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Preferred problem-solving methods employed by Grade 4 learners for measurement word problems

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Background: Problem-solving as a vehicle to develop independent thinking skills is mostly underestimated and is often either overlooked or not given adequate attention within the existing South African mathematics curriculum. Consequently, numerous learners often display limited skills or lack skills to adequately crack Mathematics problems by applying methods put forward in class. This generally results in under-achievement.

Aim: This study aims to explore and emphasise the problem-solving methods applied by Grade 4 learners involved in solving measurement word problems, and to reveal what transpires when the selected learners apply these methods to arrive at meaningful solutions.

Setting: Data were collected from a class of 42 Grade 4 learners at a primary school in Cape Town South Africa. Learners were conveniently selected.

Methods: A qualitative case study research design was adopted. Data gathering instruments of the study included observing learners solving, measurement word problem activities and focus group interviews.

Results: The study revealed that singular methods were applied by Grade 4 learners, such as, adding, multiplying, creating a sketch or diagram, grouping, dividing, subtracting, logical reasoning, guessing and tabulating values.

Conclusion: Grade 4 learners are prone to applying methods such as clustering or organising into groups, tabulating numerical values and logical reasoning were all applying mathematically sound methods. Such learners, however, needed a degree of supervision and instruction to indicate the way in which such methods were applied successfully as these methods were not necessarily dealt with in classroom context or in textbooks.

Contribution: The findings emphasise the need for tackling learners' limited problem-solving competencies and accentuate the necessity for greater attention to develop and grow methods for optimal and successful solving of problems in context.

Keywords: problem-solving methods; problem-solving competencies; measurement; word problems; Grade 4 learners.

Introduction

The performance of South African school-going children in mathematics is among the worst compared to other countries globally. This is a major concern; in fact, Mabena, Mokgosi and Ramapele (2021:451) call 'learners' mathematics performance globally and locally' to be a 'major concern'. When South African learners were involved in the Trends in Mathematics and Science Study (TIMMS) in 2011, the country ranked at the bottom of all the 21 participating countries. Even though the South African participants in the study were in a grade higher compared to those of other nations, namely Grade 4s, South Africa still performed the lowest out of a total of 38 nations who participated in PIRLS 2006 (Pooran 2011). Msimango and Luneta (2015) confirmed that South African primary school learners were outperformed by all the other countries who participated in mathematics and sciences of TIMMS 2011. In 2019, 39 countries participated in the Grade 8 TIMMS with South Africa's score second from the bottom.

The primary purpose of this study is to make a deliberate attempt to focus on a strategy of teaching mathematics that could be instrumental in improving the dire situation as spelled out earlier in the text, namely problem-solving. It is our experience and contention as educationists that problem-solving as a teaching and learning strategy seems to be neglected and not optimally engaged with in the mathematics classroom. Exploring the problem-solving methods employed by Grade 4 learners, as the initial year of the Intermediate Phase (Grades 4–6) when solving measurement word problems, would reveal how these learners choose and employ these methods to end up with meaningful outcomes or answers. This knowledge may serve as a platform to help improve the problem-solving skills of these learners. The research question emanating thus is: *What problem-solving methods do Grade 4 learners employ in solving measurement word problems?*

Rohmah and Sutiarso (2017:672) avowed problem-solving to be 'the heart of mathematics' and define mathematical problem-solving as 'the resolution of a situation in mathematics which is regarded as a problem by the person who resolves it' (2017:673). The significance of problem-solving in learning mathematics is said to be situated or positioned in the belief that mathematics principally deals with the ability to reason and is not really about memorisation (Klerlein & Hervey 2023). Problem-solving affords learners opportunities to develop insight and understanding, and also provides time and space to explain the processes utilised to come to particular solutions. Problem-solving, thus, is considered a shift away from knowledge recall or remembering and is also not the mere application of arrays of procedures.

Apart from having to consider the cognitive and social learning requirements, it is essential to also reflect on learners' reading ability (Mahlokwane 2023) as an important skill for interpreting, grasping and finding solutions to mathematical problems. In terms of reading abilities of South African learners Durbin (2023) reported that of the 57 countries assessed in the Progress in International Reading Literacy Study, South Africa was positioned at the bottom. This 2021 study tested the reading ability of 400 000 learners globally. Furthermore, the Progress in International Reading Literacy Study 2021 (PIRLS 2021) conducted by the University of Pretoria (UP) established that 81% of South African Grade 4 learners encountered difficulty in reading for comprehension at age 10. Learners need be able to read a problem with conception and understanding, that is, to have insight into what the problem requires of them. The TIMMS results reflect that South African learners' reading proficiency is well below the desired level when compared to other countries. This is indicative of the amount of input required to enhance learners' reading skills.

Many learners who underachieve in mathematics have the potential to exceed and perform well (Mabena et al. 2021; Spangenberg 2017). The concern, however, centres on the tendency of teachers to frequently compel learners to employ specific methods and methods advocated in prescribed textbooks and by teachers. Spangenberg (2017:1) stresses that 'By reinforcing or disregarding certain goals, a teacher can influence the way in which learners learn mathematics'. Many learners display a multitude of novel means and procedures to solve mathematical problems. However, provision is not always made within the traditional classroom setting to employ teaching strategies not necessarily encountered in class. Consequently, learners may be more successful in solving mathematics problems if they are afforded the opportunity to explore problems employing methods that come intuitively to each learner, individually. These methods that come intuitively to each learner could have implications for the teaching techniques used in the mathematics classroom; hence we propose that problem-solving should be increasingly integrated into the mathematics curriculum.

Literature review

This literature review is an attempt to examine and to obtain profound comprehension of what learning and teaching of problem-solving entail within a primary school mathematics context by scrutinising similar or related research studies. Many descriptions of problem-solving exist in literature. Liljedahl et al. (2016) view problem-solving as central to mathematics teaching and learning, which facilitates the linking of mathematical concepts encountered with real-world applications (Suseelan, Chew & Chin 2022; Verschaffel et al. 2010).

