Problem Posing in Mathematics Teacher Training: Developing Proportional Reasoning

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Abstract: The aim of this paper is to describe and analyze how a group of prospective teachers create problems to develop proportional reasoning either freely or from a given situation across different contexts, and the difficulties they encounter. Additionally, it identifies their beliefs about what constitutes a good problem and assesses whether these beliefs are reflected in their problem creation. This is a descriptive-qualitative study that utilizes theoretical and methodological tools from the Onto-semiotic Approach in the content analysis of participants' responses. The results indicate that the prospective teachers' beliefs about what makes a good problem do not always manifest in their practice. The prospective teachers faced challenges in inventing problems that meet the established didactic-mathematical purpose, related to insufficient didactic-mathematical knowledge of proportional reasoning, achieving better outcomes in the arithmetic context and in free creation.

Keywords: didactic-mathematical knowledge problem posing, problem solving, proportional reasoning, teacher training.

INTRODUCTION

Problem posing has emerged as a significant topic in mathematics education, with extensive research over the past few decades (Baumanns & Rott, 2021; Cai & Huang, 2020; Lee et al., 2018) underscoring its critical role in teacher training programs (Grundmeier, 2015; Malaspina et al., 2019). This approach not only introduces trainee teachers to the nuances of teaching mathematics but also deepens their understanding of mathematical content and helps identify their teaching deficiencies (Tichá & Hošpesová, 2013).

Recent studies have leveraged problem posing as a tool to develop and enhance the competencies and knowledge of mathematics teachers, particularly in the realm of proportional reasoning (Ellerton, 2013; Mallart et al., 2018). This research focuses on the outcomes of a formative intervention aimed at prospective secondary mathematics teachers, emphasizing the creation of problems that involve proportionality.

Despite the recognized importance of this approach, challenges persist in the didactic-mathematical preparation necessary for teaching proportionality. Research highlights difficulties that both trainee and practicing teachers encounter, such as teaching proportional reasoning and related concepts effectively (Ben-Chaim et al., 2012; Burgos et al., 2018; Burgos & Castillo, 2022; Burgos & Chaverri, 2022, 2023a, 2023b; Hilton & Hilton, 2019; Izsák & Jacobson, 2017; Weiland et al., 2020). Common issues include an over-reliance on arithmetical methods like the rule of three, limited development of proportional skills, underestimation of the ratio concept in favor of fractions, and confusion between linear and non-linear relationships (Cuevas-Vallejo et al., 2023).

Moreover, there is an evident gap in research addressing how teachers in training create problems in contexts beyond the arithmetic, such as functional, geometric, and probabilistic contexts where understanding proportionality is crucial. Past findings reveal that prospective teachers often struggle to devise meaningful and contextually appropriate proportionality problems, leading to tasks that may not effectively contribute to students' learning (Bayazit & Kirnap-Donmez, 2017; Tichá & Hošpesová, 2013; Xie & Masingila, 2017). Such problems are often irrelevant, improperly leveled, or flawed, highlighting a need for focused research on problem posing within diverse mathematical contexts. On the other hand, works like that of Li et al. (2020) demonstrate that teachers' beliefs about problem posing can determine how they use this strategy in their teaching practice. However, despite the importance of problem posing in mathematics teaching and learning, and of the beliefs of both preservice and inservice teachers, the study of beliefs about problem posing has received little attention (Li et al., 2020).

In the formative action described in this article, prospective teachers must develop problems in a given context (arithmetic, functional, geometric, or probabilistic) freely or from a given situation. In the first case, the goal is for future teachers to identify the complexity of the proposed problem based on the mathematical elements involved in its resolution. In the second case, teachers in training need to analyze its potential to develop essential aspects, such as the proportional nature of percentages or the properties that characterize the relationship of proportionality. The following research objectives are proposed:

1. Identify the beliefs that future secondary teachers have about what constitutes a good mathematical problem and how these influence the creation of proportionality problems.
2. Study the difficulties that future teachers manifest in posing proportionality problems freely and in a semi-structured manner in various contexts.
3. Describe the knowledge about proportional reasoning that future teachers demonstrate in problem posing.

Theoretical Framework

Practices, objects, and processes in the analysis of mathematical activity

The Onto-semiotic Approach (OSA) to mathematical knowledge and instruction, as outlined by Godino et al. (2007), presents a pragmatic and anthropological view of mathematics, focusing on the meanings of mathematical objects within systems of practices. These practices, executed either individually to highlight personal meanings or collectively within an institution to create institutional meanings, involve a variety of entities classified as mathematical objects. These objects include problem-situations, languages, concepts, propositions, procedures, and arguments.

A mathematical process is considered to be any sequence of actions activated or developed over a certain period to achieve a goal, typically the response to a proposed task subject to mathematical or metamathematical rules such as posing or solving mathematical problems or communicating solutions. The objects emerge from the systems of practices through the respective processes of problematization, communication, definition, enunciation, algorithmization, and argumentation. This framework allows the creation of onto-semiotic configurations of practices, objects and processes, forming articulated networks where objects and processes play crucial roles within their originating practice systems (Godino et al. 2007).

This structure is critical for analyzing mathematical activities, enabling educators to discern and address potential learning difficulties, evaluate mathematical competencies, and ensure timely recall of key concepts during instruction. It provides tools for both epistemic or institutional and cognitive or personal interpretations, helping to define both institutional and personal mathematical knowledge (Godino et al., 2017).

