Harmony in Teaching: Unraveling the Interplay between Pre-Service Teachers' Mathematical Knowledge Fractions and Classroom Practices

Charles Kwabena Sie[1], Douglas Darko Agyei[2]

[1]Nsawkaw College of Education, Tain District, Ghana
[2] Department of Mathematics and ICT Education, University of Cape Coast, Ghana
Siecharles78@gmail.com, ddagyei@ucc.edu.gh

Abstract: This study delves into the intricate relationship between pre-service teachers' (PSTs') Mathematical Knowledge for Teaching Fractions (MKTF) and its influence on their teaching practices. Grounded in the premise that MKTF domains exhibit interconnectivity, shaping the constructs of teaching practices, the study employed the mathematical task framework and the framework for mathematical knowledge for teaching. Utilizing the Mathematical Knowledge for Teaching Fractions test and the Teaching Practices test, data were collected from 171 PSTs. Regression analyses uncovered significant effects of MKTF domains on five teaching practice components, underscoring the pivotal role of a teacher's mathematical knowledge in effective teaching. Notably, among the six MKTF domains, the KCFS domain emerged as the most fundamental, strongly predicting various MKTF domains and influencing teaching practice constructs. This study underscores the significance of the KCFS domain in shaping both MKTF domains and instructional practices. The findings bear implications for the education of PSTs in Ghana and other nations facing similar educational landscapes.

Keywords: Mathematical knowledge for teaching, Teaching practices, Mathematical task Framework, Lesson script

INTRODUCTION

Studies have shown that mathematical knowledge for teaching influences teaching practice (Hoover et al. 2016). These influences are however not clear since there is a complex connection between mathematical knowledge for teaching and teaching practices that lead to the quality of instruction. Notwithstanding this, Hill, Umland, Litke, and Kapitula (2012) study have reported that, weak mathematical knowledge for teaching predicts low quality teaching practices, and strong mathematical knowledge for teaching predicts high quality teaching practices. On the other hand, Hill et al. (2008) suggest that there are other factors such as: professional development, supplemental curriculum materials and teacher beliefs that potentially have influence on the quality of teaching practices, but these factors may cut both ways depending on the teachers' mathematical knowledge for teaching. In addition, efforts to clarify the conceptualization of mathematical knowledge for teaching continue to be concerned with the dynamic nature of mathematical knowledge for teaching, the usefulness of knowledge, and whether, when, and how it plays in teaching (Hoover et al, 2016; Kersting et al, 2012). Although mathematical knowledge for teaching (MKT) have been shown to have influence on teaching practices, the question of what kind of teaching tasks require which domain, still require further attention (Ball et al., 2008; Markworth, Goodwin, & Glisson, 2009). These discussions suggest that along teachers' mathematical knowledge, teachers' teaching practices are key in order to produce lessons in which learners will be exposed to high quality tasks that help them to learn concepts and procedures in mathematics with understanding (Addae & Agyei, 2018). This in turn produces in learners' self-confidence to engage in challenging mathematical tasks that are provided in a rich mathematics curriculum (NCTM, 2000).

From analysis of video-recorded classroom observations and teacher interviews, Cengiz, Kline and Grant (2011) provide detailed accounts of teaching and "demonstrate that MKT matters in the way teachers pursue student thinking" (Cengiz et al., 2011). Their analysis of data from one of the participating teachers "provide evidence that a lack of certain aspects of knowledge can negatively impact a teacher's pursuit of student thinking" (p. 372). Steele and Rogers (2012) argue that the more experienced and MKT-knowledgeable teacher not only enacts a stronger and more nuanced lesson on mathematical proof, but her students end up having more mathematical authority. A study by Tanase (2011) suggests that teachers' knowledge goes beyond their own mathematical understanding. Differences are observed in teachers' ability to make connections between fraction concepts and other mathematical concepts, how they set different objectives for students as well as the extent to which they challenge students in their mathematical work (Charalambous, 2008). Tanase also observed that teachers who have strong mathematical knowledge for teaching, are able to produce lessons with high quality of instruction. Johnson and Larsen's (2012) study posit that teachers need not only knowledge of students' misconceptions, but also knowledge of when and why students are likely to be confused and display misconceptions and of the consequences of such misconceptions when students engage in new activities. In his study of mathematics teacher knowledge and its impact on how teachers engage students with challenging tasks, Choppin (2011) noted that teacher's knowledge appears to influence teaching in the adaptation of tasks. Engaging students with challenging tasks is an important component of the work of teaching mathematics, and so is the selection and use of appropriate examples. Charalambous (2008) explored the relationships between pre-service teachers' MKT and their five teaching practices (selecting and using tasks, using representations; providing explanations, responding to students' requests for help and analysing student's work/contributions) required for quality teaching. Charalambous study did not show the impact of the domains of MKT on the five teaching practices. Additionally, his study considered only two mathematical knowledge for teaching domains (common content knowledge and special content knowledge).

