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The Influence of Decisions Planning an Out-of-Class Academic Festival on Statistics Teaching Practices

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Abstract

Some researchers in mathematics education have studied the decisions professors make during classroom context activities. These decisions are explained by several factors: 1) how professors conceive the structure of mathematics; 2) what they know about the mathematical content; and, 3) what they consider the best didactic strategies to improve students' learning. Based on these studies, this article explores how two professors' decisions, made beyond classroom contexts, influence their teaching practice. Two professors, who were teaching the same probability course, participated in this study. We apply Wenger's theory of Communities of Practice to analyze such influence in terms of the way these professors participated in, and reified, their out-of-classroom decisions. This study allowed us to perceive that decisions taken at faculty meetings are not always relevant to students' understanding of concepts, however, it did not enlighten us as to the process that led the professors to make their decisions.

Keywords
Communities of practice, instructors' meetings, statistics teaching practice, in-class decision, professors' decisions

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La Influencia de las Decisiones fuera del Aula de Clase de una Feria Académica en la Práctica de la Enseñanza de la Estadística

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Resumen

Algunos investigadores en educación matemática se han dedicado al estudio de las decisiones que toman los profesores en el aula de clase. Éstas se explican por varios factores como 1) cómo conciben los profesores la estructura de las matemáticas; 2) qué saben sobre el contenido matemático; y, 3) cuáles consideran las mejores estrategias didácticas para mejorar el aprendizaje de los estudiantes. A partir de estos estudios, esta investigación explora cómo las decisiones de dos profesores, tomadas fuera del aula, influyen en su práctica docente. En este estudio participaron dos profesores que impartían el mismo curso de probabilidad. Se toma como marco referencial la teoría de Comunidades de Práctica de Wenger para analizar dicha influencia desde la participación y cosificación de las decisiones de los profesores fuera del aula. Esta investigación permitió observar que las decisiones tomadas en las decisiones tomadas en reuniones de academia no siempre son relevantes para la comprensión de los conceptos por parte de los estudiantes, sin embargo, el análisis dejo entrever el proceso que llevó a los profesores a tomar sus decisiones.

Palabras clave

Comunidades de práctica, reuniones de academia, práctica de la enseñanza de la estadística, decisiones en el aula de clase, decisiones de profesores

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Since Bishop (1976), Shavelson (1973) and Shulman and Elstein's (1975) several influential works on the decision-making processes of mathematics professors and recently Borko, Roberts and Shavelson (2008), some mathematics educators have focused their research on the factors that influence what professors do in their instructional practice. This interest is manifested in Ponte and Chapman's (2006) review of the studies reported to the Psychology in Mathematics Education community concerning professors' knowledge and practice during the period 1977-2005. Among their findings, these authors noted that in many studies researchers have implicitly or explicitly established a relationship between professors' beliefs and their decisions (see also other research reports published in specialized journals: Beswick, 2007, 2012; Cross, 2009; Ernest, 1989; Lloyd, 2002; Thompson, 1992). Although beliefs are one of the factors that most heavily influence what professors do in their classrooms, Ponte and Chapman (2006) recognized the need to study how these factors are manifested in real contexts: "overall, the findings in the mathematics knowledge and beliefs/conceptions categories tend to be about professors' knowledge independent of consideration for situatedness in practice" (p. 485).

It is in this practical context that other scholars have developed theoretical frameworks, or proposed concepts, with which they seek to understand how professors make decisions. One example of how theory helps elucidate the role played by beliefs, goals and knowledge in the act of teaching is Schoenfeld's (1998) cognitivist Theory of Teaching-in-context. Stockero and Van Zoest (2013), meanwhile, conducted research that exemplifies how concepts may shed light on the nature of professors' decisions. Using the concept of the pivotal teaching moment, defined as "an instance in a classroom lesson in which an interruption in the flow of the lesson provides the teacher an opportunity to modify instruction in order to extend or change the nature of students' mathematical understanding" (p. 127), these researchers suggest that professors' decisions in response to these moments impact students' learning. Similar to Stockero and Van Zoest, Ponte and Quaresma (2016) claim that two key elements that prompt professors to make decisions are the tasks assigned to students and how professors manage communication in the classroom. They propose that students' wrong answers and difficulties in finding the proper words to explain their answers are two actions that occur during their classroom practice that professors can use to elaborate actions that foster students' reasoning; in other words, the decisions professors make are rooted in both students' wrong answers and their difficulties in communicating their reasoning. In the context of statistics professors, Eichler (2011) used Stein, Remillard and Smith's (2007) model of the curriculum as a theoretical framework to explore the relationship among what professors plan to teach, what they actually do in their classrooms, and what students learn from that planning.