Meanings of proportionality

The objects and processes involved in problem-solving practices that involve proportionality depend on the contexts of application, as demonstrated by numerous studies on the nature and development of proportional reasoning (Ben-Chaim et al., 2012). Therefore, specific meanings can be delineated for different fields of application of proportionality: arithmetic, algebraic-functional, geometric, probabilistic, etc.

The arithmetic approach, centered on the notion of ratio and proportion, has been predominant in curriculum developments and research proposals. A ratio is an ordered pair of quantities of magnitudes (homogeneous or heterogeneous) compared multiplicatively. Each of these quantities is expressed using a real number and a unit of measure. A ratio may appear as a fraction when the units of measure of the related quantities are disregarded; in this case, a proportion is the equality of two equivalent fractions.

This approach essentially distinguishes two categories of proportionality problems: comparison and missing value (Cramer & Post, 1993). In the arithmetic approach, various constructs associated with the rational number, particularly the percentage, are of special importance. Although the meanings of percentages are diverse, such as a number (which can be written as a fraction or decimal), as an intensive quantity, part-whole relationship, part-part relationship, or as an operator, it is fundamentally based on the need to compare two quantities not only in an absolute manner but also relatively. Percentage allows for the concise expression of proportionality relationships (Parker & Leinhardt, 1995). Knowledge about percentages involves much more than conversions, calculations, and applications—it implies seeing the percentage as a proportion (Dole, 2010).

In the algebraic approach, proportionality is recognized as a situation in which there is a constant multiplicative functional relationship between two covarying magnitudes. Problems in an (algebraic-)functional context are characterized by the application of the notion of linear function and resolution techniques based on the properties of such a function (additive, $f(x_1 + x_2) = f(x_1) + f(x_2)$, scalar-multiplicative $f(\lambda x) = \lambda f(x)$, functional-multiplicative $f(x) = kx$, for any real number $x, x_1, x_2, \lambda; k$ being the constant of proportionality). A translation of a linear function $f(x) = mx$ can lead to the affine function $g(x) = mx + n$, or conversely, a linear function is a case of the affine function $g(x) = mx + n$ such that $g(0) = 0$. Linear and affine functions are the first examples of real functions of a real variable in Secondary Education.

The geometric approach is based on the notion of similarity of figures and scales in which the ratios and proportions are established between segments. Tasks related to the application of the Theorem of Thales, scales, enlargements, and reductions of figures while preserving shape, particularly reproducing a puzzle at a different scale, fall under the geometric approach (Aroza et al., 2016; Ben-Chaim et al., 2012).

Finally, works such as those by Bryant and Nunes (2012) show that proportional reasoning is a key factor in children's ability to understand and apply probabilistic concepts. Proportional reasoning is part of the analysis of the sample space, the quantification of probabilities, the study of the random variable and sampling, and the understanding and use of correlations (Bryant & Nunes, 2012), making it an essential element of probabilistic reasoning. Moreover, insufficient proportional reasoning and the close cognitive and intuitive connection between the notions of chance and proportion may underlie many of the conceptual and procedural errors in the field of probability (Bryant & Nunes, 2012), emphasizing the importance of providing opportunities for students to develop proportional reasoning in the probabilistic context (Begolli et al., 2021)

Didactic-Mathematical Knowledge and Competence Model

The Didactic-Mathematical Knowledge and Competence model (DMKC) for teachers, developed within the Onto-semiotic Approach (OSA) framework, articulates the categories of knowledge and competencies of mathematics teachers, through the facets and components of the processes of mathematical study considered in this framework (Godino et al., 2017). Thus, it is accepted that the teacher must possess mathematical knowledge per se, which is common, relative to the educational level where they teach (shared with their students and sufficient to solve problems and tasks proposed in the curriculum), and expanded, which allows them to link it with higher levels. Additionally, as any mathematical content comes into play, the teacher must have didactic-mathematical knowledge of the various facets that affect the educational process: epistemic, ecological, cognitive, affective, mediational, and interactional. These facets interact in any process aimed at teaching and learning mathematical content.

In addition to possessing these knowledge bases, the DMKC model proposes that the teacher should be competent to describe, explain, and judge what has happened in the study process and make improvement proposals (Godino et al., 2017). Specifically, the competencies in *analyzing global meanings*—identifying and describing operative and discursive practices in mathematical activity—and in *onto-semiotic analysis of practices*—recognizing the configurations of objects and processes emerging from mathematical practices—are fundamental for problem posing with didactic purposes. In turn, problem posing serves as a means to develop these competencies, as it requires: reflecting on the global structure of the problem to understand its objectives and assess if the provided information is sufficient; analyzing potential solutions and the interrelationships of mathematical objects and processes involved; and identifying potential difficulties students may encounter, along with strategies to address these in new problem formulations.

Problem Posing in Teacher Training

Mathematical problem posing is a fundamental skill for teachers, not just in solving problems but in selecting, modifying, and designing them with an educational goal (Malaspina et al., 2015). This skill enhances students' conceptual understanding and is a vital tool for assessing their knowledge and gaps (Kaur & Rosli, 2021; Kılıç, 2017; Lee et al., 2018; Kwek, 2015).

The literature describes various methodologies and categories for problem posing, as highlighted by Stoyanova and Ellerton (1996). They categorize problem creation into three types: free, semi-structured, and structured. In free situations, students create problems without restrictions, drawing from personal experiences. Semi-structured situations provide a partial framework that students complete using mathematical knowledge and experiences. Common tools in semi-structured problem posing include images, graphs, and tables (Silver & Cai, 2005). Akay and Boz (2010) distinguish various semi-structured situations based on the starting scenario. Finally, structured situations are based on reformulating existing problems, altering their conditions or questions to fit new contexts (Prabhu & Czarnocha, 2013; Van Harpen & Presmeg, 2013).