Similar to Charalambous (2008) study, our previous study (Sie & Agyei, 2023) also identified significant relationships between pre-service teachers' MKTF domains and five teaching practices constructs. The present study, unlike the previous studies (Charalambous, 2008; Sie & Agyei, 2023) focused on examining the impact of the domains of MKT on the five teaching practices constructs. Applying the mathematical knowledge for teaching domains and the mathematical tasks framework we hypothesis that mathematical knowledge for teaching domains contribute to the ability to perform the five teaching practices.

Conceptual Framework for the Study

Ball et al. (2008) developed their conceptualization of MKT, which is based on the actual teaching practices of mathematics teachers, to include six knowledge domains: Common Content Knowledge (CCK), Specialized Content Knowledge (SCK), Horizon Content Knowledge (HCK), Knowledge of Content and Students (KCS), Knowledge of Content and Teaching (KCT), and Knowledge of Content and Curriculum (KCC). Researchers have recognized the following issues that still need more research on MKT as a common framework for understanding teachers' mathematical knowledge for teaching: (a) Which domain is required for what kinds of teaching tasks? (b) What connection exists between the MKT domains? (c) What are the MKT domains' actual definitions? (Ball et al., 2008; Markworth, Goodwin, & Glisson, 2009).

The design, presentation, and execution of tasks—the three phases of the instructional process—are analyzed using the Mathematical Task Framework (MTF) (Stein & Smith, 1998; Stein et al., 2000). According to Stein and Smith (1998), students engage in one of two forms of thinking depending on whether they are required to memorize methods in a systematic way (instrumental thinking) or think conceptually and draw connections during a task (relational thinking). This indicates that how tasks are selected and carried out during instruction has an impact on what students learn.

Based on the MTF, Charalambous (2008) suggested a few teaching practices under each of the three stages that instructional tasks go through. These teaching practices, which are thought to improve the quality of mathematics instruction, were used in the study. They included choosing and using tasks, using representations, giving explanations, responding to students' direct or indirect requests for help, and analyzing students' work and contributions. Teachers are expected to carry out specific duties (such as selecting instructional tasks, modifying/adapting instructional tasks, sequencing instructional tasks, and anticipating students' faults or difficulties) and create lesson plans during the planning phase. During the presentation phase, teachers are expected to present definitions, explain concepts, give examples and counterexamples, use analogies, use representations and manipulatives, establish connections between various concepts and representations, and simplify tasks to support student success. Additionally, during the enactment phase, teachers work alongside students on assigned activities or tasks while implementing specific teaching strategies (e.g., responding to students' help requests, monitoring and analyzing students' thinking, spotting mistakes, appreciating students' alternative approaches, posing probing questions, and facilitating the exchange of multiple ideas or solutions) (Charalambous, 2008; Fumador & Agyei, 2018). Despite being discussed and presented individually, these three phases have no distinct limits, according to Charalambous. Charalambous noted that though these three phases are discussed and presented separately, there are no clear boundaries between them. This study adopted a conceptual framework based on the earlier work of the authors (Sie & Agyei, 2023) that is depicted in Figure 1 by combining the theory of MKT with the mathematical task framework.

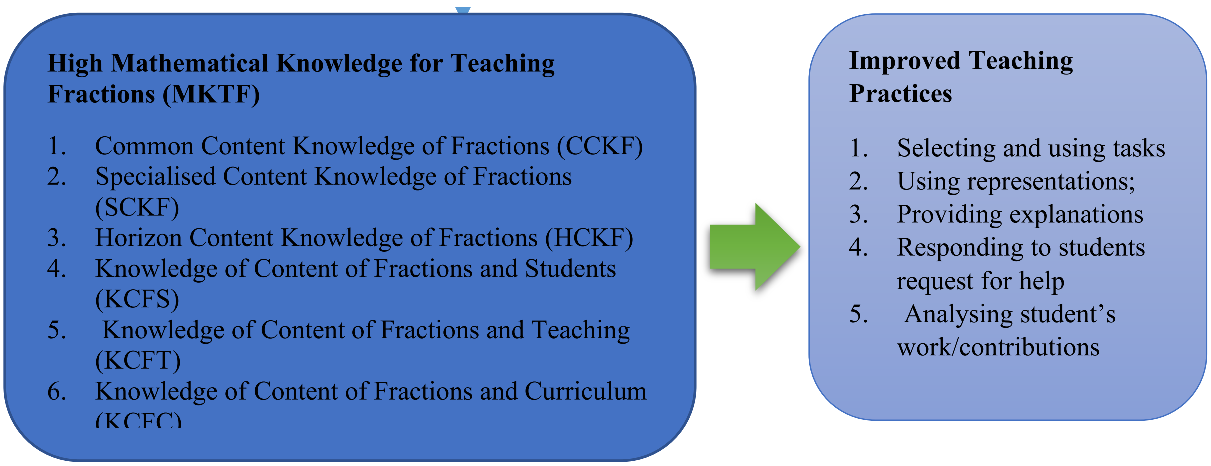

Figure 1: Conceptual framework of the study

From the figure, the study hypothesizes that mathematical knowledge for teaching domains have great impact on the five teaching practices constructs and that MKTF domains impacts positively to influence each other and influence the teaching practices constructs.