Pertinent issues relating to the education of problem-solving are focussed on, namely support required to employ appropriate methods to problem solve (Anderson 2005); ascertaining what methods learners use when they problem solve (Aydogdu & Kesan 2014); to develop insight into learners' understanding of word problems (Cummins et al. 1988); to understand the processes involved in problem-solving in a mathematics class (Brijlall 2015); to ascertain if teaching strategies, emphasising the teaching of general heuristics for mathematical problem-solving would improve learners' performance (Kaitera & Harmoinen 2022) and to look at problem-solving activities used in a mathematics classroom (Lesh et al. 2013). Suseelan et al. (2022) conducted a literature review on mathematics problem-solving in primary education research that took place from 1969 to 2021. According to them the research was based mostly on the following aspects:

> (1) problem-solving involving arithmetic and mathematical representations, (2) mathematics teaching and learning based on word problems, (3) cognition of pupils and affective domains in mathematics problem-solving, and (4) problem-solving involving algebra and teachers' role in problem-solving learning. (Suseelan et al. 2022:1993)

According to Anderson (2005), teachers of mathematics require built-in structures to render support towards successfully implementing a problem-solving teaching approach. A general pattern emerged from the research results conducted by Anderson (2005), which indicated that the majority participating teachers were receptive and supportive of teaching mathematics through teaching methods that involve problem solving. Significantly, participant teachers admitted to relying heavily on using the prescribed textbook as the principal resource. In addition, it was found that the teachers expressed openness and willingness to incorporating problem-solving should the use of textbooks be modified to accommodate that particular teaching and learning approach. Another important finding from Anderson (2005) was that participants readily expressed willingness to expose themselves to further training to become proficient, skilful, and knowledgeable to teach by means of problem-solving methods.

A qualitative investigation to ascertain the diverse methods applied for solving geometry problems (Aydogdu & Kesan, 2014) revealed that the majority of the participating learners resorted to making drawings at least once as an aid or strategy. This implies that learners are more prone to solving a problem through visualising it. It emerged from the same study that upon having been exposed to a particular problem-solving approach, participants were inclined to accept that approach unconditionally, applying it during lessons, and when indulging in homework exercises.

Erdogan (2015) investigated which problem-solving methods Grade 6 learners employed when problems were unfamiliar to them. Learners had to complete a range of problems over a period of 5 weeks. Upon analysis of learners' calculations and solutions of the given tests, the researchers discovered that the majority of participants resorted to identifying a pattern upon which generalisations were made. Mathematics problems demanding learners to apply manifold or complex methods such as those that compel them to decompose the problem into its constituent parts and then attempt to solve each part employing distinct methods were perceived by learners to be quite demanding and challenging, and they struggled as a consequence.

Cummins et al. (1988) explored Grade 1 learners' grasp or sense-making of word problems. The participant learners had to solve a certain type of arithmetic problem and all of them successfully solved them. Upon changing the same problem to a contextual (word) problem a mere 29% managed to successfully solve it. That could be ascribed to learners lacking sufficient exposure to or rarely engaging with word problems. They were much more at ease solving problems devoid of contexts, that is, mathematics problems that were expressed using only mathematical symbols and numerical values. According to Cummins et al. (1988) learners had been extensively exposed to those types of problems in class. Thus, word problems or problems in contexts fall outside of their frame of reference, which hampers understanding or making sense of a word problem, or what it requires of them.

It is claimed that learners' performance on contextual problems tends to be 30% lower compared to when they encounter problems that are numerical only and devoid of context. Erdogan (2015) maintains that these restricted abilities to solve problems could be blamed on the current curriculum, which puts a lot emphasis on identifying patterns as its sole problem-solving strategy (CAPS 2011). Furthermore, it is advised that subjecting learners to a wider set of problem-solving methods would afford them the opportunity to explore different methods and discover methods that would better suit their individual learning styles.

Theoretical framework

Efforts to develop a solid understanding of how learners grow into efficient problem solvers compel us to consider appropriate theories that adequately underpin learning in mathematics (Klerlein & Hervey 2023). The Constructivist Learning Theory (Piaget 1977; Vygotsky 1978), merged and integrated with Polya's (1957) problem-solving model, is considered as effectively underpinning this particular research. While constructivism focusses on learners thought processes and the social aspects, Polya's problem-solving model deals with methods employed in solving the measurement problems by participant Grade 4 learners.

Constructivism was formally documented in the 1990s. Mathematics education was to a great extent influenced by Piaget and Vygotsky (Vintere 2018). According to constructivism during cognitive development, learners are themselves constructing their own knowledge through assimilation and accommodation processes (Piaget 1977). Vygotsky, a Russian psychologist accentuates not only the significance of social influences in cognitive development but also that of culture and other citizens as the most crucial factors in a learner's development (Vintere 2018). He showed that humans could only develop by interacting with the environment, including other humans (Vygotsky 1978).

Constructivism maintains that knowledge is constructed by means of personal observation and investigation. Knowledge is claimed to be produced and grounded in the individuals' own experiences and (social) interactions with the environment within which they found themselves (Major & Mangope 2012). Constructivism thus holds that individuals have the potential or ability to manifest their own understanding of various or multiple thoughts and perceptions by increasing or growing on prior knowledge sets and experiences (Surgenor 2010). Furthermore, Bada and Olusegun (2015) assert that newly found knowledge of concepts as a consequence is assimilated into learners' existing knowledge set. These processes of accommodation and assimilation (Piaget 1977) allow learners to acquire and grow a meaningful grasp or sense of the world within which they operate and find themselves.