The Cruz model (2006) guides teachers through posing new problems in educational settings. This process starts with selecting a mathematical object based on didactic needs, followed by analyzing and possibly transforming this object to *posing* a new problem, emphasizing the relationship between mathematical concepts and their properties.

Grundmeier (2015) explores problem posing in teacher training from two angles: *reformulation* and *generation*. Reformulation allows for various modifications to a problem, such as swapping the given and required elements, altering the context while keeping the structure, adding data, changing wording, or expanding the problem to broader contexts. Structural reformulation demands greater creativity and a deeper grasp of the mathematical content, distinguishing it from more superficial techniques like merely adding information or modifying the wording of the problem.

Lee et al. (2018) differentiates between generation and reformulation by their purposes: generation fosters creativity by connecting mathematics with real-life situations, while reformulation encourages a deeper reflection on existing problems, enhancing the student's creative and analytical skills.

Malaspina (2013) defines the elements that characterize a mathematical problem: information, requirement, context, and mathematical environment. The context could be intra-mathematical or extra-mathematical, focusing on how the problem is framed within a specific mathematical domain, such as algebraic or geometric contexts. The process of creating a new problem can occur through variation—modifying elements of an existing problem—or elaboration—constructing a problem freely from a situation, focusing on the relationships between the information provided and the mathematical framework involved.

This detailed approach to problem posing is crucial for teachers, as it not only helps in understanding how to construct educational and challenging problems but also aids in evaluating the didactic-mathematical knowledge required for effective teaching.

METHOD

The study is framed within a descriptive research approach that is essentially qualitative, as it aims to describe and interpret how prospective teachers create problems to develop proportional reasoning and the difficulties, they face in creating problems, as well as their beliefs about what constitutes a good problem and how to create one. Content analysis (Cohen et al., 2018) is used to examine the response protocols of the prospective teachers who participated in the formative intervention.

Research Context

The formative action that frames the research was conducted with 16 students training to be teachers (5 women and 11 men) in the context of a mathematics specialization course of the Master's in Secondary Education Teaching at a Spanish university (March 2023). The prior degrees of the participants entering the Master's program include Bachelor's in Mathematics (11) and Bachelor's in Physics (5). As part of their training in the master's program, the prospective teachers (PTs henceforth) had also received training in another course on rich tasks to be developed in the secondary education classroom (Arce et al., 2019), as well as aspects of teaching and learning proportionality based on reading various research articles such as the historical phenomenology, types of problems and different solving strategies, students' learning difficulties, and the common treatment of proportionality in textbooks (Cramer & Post, 1993; Fernández & Llinares, 2010; Martínez-Juste et al., 2014; Oller-Marcén & Gairín, 2015; Steinthorsdottir, 2006 ; Tinoco et al., 2021).

Once informed of the research purposes, the PTs signed a consent form and individually solved the proposed assessment tasks over two hours. In addition to individual work reports, evaluations made by the PTs about some of the assigned tasks and the first impressions made by some of the participants upon submission are also available. Finally, they were encouraged to attend the next class sessions, after the practicum period, to share insights on the questionnaire responses and complete the module by presenting research findings and the didactic proposal on proportionality included in Martínez-Juste (2022).

Data Collection Instrument

The questions and tasks proposed as part of the data collection instrument were designed by the researchers, taking into account the results obtained in previous studies (Burgos et al., 2018; Burgos & Chaverri, 2022; Mallart et al., 2016). These tasks were applied to a pilot group of preservice primary education teachers. To understand the participants' beliefs about problem creation, they were initially asked to respond to the following questions:

- *What characteristics do you think a good mathematical problem should have?*
- *What factors do you believe determine the complexity of a mathematical problem?*
- *What knowledge and skills do you think a teacher needs to create a good mathematical problem?*

Subsequently, two tasks were proposed. In the first task (Figure 1) students were required to freely create a problem (without a starting situation) involving proportional reasoning in three different contexts: arithmetic, geometric, and probabilistic.

TASK 1: Create a problem that is solved by applying proportional reasoning for each of the following three contexts: arithmetic, geometric, and probabilistic. Then, solve it.

In each problem:

- Indicate how proportional reasoning is used.
- Identify the objects (concepts, procedures, properties, and arguments) and the mathematical processes (enunciation, signification, algorithmization, argumentation, representation, particularization, generalization, etc.) that are involved.
- Justifiably identify the degree of complexity involved in solving each one.

Figure 1: Free creation of proportionality problem

After creating each problem, the participants must justify how proportional reasoning is involved, identifying the network of objects and processes, so as to verify the relevance of the problems they have devised in meeting the established purpose. Additionally, this process is expected to help them recognize the level of complexity involved in their problems.

In the second task (Figure 2), three scenarios were proposed: an image about percentages and discounts in stores (Situation A), a graph of two functions (linear and affine) (Situation B), and a puzzle with given measurements of the pieces (Situation C).