The MKTF domains are described in this study as follows:

- *Common Content Knowledge of Fractions (CCKF*) is the level of proficiency in fractional knowledge that comprises an understanding of the ideas, terms, definitions, rules, and symbols used in fractions that all workers who use fractions must possess.
- *Specialized Content Knowledge of Fractions (SCKF)* is the knowledge of: multiple solution strategies, generalizations, figuring out why an algorithm works or makes sense, explaining ideas by using appropriate examples and representations to visualize fractions, making connections between various representations, and figuring out actual definitions.
- *Horizon Content Knowledge of Fractions (HCKF)* refers to knowledge and awareness of how topics in the mathematics curriculum are related so that teachers can draw connections to topics while teaching fractions.
- *Knowledge of Content of Fractions and Students (KCFS*) refers to understanding of how students learn fractions, including understanding of frequent mistakes, common misconceptions, and challenges that students have in learning fractions.

- *Knowledge of Content of Fractions and Teaching (KCFT)* refers to understanding the fundamental concepts behind fractions as well as the various instructional approaches that can be utilized to teach them.

- *Knowledge of Content of Fractions and Curriculum (KCFC)* is the understanding of the contents as well as their organization that is required for teaching fractions at a particular level and at different levels.

The teaching practices constructs are also described as follows:

- *Selection and using instructional tasks* refer to the capacity to select, modify, and arrange instructional tasks in a way that challenges students' cognitive abilities to learn and establish connections for conceptual understanding.
- *Providing explanations* refers to the capacity of a teacher to give concise explanations that aid pupils in understanding the mathematics being taught. Here, a teacher creates and provides simple, student-understandable mathematical examples, counterexamples, and analogies.
- *Using representations* is the ability to enhance student learning by working with and around representational modes.
- *Analysing students' work and contributions* is the ability to evaluate student explanations and decipher what they say, to assess the validity of students' mathematical strategies and non-routine approaches to problem-solving, to assess what students know and their knowledge gaps based on their work and contributions or their errors
- *Responding to student's requests for help* refers to the ability of a teacher to respond to and attend to students' requests, either directly or indirectly.

Research Design and Questions

The study employed a correlational research design in which the researchers sought to identify the relationship between pre-service teachers MKTF and their teaching practices and identify how PSTs MKTF predict their teaching practices. The study addressed two main research questions: (i) what is the impact of each MKTF domains on the other domains? And (ii) what is the impact of each of the six MKTF domains (*CCKF, SCKF, HCKF, KCFT, KCFS, and KCFC*) on the five constructs of teaching practices?

METHOD

Respondents

The study's population was pre-service mathematics teachers at Ghana's 46 public colleges of education. To avoid the dangers of long-distance travel during the COVID outbreak, the available population of the study was chosen to include pre-service mathematics teachers from five colleges which were conveniently sampled. Out of 1445 prospective mathematics teachers from the five colleges of education, 171 were chosen at random to make up the study's sample using the stratified random sampling method.

Instruments

The MKTF domains and teaching practices components of the 171 PSTs were assessed using two tests, the Mathematical Knowledge for Teaching Fractions Test and the Teaching Practices Test, respectively. The sections below that follow discuss these instruments in more detail.

Mathematical Knowledge for Teaching Fractions Test

To assess PSTs' MKT in fractions, the researcher modified the online sample of the Learning Mathematics for Teaching (LMT) test items by Hill, Schilling, and Ball (2004). There are 64 test items on number, algebra, and operations in the online LMT sample test items. Several of the items on this instrument were found to be unrelated to the current study after analysis. As a result, 11 fraction-related items were chosen, altered, and used in the research. The online LMT test questions include questions that could assess knowledge in the CCK, SCK, KCT, and KCS knowledge domains of MKT. By reviewing earlier studies (Shulman, 1986; Ball, et al., 2008; Cole, 2012; Sugilar, 2016; Avcu, 2019) that highlighted the concepts and skills that instructors must possess in order to teach fractions properly, we were able to expand the LMT items by 33 to adequately cover all six knowledge domains of MKT (Ball, et al., 2008).

The Mathematical Knowledge for Teaching Fractions Test, which was composed of redesigned and modified LMT test items, had closed-ended questions. The responses of pre-service teachers to each question on the MKTF test were dichotomously evaluated on a 2-point scale: 0 for an incorrect response and 1 for an appropriate one. A total of 31 out of the test's 44 questions were scored and categorized into the six MKTF domains: CCKF (8 questions), SCKF (6 questions), HCKF (3 questions), KCFT (3 questions), KCFS (4 questions), and KCFC (7 questions). For ease of comparison, the total score for each MKTF domain was standardized to a maximum value of 8 points. A score of 4 was considered as the average score point value. A score of 4 or higher was regarded as a high MKTF score, while a score of lower than 4 was regarded as a low MKTF score. The MKTF domains' Kuder-Richardson reliabilities were higher than the acceptable cutoff value of 0.60, ranging from 0.64 to 0.82 (CCKF, =0.72; SCKF, =0.70; HCKF, =0.82; KCFT, =0.64; KCFS, =0.73; and KCFC, =0.79).