Regarding what professors do in their classrooms, Eichler claims that it is associated with how they interpret the curricula prescribed by national governments (according to Stein et al., 2007), such interpretation is conceived as 'professors' intentions to teach'). He asserts, however, that this association is not straightforward: "Within the professors' intentions and classroom practice, the role of context seems to play a significant role in explaining differences among professors" (p. 183). Eichler's assertion concurs with, on one hand, what some researchers have documented regarding the different ways in which two professors teach the same task (Burgess, 2008; Sensevy, Schubauer-Leoni, Mercier, Ligozat & Perrot, 2005; see also Morgans' analysis (2014)) and, on the other, with Sullivan and Mousley's (2001) observation: "that in the classroom too much is happening for professors to make new decisions in response to every circumstance" (p. 147). In this respect, Schoenfeld (1998) affirms that "what takes place in classrooms [...] is shaped in fundamental ways by social, economic, and organizational, and curricular factors" (p. 5).

It is this contextual characteristic of classroom practice that led Sullivan and Mousley (2001) and other researchers (Jaworski, 2004, 2006; Potari & Jaworski, 2002) to adduce that professors' decision-making in classroom practice is a complex research topic (see also Konrad Krainer's commentary in Adler, Ball, Krainer, Lin and Novotna's survey (2005, p. 369) on the complexity of research in teacher education). One attempt to resolve this complexity entails focusing on relationships distinct from those between students and professors. As Morgan (2014) pointed out: "in order to understand the practices of individuals, it is necessary to understand how those practices and individuals relate to the social structures within which they are situated" (p. 131). Here, Morgan echoes social perspectives that posit the possibility of studying the influence of social structures on the practices of different individuals.

Taking into account Morgan's statement (2014), we assume that instructor meetings may exemplify a social practice in which professors make decisions that are important in terms of enhancing their instructional practices. Based on this assumption, and on the fact that Franke, Kazemi and Battey (2007) "consider the act of teaching as involving a number of different practices, with different people, which go beyond classrooms" (p. 226), this article addresses the following research question: How do the decisions taken by mathematics professors, in a beyond- classroom practice context, influence their instructional practice? Specifically, it explores the relationship between the decisions made by two probability professors during instructor meetings and their teaching practice. Our objective is that the results of this study will provide elements that will improve our understanding of the complexity of professors' intentions in relation to improving students' mathematical learning.

Theoretical Framework: Instructors Meetings as A Community of Practice

In addition to the various studies in which Wenger's social theory has been used to explore teaching practice (see Buysse, Sparkman & Wesley, 2003; Goos & Bennison, 2008; Llinares, 2002), we believe it is useful for analyzing what professors decide in instructors' meetings in relation to what they actually teach. As there is no consensus among mathematics educators as to what the term 'practice' actually means -as Ponte (2011) has emphasized- we draw specifically on Wenger's concepts of practice as reification and participation to interpret professors' decisions and their connection with their teaching activities. In choosing a social theory, this paper intends to show how such an approach may shed light on our understanding of professors' practice as we can see in Goos (2013) and Lerman´s (2001) analysis of social theories in research on mathematics professors. The following section explains the concepts of 'community of practice', participation, and reification as they apply to this study.

Practice in the Theory of Communities of Practice

Although the concepts of community and practice can be defined independently, in Wenger's (1998) social theory they intertwine in such a way that community of practice is understood as unique. In this sense, practice always occurs in a community: it "is the source of coherence of a community" (p. 72). Wenger describes three dimensions in which practice and community are interrelated: mutual engagement, joint enterprise, and a shared repertoire. Mutual engagement exists when people (eg. professors) are involved in actions (eg. teaching) that require establishing meanings (eg. what to learn, how to evaluate, and how to enhance learning) that must be negotiated. Since practice does not exist in isolation, or without reference to specific actions, mutual engagement gives a community coherence. A joint enterprise, meanwhile, is defined as a group's negotiated response to specific circumstances. For example, when probability professors at an instructor's meetings discuss what teaching strategies are appropriate for improving students' learning, they engage in the joint enterprise of making learning more accessible to students. Since "a joint enterprise is a process [and] not a static agreement" (Wenger, 1998, p. 82), the professors' joint enterprise does not end when they reach an agreement at their meeting. A shared repertoire, finally, consists of "routines, words, tools, ways of doing things, stories, gestures, symbols, genres, actions, or concepts that the community has produced or adopted in the course of its existence, and which have become part of its practice" (Wenger, 1998, p. 83). In the context of this study, an example of a shared repertoire is the meaning of teaching certain topics in probability courses.

Participation And Reification: The Duality of Meaning

The concepts of participation and reification are also interrelated. The former "refers to a process of taking part and also to the relations with others that reflect this process" (Wenger, 1998, p. 55), while the latter refers "to the process of giving form to our experience by producing objects that congeal this experience into 'thingness'" (p. 58). We may say that, on the one hand, reification makes participation palpable; while on the other, participation makes reification interpretable. "Reification and participation enrich each other, they are not the opposite of each other" (Farnsworth, Kleanthous & Wenger, 2016). The interplay between participation and reification is a duality that, as such, cannot be separated. This duality is what Wenger calls the *negotiation of meaning.*

> The duality of participation and reification is […] a complexifying distinction, it is meant to enrich the notion of negotiation of meaning, not to classify meaning as one or the other [process]. It is not that when you have more of one, you have less of the other. On the contrary, the negotiation of meaning always entails both in interplay. So, you always have to look for both processes whenever you try to understand a moment of meaning making (Farnsworth, Kleanthous & Weng., 2016, p. 146).