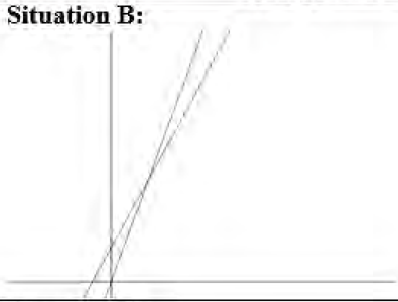
TASK 2: Below, some situations described through images are proposed. For each situation, you must create a problem that involves proportional reasoning in the given context and meets the established requirement.

Situation A:

50% off on the 2nd unit | *3 for 2 on over 1500 products* | *2nd unit at 70% off*

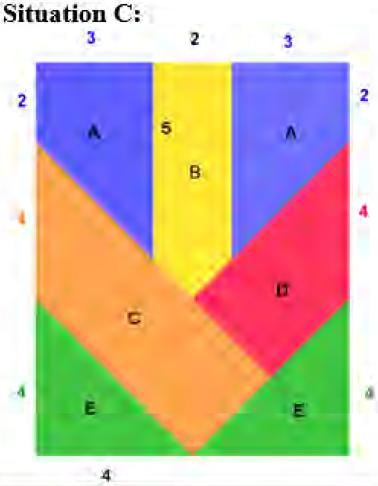
Context: Arithmetic.
Requirement: The problem should motivate the student to understand the proportional nature of percentages.
Solve the problem, highlighting how these properties are used.

Situation B:



Context: Functional.
Requirement: The problem should encourage the student to use the properties of the direct proportionality relationship (additive, multiplicative: scalar relationship and functional relationship).
Solve the problem, highlighting how these properties are used.

Situation C:



Context: Geometric.
Requirement: The problem should encourage the student to use the properties of the direct proportionality relationship (additive, multiplicative: scalar relationship and functional relationship).
Solve the problem, highlighting how these properties are used.

Figure 2: Creation of proportionality problems from situations

Students were asked to create and solve a problem for each of the associated contexts (arithmetic, functional, and geometric, respectively) with a given purpose related to the proportional nature of percentages or the properties of the direct proportionality relationship.

Analysis Guidelines

The responses from the PTs were analyzed by two authors based on pre-established criteria by the research team.

To analyze the responses regarding beliefs about what constitutes a good problem or what determines its complexity, content analysis was applied to the participants' written responses, identifying and isolating descriptions that could be associated with characteristics related to the problem statement itself, the activity it develops, or the potential solvers.

The analysis assessed the significance and pertinence of the posed problems. A problem is *significant* if it clearly establishes a mathematical challenge that is solvable, the solution is not implicit, the wording is clear and unambiguous, and its elements (context, information, requirement, environment) are clearly identified.

A problem may be significant but not *pertinent*. For example, in Task 1 (Figure 1), non-pertinence occurs if the problem lacks proportional reasoning or fails to match the requested context. In Task 2 (Figure 2), a problem is non-pertinent if it deviates substantially from the given situation or overlooks the intended context or the didactic-mathematical requirement. Conversely, a pertinent problem accurately reflects the situation and addresses the requirements.

Task 1 demands a precise description of how the problem involves proportional reasoning. We aim to understand the factors PTs consider when evaluating the complexity of their problems, using Stein et al.'s (1996) model to categorize them as memorization, procedures without connection, procedures with connection, or doing mathematics. Task 2 analyzing how PTs recognize the proportional nature of percentages in Situation A and apply proportionality properties in Situations B and C.

RESULTS

Beliefs

The reflections of the PTs about the characteristics that a good mathematical problem should have can be organized around three dimensions: the statement of the problem, the mathematical activity it motivates, and the demands it places on students.

According to Table 1, for the PTs, a good mathematical problem is characterized by a clear and unambiguous statement, a motivating context, as well as a requirement that stimulates reasoning and allows reaching its solution in an accessible way and by different strategies. These ideas coincide with and expand the results obtained by Mallart et al. (2016).

Characteristic	Frequency
The statement	
Clear, unambiguous, includes necessary information	7
Appropriate, enriching, motivating context	5
Coherent with the mathematical object being taught	2
The mathematical activity promoted	
Allows different resolution strategies	7
Questions encourage reasoning, checking, reflection	5
Mobilizes competencies (non-algorithmic)	3
Motivates the need for mathematical objects, fosters questioning and the extension of its solution to other settings	3
The students	
Accessible to all students, not difficult to pose and begin to solve	4

Table 1: Characteristics of a Good Problem

When referring to the factors that determine the complexity of a problem (second question, Table 2), they again referred to conditions about the formulation of the problem, highlighting how data and questions appear ordered and related, as well as the "scaffolding" (PT3) of these, about the mathematical activity they involve, specifying that the level of abstraction or the deployment of "happy ideas" (PT16), that is creativity and ingenuity, raises the complexity of a problem. Finally, they considered the knowledge and skills that students need to face them as an aspect that influences their complexity.

Factors	Frequency
About the problem statement	
Information. Amount of data, order and relation between questions	4
Requirement. Difficulty in interpreting the statement, what it requires, selecting what's relevant	6
Context. Connection with other fields	2
Environment. Complexity of the concepts and processes involved	5
The mathematical activity promoted	
Existence of more than one way to solve	3
Abstraction and creativity	3
About the Students	
Required knowledge and competencias	7

Table 2: Factors determining the complexity of a problem

PTs found it more complex to specify the knowledge and competencies that a teacher requires to create good problems (Table 3), which may be justified by their lack of training and experience in this area, as they themselves acknowledged. In fact, of the 16 participants, two did not respond to this question.