Teaching Practices Test

The five teaching practices of selecting and using tasks, using representations, providing explanations, responding to students' requests for assistance, and analyzing student work/contributions were all included in the measurement of teaching practices. For the study, the Teaching Practice Test was modified from Charalambous (2008) teaching practices interview guide. Charalambous used a 24-item interview guide to examine the effectiveness of 20 pre-service teachers in five different teaching practices. However, the adapted teaching practices test in this study included 27 test items, to which respondents were required to answer at various stages with information regarding what they had observed, how they had interpreted it, and how they would have carried out such activities. A lesson script containing the five teaching practices a teacher might employ in a lesson on fractional division was provided along with the test. The PSTs were required to read the lesson script and respond to test questions on the teaching practices they observed, how they perceived or assessed them, and how they would put those interpretations or evaluations into practice.

The test, referred to as the "teaching practices test," was made up of closed-ended questions, and responses from pre-service teachers were graded on a dichotomous 2-point scale of 0 and 1, where a score of zero (0) indicated an incorrect response and a score of one (1) indicated a correct response. Twenty of the test's twenty-seven items were scored, and they were categorized into groups according to the five teaching practices constructs: selecting and using tasks (6 items), using representations (4 items), providing explanations (4 items), responding to students' requests for help (3 items), and analyzing students' work and contributions (3 items). The overall score for each of the teaching practices constructs was standardized to the same scale maximum value of 6 points to make scoring easier to compare. Obtaining a score of 3 was regarded as the average point value. Thus, a high score in teaching practices was understood to mean having a score of 3 or higher, while a low score in teaching practices was understood to imply having a score lower than 3. Three of the teaching practices constructs had Kuder-Richardson reliabilities that ranged from 0.61 to 0.81, exceeding the acceptable threshold value of 0.60 (providing explanations, α =0.61; analyzing student work/contributions, α =0.81; and using representations, α =0.68), whiles the Kuder-Richardson reliabilities for the remaining two teaching practices constructs (selecting and using tasks, $\alpha = 0.54$; and responding to students' requests for help, $\alpha = 0.51$) which did not meet the acceptable threshold of 0.60; where later accepted by the researchers as having moderate reliabilities based on Hinton et al.'s (2014) guidance on appropriate cut-off points for reliability coefficients.

Data analysis

The positivist approach was used in this study to analyze numerical information about MKTF and teaching practices from a sample of 171 PSTs. The data was analyzed using both descriptive (mean and standard deviation) and inferential (regression) statistical methodologies.

RESULT

We conducted descriptive analyses to determine the mean and standard deviation of the mathematical knowledge for teaching fractions domains and the teaching practices constructs before determining the effects of pre-service teachers' MKT on their teaching practices. In tables 1 and 2, the outcomes of the descriptive analyses are displayed. The descriptive statistics for each of the six categories of mathematical knowledge for teaching fractions are shown in Table 1 below.

MKT Domain	Mean	Std. Deviation
Horizon Content Knowledge of Fractions (HCKF)	4.83	3.342
Knowledge of Content of Fractions and Curriculum (KCFC)	4.54	2.350
Knowledge of Content of Fractions and Teaching (KCFT)	3.58	2.020
Special Content Knowledge of Fractions (SCKF)	3.36	2.109
Common Content Knowledge of Fractions (CCKF)	3.22	2.381
Knowledge of Content of Fractions and Students (KCFS)	2.91	2.051

Table 1: The descriptive statistics of the six MKTF domains (N = 171)

The range of MKT domains' average scores was 0.291 to 4.83. The findings reveal a variability in the MKTF domains' ratings, which went from 2.020 to 3.342. In comparison to the average score point value of 4, the PSTs' mean scores (HCKF, M = 4.83; KCFC, M = 4.54) showed that PSTs generally performed well in these two MKTF domains. The PSTs mean scores in these four domains (KCFT, M = 3.58; SCKF, M = 3.36; CCKF, M = 3.22; and KCFS, M = 2.91) were, nevertheless, low when compared to the average score point value of 4. Additionally, PSTs scored the lowest on average in the KCFS.

For the pre-service teachers teaching practices, the descriptive statistics for the five constructs of teaching practices are shown in Table 2.

Teaching Practices	Mean	Standard Deviation
Tasks	2.17	1.371
Explanations	2.04	1.441
Analysing	2.03	1.708
Representations	1.48	1.699
Requests	1.26	1.320

Table 2: The descriptive statistics of the five teaching practices (N = 171)

Table 2 shows the average scores pre-service teachers obtained for the five components of teaching practices, which ranged from 1.26 to 2.17. The PST results revealed a spread in scores for the five teaching practices components, ranging from 1.320 to 1.708. The PSTs' mean scores (Tasks, M = 2.17; Explanations, M = 2.04; Analyzing, M = 2.03; Representations, M = 1.48; and Requests, M = 1.26) showed that, on average, PSTs performed poorly in all five teaching practices components when compared to the average score point value of 3. Responding to students' requests for help received the lowest average score among the PSTs' teaching practice constructs.

