure, mathematization, meta-mathematization, mathematical analysis and reporting. Rabbitt (1997) emphasizes that the errors made are noticed by individuals and this situation shows that individuals monitor their thoughts instantaneously. This suggests the possibility that debugging may also be encountered in other steps of mathematical modeling.

The mental actions related to the debugging skill that emerged in the technology-supported mathematical modeling process occurred not only at the end of the process but also throughout the process (with irregular transitions). Similarly, Gehring et al. (1993) found that differences occurred in the regions of the brain related to metacognition with error recognition and emphasized the relationship between the two. According to Hıdıroğlu (2015), Maaß (2006), and Pugalee (2001), metacognitive actions in the mathematical modeling process can act as stimuli that provide irregular transitions between basic steps (Fernandez et al., 1994; Hıdıroğlu, 2015; Maaß, 2006; Stillman et al., 2007) or in an organizing and structuring role (Hıdıroğlu, 2015; Lesh & Doerr, 2003). The fact that there were irregular transitions between the basic steps in the technology-supported mathematical modeling process in cases where there were debugging actions in the study is in line with other ideas about the relationship between debugging and metacognition.

After the errors noticed in the solutions were corrected, the solution was continued by returning to the previous basic steps. In the technology-supported mathematical modeling process, the debugging skill was revealed less intensively than the researchers expected. This does not coincide with the suggestion of Shute et al. (2017) that debugging is one of the most intensively used skills in computational thinking. The reasons for this difference may be student competencies and the nature of mathematical modeling problems. In order to better understand this situation, studies can be conducted with teachers, prospective teachers, and students with different mathematical modeling competencies. In addition, similar studies can be conducted with other mathematical modeling problems suitable for Berry and Houston’s (1995) mathematical modeling types.

Errors made during the technology-supported mathematical modeling process were focused under two headings. These are: (1) errors arising from simple assumptions made at the beginning of the process, and (2) errors arising from the GeoGebra software language (notation). In order to detect errors, the prospective mathematics teachers went to the place where they thought there might be a mistake, checked their operations and thoughts, and made the necessary corrections. In cases where the errors were caused by the software, the prospective mathematics teachers made the necessary corrections in their technology-based notations. They corrected their initial assumptions and created alternative high-level assumptions. In some cases, prospective mathematics teachers realized that there was an error, but they could not detect it and continued their solutions with a different solution strategy.

In the solution of mathematical modeling problems, prospective mathematics teachers experienced the process with the role of the computer as a learner (tutee) from Taylor's (1980) perspective. As stated by Taylor (1980), in the technology-supported mathematical modeling process, the solvers played the role of both software developers and mathematicians. The errors made by the prospective mathematics teachers in solving the same problems were different from each other (Hıdıroğlu & Bukova-Güzel, 2014). It can be stated that the errors arising from GeoGebra are related to the software knowledge of the solvers. It is important for prospective mathematics teachers to relate mathematics and computer language effectively and this is related to the role of technology as a semiotic mediator (Bartolini & Mariotti, 2008). In this study, GeoGebra can be considered to be an effective semiotic mediator in this process as students transfer their mathematical thoughts to GeoGebra and the feedback they receive from the software supports them to change and improve their thoughts. Prospective mathematics teachers made many transitions between technological language and mathematical language in the process of technology-supported mathematical modeling. Sometimes they had no difficulty during these transitions, while sometimes they experienced difficulties. It can be stated that technology creates mental environments that will eliminate the difficulties that may be experienced in these processes and reduces the mental load by making it easier to recognize the mistakes made. This is in line with the amplifier and reorganizer roles of technology in the process emphasized by Pea (1987). GeoGebra's ability to shape students' solution processes and provide opportunities to detect errors is related to its reorganizing role. The dynamic models obtained with technology could be organized with different assumptions and transformed into different problem situations. In this sense, it can be said that technology increases the variety of problems that can be overcome (Ball & Stacey, 2005). This is related to the elevating role of GeoGebra from Pea's (1987) perspective. According to Drijvers et al. (2005), GeoGebra can also be seen as an artifact in this study. It can be claimed that GeoGebra has turned into an instrument with the use of prospective teachers in the problem-solving process. The interaction between prospective mathematics teachers' experiences in GeoGebra application and their mathematical knowledge and GeoGebra revealed different solution strategies and caused GeoGebra to take place as a different instrument in each prospective teacher. In future studies, the role of technology in the process will be understood more deeply if the instrumental construction theory is included in the study in addition to the examination of computational thinking skills in the process of technology-supported mathematical modeling.

Debugging skills emerged only in cases where errors were detected or corrected and in mental actions where the consistency of real world situations and mathematical results were examined. However, it can be stated as a limitation that the actions of making the solution better or presenting a different solution can be considered outside of debugging although the solution is correct. In this sense, Csizmadia et al. (2015) considered the evaluation dimension as a component of computational thinking instead of debugging. In this sense, studies conducted with different computational thinking frameworks in technology-supported mathematical modeling processes gain importance.