Teaching within a constructivist paradigm shifts the emphasis away from the teacher, towards the learners, Taber (2011) maintains. This means that applying constructivism to the mathematics teaching environment shifts the focus of the lesson mostly towards the learners and away from the teacher. This implies that mathematics teaching has the potential to become more learner-centred as opposed to being teacher-centred. As a consequence, learners are afforded the opportunity to participate more readily in lessons and tasks, thereby allowing them to be the prime focus of the lesson. Learners thus can increasingly rely on, and apply what is already known to them, as well as past experiences, which would facilitate independent learning, and 'construct their own meaningful methods' (Brijlall 2015:292). Through their own observations, learners are empowered to build unique associations (Surgenor 2010) that permit them to create a personal grasp of the mathematics content they come across. Gray (2005) likewise claims that a constructivist arrangement facilitates dynamic learner involvement and allows learners to hypothesise, collaborate and utilise their imaginations. Being actively involved is said to serve as an additional motivation for learners to reflect on past lived encounters and connect those to novel ones, thereby expanding conceptions of the world (Gray 2005).

Teachers generally need to ensure that the lesson structure allows learners to investigate the problems at hand independently in terms of making deductions and drawing conclusions. The teacher should mostly act as a facilitator, and occasionally take on the part of moderator and mediator, moving about the classroom posing leading questions and making guiding statements if required. Learners are thereby encouraged to construct their own knowledge. Brijlall (2015:292) agrees that 'In the constructivist approach, learners are assumed to construct their own mathematical conceptual understanding as they take part in cultural practices, and whilst they interact with each other'. The teacher in addition encourages learners to make comparisons through investigation and to formulate hypotheses allowing them to uncover relationships among ideas and ultimately develop a novel understanding of the mathematics content or relevant concept (Brooks & Brooks 1999). Consequently, the impression may be created within learners as they exerted better control and developed a higher degree of confidence in respect of their own competencies.

Social constructivism, as mentioned before, puts emphasis on social associations and connections. Vygotsky (1978) stresses the effects that social and cultural contexts exert on learning (Surgenor 2010). In addition, Vygotsky was all for a learning model with discovery as the foundation as he vehemently advocated that learners should be armed with the intellectual devices crucial for development by interacting with their culture. The discovery model permits the cognitive abilities of learners to naturally grow and progress, allowing them to discover by means of making comparisons and trading thoughts with equals. Vygotsky's (1978) social constructivism theory centres on what he calls the Zone of Proximal Development (ZPD). The ZPD is described as 'an integral part of Vygotsky's sociocultural theory of learning, which explains how learning and development is the result of social and cultural influences' (Abtahi 2021:7). Vygotsky views the ZBD as 'the distance between the actual developmental level (independent problem-solving) and the level of potential development (problem-solving under adult guidance or in collaboration with more capable peers)' (Vygotsky 1978:69). So, the ZBD alerts towards the vacuum that exists between the level that a learner can potentially accomplish individualistically. Thus, the ZPD is described as the region in which learning occurs optimally. According to Vygotsky (1978) guidance and mentoring from, and collaboration with more informed learners and grown-ups, can help learners accomplish activities successfully as individuals, and exceed expectations (Taber 2011).

Polya's problem-solving model

The most extensively applied work on problem-solving strategies is that of George Polya. In 1945, Polya published the text entitled, 'How to Solve It'. In this book he summaries four basic stages of problem-solving (Aljaberi 2015),which are discussed in next section.

Understand the problem (making sense)

This initial step is crucial. Inability on the part of learners to comprehend what the problem entails and requires from them, implies inability or failure to solve it. Shirali (2014) claims that when the problem at hand is fully grasped, there are certain aspects that the learners should have mastered. Proust (2014) maintains that having group discussions and collaboration with fellow learners is an effective strategy to share ideas and validate thoughts. By employing this strategy, they actively engage in helping peers to generate a deeper understanding of the problem. Initial understanding of the meaning assigned to words contained in the problem is considered essential. A dictionary should be consulted to familiarise themselves with new words or phrases never encountered before. A more knowledgeable person may also be consulted to explain or clarify meanings. Learners need to identify accurately what the problem requires of them in terms of what to determine or what to explain (Kaufman 2010).

At this stage learners should have the ability to paraphrase in their own words. The problem is now expressed in words learners are familiar with and which constitute a significant fraction of their active vocabulary. Shirali (2014) claims that learners would now have a greater degree of familiarity of the problem content, enhancing learners' comprehension of the problem on their own cognitive level. For those learners who are unable to interpret the problem in its entirety at this juncture, creating a sketch may assist in achieving a more holistic conceptualisation of the problem (Kaufman 2010).

Devise a plan (translate)

Upon having interpreted the problem and establishing what is required, an all-inclusive designed plan needs to be established according to which the learner will approach any problem. In'am (2014) asserts that numerous problem-solving methods exist from which to pick when attempting to solve mathematical problems. In addition, Kaufman (2010) maintains that suitable methods are determined by the context and facts provided in the problem, The omitted chunks of information the learner requires to ascertain how these sets of familiar and unfamiliar information are related and connected.

Carry out the plan (solve)

Polya (1957) regards carrying out of the plan to be the most demanding of the four steps as learners may easily be bogged down and may experience difficulty in modifying or adapting their strategy. Executing the strategy should be accompanied by meticulous record-keeping of the steps employed and findings obtained in order to clearly identify what was done incorrectly in cases where they were unsuccessful. Consequently, alterations can be made or alternative methods can be employed until valid solutions are arrived at. In'am (2014) maintains that the designated strategy frequently fails to take learners towards a solution. At this juncture, learners need to devise a different or more effective strategy devoid ofthe same deficiencies as their first strategy. Learners may experience increased frustration states Aljaberi (2015), if after having attempted numerous different methods they are still incapable of arriving at a rational or realistic solution. Learners frequently, and erroneously, make the assumptions that they are unable to solve never-before-encountered problems. This is a common misconception, which Proust (2014) refers to as a false belief as learners do not always realise that each problem is in fact unique and may be successfully solved through different methods.