The researchers performed multiple linear regression analyses to explore the impact of each MKTF domain on the other domains (*CCKF, SCKF, HCKF, KCFT, KCFS, and KCFC*). Table 3 below shows the results of these analyses.

Independent Variables		Dependent Variable					
		CCKF	SCKF	HCKF	KCFT	KCFS	KCFC
CCKF	Beta		0.144	-0.028	-0.050	0.133	0.076
	(Sig)		(0.031)	(0.694)	(0.524)	(0.039)	(260)
SCKF	Beta	0.194		-0.054	0.150	0.381	0.255
	(Sig)	(0.031)		(0.505)	(0.097)	(0.000)	(0.001)
HCKF	Beta	-0.034	-0.050		0.320	0.224	0.343
	(Sig)	(0.694)	(0.505)		(0.000)	(0.002)	(0.000)
KCFT	Beta	-0.049	0.111	0.257		-0.228	0.018
	(Sig)	(0.524)	(0.097)	(0.000)		(0.000)	(791)
KCFS	Beta	0.192	0.410	0.263	-0.333		0.147
	(Sig)	(0.039)	(0.000)	(0.002)	(0.000)		(0.070
KCFC	Beta	0.101	0.251	0.368	0.024	0.134	
	(Sig)	(0.260)	(0.001)	(0.000)	(0.791)	(0.070)	

Table 3: Predicting each MKTF domain by other MKTF domains

From table 3, the results of the multiple linear regression analyses reveal significant positive impact of: SCKF (β = 0.194, P = 0.031); KCFS (β = 0.192, P = 0.039) in predicting PSTs *CCKF*. However, the results did not reveal any significant impacts of: HCKF (β = -0.034, P = 0.694); KCFT (β = -0.049, P = 0.524); and KCFC (β = 0.101, P = 0.260*)* in predicting the PSTs CCKF. The result therefore shows that two other MKTF domains of PSTs (SCKF and KCFS) impacted positively to predict their CCKF domain of MKT which appears to suggest that the PSTs in these two knowledge domains have direct influence on their development of the CCKF knowledge domain. Of the two MKTF domains (SCKF and KCFS) that significantly predicted PSTs CCKF domain of MKTF, the impact of SCKF was a little higher than the impact of KCFs. Thus, the study identified the PSTs' SCKF domain as having the greatest influence on their development of CCKF domain.

With regards to the SCKF domain, the results showed significant positive impacts of three other MKTF domains: CCKF (β = 0.144, P = 0.031); KCFS (β = 0.410, P = 0.000); and KCFC (β = 0.251, P = 0.001) in predicting the PSTs SCKF domain. On the other hand, the results for two other MKTF domains: HCKF (β = -0.050, P = 0.505); and KCFT (β = 0.111, P = 0.097) were not significant in predicting the PSTs SCKF domain. Of the three MKTF domains (CCKF, KCFS and KCFC) that significantly predicted PSTs SCKF knowledge domain, the impact of KCFS was the highest. Thus, identifying PSTs' KCFS as the best strongest predictor of their SCKF knowledge domain, and hence having a pronounced positive influence on the development of the PSTs SCKF knowledge domain.

With regard to PSTs other MKTF domains predicting their HCKF domain, the results have shown significant positive impacts of three MKTF domains: KCFT (β = 0.257, P = 0.000); KCFS (β = 0.263, P = 0.002); and KCFC (β = 0.368, P = 0.000*)*, at α=0.05. However, the impacts of: CCKF (β = -0.028, P = 0.694); and SCKF (β = -0.054, P = 0.505) did not show significant predictions of the PSTs HCKF knowledge domain. Of the MKTF domains that significantly predicted PSTs HCKF domain, the impact of KCFC seems to be very pronounced compared to the others, followed by KCFS and least by KCFT. Thus, the result has shown that the impact of PSTs KCFC was most pronounced on their HCKF domain, an indication that the PSTs KCFC knowledge domain has the greatest influence on the development of their HCKF knowledge domain.

With respect to PSTs other MKTF domains predicting KCFT domain, the results have shown a significant positive impact of: HCKF (β = 0.320, P = 0.000); and negative impact of KCFS (β = -0.333, P = 0.000) in predicting the PSTs' KCFT. However, the results did not show significant impacts of: CCKF (β = - 0.050, P = 0.524); SCKF (β = 0.150, P = 0.097); and KCFC (β = 0.024, P = 0.791) in predicting the PSTs KCFT. Of the two MKTF domains that significantly predicted the PSTs KCFT domain, the impact of KCFS was negative whiles the impact of HCKF was positive, suggesting that a PST KCFS has negative influence on the development of their KCFT whiles PSTs HCKF has positive influence on the development of their KCFT knowledge domain.