In placing the processes of reification and participation in a mutual relationship, Wenger (1998) uses the concept of negotiation of meaning as a process characterized by a constant doing, thinking, re-doing and re-thinking of our daily actions. "All that we do and say may refer to what has been done and said in the past, and yet we produce again a new situation, an impression, an experience" (Wenger, 1998, p. 52). Thus, Wenger's social theory makes it impossible to analyze how people make meaning if only one process (reification or participation) is considered.

In applying this concept to the topic of this article, we may say that to understand how two probability professors make meaning of their decisions beyond the classroom-practice level (reification), we must determine how they implement (participation) their teaching strategies in the classroom. Hence, in order for university professors to make their teaching practice meaningful, they must reify their previous agreements in classroom practice. In this sense, we conceive professors' decisions as a result of their negotiation of meaning, whose constituent processes –participation and reification– do not always concur during classroom practice.

More specifically, professors' participations in instructors' meetings take place outside classroom practice, whereas what they do in classrooms entails actions that reify those participations. It is through this reification that professors are able to negotiate what they previously decide outside classroom practice.

Methodology

Study Participants

Because we are interested in the influence of the decisions that professors make outside classroom practice on their teaching acts, this study was conducted using both qualitative methodology and the interpretative tradition (Travers, 2001). It is a case study with which we offer "a unique example of real people [e.g. university professors] in real situations" (Cohen, Manion & Morrison, 2007, p. 253). Two university probability professors participated: Pedro (male) and Claudia (female). Both have taught mathematics courses for 10 years. During the study period, both were teaching a Probability and Statistics course to undergraduate international business students (20-21 years of age).

Claudia had taught this course three years earlier (at that time her colleague was a different professor), but Pedro had never taught it before. In light of Claudia previous experience, and that two professors would be teaching the same course, we asked Claudia to encourage Pedro to attend regular instructors' meetings where they could discuss all matters related to their teaching (such meetings were not always feasible, primarily because at this university two professors rarely teach the same course).

Data Collection

Data were gathered at three moments. Moment one was during the first two instructors' meetings organized by Pedro and Claudia; the second occurred in their classes after the instructors' meetings; and the third moment was during the development of a Probability Festival that they organized (see the following section: Claudia's previous experience:

Proposing a Festival). The professors allowed us to video-record events at the festival, but not the meetings or classes.

There, we had to depend on field notes to record information on what they discussed in the meetings and what they taught in their classes. They also provided documents produced during the instructor's meetings. We interviewed Pedro and Claudia to clarify what they had discussed or taught. We were interested specifically in determining certain features of their classes after the second meeting. All interviews were semi-structured; that is, they were used as "an explanatory device to help identify" (Cohen, Manion & Morrison, 2007, p. 351) what Pedro and Claudia thought was the best way to teach.

Since Claudia´s previous experience played an important role in the decision-making process involving both professors, we asked her to explain in detail how she and previous professor had assessed students' learning when they taught the Probability and Statistics course three years earlier. In the following section we describe the organization of a Probability Festival.

Claudia´s Previous Experience: Proposing a Festival

Claudia recalled that when she and previous professor taught Probability and Statistics to undergraduate administration students (18-19 years old), they were concerned about their failure rate in the first two units of the course. As a result, Claudia proposed to previous professor that they assess the students' learning of the contents of Unit 3 (The concept of probability and the calculus of probabilities) with a strategy distinct from the written test format they had implemented in the previous two units. The new strategy consisted in dividing their respective groups into teams of 4-5 students (n=30 (Professor group) of N=300 (Academic Program)). Each team was instructed to freely choose games of chance involving risk. The main pedagogical goal of this activity was to have them calculate the probabilities of the winning odds involved in those kinds of games. No two teams could choose the same game. Claudia and previous professor also decided to organize what they called the First Probability Festival (PF-1), where each team would showcase its game. PF-1 was open to the entire university community. In order to bet in any of the games, players had to buy tokens. Claudia was sure that this new strategy would lower the failure rate.