Look back (check and interpret)

The salience of critically reflecting and evaluating the credibility and dependability of the outcome after completion is emphasised by Polya (1957). The procedure of reflecting allows learners to verify their solutions to determine whether their solutions make sense and also if it adequately adheres to all the required stipulations of the problem. It is beneficial for learners to scrutinise their calculations for possible inaccuracies when attempting to solve the problem (Shirali 2014). Mistakes involving basic operations such as addition, division or subtraction are quite easy to make. Using unsuitable operations at times, would result in incorrect solutions according to Aljaberi (2015).

Research methods and design

An exploratory case study was deemed suitable and a fitting research plan. Yin (2014) maintains a case study to be employed to investigate current phenomena (Yin 2014) that occur in bounded, real-world situations. Understanding the said phenomena, is to a significant extent dependent on the contextual factors within which those phenomena occur (Seshea 2017). Case study design facilitates finding valid answers to research questions such as: 'How?', and 'Why?' Where total control over selected participants is impossible, and with limited control in terms of ways in which participants behave, case study is preferred.

This exploratory case study focuses on the types of problem-solving methods applied by Grade 4 learners within the context of measurement contextual problems. The study expected learners to write a test based on selected measurement word problems. The trustworthiness of the investigative tool was determined through a peer review process to ensure that the questions were fair, dependable and aligned to the research questions. All of 42 Grade 4 learners, conveniently selected, had to be exposed to an identical activity that served as one of the information generated tools. No explicit guidelines in terms of the methods to be used were provided. This technique allowed learners the freedom to propose their own individual problem-solving methods. Data collected from learners' responses were tabulated, and comparisons were made and used to compile representational information to detect the various problem-solving methods used. Additional data were generated by means of document analysis, observation and focus group discussions.

In essence, document analysis involved a short, written activity consisting of six measurement word problems. Bowen (2009) expresses document analysis as a way of interpreting and reviewing data. In comparison to other research methodologies, document analysis necessitates data to be analysed and interpreted into information, which is expressive. Learners were observed and observations recorded in an effort to directly address the research questions. Learners were then divided into focus groups after having completed the activity upon which interviews were conducted with each group. The aim was for learners to reflect and provide first-hand feedback on what transpired during the problem-solving process. It was implicit that all participants could read with comprehension and possessed the ability to unravel mathematics word problems independently to some degree.

Ethical considerations

Ethical clearance to conduct this study was obtained from the University of the Western Cape Research Ethics Committee (HS19/2/14).

Research findings

The findings state the way in which the six questions are structured together with Table 1, Table 2, Table 3, Table 4, Table 5 and Table 6. These tables provide insight into the problem-solving methods learners used or applied for six

TABLE 1: Problem-solving methods employed for solving Problem 1.

	Methods employed by participants												Solutions		
	Just adding	Creating a sketch and adding	Grouping and adding	Multiplying and adding	Total	Just multiplying	Multiplying and dividing	Total	Just grouping	Creating a sketch and grouping	Total	Just creating a sketch	Correct	Incorrect	Total
Total	12	2	4	1	19	9	1	10	1	7	8	1	12	26	38

TABLE 2: Problem-solving methods applied for solving Problem 2.

	Methods employed by participants													Solutions		
	Just adding	Creating a sketch and adding	Dividing and adding	Total	Just dividing	Creating a sketch and dividing	Total	Just subtracting	Working in reverse and adding	Working in reverse, adding and dividing	Total	Grouping	Guessing	Correct	Incorrect	Total
Total	3	1	1	5	15	4	19	3	3	2	5	2	4	24	14	38

TABLE 3: Problem-solving methods applied for solving Problem 3.

	Methods employed by participants									Solutions					
	Just adding	Logical reasoning and adding	Total	Just dividing	Logical reasoning and dividing	Total	Subtracting	Logical reasoning and subtracting	Total	Just logical reasoning	Just tabulating	Guessing	Correct	Incorrect	Total
Total	5	1	6	8	3	11	3	1	4	11	2	4	7	31	38

TABLE 4: Problem-solving methods applied for solving Problem 4.

	Methods employed				Solutions (33)		
	Just adding	Just multiplying	Just dividing	Guessing	Correct	Incorrect	Total
Total	10	17	4	2	8	25	33

TABLE 5: Problem-solving methods applied for solving Problem 5.

	Methods employed			Solutions (33)		
	Just adding	Just multiplying	Guessing	Correct	Incorrect	Total
Total	29	1	3	8	25	33

TABLE 6: Problem-solving methods applied to solve Problem 6.

	Methods employed by participants								Solutions (33)		
	Just adding	Adding and multiplying	Add dividing	Total	Add and tabulating	Just multiplying	Just dividing	Guessing	Correct	Incorrect	Total
Total	6	1	5	14	2	2	10	7	11	22	33

problems in the test. A few images are included to provide a better sense of the problem-solving methods that learners applied. Next to each image appears a description of what the image depicts, which integrates the learners' responses on the task sheet (document analysis) with the responses from the interviews. The following methods are depicted in Figure 1, Figure 2, Figure 3, Figure 4 and Figure 5: (1) adding, (2) amalgamating dividing and creating a sketch or diagram, (3) dividing, (4) multiplying, (5) merging dividing and adding:

Problem 1: Sipho possesses 14 bits of rope. Each bit is 8 m long. What is the total length of rope Sipho has?

Table 1 specifies that 19 learners applied addition as a means of calculating problem 1. From the 19 addition methods, 12 involved just adding of numerical values, two solutions combined adding with creating sketches, four solutions combined adding with grouping. Only one solution combined adding with multiplying.