With regard to predicting the PSTs KCFS domain, the results have shown the impacts of three other MKTF domains: CCKF (β = 0.133, P = 0.039); SCKF (β = 0.381, P = 0.000); HCKF (β = 0.224, P = 0.002)*;* as positive and significant and one other MKTF domain: KCFT (β = -0.228, P = 0.000) as negative and significant. However, the impact of KCFC (β = 0.134, P = 0.070) was not significant indicating that the PSTs knowledge in this domain did not significantly predict their KCFS knowledge domain. Of the four MKTF domains that significantly predicted the PSTs KCFT domain, the results showed negative influence by the PSTs KCFT domain and showed positive influence by the other three MKTF domains (CCKF, SCKF and HCKF). The results also revealed that, of the MKTF domains that significantly predicted PSTs KCFS, the impact of SCKF appears to be very pronounced compared to the others. The finding therefore show that the PSTs SCKF knowledge domain has the greatest influence on the development of their KCFS knowledge domain.

Regarding the PSTs other MKTF domains predicting the PSTs KCFC domain, the results have shown significant positive impacts of: SCKF (β = 0.133, P = 0.039); and HCKF (β = 0.224, P = 0.002) in predicting PSTs KCFC domain. However, the impact of: KCFC (β = 0.134, P = 0.070); KCFC (β = 0.134, P = 0.070); and KCFC (β = 0.134, P = 0.070*)* were not significant indicating that the PSTs knowledge in these domains did not significantly predict their KCFC knowledge domain. Of the MKTF domains that significantly predicted PSTs KCFC, the impact of HCKF appears to be very pronounced compared to the impact of SCKF. The finding therefore show that the PSTs HCKF knowledge domain has the greatest influence on the development of their KCFC knowledge domain.

To sum up, the results showed that each of the PSTs MKTF domains was significantly predicted by at least two other MKTF domains an indication that the PSTs MKTF domains influence each other. The results have shown the PSTs KCFS as a very influential knowledge domain as it significantly predicted most of the MKTF domains of the PSTs.

The researchers performed multiple linear regression analyses to explore the effect of each of the six MKTF domains (CCKF, SCKF, HCKF, KCFT, KCFS, and KCFC) in predicting each of the five constructs of teaching practices. Table 4 below provides the summary of the results.

MKTF Domain	Standardised Coefficients (Beta)				
	Tasks	Representations	Explanations	Requests	Analysing
CCKF	0.086	0.174	0.252	-0.020	0.251
(Sig)	(0.266)	(0.013)	(0.001)	(0.786)	(0.001)
SCKF	-0.080	0.129	0.084	0.059	0.162
(Sig)	(0.372)	(0.110)	(0.337)	(0.498)	(0.056)
HCKF	-0.031	0.078	0.164	0.051	0.159
(Sig)	(0.722)	(0.314)	(0.052)	(0.537)	(0.049)
KCFT	0.005	0.048	0.143	0.142	0.254
(Sig)	(0.945)	(0.484)	(0.059)	(0.054)	(0.001)
KCFS	0.349	0.314	-0.248	0.422	0.116
(Sig)	(0.000)	(0.000)	(0.007)	(0.000)	(0.187)
KCFC	0.138	0.095	0.189	0.033	-0.070
(Sig)	(0.122)	(0.237)	(0.031)	(0.700)	(0.403)

Table 4: The effect of each of the six MKTF domains in predicting each of the five constructs of teaching practices

From Table 4, the results have shown a significant positive impact of KCFS (β = 0.349, P = 0.000) in predicting PSTs selection and using tasks. The finding showed that there was a significant increase of 0.349 units in PSTs' practice of selection and using tasks for every unit change in PSTs' KCFS, when the variance explained by all other MKTF domains in the model is controlled for. However, the results did not show significant impacts of: CCKF (β = 0.086, P = 0.266); SCKF (β = -0.080, P = 0.372); HCKF (β = -0.031, P = 0.722); KCFT (β = 0.005, P = 0.945); and KCFC (β = 0.138, P = 0.122*)* in predicting their selection and using tasks. The result therefore shows that, of the six MKTF domains of PSTs, only KCFS significantly impacted positively to predict their practice of selection and using tasks which appears to suggest that a pre-service teacher with a very strong KCFS has the potential to perform well in the practice of selecting and using tasks.

With regards to the domain on representation, the results showed significant positive impacts of two of the MKTF domains: CCKF (β = 0.174, P = 0.013); and KCFS (β = 0.314, P = 0.000) in predicting the PSTs practice of using representations. The results: SCKF (β = 0.129, P = 0.110); HCKF (β = 0.078, P = 0.314); KCFT (β = 0.048, P = 0.484) and KCFC (β = 0.095, P = 0.237) however, were not significant for four of the domains in predicting the PSTs practice of using representations. Of the two MKTF domains (CCKF and KCFS) that significantly predicted PSTs use of representations, the impact of KCFS was higher compared to the impact of KCCF. Thus, identifying PSTs' KCFS as the best strongest predictor of their practice of using representations, suggesting that a pre-service teacher with a strong KCFS has the potential to perform well in the practice of using representations.