Since debugging skill is an important metacognitive skill, a similar study can be conducted by considering metacognition-based theoretical frameworks. Studies can be conducted to reveal the relationship between debugging skill and other computational thinking skills. With a theoretical framework based on computational thinking in which the debugging skill is expressed more comprehensively, that is, even if the solution is correct, including the improvements made in the solution, problem solvers' debugging skills in mathematical modeling processes in technology-supported or unplugged environments can be examined. The data collection process of this study was conducted online-individually. Considering the limitations of the study, it is recommended that further studies addressing different theoretical frameworks be conducted with face-to-face interviews with collaborative working groups.

5. Limitations and Future Implications

This study elucidates the 'debugging' actions of three prospective mathematics teachers, which can be defined as a sub-dimension of computational thinking in the context of technology-supported mathematical modeling. In this sense, the study contributes to the existing literature on the effective collaboration of mathematical modeling and computational thinking by offering a different perspective to researchers. As a result, the study argues that mathematical modeling and computational thinking are important skills that trigger each other. The study exemplifies how 'debugging', one of the subcomponents of computational thinking, is revealed in technology-supported mathematical modeling and how it triggers cognitive/metacognitive actions. The limitations of this study are that it deals with one dimension of computational thinking (debugging), it was conducted with three prospective mathematics teachers, two mathematical modeling problems were used, and it was based on GeoGebra software. Considering that the effect of technology on the mathematical modeling process needs to be explained in detail and that such research is not sufficient (Ang, 2020), it can be said that this study will be one of the pioneering sources in establishing the competencies framework based on the computational modeling (Hıdıroğlu, 2022) approach.

For future studies, there is a need for studies describing possible student learning paths/roadmaps in technology-supported (computational thinking-centered) mathematical modeling at different grade levels. Competency frameworks and rubrics should be developed to describe the expectations of students in mathematical modeling and computational thinking at different grade levels and to categorize their mental actions into levels. This study is expected to contribute to the development of interdisciplinary learning environments such as future mathematics or integrated STEM with the sample integration strategy it presents. In future studies, the existing literature on this topic can be expanded by addressing mathematical modeling with a variety of problems and theoretical frameworks. Further research could focus on the differences between different problem types in mathematical modeling, GeoGebra 3D-based applications and different software, including Scratch, CODE.org and CODAP. Further research could be conducted to enhance comprehension of technology integration in mathematical modeling by examining diverse integration theories, including instrumental genesis theory, microworld of Papert (1996), and semiotic mediation theory.

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Appendix 1. Modeling tasks

1) Ferris Wheel Problem

Mustafa is on vacation in Las Vegas and goes to "The High Roller" to ride the biggest Ferris wheel in the world. Mustafa parks his car at the place indicated by the red dot in Figure 1 and gets on the Ferris wheel (The High Roller). The location where Mustafa parked his car is also shown on Google Maps in Figure 2. What can you say about the distance to Mustafa's car when the Ferris wheel is in motion? Express your thoughts mathematically.

Figure 1. *Mustafa's Car and The High Roller*

Figure 2. *Google Maps Image*

(For photos and video films: *https://drive.google.com/drive/folders/1X9qu04LrnR27dcAKGnd57A2jVhIHBxgc?usp=sharing*)

2) 200 Meter Run Records Problem

Table 1
Best 200 Meters Male and Female Results by Year

Year	*Men*	*Country*	*Time*	*Women*	*Country*	*Time*
2020	N. Lyles	USA	19.76	S. Miller-Uibo	BAH	21.98
2019	N. Lyles	USA	19.50	S. Miller-Uibo	BAH	21.74
2018	N. Lyles	USA	19.65	D. Asher-Smith	GBR	21.89
2017	I. Makwala	BOT	19.77	T. Bowie	USA	21.77
2016	U. Bolt	JAM	19.79	E. Thompson	JAM	21.93
2015	U. Bolt	JAM	19.55	D. Schippers	NED	21.63
2014	J. Gatlin	USA	19.68	A. Felix	USA	22.02
2013	U. Bolt	JAM	19.66	S. A. Fraser-Pryce	JAM	22.13
2012	U. Bolt	JAM	19.32	A. Felix	USA	21.88
2011	Y. Blake	JAM	19.26	S. Solomon	USA	22.15
2010	U. Bolt	JAM	19.56	V. Campbell-Brown	JAM	21.98
2009	U. Bolt	JAM	19.19	A. Felix	USA	21.88
2008	U. Bolt	JAM	19.30	V. Campbell-Brown	JAM	21.74
2007	T. Gay	USA	19.62	A. Felix	USA	21.81
2006	X. Carter	USA	19.63	S. Simpson	JAM	22.0
2005	W. Spearmon	USA	19.89	A. Felix	USA	22.13
2004	S. Crawford	USA	19.79	V. Campbell	JAM	22.05
2003	B. Williams	USA	20.01	A. Felix	USA	22.11
2002	K. Kedéris	GRE	19.85	D. Ferguson-McKenzie	BHS	22.20
2001	J. J. Johnson	USA	19.88	L. Jenksin	USA	22.39
2000	K. Kenteris	GRE	20.09	P. Davis-Thompson	BAH	22.27

Table 1 shows the countries and records of the men and women who won gold medals in the 200-meter run annually from 2000 to 2020. Based on this data, what can you say about the records of male and female runners.