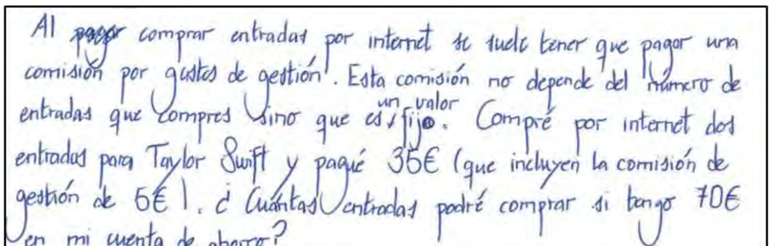
Knowledge and Skills	Frequency
Related to Content	
Knowledge of the content and problem field	5
Knowledge of the goals pursued and how to achieve them	4
Creativity	6
Related to Students	
About students' prior knowledge	3
About potential difficulties, frequent errors	4
About students' tastes and interests	5

Table 3: Knowledge and skills required for a teacher to create problems

For the PTs, a "deep knowledge of mathematics" (PT9) is necessary, including understanding of the mathematical objects and how they are used to solve problems. They also considered it important to be creative in proposing attractive problems in useful and real contexts, taking into account the goal being pursued ("knowing what you want to achieve with the problem and how to generate new knowledge from it", PT16). Only some participants recognized as important having experience with problem creation (PT6, PT16) as well as "writing skills" (PT8).

Free Creation of Proportionality Problems

Out of the 16 participants, two did not create problems in the arithmetic context, and two others proposed non-significant problems, as they did not establish the regularity needed to apply a proportionality relation. The remaining PTs created relevant problems, mostly (nine) involving missing values (Figure 3), although they also formulated problems on comparing ratios, proportional distribution, or compound proportionality.


Al comprar entradas por internet se suele tener que pagar una comisión por gastos de gestión. Esta comisión no depende del número de entradas que compres sino que es un valor fijo. Compré por internet dos entradas para Taylor Swift y pagué 35€ (que incluyen la comisión de gestión de 5€). ¿Cuántas entradas podré comprar si tengo 70€ en mi cuenta de ahorro?

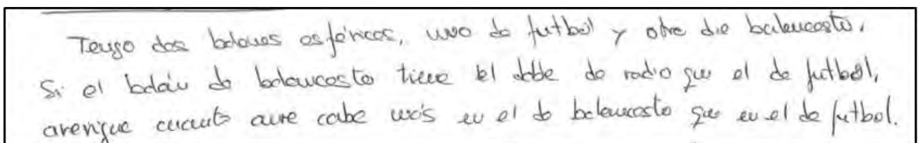
When purchasing tickets online, a fixed service fee is typically charged. This fee does not depend on the number of tickets purchased. I bought two tickets for Taylor Swift online and paid 35 euros (which includes a 5-euro service fee). How many tickets can I buy if I have 70 euros in my savings account?

Figure 3: Proportionality problem in arithmetic context (PT6)

For instance, in the missing value problem included in Figure 3, a direct proportionality relationship is established between the number of tickets for the concert and the price paid for them (excluding the service fee).

Participants encountered significant challenges when posing proportionality problems in the geometric context. Four PTs did not propose any problems, indicating uncertainty about applying proportionality in geometry compared to arithmetic, and a general aversion to the subject. Additionally, two submitted non-significant problems (ambiguous, lack of regularity conditions), and another provided a significant problem that, nonetheless, failed to incorporate proportional reasoning. Successful submissions typically involved the similarity of geometric figures, primarily triangles and, in one case, squares, aiming to calculate lengths (in three instances) or areas (in another three).

They also proposed two statements on scales (determining length from scale, or the relationship between scales) and one statement that demanded the relationship between the volumes of two spheres (Figure 4).


Tengo dos balones esféricos, uno de fútbol y otro de baloncesto. Si el balón de baloncesto tiene el doble de radio que el de fútbol, averigüe cuanto aire cabe más en el de baloncesto que en el de fútbol.

I have two spherical balls, one soccer ball and one basketball. If the basketball has twice the radius of the soccer ball, find out how much more air the basketball can hold than the soccer ball.

Figure 4: Proportionality problem in geometric context relationship between volumes of spheres (PT1)

Posing problems that involve proportional reasoning in the probabilistic context also proved complex. Four PTs did not respond to this task (three of whom also did not do so in the geometric context and two also not in the arithmetic), another four proposed non-significant problems, due to the information provided not allowing the problem to be solved, and one proposed a significant problem, but not pertinent as it did not involve proportional reasoning ("In the lottery the probability of winning is 1/1000. If you play every week, does it increase?", PT2).

In this context, the pertinent problems primarily required calculating and comparing simple probabilities, with exceptions including a fair play scenario, determining the composition of an urn based on a given probability, and a compound experiment. For example, PT3's problem asked, "If a basketball player makes 3 out of every 10 shots, how many shots must he average to make at least 17?" The ratio was interpreted as the probability of success, fitting a missing value problem of direct proportionality, where "on average" suggests uncertainty. In this case, PT3 considered that the difficulty was due to "the number of shots will not come out whole, hence the word at least in the statement, because they will have to add 1 to the integer part of that number."