To evaluate the teams, Claudia and previous professor elaborated an assessment worksheet, which they gave to a specialized committee (three other professors from the university) that would evaluate the performance of the teams during PF-1. This worksheet was divided into three main segments (see Appendix I in this document for details): 1) General information (0 points), 2) Setting of the game (maximum value: 80 points), and, 3) Presentation (maximum value: 20 points). The second segment had five items. The description of one of these items read: "The explanation of the probabilities is clear" (hereafter the explaining-the-probability-game item). Reflecting the pedagogical purpose of PF- 1, this item had the maximum value (25 points) on the worksheet. To obtain all 25 points, the team had to explain how to calculate the probabilities involved in the game. Committee members visited each team to fill out the items on the worksheet. Specifically, when assessing the explaining-the-probability-game item, they had to pay tokens to the team just like any other student who wanted to play. The third segment, Presentation, had two items (10 points each). The description of one item was: “The explanation [of the game] is clear” (hereafter the clarity- of-the-game item). It evaluated the clarity of the team’s explanation of the entire game. The main difference between the explaining-the-probability-game item and the clarity-of-the-game item was their respective content. In the former, students had to mention the possibilities of the winning odds; in the latter, they had to clearly explain the rules of the game.

The analysis outside the classroom is carried out by collecting data in academy meetings. At the university there are two types of meetings: the academy meetings by coordination and the academy meetings of basic sciences for administrative careers. From now on we will designate an instructor meeting to the boards of the professors that make up basic sciences. They are carried out between 2 and 5 times per semester in order to make an improvement plan in the area of mathematics and statistics. The administrative committee is made up of Claudia, Pedro and another professor of statistics where Claudia is responsible for raising the notes of the agreements at each meeting. In these meetings, an activity plan is designated at the beginning of the semester, which is subsequently submitted to the coordination.

First Instructors´ Meetings

During Pedro and Claudia´s first instructors’ meetings, Claudia shared with Pedro her previous experience in organizing PF-1. She told Pedro that at the end of that event students’ understanding of the concept of probability had improved. Claudia then proposed organizing a Second Probability Festival (PF-2, see Appendix II). Pedro agreed and, to gain a clearer idea of how Claudia and previous professor had evaluated each game, he asked Claudia to send her an email with the PF-1 assessment worksheet file so that he could become familiar with its criteria.

The first instructors’ meetings also allowed Pedro and Claudia to agree on how to organize their groups. As at PF-1, Pedro and Claudia’s groups would be divided into teams of 4-5 students. They agreed to begin preparations for PF-2 at their next faculty meeting, before working with students on Unit 3. Like Claudia and previous professor’s students earlier, the current students had to freely choose a game of chance that involved risk and present it to the university community.

Second Instructors´ Meetings: Changing the PF-1 Assessment Worksheet

Before the second instructors’ meetings, Pedro had read the PF-1 assessment worksheet (Appendix I). At that meeting, Pedro told Claudia what he thought about the format and proposed four changes, though without modifying the names of the segments. The first two changes involved the value assigned to the explaining-the-probability- game item and the description of the clarity-of-the-game item. With respect to the value, Pedro considered 15 points (instead of 25) sufficient, but added as second change the following sentence: “and a lunch was offered to the special committee” (the value of this item, 10 points) (As see in Appendix II).

The third modification adjusted the value (Team organization: clothing, punctuality and organization in the presentation of the game) of an item in the Presentation segment from the original 10 points to a maximum of 15. The fourth modification consisted in incorporating a "Maximum profit in the game" item (hereafter the money item, Appendix II) with a maximum value of 5 points. In short, the PF-2 worksheet had the same segments as the original, but assigned different maximum values to segments 2 and 3. By the end of the second meeting, Claudia had accepted all the proposed changes. Unlike PF-1, where the university community had bought tokens, at PF-2 community members bet real money in every game.

Each team began with the same amount (2.5 US dollars). This condition was a result of incorporating the money item. Table 1 simplifies the PF assessment worksheets and shows the four modifications made by Pedro.

Table 1
Differences Between the two PF Assessment Worksheets

Goal			
The goal of the Probability Festival is to engage students in creating and calculating probabilities in an easy and fun way by applying what they learned in class.			
Segment	Value (points) PF-1	Description	Value (points) PF-2
Setting of the game	25	"The explanation of the probabilities is clear"	15
Presentation	10	Team organization: clothing, punctuality and organization in the presentation of the game	15
Presentation	10	The explanation of the game is clear and a lunch was offered to the specialized committee	10

Data Analysis

We divided the analysis into two parts. In the first, we examine the production of the PF- 2 assessment worksheet; while in the second, we discuss the influence of this production on Pedro and Claudia's classes. More specifically, we deal first with Pedro and Claudia's decision-making process as a result of their first two meetings, and then discuss how this process impacted their teaching practice. Both the production of the PF-2 assessment worksheet and the decision-making process are analyzed on the basis of Wenger's concepts of participation and reification.

Data Collection Techniques

The data collection techniques in this research were: field notes, videotapes and interviews to support the methodology proposed by Travers (2002). In the academy meetings 3 field notes were taken, the dialogues were videotaped and transcribed. The professors participated in the academy meetings. In total, five videos of the meetings were recorded in which the theme of the probability festival and the modifications to the assessment worksheet were discussed in order to observe the meanings that are negotiated through the participation of teachers. Three interviews were conducted with the professors. All of these were semi-structured (Cohen, Manion and Morrison, 2007).