In addition, Table 1 reflects nine solutions based on just multiplying as the method, one solution relied on combining multiplying and dividing numbers. Seven of the solutions involved combining both grouping and creating sketches, while one solution applied grouping as the method. Just one applied a sketch as method. What is also evident is that for Problem 1 just 12 from 38 solutions were found to be accurate:

1. For problem 1, Learner-participant (LP) 28 just applied adding as a method as can be observed in Figure 1.

Problem 2: Kate possesses 100 cm of ribbon. She cuts it into four congruent pieces. What is the length of each piece of ribbon?

Table 2 reveals that three of the solutions just involved adding of values, whilst one solution is a combination of adding values and creating a sketch. One solution amalgamates adding and dividing as methods. Furthermore, 15 solutions exclusively used dividing, whilst four solutions merged dividing with creating sketches. Three solutions just relied on subtracting; three solutions amalgamated working in reverse and adding, and two solutions merged working in reverse with adding and dividing. Two solutions relied on grouping methods and four solutions applied only guessing as the method. Finally, 14 of the 38 solutions were found to be inaccurate:

2. LP 4's method: an amalgamation of dividing and a diagram as depicted in Figure 18.

Problem 3: Ciaran walked 1/2 km, Alex walked 1/5 km and Calum walked 1/4 km. Calculate the distance walked by each of the boys, in metres?

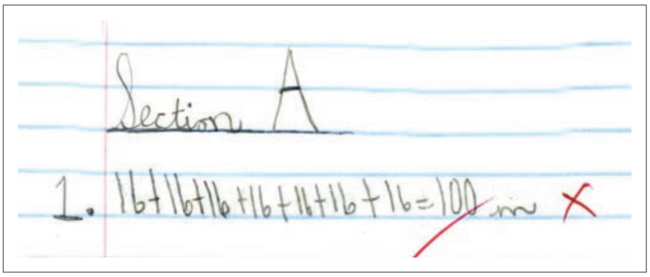
According to Table 3, five of the solutions applied addition as the only method. One solution reflected a combination of adding with logical reasoning, while eight solutions just applied dividing methods. Only one solution merged dividing and logical reasoning. Eleven solutions prove that logical reasoning was the only problem-solving method applied.

In addition, Table 3 reveals that just two of the solutions relied on tabulation of values. Four solutions applied methods involving guessing and from 38 solutions, 31 were incorrect:

3. LP 21's method of dividing as depicted in Figure 3.

Problem 4: A pencil has a length of 130 mm, and a ruler is three times longer than the pencil. What is the length of the ruler in centimetres?

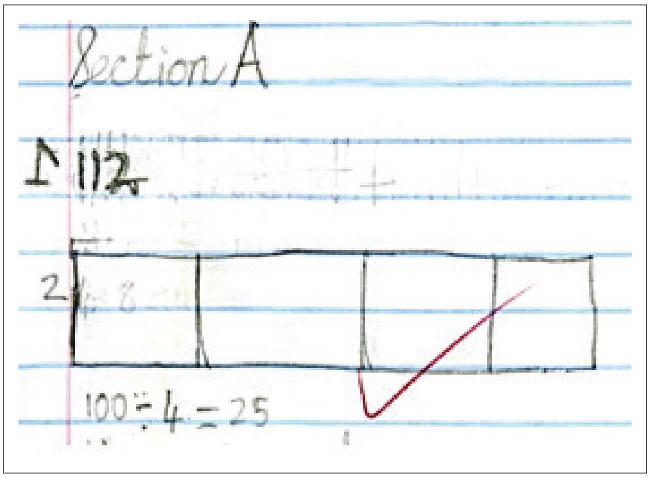
From Table 4 it is clear that 10 solutions applied adding of numbers exclusively, 17 used multiplying, while four solutions resorted to dividing methods only. Two solutions evidenced guessing as method only. Based on Table 4, out of the 33 solutions, 25 learners were successful:



Source: Learner 28

Note: LP 28 claimed that she grasped what the problem required, but simultaneously admitted it to have been quite demanding as she considered the numerical values used as big. Instead of adding 14 groups of 8, LP 28 used 7 groups of 16, *arguing that that way she had fewer values to add*.

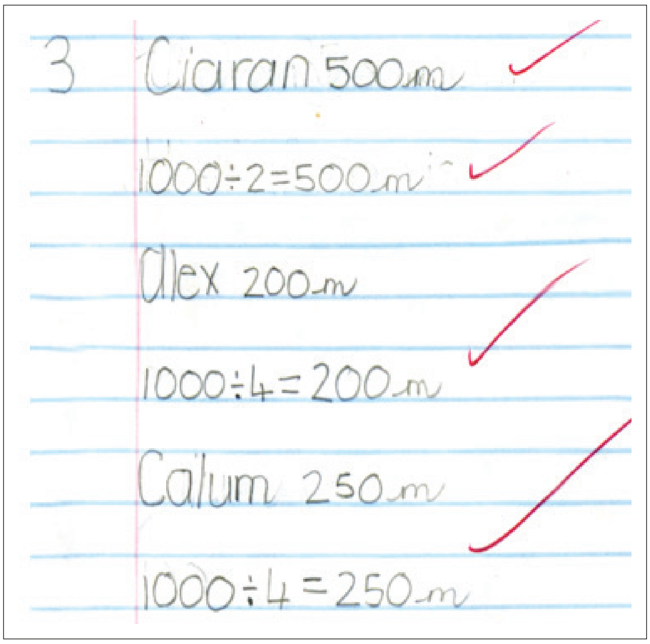
FIGURE 1: Application of adding as method by LP 28 to solve Problem 1.



Source: Learner 28

Note: Figure 2 depicts a diagram put forward by LP 4, in combination with dividing method. Starting with a rectangular shape which is then divided into 4 parts, *LP 4 said in the interview she recognised that the problem necessitated that the 100 cm of ribbon to be partitioned into 4 congruent parts*. The diagram thus facilitates the realisation as to which method was required. LP 4 managed to solve this problem.