Participants needed to solve the problems they created and identify the involved mathematical objects and processes. Out of 14 arithmetic context problems, seven were correctly solved, identifying proportional reasoning through the direct proportionality relationship between the magnitudes. PT6, for instance, identified in relation to the problem included in Figure 3, "Proportional reasoning appears in the relationship of Dir. Prop [direct proportionality] between Entries and money", indicating later as objects the "ratio €/entry, direct proportionality between € and entries" and as mathematical processes, "signification", which links to interpreting the commission, "algorithmization", which he understands as applying the unit reduction technique to solve the problem and "argumentation". However, the PTs generally did not identify objects beyond basic concepts of ratio, fraction, and direct proportionality, nor did they recognize processes beyond basic arithmetic operations and rule of three (the routine procedure to find the fourth term of a proportion when the other three are given, based on the equality of the cross product of the means and extremes of the proportion) (Arican et al, 2023), except PT3 who mentioned "generalization" relating to regularity.

The PTs assessed problem complexity based on the difficulty level for students. The 10 participants who made some reflection on this point highlight aspects that affect the greater or lesser difficulty of one task compared to another. Most found their problems to be of low difficulty, citing “clear language” and straightforward steps, although PT6 and PT11 noted medium-high difficulty due to the need to distinguish between different types of relationships from the additive (the service fee) and handle non-whole ratios when the magnitudes were discrete ("fraction of an egg in a cooking recipe", "contradictions may arise").

From an expert viewpoint, all the proposed problems corresponded to a level of cognitive demand for memorization or procedures without connection according to Stein et al. (1996), except for a couple of problems in the level of procedures with connection where it was necessary to compare ratios in compound proportionality (PT16) or obtain the relationship between flows of two hoses knowing the relationship between the volume (PT1). In both cases, the PTs indicated that the problems were not complex.

In the case of problems in the geometric context, only five PTs solved their problem and correctly identified proportional reasoning. Two others did not solve it but did indicate how proportional reasoning was involved: through the use of scale or the similarity relationship between figures. In this case, only four PTs identified some concepts and properties (right triangle, similar triangles, proportionality, area, similarity of triangles) and only one PT also mentioned the processes of modeling, representation, and argumentation. Regarding the complexity of the problems posed in this context, only nine of the participants made any mention of it. It is observed that, although they mostly still think that their problems are not difficult, they consider them more complex than those posed in the arithmetic context, referring to the context itself ("relate the proportion to the scale characteristic", PT10; "use concepts of geometry and be less explicit the method of solution", PT8) or the use of new magnitudes. For example, PT3 considers that his/her problem "is difficult due to the paradigm shift that occurs when changing from direct to quadratic proportionality" and PT1 considers that in his/her problem (Figure 4).

> "*Students must find proportional relationships among 3D figures. They need to understand that they are asked to find the relationship between the volumes of the spheres, disguised as elements common to their environment. That is, finding that magnitude may pose an additional problem*".

In this case, the cognitive demand of the proposed problems is higher than those posed in the arithmetic context (only one of them is memorization and there are five that involve procedures with connection, for example, the one included in Figure 4).

Only three PTs solved the proposed probability problem and another two, although did not solve it, specified that proportional reasoning was involved in the calculation and comparison of probabilities or in the use of Laplace's Rule. The objects indicated by these five PTs included the concepts of probability, favorable cases, possible, simple probability, conditioned probability, frequentist probability and only PT3 indicates as a process the generalization (linked to the assumption of equiprobability).

Of the eight PTs who reflected on the degree of complexity of their problem, three indicated that it was low, considering that only "basic concepts" are used (PT14), that the statement is clear (direct) and few steps are needed (PT13), while the others considered that the complexity is greater due to the context itself and the implications it has on the relationships between the numbers ("thinking if a fraction that expresses a probability makes sense", PT2; "understanding the frequentist meaning of probability", PT11). In this respect, except for one problem at the level of memorization and another of procedure without connection, all the problems created in the context of probability responded to the level of procedure with connection.

Creation of Proportionality Problems from Situations

In Task 2 (Figure 2) the PTs had to create proportionality problems from situations described by images, in different contexts and with the intention that their solution involve certain fundamental knowledge of proportional reasoning.

In situation A, which describes three common commercial offer options (50% discount on the second item, buy 2 and get 3 products, and 70% discount on the second item), the given context is arithmetic and the problem posed should motivate reflection on the proportional nature of percentages. In this case, two PTs did not propose any problem; another three formulated non-significant problems (lack of requirement or incomplete information) and another participant posed a problem that although it was significant substantially changed the information (offer 3x2 and second unit 60%).

The rest (ten PTs) created pertinent problems, that is, significant, elaborated from the situation and involving the proportional nature of the percentage. Most of these problems (seven) lead to deciding which is the best offer in the purchase of a given number of units assuming that the price of the unit is always the same, known (in three cases) or unknown (in the rest) (see, for example, Figure 6). In two other cases, it must be decided in which offer less is paid per unit ("which offer implies a lower cost per product", PT5; understanding that two units or three units are bought, depending on the offer).

A Me planteo comprar 6 cajas de leche en uno de los tres supermercados de que se muestran ofertas arriba. Si la unidad suelta vale lo mismo en cada uno (1'50 €) ¿dónde me saldrá más barato? ¿Depende del precio?
I am considering buying 6 boxes of milk from one of the three supermarkets showing offers above. If the price per individual unit is the same at each one (1.50 euros), where will it be cheaper? Does it depend on the price?

Figure 5: Elaboration from situation A (PT11) pertinent problem

In three statements, as shown in Figure 5, the PTs asked how the price of an item influences which offer is superior. Although they solved the problems, they seldom discussed how proportional reasoning and percentages are involved, except for PT11 (problem posed in Figure 5) who associated proportionality with discounts, noting, "It does not depend on the price. Note that the proportionality of these discounts is used where it is maintained (in pairs or trios)," and PT4 who stated, "the ratio/relationship of what you pay per unit is used." Generally, the PTs used the percentage as an operator to calculate and determine the proportional part of the unit price for each offer to then compare them.