The data collection had three moments. The first moment was during the academy meetings with professors. The second moment was during unit three classes. The topics were covered in 3 weeks. Finally, the third moment during the development of the probability festival. The techniques used are detailed below.

Field Notes

The field notes were organized in logs. They described the agreements and suggestions reached by professors related to the relevance of the subjects of unit 3, the depth with which the concepts had to be seen and the organization of the probability fair.

The field notes allow collecting data for the work because record the activity that took place in the academy meetings in a detailed way and allow comments on the records that were taken. Thus, even if not all the activity or textual dialogues are recorded, the notes provide the elements of the dialog to make an analysis of professor´s negotiation considering the topics discussed.

Video Tapes Recording

The Claudia and Pedro´s classes were video-recorded. The principal aim was to observe if there was an influence of the agreements of the professor´s meetings in teaching and the changes that were made. The video recorded classes corresponded to the following topics in unit 3:

1. Define probability
2. Rules of addition
3. Describe the classical, empirical, and subjective approaches to probability
4. Explain the terms experiment, event, outcome, permutations, and combinations
5. Define the terms conditional probability and joint probability
6. Calculate probabilities using the rules of addition and rules of multiplication
7. Apply a tree diagram to organize and compute probabilities
8. Calculate a probability using Bayes' theorem

These topics have been chosen to see if there are any changes in the way they are taught. It is necessary to know if the content is modified by the professor so that the student understands more easily the concepts that are related to chance. Even though the content is the same, the professors make their own decisions in class when they were teaching the topics and these are closely related to the participation and commitment that professors have to carry out the probability festival.

Professor´s Interview

After the festival were held, Claudia and Pedro were interviewed. A semi-structured interview was done for each one. These were done individually. The principal aim of the interview was to inquire about the positive and negative experiences that occurred in the development of the festival and about the assessment worksheet.

A second aim was to complete the information collected in the videotapes of academy meetings, in the notes taken and in the classes that were recorded to be able to describe and analyze the decisions that the professors take.

Data Interpretation

Pedro and Claudia´s Decision to Produce the PF-2 Assessment Worksheet

Pedro and Claudia's processes of participation and reification during their first meeting can be explained in terms of, on the one hand, their mutual engagement and, on the other, the joint enterprise they created. It may be obvious that as university professors Pedro and Claudia were implicitly engaged in improving students' learning; however, the specific mutual engagement in which they became involved was created mainly by Claudia at the moment when she decided to share her previous experience with her colleague. Her experience in observing students' understanding of ways to calculate probabilities led her to show Pedro what her participation in PF-1 was all about, and what she knew about that festival. She then had to negotiate the meaning of organizing another Probability Festival on the basis of her participation three years before. Turning to Pedro´s participation in the first meeting, he needed to construct the meaning of a Festival in the context of his first experience in teaching the Probability and Statistics course. His lack of experience in teaching probability does not mean that he was incapable of creating a mutual engagement with Claudia, but that accepting her proposal obliged him to take certain actions that had to be negotiated with her. One of those actions was to review the PF-1 assessment worksheet. At that point, Pedro not only examined the past of Claudia's teaching practice, so to speak, but also pondered how he would need to be involved in this new practice. The assessment process served as the point of departure from which Pedro was able to create a mutual engagement with Claudia, and so negotiate the meaning of assessing students' understanding of the concept of probability. Paraphrasing Wenger's quote (1998, p. 52), all that Pedro and Claudia did and said regarding the organization of PF-2 must have referred to what Claudia had done and said three years earlier. Looking at the past, they produced new situations and experiences related to organizing the upcoming PF-2. Here again, their process of reification, exemplified in the PF-1 assessment worksheet, was used by Pedro and Claudia to begin the former's process of participating in the community of practice they had just created. Once Pedro was engaged in what Claudia had proposed, he also created a joint enterprise that consisted in making PF-2 possible and visible to students.

The duality of meaning in Pedro and Claudia's community of practice is observable in the way that both modified the PF-1 assessment worksheet to elaborate the version used at PF-2. The main modifications Pedro proposed –on the clarity-of-the-game and money items– were based on his way of representing students' learning. Although the idea of offering lunch may not be related to the understanding probability, Pedro may have perceived an opportunity to evaluate something distinct from simple learning at PF-2; namely, that business students should be courteous. Regarding the inclusion of the money item, Pedro believed that in the context of games of chance involving risk the more money students make, the better their understanding of how to calculate probabilities. Pedro's participation in the new PF-2 design captured his experience of conceiving the use of the concept of probability in real life. From Pedro's point of view, casino owners are interested in making money and in using the concept of probability to their own benefit. Since students were to elaborate games of chance involving risk, it would be useful to ask them not only to explain how the concept of probability was involved in their games (the explaining-the-probability-game item) but also to attempt to make money (the money item).