With regard to PSTs MKTF domains predicting their practice of providing explanations, the results have shown significant positive impacts of two domains: CCKF ($\beta = 0.252$, $P = 0.001$); KCFC ($\beta = 0.189$, $P = 0.031$), and significant negative impact of one domain: KCFS ($\beta = -0.248$, $P = 0.007$), at $\alpha=0.05$. However, the impacts of: SCKF ($\beta = 0.084$, $P = 0.337$); HCKF ($\beta = 0.164$, $P = 0.052$); and KCFT ($\beta = 0.143$, $P = 0.059$) did not show significant predictions of the PSTs practice of providing explanations. Of the MKTF domains that significantly predicted PSTs practice of *providing explanations*, the impact of CCKF seems to be very pronounced compared to the others, followed by KCFS and least by KCFC. Thus, the result has shown that the impact of PSTs CCKF was most pronounced on their practice of providing explanations, an indication that a pre-service teacher with a strong CCKF has the potential to perform well in providing explanations as a teaching practice.

With respect to PSTs MKTF domains predicting their practice of responding to students' requests for help, the results have shown a significant positive impact of KCFS ($\beta = 0.422$, $P = 0.000$) in predicting the PSTs' practice of responding to students' requests for help. This showed that there was a significant increase of 0.422 units in PSTs' practice of responding to students' requests for help for every unit change in PSTs' KCFS, when the variance explained by all other MKTF domains is controlled for. However, the results did not show significant impacts of: CCKF ($\beta = -0.020$, $P = 0.786$); SCKF ($\beta = 0.059$, $P = 0.498$); HCKF ($\beta = 0.051$, $P = 0.537$); KCFT ($\beta = 0.142$, $P = 0.054$); and KCFC ($\beta = 0.033$, $P = 0.700$) in predicting the PSTs practice of responding to students' requests for help. The finding therefore, show that, of the six MKTF domains of PSTs, only KCFS significantly impacted positively to predict their practice of responding to students' requests for help, suggesting that a PST with a strong KCFS has the greatest potential to do well in the practice of responding to students' requests for help.

With regard to predicting the PSTs practice of analyzing students' works and contributions by their MKTF domains, the result has shown the impacts of: CCKF ($\beta = 0.251$, $P = 0.001$); HCKF ($\beta = 0.159$, $P = 0.049$); and KCFT ($\beta = 0.254$, $P = 0.001$) as positive and significant in predicting PSTs practice of analyzing students' works and contributions. However, the impacts of: SCKF ($\beta = 0.162$, $P = 0.056$); KCFS ($\beta = 0.116$, $P = 0.187$); and KCFC ($\beta = -0.070$, $P = 0.403$), were not significant indicating that the PSTs knowledge in these three MKTF domains did not significantly predict their practice of analyzing students' works and contributions. Interestingly, the impact of SCKF ($\beta = 0.162$) was higher than the impact of HCKF ($\beta = 0.159$) in predicting the practice of analyzing students' works and contributions, yet the impact of HCKF was significant while the impact of SCKF was not. This might be due to the fact that the PSTs SCKF did not correlate with their KCFT as they were too far apart uncorrelated (see Table 4). Of the MKTF domains that significantly predicted PSTs practice of analyzing students' works and contributions, the impact of KCFT appears to be very pronounced compared to the others, followed by CCKF and least by HCKF. The finding therefore show that the impact of PSTs KCFT was most pronounced on their practice of analyzing students' works and contributions, an indication that a PST needs a strong knowledge in KCFT to be able to perform well in the practice of analyzing students' works and contributions.

To sum up, the results have shown significant predictive models of PSTs teaching practices constructs by their mathematical knowledge for teaching fraction domains an indication that MKTF domains have impact on teaching practices. The results have also revealed that the impact of PSTs KCFS was most pronounced in predicting three constructs of teaching practices (selection and using tasks, using representations, and responding to students' requests for help). Moreover, the results have shown that the PSTs CCKF was the strongest predictor of their practice *of* providing explanations. Again, the results have also shown that the impact of PSTs KCFT was most pronounced in predicting their practice of analyzing students' works and contributions. Interestingly, the results have shown the impact of SCKF higher than the impact of HCKF in predicting PSTs practice of analyzing students' works and contributions, yet the impact of HCKF was significant whiles the impact of SCKF was not. The results have shown from the perspectives of the pre-service teachers that KCFS knowledge domain has great impact on teaching practices as it significantly predicted majority of the pre-service teaching practice.

DISCUSSION

This research aimed to assess the influence of pre-service teachers' Mathematical Knowledge for Teaching (MKTF) domains on their instructional methods. The study employed the Mathematical Knowledge for Teaching framework and the Mathematical Task framework to identify six domains of teacher knowledge related to fractions (CCKF, SCKF, HCKF, KCFT, KCFS, and KCFC), along with five essential teaching practices: selecting and using tasks, utilizing representations, providing explanations, responding to students' requests for help, and analyzing students' work and contributions. Specifically, the investigation focused on understanding how pre-service teachers' mathematical knowledge for teaching fractions affects their actual teaching practices in the classroom.