Posing problems from situation B, featuring a joint graph of a linear and an affine function, proved challenging for the PTs. It was asked that the problem inspire students to use proportionality properties. Five PTs failed to devise any problems, four presented non-significant problems (ambiguous or insufficient information), two offered significant but unrelated problems (involving shopping, comparing two affine functions), and one created a relevant but not proportionality-focused problem (calculating function intersections). In the pertinent responses, the PTs needed to align the functions to the graphical representation, interpreting slope and the "additive part" in the affine function, as shown in Figure 6.

Teniendo en cuenta que A y B representan la distancia recorrida al caminar de Álvaro y Berta respectivamente, podemos decir que Berta ha empezado antes a caminar, pero va más lenta. Es por ello, que Álvaro la alcanza en algún momento, y después la va superando cada vez más en velocidad. La relación aditiva se identifica por el punto de corte de las rectas con el eje de abscisas, y la multiplicativa por la pendiente de las rectas. 10	Considering that A and B represent the distance walked by Alvaro and Berta respectively, we can say that Berta started walking earlier, but she is slower. That is why Alvaro catches up with her at some point, and then increasingly surpasses her in speed. The additive relationship is identified by the point where the lines intersect the x-axis, and the multiplicative relationship is identified by the slope of the lines.

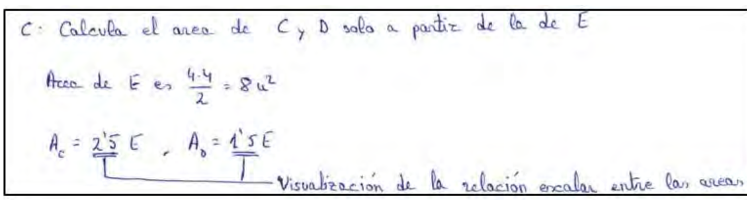
Figure 6: Problem created by elaboration from situation B and solution (PT14)

When solving the problem, on rare occasions did they mention how the didactic-mathematical purpose is contemplated. Nevertheless, it is possible to observe some confusion with the additive and multiplicative properties of the proportionality relation. Either they associated the multiplicative property with the slope or growth of the lines (Figure 6), or they consider that this is a possible relationship between the slopes ("the slopes are proportional"; PT11, PT16). As for the additive property, it seems to be interpreted (Figure 4) as the point of intersection with the abscissa axis (so that in a linear function that additive part would be 0) or also as the difference between the functions for *x*=0 ("original difference", PT11).

Finally, only six PTs posed a problem for situation C, openly stating their inability to create a proportionality problem from this situation (PT3) or showing their difficulty in specifying the ideas in a statement:

> *"I can't think of anything beyond asking them to relate quantities such as the areas of the polygons shown in the image. It would be interesting to ask them to explain how the area of one of them would vary if the numerical values shown in the figure were modified*." (PT4)

Of the six problems posed, all significant non-pertinent, two did not take into account the starting situation and the other four, although they did incorporate the puzzle of C in their approach, did not consider the intended didactic-mathematical purpose or the context was arithmetic (Figure 7).

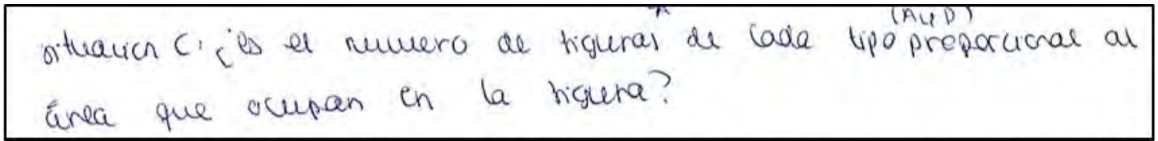


Calculate the area of C and D based solely on E.
Area of E is $\frac{4\cdot 4}{2} = 8u^2$
$A_C = 2.5\ E\ ; A_D = 1.5\ E$
Visualization of the scalar relationship between the areas.

Figure 7: Problem elaborated by PT4 from situation C

As shown in Figure 7, PT4 asked to determine the area of two pieces of the puzzle (C and D) using as a unit of measure the piece E, obtaining "how many times E fits" in C and D (two and a half times, as a decimal 2.5 in C and one and a half times, as a decimal, 1.5, in D). Regarding the properties of proportionality, PT4 mentioned the "scalar relationship between areas", however, no proportionality relation between magnitudes is established.

While in the previous task of free creation, the PTs utilized the concept of similarity of figures (relationship between lengths or areas of similar figures), the notion of scale, or the application of Thales' theorem, they did not resort to these concepts or know how to incorporate them into the established purpose when starting from Situation C. As shown in Figure 8, PT12 used Situation C to formulate the problem; however, it does not represent a geometric meaning of proportionality, as it proposes a relationship between the number of figures and the area they occupy.



Situation C: Is the number of figures of each type proportional to the area they occupy in the figure?