Unfortunately, Pedro and Claudia did not plan a strategy to determine which team actually made more money. Though they verified that each team began with the same amount (2.5 U.S. dollars), they did not constrain the number of players who participated, so it was impossible to apply the uniform criteria that would have allowed them to compare the respective team's winnings (or losses).

Instead of arguing over who was responsible for these details, we claim it is an example of how a decision depends on the context in which is made and, moreover, on how it involves the participation and reification processes in order to make it meaningful. On the one hand, we insist, Pedro's decision to incorporate the money item was determined by his way of conceiving the real context in which a game of chance involving risk is useful. This particular context was not considered by Claudia or previous professor three years earlier and was not reified in the PF-1 assessment worksheet. On the other hand, by deciding to evaluate how much money each team would make, Pedro and Claudia were impelled to take certain actions to make that decision meaningful. Those actions were the participation and reification processes involved at the very moment when they decided to give the 5 points of the money item to the teams. Since that decision was made at the beginning of the course and PF-2 was held after Pedro and Claudia had finished teaching the contents of Unit 3 (calculate probabilities using the rules of addition and multiplication, describe the classical, empirical and subjective approaches to probability and combinatory analysis), the act of becoming aware of their lack of planning is explained in terms of their participation related to what they reified in the money item. Paraphrasing Wenger's statements (1998, p. 68), participating in their own decision renegotiated the meaning of the "Maximum profit [obtained] in the game" description in a new context: the start-up of PF-2.

In the same vein, by reifying their decision they created conditions for the new meanings of the money item. In Pedro and Claudia's intention to make the money item possible, participation and reification processes interplayed with each other such that the meaning of assessing student's learning "seems to have its own unitary, self- contained existence" (Wenger, 1998, p. 63); that is, in the context of Pedro and Claudia's teaching practice, an important characteristic of the meaning of assessing student's learning has to do with the capacity to make money.

Pedro and Claudia's decision to incorporate the money and clarity-of-the-game items influenced not only their conception of what learning probability may mean, or the changes made to the other two items (see Table 1) in order to adjust the 100 total points of the PF-2 assessment worksheet, but also how they changed their way of teaching the contents of Unit 3.

Pedro and Claudia´s Decisions and Their Influence on their Teaching Practice

To clarify how decisions made outside the classroom context have an effect upon teaching practice, let us begin by exemplifying how Claudia and previous professor's decision to organize PF-1 pushed Claudia to modify her teaching practice. Before the first faculty meeting, we asked Claudia if the strategy of incorporating PF-1 into the contents of Unit 3 had had any repercussions in the way she taught her classes. Her answer was as follows:

> When I taught [the Probability and] Statistics [course] before the PF-1 idea came through, the topic Combinatory Analysis was included in the study program at the end of Unit 3. The topic was included just like counting rules. Permutation counting rules were defined by the teacher as any arrangement of objects selected from possible objects (the order of arrangement is important in permutations…) and Combination counting rules as the number of ways to choose objects from a group of objects without regarding the order… At that moment [before PF-1], as a professor, I did not see Combination counting rules as a tool for calculating probabilities. PF-1 changed my way of teaching the topic of Combinatory Analysis. I included Combinatory Analysis in another context. It helped the students increase their own probability of winning in a gambling game. I used the classical probability formula to calculate the probability of an event by dividing the number of favorable outcomes by the total combinations of possible outcomes [Interview 1].

Thus, Claudia recognized that planning PF-1 as a new strategy for assessing student's learning forced her, first, to understand ("At that moment, as a professor, I did not see Combination counting rules as a tool for calculating probabilities") and, second, to teach ("I included combinatory analysis in another context") the concepts of permutation and combination in different ways from the ones she had been using ("The topic was included just like counting rules"). Although Claudia did not say whether she modified her didactic strategies, she made it clear that PF-1 kept her from teaching Analysis Combinatory as a topic unrelated to the concept of probability. The decision to organize PF-1 made her realize that it is possible to use combinations and permutations as tools to calculate probabilities ("I used the classical probability formula to calculate the probability of an event by dividing the number of favorable outcomes by the total combinations of possible outcomes").

Let us now return to the decision of organizing PF-2 and its effects upon Claudia's teaching practice. Two weeks after the second professor meeting, in a regular class, and due to the urgency to propose a game of chance for the PF-2, a 4-student team asked Claudia how to calculate the probability of obtaining a certain sum of two dice, when rolling them twice. The

complete description of the game of chance this team had in mind was as follows, as they explained to Claudia and the entire class:
The numbers in the following table correspond to all the values of the sum of two dice if they are thrown once:

Table 2
Sum of Two Dice

2	3	4
5	6	7
8	9	10
11	12	

The cost of the game is 0.1 American dollars. The player shall place a token on the sum she/he desires. Let us say she/he chose the number 9. The bettor then rolls two dice twice. If the sum of the dice is 9 on at least one roll, the bettor wins 0.5 dollars. If the sum of the dice is 9 on both rolls, the bettor wins 1 American dollar.