The research revealed a significant correlation between each of the pre-service teachers' (PSTs) Mathematical Knowledge for Teaching (MKTF) domains, with each domain being notably predicted by at least two other MKTF domains. This finding suggests a mutual influence among the PSTs' MKTF domains, aligning with the perspective of Fennema and Franke (1992), who asserted that teacher knowledge domains have interrelated effects. Understanding these interactions is crucial, as it is challenging to comprehend the role of a single MKT domain in the broader context of teacher knowledge without insight into how these domains mutually affect each other. From this study, we identified the domain of KCFS as being a fundamental knowledge among the MKTF domains since it significantly predicted majority of the PSTs MKTF domains. This means that KCFS played a significant role in relation to the totality of the MKTF domains.

The study has found that the impact of PSTs KCFS was most pronounced in predicting three constructs of teaching practices (selection and using tasks, using representations, and responding to students' requests for help). This means KCFS was fundamental (foundational knowledge) to PSTs' ability to uniquely perform majority of mathematics teaching practices. This means that as a teacher, the ability to know the subject matter of mathematics and know your students are key to be able to teach and perform majority of mathematics teaching practices in order to bring about instructional quality. The result has shown that the PSTs CCKF was the best strongest predictor of their practice of providing explanations. This means that PST CCKF was a foundational knowledge needed to uniquely perform the task of providing explanations in the instructional process. The finding shows that the impact of PSTs KCFT was most pronounced in predicting their practice of analysing students' work and contributions. This contradicted Rosebery's (2005) assertion that a thorough understanding of the topic and students is necessary for analysing student errors or challenges, which in turn allows the instructor to plan ahead of time in order to maintain a cognitive level of tasks and support students so that they remain engaged. Though, as established in a previous study (Sie & Agyei, 2023), KCFT did not correlate with majority of PSTs' teaching practices, the results of the multiple regression performed in the present study revealed that it was still a significant foundational knowledge requirement of PSTs' to be able to effectively analyse students' errors in mathematics. This means that a PST needs to uniquely show sufficient knowledge of contents of mathematics and knowledge of teaching the contents in order to effectively analyse students work and contributions when teaching mathematics.

The results showed that KCFS was the most foundational knowledge as it significantly predicted majority of the PSTs MKTF domains and imparted on majority of the teaching practices constructs. This means that KCFS played a significant role in relation to the totality of the MKTF domains and teaching practices constructs. Interestingly, the result has shown the impact of SCKF higher than the impact of HCKF to the model that predicted PSTs practice of analysing students' work and contributions, yet the impact of HCKF was significant whiles the impact of SCKF was not. This was attributed to the fact that the PSTs SCKF did not correlate with their KCFT as they were too far apart uncorrelated (see Table 4). This is consistent with researchers (Pedhazur, 1997; Pedhazur & Schmelkin, 1991) who describe beta weight values as partly a function of the correlations between the predictors themselves. That means a certain predictor variable may have a high beta coefficient than another variable in a model but may fail to be significant to the model because of how weak it correlates with other predictors, whiles the other variable with less beta coefficient but correlate well with other predictors in the model would instead be significant. Thus, the results therefore failed to provide evidence to support that teachers SCK helps them to uniquely teach mathematics (Ball et al., 2008; Hill & Ball, 2004).

CONCLUSIONS

This study was not without limitations. Teaching practices in this study were explored using teaching approximations, thus identifying and interpreting teaching practices contained in lesson scripts. It would have been more appropriate to use observation data instead of using tests in which PSTs were made to read, identify and interpret the appropriateness of the teaching practices contained in lesson scripts. However, the use of a test enabled us to obtain data from PSTs about the same teaching practices for easy comparison that would have been difficult if observation data was used. Future research is therefore needed to use both observation data and lesson scripts with accompanying tests to explore the teaching practices of PSTs in order to compare whether PSTs' performance in the test is similar to their performance in the observation of teaching data. The study only explored MKT and teaching practices in fractions. It is not clear if the same results will be obtained using different topics in mathematics. This, to some extent limits the generalization of the results to other areas in mathematics. Future research is needed using other topics in mathematics to examine the relationship between teachers' MKT and their teaching practices.

Notwithstanding these limitations, the findings provide some insights into how pre-service teachers in Ghana and countries with similar contexts could conceptualise MKT and teaching practices. The findings show that the domains of MKTF have great influence on teaching practices constructs which suggest that teacher knowledge is key to the enhancement of quality mathematics teaching which consequently, could lead to improved students' performance. The findings here confirm the hypothesis that mathematical knowledge for teaching domains contribute to the ability to perform the five teaching practices. The findings implies that teachers need MKT domains to be able to teach to bring instructional quality that involves *selecting and using tasks; using representations; providing explanations; responding to students' direct or indirect requests for help; and analysing students' works and contributions*. These findings suggest that teacher training institutions should focus on training PSTs to explicitly acquire the six knowledge domains of MKT that will have great influence on their ability to perform mathematics teaching practices that bring about instructional quality and consequently lead to quality learning. The findings further implied that in the training of prospective teachers these three knowledge domains (CCK, KCT, and KCS) are foundational to the acquisition of their knowledge as they will help them to uniquely perform specific tasks of teaching mathematics.

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