Figure 8: Proposal by PT12 of a problem from situation C

DISCUSSION AND CONCLUSIONS

Teachers, in addition to being competent in solving the problems they propose to their students, must also know how to choose, modify, and create them with an educational purpose (Malaspina et al., 2015). The creation of problems enhances students' conceptual understanding (Kaur & Rosli, 2021; Kılıç, 2017; Lee et al., 2018) and serves as a crucial tool for assessing what students know and do not know (Kwek, 2015). Furthermore, problem posing contributes to the development of mathematical knowledge during initial teacher training, prompting them to "rethink" the nature of mathematical objects before explicit instruction (Kılıç, 2017). Achieving this competency requires that teacher training includes the design and implementation of specific actions to ensure the necessary didactic-mathematical knowledge. When the goal is epistemic in nature, these knowledges involve aspects specific to the mathematical content, in this case, proportional reasoning. Since different application contexts of the notions of ratio and proportion involve the participation of specific objects and processes from those fields in the corresponding practices, ideal teaching of proportionality must consider the different meanings—arithmetic, algebraic-functional, geometric, probabilistic—in an articulated manner. The teacher must thus be able to pose proportionality problems in different contexts, motivating students to learn through their resolution the essential properties of the proportionality relationship.

In this article, we have reported the results of a formative experience with a group of prospective secondary teachers focused on the creation of proportionality problems with a didactic purpose. The findings described contribute to understanding what they value in a good problem, the difficulties they face in posing problems, and diagnosing their didactic-mathematical knowledge.

Regarding our first objective, the PTs' beliefs about what constitutes a good mathematical problem and what skills they need to develop them are not consistent with each other or with their proposed statements. For example, from an instructional perspective, PTs consider one of the characteristics of a good mathematical problem to be a clear and unambiguous statement (Table 1), yet they designed non-significant problems (ambiguous) in both tasks. Similarly, PTs believe a good problem allows for various solution strategies, however, there is no evidence that they verified the possibility of addressing the requirement in their problems in more than one way, or that the problems could indeed be solved. This coincides with findings in research such as that of Bayazit and Kirnap-Donmez (2017). From a cognitive-affective standpoint, although they consider students' prior knowledge as one of the essential aspects for determining the complexity of the problems (Table 2), they prioritize creativity or the ability to cater to their interests as necessary skills for creating good problems (Table 3).

Concerning the second and third objectives, the PTs' competency to create pertinent problems is adequate in the arithmetic context but very limited in the functional, geometric, and probabilistic contexts. This reveals that the knowledge required to create a mathematical problem in one context does not transfer to another and that the success in developing problems is determined by the field in which they are framed. It is significant, however, that the complexity of the problems created was higher in the probabilistic and geometric contexts than in the arithmetic one.

Like in previous research (Burgos et al., 2018; Burgos & Chaverri, 2022; Mallart et al., 2016), PTs (despite their higher mathematical education) had limitations in identifying the mathematical objects and processes emerging from the solution to the proposed problems. This deficiency could explain why some significant problems were not pertinent as they did not respond to the epistemic purpose established in the task. Despite this, the few PTs who reflected on the complexity of their proposed problems explicitly mentioned objects and processes (even though they had not referred to them before). This suggests a need in secondary PT training to "strengthen the study of task complexity based on the analysis of the mathematical objects and processes involved in their resolution" (Burgos & Chaverri, 2022).

Although previous studies like those of Şengül and Katranci (2015) or Bayazit and Kirnap-Donmez (2017) observed that teachers in training had less success in freely creating problems than when doing so in a semi-structured manner (in an arithmetic context), our research shows better performance in the free case than in the semi-structured. In this case, responding to the didactic purpose established in the guideline could have been the biggest obstacle. The PTs need to recognize from the information provided which magnitudes can be related proportionally, know and recognize in the situation the properties of the functional relationship, and establish the requirement in such a way that solving the problem requires using that relationship and its properties. As observed in previous research with primary school teachers in the arithmetic context (Burgos & Chaverri, 2022), behind this difficulty could be an insufficient knowledge of proportional reasoning in the probabilistic and geometric contexts (Batanero et al., 2015; Copur-Gencturk, et al., 2023).

Aware of the limitation that the sample size imposes for generalizing the results, it is necessary to continue researching and making didactic proposals for problem posing, paying attention to the influence of context, educational purpose, and the involved didactic-mathematical knowledge. Considering the results obtained, in new interventions it would be necessary to know the mathematical and didactic-mathematical knowledge of prospective teachers, since the training received in their undergraduate and master's degrees in teaching might not be as expected. Similarly, it would be advisable to reinforce their knowledge of proportional reasoning in the geometric context and even more so, the probabilistic one. Also, dealing with structured situations, which were not considered in this research, and incorporating inverse proportionality. In this last case, there might be a lower success rate, given the more extensive and cross-sectional presence of direct proportionality in the curriculum compared to the inverse, and the potential interferences between both structures caused by prior knowledge (Parameswari et al., 2024). It would be desirable to design tasks that focus on a single context, for example, functional, geometric, or probabilistic, since they were the least successful. It would also be advisable to address cognitive-type purposes, for example, creating problems with a certain complexity or cognitive demand. As suggested by Crespo and Harper (2020) "promoting students' reasoning and problem-solving depends on the teacher's ability to identify, construct, and pose mathematically rich problems from a cognitive perspective" (p. 1).

The flexibility of the methodology allows designing and implementing new interventions with practicing teachers from different educational stages, in which we would expect better results than the current ones, although research suggests that, despite their experience, they have difficulties in creating problems (Kaur & Rosli, 2021). The integration or expansion of spaces where PTs can practice and develop the competency of problem creation helps to bridge the gap between teachers' knowledge and their teaching practices (Lee et al., 2018).

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