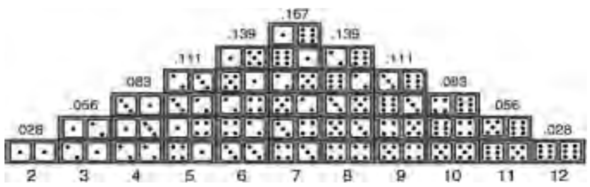
After listening to the team's explanation, Claudia drew a table and a picture on the blackboard (see Figure 1). She then asked the class to pay close attention to what she was about to say:

Listen up! Here [she refers to Figure 1] both the table and the drawing are called the distribution of probabilities of the sum of two dice. There are two ways of calculating the probability of getting the sum of a number if you roll two dice twice. In the table [she refers to the left column of Figure 1], x is the sum of the two dice: two, three, four and so on. Here [she refers to the right column of the Figure 1] P(x) is the probability of that particular sum. For example, the probability to obtain the sum of 9 is 4/36. In this picture [she points to the drawing] you can see that there are four ways to obtain 9 by rolling two dice: 5-4; 4-5; 6-3; 3-6. Now, if you want to know the probability to obtain the sum of 9 both times, you have to multiply (4/36) (4/36). [Field notes]

Figure 1

Claudia's Two Ways of Representing the Probability Distribution of the Sum of Two Dice

x	*P (x)*
2	1/36
3	2/36
4	3/36
5	4/36
6	5/36
7	6/36
8	5/36
9	4/36
10	3/36
11	2/36
12	1/36



Note. Right: the table with the probabilities expressed in rational numbers. Left: A well-known picture of the probability distribution drawn on the blackboard by Claudia. The probabilities are expressed in decimal numbers

The second approach that Claudia mentioned consisted in applying the concept of binomial distribution. On the blackboard, she wrote the mathematical expression of this concept: $P(x) = \frac{n!p^x(1-p)^{n-x}}{x!(n-x)!}$ and, considering the specific dice game, explained the meaning of every letter in the expression: P(x) means the probability of x events out of n trials; p is the probability of obtaining the number 9 (from Figure 1, students could see that p is 4/36).

Claudia continued:

For example, if you want to calculate the probability of obtaining the sum 9 in two consecutive rolls, x is 2; n is always 2 in this example, since the game proposed by your teammates consists in rolling the dice twice. Thus $P(2) = \frac{2!\left(\frac{4}{36}\right)^2\left(1-\frac{4}{36}\right)^{2-2}}{2!(2-2)!}$. After briefly explaining the meaning of factorial function, Claudia proved that the value ofP(2) is the same as that calculated by multiplying (4/36) (4/36). She then went on to give more examples of how to calculate the probability of obtaining the other values of x (0 or 1).

Since we knew that she had not taught the concept of binomial distribution three years earlier (according to what she told us before the first faculty meeting), we asked to explain why she had decided to include this concept in her explanation:

> I included the binomial distribution formula as a new option for calculating these kinds of probabilities. Since in this festival [PF-2] we are going to measure the money made by each team, I took advantage of the game to give my students mathematical tools so they would know how to modify or create their games […] Before PF-2, I taught the concept of Combination in the same way [referring to how she taught after PF-1, see Interview 1]. Moreover [with PF-2], I think I am able to modify my class by making experiments that consist in rolling two dice and proposing different events. [Interview 2]

In a sense, the 4-student team's proposal triggered Claudia's new way of teaching how to calculate probabilities. However, not only this fact, nor the decision to organize PF-2, but also the idea of including the money item pushed Claudia to modify the approach she had used three years earlier. Because making money was to be evaluated, she believes that teaching the binomial distribution formula would give the students an efficient mathematical tool for calculating probabilities. By doing so, Claudia expected her students to be eager to change the rules of any game of chance that involves risk to ensure that the odds would not be in the bettor's favor and, therefore, increase the amount of money their team could make. It is in the context of PF-2 that Claudia's decision to incorporate the binomial distribution formula makes sense. She could have continued teaching Combinatory Analysis in the same way as before, when she showed students how the concepts of combination and permutation could help them calculate probabilities; however, the need to evaluate the amount of money made put pressure on Claudia in terms of what she had to teach to show students how to use probability for their own benefit. The binomial distribution concept was the probabilistic tool that Claudia found useful for achieving her pedagogical goal.

By incorporating binomial distribution into her classes, Claudia reified (in Wenger's terms) what she and Pedro had decided at the beginning of the second meeting. In other words, in taking the decision to evaluate how much money each team would be able to make, Claudia needed to reify that decision in order to signify the meaning of the money item. The binomial distribution concept was, so to speak, the appropriate way that Claudia found to make the money item meaningful. In terms of Wenger's (1998, p. 58) definition of reification, the binomial distribution congealed Claudia's experience into 'thingness'. However, the concept of binomial distribution could not have been the 'thingness' of the money item if Claudia had not participated in including the money item in PF-2; that is, without her input on including the money item in the PF-2 assessment worksheet.