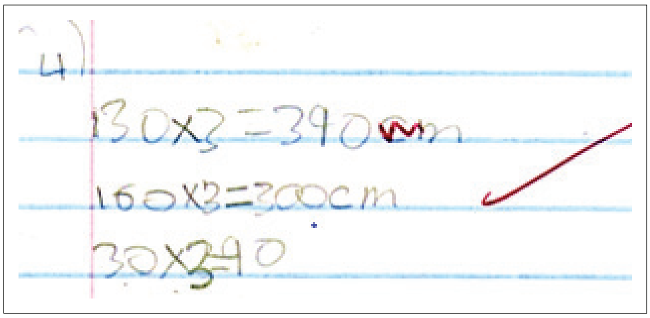
FIGURE 2: LP 4 making a drawing when solving Problem 2.



Source: Learner 21

Note: From Figure 21 it is evident that LP 21 applied dividing to obtain distances travelled by each learner in metres. LP 21 obtained the actual distances for all three children.

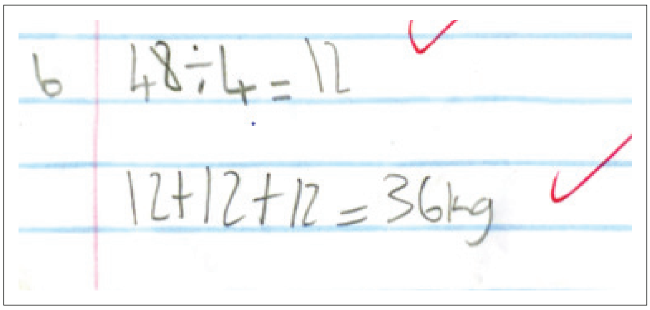
FIGURE 3: LP 21 using division when solving Problem 3.



Source: Learner 34

Note: Figure 4 is indicative of how an expansion method is applied by LP 34, namely multiplying 130 mm by 3 to arrive at 390 mm; LP 34 decomposes 130 into 100 and 30, each of which is multiplied by 3 prior to adding the two products to arrive at 390 mm. The final solution is not changed to centimetre as the problem demanded.

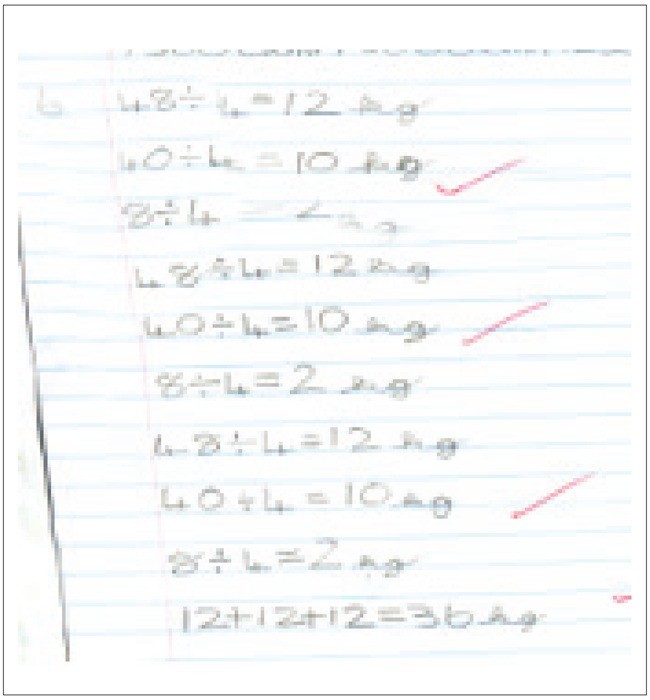
FIGURE 4: LP 34 using multiplication when solving Problem 4.



Source: Learner 28

Note: In Figure 5 LP 28 divides Mary's mass of 48 kg by 4 to obtaining 12 kg. Upon finding of the mass, LP 28 finds the sum of the three masses of 12 kg, arriving at of Mary's mass amounting to 36 kg. This was accurate.

FIGURE 5: LP 28 using both division and addition when solving Problem 5.



Source: Learner 21

Note: Figure 6 indicates that LP 21 initially employed division to acquire a quarter of 48 kg. After havening obtained value/quantity of 12 kg, she consequently added three x 12 kgs together to determine three quarters of 48 kg, upon which she obtained a final correct solution of 36 kg, LP 21 comprehended the problem well and explained her method, which led to an accurate solution during post-test interview:

LP 21: I divided 48. I said 40 divided by 4 equals 10. 8 divided by 4 equals 2.

LP 21 repeated these steps 3 times to get three 12s, then added them together to get 36.

FIGURE 6: LP 21 using division and repeated addition when solving Problem 6.

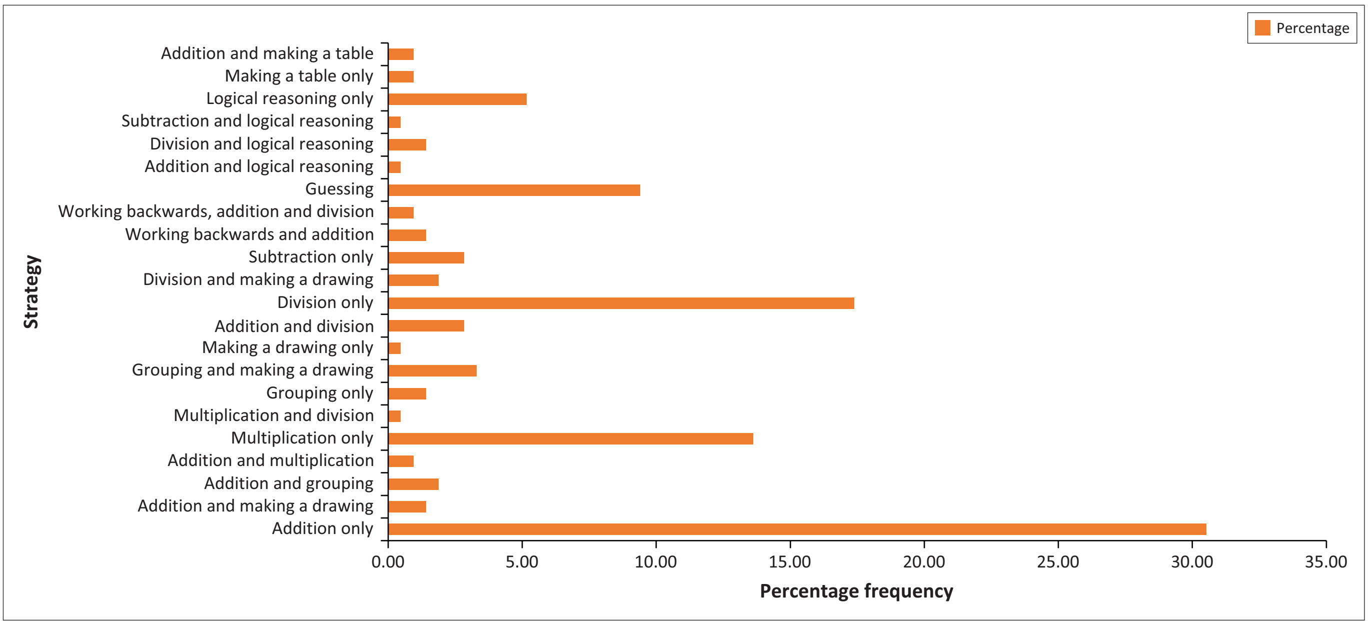


FIGURE 7: Percentage frequency of all methods used to solve the problems in Section A.

4. LP 34's multiplying method as depicted in Figure 4.

Problem 5: On day one John drove his motorcar a distance of 75 km. On day two he drove 12 500 m and on the third day 13 km. Calculate the total distance driven in metres?

According to Table 5, 29 of the solutions involved just adding of numbers, a single solution involved just multiplying, whilst three involved guessing methods. From the 33 solutions, the majority namely, 25 out of 33, contained mistakes:

5. LP 28's method involving merging dividing and adding is evident in Figure 5.

Problem 6: Mary weighs 48 kg. Paula, her younger sibling, has a weight three-quarters that of Mary. How much does Paula weigh?

Table 6 shows that 14 of learners' solutions involved adding as method. Six exclusively involved adding as a method. One learner combined adding and multiplying methods, while five merged addition and division methods. Only two amalgamated addition with creating a table:

6. L21's method of joining division and repeated addition is evident in Figure 6.

Furthermore, two solutions used only multiplication methods, 10 solutions used only division methods, while seven solutions used guessing. According to Table 6, out of the 33 solutions, 11 were correct and 22 were incorrect.

From Figure 7, it is evident that learners applied diverse problem-solving methods in Section A. As indicated by Figure 7, methods involving addition alone, were observed in 30.52% of the solutions. Methods comprising division solely, were found in 17.37% of all solutions, while 13.62% of all solutions contained only multiplication methods.

A fascinating result is linked to the application of methods by participants never dealt with in a classroom setup before. These include methods such as logical reasoning found in 5.16% of the total solutions. More riveting was the array of combination methods applied by participants. Methods of merging addition with creating a sketch, and merging addition with working in reverse, each appeared in 1.41% of the total solutions. These findings reveal learners' degree of innovation and the fact that they possessed the skill of acknowledging the necessity of applying diverse aspects of mathematics to solve problems.

Discussion of the findings

Observations of how participant learners went about solving measurement word problems were analysed, with specific emphasis on the methods those learners employed. These methods were identified as singular methods, and combination methods.

Singular methods involved: adding; multiplying; sketches or diagrams; grouping or clustering; dividing; subtracting; rational reasoning; predicting or conjecturing and tabulating values. Combination methods identified, were: adding values and constructing sketches and diagrams; adding and grouping values; adding and multiplying; multiplying and dividing; grouping and constructing sketches; adding and dividing; dividing and constructing sketches; working in reverse order and adding; working in reverse order, adding and dividing; adding and rational reasoning; dividing and rational reasoning; subtracting and rational reasoning; adding and tabulating; adding and dividing; dividing and grouping; subtraction and grouping; multiplying and constructing sketches; adding and working in reverse order; multiplying and adding; multiplying and subtracting; multiplying, adding and constructing sketches; multiplying, adding and decomposing the problem; multiplying, constructing sketches and decomposing the problem; multiplying, adding, constructing a sketch and decomposing the problem; adding and decomposing the problem and multiplying, subtracting and constructing a sketch.

It is noticed that singular methods were mainly dealt with in the Foundation Phase, which implies that learner-participants were familiar with only the four basic operations, namely adding, subtracting, multiplying and dividing, as well creating sketches. Evidently, participants tended to fall back on methods and operations they were familiar with. Furthermore, learners were inclined to resort to the construction of sketches as method, which was frequently applied in the Foundation Phase as the underpinning of their solutions. They would then build on these sketches to incorporate them into the said problems. Research by Bruun (2013) similarly revealed that constructing sketches was the most frequently utilised method. A similar result was evident in the study by Yasmeen (2019), who stated that learners achieved reasonably well when incorporating sketches or diagrams into their methods to solve problems.

Arithmetically based methods, and resorting to diagrams, sketches or illustrations were applied most often across all problems. Participants commented that they liked applying the said methods alongside each other. Learners capitalised on a variety of methods in an effort to find the correct solution. A past study undertaken by Sulak (2010) revealed comparable outcomes. Sulak's (2010) findings revealed that learner-participants employed methods such as: illustrations or sketches, numerical tables, written sentences in pursuit of possible patterns, creating word lists, applying logical reasoning and the guessing-and-checking or trial-and-error method.