Through processes of reification and participation, Claudia learned a new way to teach the concept of probability.

As to the influence of the PF-2 on Pedro's teaching practice, while the PF-2 was taking place, we asked him to share his impression of this event with us:

> The PF-2 had a good impact if it is compared to traditional assessment methods; for it helped us [Pedro and Claudia] to better measure the students' understanding in practice. Students started focusing on understanding rather than on memorizing probability concepts (…) As professors, we need extra time to prepare the activity, for it takes more dedication [to evaluate students' learning] than [to evaluate it with] regular exams. This is because [the act of] assessing using this methodology entails participation in meetings; it changes the way of teaching probability and it involves students in a new way to learn the concepts. [Interview 2]

By not having a point of reference from which Pedro could compare his own teaching practice of the Probability and Statistics course, he highlighted the advantages of PF-2 in assessing students' learning of the concept of probability. Although he had no previous experience in organizing such a Festival, his teaching experience, which consisted in evaluating students' learning by means of written exams, gave him an important reference point around which he could develop PF-2 as a pedagogical tool. At the end of the event, Pedro congealed (in Wenger's terms) his participations during the faculty meetings; furthermore, he was then able to signify his decision to agree to incorporate PF-2 into the Probability and Statistics course. Now, through this duality of meaning (characterized by the act of congealing his decision), Pedro lived a new experience from which he was able to learn. For example, the experience of organizing PF-2 inevitably forced him to compare his traditional way of assessing student's learning (written exams) with how he assessed this by interpreting the PF-2 assessment worksheet. The new information acquired gave him new knowledge that he could use to make decisions in relation to the best way to favor and assess student's learning. If Pedro wishes to organize an activity similar to PF-2 in the future, he may consider the amount of time required ("it takes more dedication") and, if he decides to proceed, he will be aware beforehand that he will need to modify his teaching practice.

Conclusion

This study examined how out-of-classroom decisions by two university professors influenced their act of teaching. Drawing from Wenger's CoP Theory, we showed that this decision was rooted in the intertwining of Claudia's experience and Pedro's participation in the instructor´s meetings they organized. With Pedro's initiative to organize a second Probability Festival, she and Pedro discovered a commonality while discussing similar topics involved in teaching their respective courses. The community of practice they created during the faculty meetings played an important role in developing strategies to: 1) assess students' learning by elaborating the PF-2 assessment worksheet; and, 2) teach the concepts of the course. We sustain that Pedro and Claudia "congealed" all their faculty agreements in the PF-2 assessment worksheet. In order for this worksheet to make sense in their teaching practice, they had to take part in that decision; that is, they needed to participate and reify their decision in order to determine whether or not it produced a favorable result in regard to their students' learning process. Based on Wenger's CoP (Wenger, 1998), we claim that without the processes of participation and reification, Pedro and Claudia would not have been able to learn from their decisions.

Although we agree with Shavelson (1973) that "the basic teaching skill is decision- making" (p. 144), in this article we set out to show that the decision-making process occurs not only in the classroom, but also outside it. In the case of Pedro and Claudia, we noted that their decision to organize PF-2 emerged from the need to develop actions that would, simultaneously, make the PF-2 a reality (process of participation) and create benchmarks (eg. the PF-2 assessment worksheet) around which they could evaluate their decision (process of reifying). Thus, we may say that two additional teaching skills are *participating in* and *reifying* the decision-making process. Had Pedro and Claudia not participated and reified PF-2, they would not have identified the pros and cons of their decision. In fact, in participating and reifying that decision, they were, in a sense, responsible for the way they: 1) assessed students' learning; 2) changed the contents of the Probability and Statistics course; and, 3) taught the calculus of probabilities. These three responsibilities, we believe, are examples of how an out-of-classroom decision influenced teaching practice.

This study allowed us to perceive that decisions taken at faculty meetings are not always relevant to students' understanding of concepts, however, it did not enlighten us as to the process that led the professors to make their decisions. More research is needed to explore how professors persuade each other during faculty meetings, and the kinds of arguments they use to create mutual engagements.

Future research may also focus on the essence of those decisions that initially may appear to be related to students' learning, though later it may prove difficult to determine whether they produced a favorable result (e.g. including the money item was designed to encourage students to think about a game in which the odds should be in their favor; however, at the moment Pedro and Claudia counted the money made by each team, they did not consider other variables that may have influenced the money-making process). From a theoretical point of view, we believe that the analysis presented herein may lead to explorations of how other out-of-classroom decisions (aside from those taken at faculty meeting) are intertwined with instructional decisions, and how educational institutions modify their policies in light of the influence of decisions made at instructor meetings.

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