It is evident that learners conceptualised the meanings conveyed within word problems confronted within the study, and mentally constructed apt or fitting methods, but often arrived at wrong outcomes because of calculation errors. Calculation challenges can be countered or remedied through constructive practise, thereby guaranteeing greater success at solving word problems. Lubin, Houde and De Neys (2015) agree with this, advocating that it is crucial and a prerequisite for learners to possess basic arithmetic calculation skills as these are generally inherent to word problems encountered in the mathematics curriculum. One is obliged to applaud learners for their vision and ability to use such a wide range of methods. These results concur with those of Intaros, Imprasitha and Srisawadi (2014), who established the fact that learners demonstrated competencies to construct their own problem-solving methods, with specific reference to solving open-ended word problems.

The phenomenon of employing a combination of methods for solving word problems that was alluring and impressive was the variety of differing combination employed when engaging in specific calculations. Certain combinations merged two, three or more methods to solve a single word problem. Saygili's (2017) research findings confirm this when stating that learners resort to a multiplicity of methods when solving non-routine and unfamiliar problems. Upon executing multiplication as an operation, certain learners resorted to decomposing the larger numbers into smaller constituent numbers. Each (constituent) smaller numeric value is then multiplied by the specific number, the answers are regrouped, and through addition one can arrive at the final answer. Likewise, learners multiply large numeric values by means of recurrent addition. This method was preferred by those who felt more at ease applying addition, rather than multiplication.

An additional noteworthy observation was learners' ability of employing particular arithmetic methods jointly with drawings (or diagrams). Upon planning their methods, certain learners merged different methods to facilitate the problem-solving process. This is indicative of the fact that learners recognised that it was possible to amalgamate methods to facilitate the problem-solving procedure. As alluded to formerly, the predominant couple of methods that were amalgamated were identified as simple arithmetic calculations and sketches or diagrams. The constructed sketches would reflect learners' grasp or their conception of what the problem expects them to do. The type of sketch relates to the suitable arithmetic operations that would generate the solution. This finding aligns with Mudaly (2012) who maintains that the use of drawings when solving problems actively involves learners in constructing meaning.

Conclusion

Within the realm of constructivism, knowledge is generally constructed by means of observing other human beings and occurrences, and other individuals' experiences, according to Major and Mangope (2012). This study offered learners the opportunity and space to operate individualistically, empowering them to construct a unique conceptualisation of problems encountered or confronted with. The learners forged sensible associations and devised authentic methods in accordance with their conceptions of each problem; Creating opportunities for learners to thrive in a constructivist setting, not only revealed a multitude of methods learners put forward, but similarly showed a varied set of methods they used to manifest their chosen methods. Operating in a constructivist setting facilitates reflection, and remedying of own mistakes, which is linked to learner's unique conceptions of measurement. This, in all likelihood compels learners to become increasingly engrossed in the problem-solving process and achieve deeper insight into what the questions required (Shah 2019).

The focus group interviews served as a means for participant learners to engage in self-reflection and self-critique not only in terms of their solutions but sharing their opinions as well. Communicating perceptions and opinions in groups allowed learners to deliberate with peers on matching cognitive levels. Consequently, this facilitates interpreting communication and conceptualisation by making it much easier as they used language and vocabulary that all can comprehend (Polya 1957).

Most learner-participants made perfect sense of the problems as they could accurately relay – in their own words – what questions required them to do. Learners frequently successfully solved problems that were structured in ways they encountered before. Such word problems are said to contain 'sight words', which are ordinary and familiar words that children recognise instantaneously. These sight words assist in identifying, which operations are more suitable and would ensure success.

Less than 50% of the learners managed to arrive at the correct solution because of incorrect application of problem-solving methods. This state of affairs could be ascribed to careless or computational errors made by learners who applied arithmetic calculations as their problem-solving methods. Again, it accentuates the claim that more practice and re-enforcement exercises are needed, with specific emphasis on number manipulation skills. An additional reason said to cause faulty solutions is blamed on incorrect application of diagrams or sketches. Indecisiveness of how such sketches should be optimally employed to facilitate or enhance the problem-solving process was evident. A further observation not only centred on certain learners' precise graphic representations of the problems confronted with, but on their stark inability to apply the proper mathematical methods required. These results are in accordance with findings presented by Peranginangin (2017), which claimed that learners found it to be challenging to execute their intended methods as well as to reason meta-cognitively when executing their methods.

It is noticed that some of the broad variety of sketches and diagrams employed to solve measurement contextual problems were either inaccurate or not corresponded to the problem. These findings resemble those in the study by Tambychik and Meerah (2010), who maintained that learners are incapable of constructing precise or fitting graphic illustrations of word problems or they could not comprehend problems efficiently.

During self-critique sessions learners offered truthful, candid and reasonably realistic responses. Learners, in retrospect, generally expressed satisfaction with methods they selected and how those were applied. Interestingly, there were those learners who expressed dissatisfaction and regretted not having used alternative methods instead. This realisation emerged during peer group discussions upon learning what others did in similar situations. The phenomenon of discussing what worked and what did not, could be interpreted as reflecting metacognition in action, where leaners ponder on what their strengths and weaknesses were (Alzahrani 2017). This accomplishment where learners think about their thinking, which emanated from learners' discussions is considered a principal aim of this study.

The contribution of this study to the research field is situated in the insights gained on the methods learners in the early grades employed to solve contextual mathematical problems. The analysis of the methods used by Grade 4 learners provides insight into learners' sense-making, and their approaches employed in solving the given problems. In addition, the findings give teachers' ideas to learners' shortcomings, and limitations of existing misconceptions and limitations in terms of logical thinking and reasoning.

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Competing interests

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Authors' contributions

R.G. and S.R. were part of the team that conceptualised the study and the article, invoked applied methodology and conducted the formal analysis. S.R. carried out the investigation and curated the data. R.G., S.A.A., and S.R. prepared the draft manuscript and through several iterations of review and editing produced the final article for submission.

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Data availability

The data that support the findings of this study are available from the corresponding author, R.G. upon reasonable request.

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