Investigating Pre-Service Primary School Teachers' Difficulties in Solving Context-Based Mathematics Problems: An Error Analysis

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***Abstract**: The issue of students' difficulties in solving context-based mathematics problems has been extensively investigated by numerous studies. However, limited study focus on how pre-service primary school teacher (PSPSTs) encounter difficulties in solving context-based fundamental mathematics problems. To fill this gap, this study aims to investigate PSPSTs' difficulties encountered while solving context-based mathematics tasks by identifying the error type they made based on the error classification proposed in Newman Error Analysis (NEA). This is an error analysis study with a summative qualitative content analysis approach involving 87 PSPSTs in an Indonesian Islamic University. Data were collected through a test, in-depth interviews and document analysis of the PSPSTs' responses. Data were analyzed qualitatively and quantitatively. Qualitative content analysis was performed using Atlas.ti software. The findings revealed that many PSPSTs encounter difficulties in solving context-based problems. Approximately 22.1% of PSPSTs committed errors in comprehension, 17.5% each in reading and encoding, 14.7% in transformation, and 8.7 % in process skill. Furthermore, the findings indicated a hierarchical structure in the occurrence of errors. Errors in the early stages have a high potential to cause errors in subsequent problem-solving stages. All the results are discussed, along with their implications for practice and suggestions for future research.*

Keywords: solving context-based problems, pre-service primary school teachers, error analysis

INTRODUCTION

Context-based mathematics problems have become increasingly popular in recent decades as they represent the importance of learning mathematics in a real-world context (Barcelos-Amaral & Hollebrands, 2017, Widjaja, 2013). A context-based problem refers to a mathematical task that employs situations derived either from real-life or non-real-world settings, providing related and meaningful connections with the learner's daily life experience (Ilhan & Akin, 2022, Kohar et al., 2019). These contextual problems might present the relevance of mathematical concepts in students' everyday lives, which allows them to see how the concepts they are learning can be applied in practical situations. However, while context-based problem is essential in mathematics instruction, it also remains many challenges. Several studies have indicated that contexts can also be problematic when they are used in mathematics tasks (Greer et al., 2007, Hoogland et al., 2018, Wijaya et al., 2014). In another study, Can and Yetkin Özdemir (2020) revealed that students were more successful on non-context-based tests than context-based mathematics tests.

Many students encountered difficulties in solving context-based mathematics tasks (Fonteles Furtado et al., 2019; Wijaya et al., 2014). To address this issue, teachers undoubtedly play a significant role in enhancing students' understanding of mathematics concepts and problem-solving skills. Unfortunately, one of the common problems is that many teachers are also struggling with context-based mathematics problems (CbMP), either in creating problems or even solving CbMP. This will surely be a serious problem as how could teachers employ student problem-solving improvement strategies if they themselves lack of that skill.

The use of context-based problems is not restricted to the topics associated with the advanced-level mathematics. Ideally, fundamental mathematics concepts should be represented in CbMP to create more meaningful learning. Arithmetic, including numbers and their properties and operations (Guberman, 2016), is one of fundamental mathematics concepts (Chin & Zakaria, 2015) that is taught in primary school. A deep conceptual understanding of numbers is arguably one of the most foundational mathematics learning goals for students at all levels of schooling and beyond (Elias et al., 2020). Primary teachers must have good content knowledge and pedagogical skills to teach them accurately. Despite the perceived simplicity of this mathematical concept, Butterworth et al. (2011) argued that arithmetic difficulty is a common problem for children and adults. It is crucial to investigate whether pre-service primary school teachers still encounter difficulties in solving arithmetic problems.

It is widely emphasized that teachers' mathematical knowledge is crucial in enhancing teaching competencies related to mathematical content organization (Burgos & Godino, 2022). PSPSTs are individuals who are prepared to become teachers, those who are prepared to teach mathematics to primary school children in the future. However, several prior studies have indicated that a significant number of pre-service teachers possess inadequate mathematics content knowledge (Blömeke et al., 2013, Guberman, 2016). If they themselves experience misconceptions or struggle in solving context-based fundamental mathematics concepts, this raises the concern of how these challenges may trickle down to their future students. Therefore, it is essential to examine the accuracy of PTPSs' solutions and identify their errors in solving context-based arithmetic problems. Analyzing test takers' errors can help identify the difficulties they encounter in solving challenging mathematics problems (Abdullah et al., 2015, Hadi et al., 2018). Identifying difficulties as well as the underlying factor structure of these errors will be valuable for a better understanding of future educators when teaching students in real school settings (Khalo et al., 2015).

Newman's Error Analysis (NEA) is one of the most commonly used methods for identifying errors when solving mathematical problems. This framework includes five cognitive processes involved in solving mathematical problems: reading, comprehension, transformation, ability, and encoding (White, 2010). These five processes are also required to solve context-based mathematical problems. In addition, NEA is an appropriate approach for scrutinizing errors in written sentence problems (Clements & Ellerton, 1996, Prakitipong & Nakamura, 2006), including word problem and context-based problem. Therefore, this study uses NEA to identify PSPSTs' errors and consider the reasons that underlie the difficulties experienced with context-based arithmetic problems. Each type of PSPSTs' error is then classified based on its type, such as reading, comprehension, transformation, process skills, and encoding errors.

Numerous studies have undertaken error analysis on wide-ranging mathematical concepts, such as students' errors in geometry (Chiphambo & Mtsi, 2021, Sumule et al., 2018, Zamzam & Patricia, 2018), algebraic (Fitriani et al., 2018), set (Noutsara et al., 2021), integral calculus (Angco, 2021), and several error analysis studies related to the various aspects of students' mathematical ability, such as modelling (Kotze, 2018) and evaluating and creative thinking skills (Alhassora et al., 2017). In particular, several studies have also pointed out the students' difficulties in solving context-based mathematical problems (Wijaya et al., 2014). However, limited studies have investigated PSPSTs' difficulty in solving context-based arithmetic problems as fundamental mathematical concepts. Therefore, this study aims to address this research gap.

This study aims to investigate the potential difficulties that PSPSTs may encounter while solving context-based fundamental mathematics problems. This article identifies the most common misconceptions and errors made by PSPSTs when solving context-based mathematics problems, which can then be addressed to improve their understanding of the subject. This study is essential to ensure that future mathematics educators in primary schools have a strong foundation in content knowledge mastery. The results of this study provide insights into improvement interventions for pre-service primary school teachers that would be beneficial for enhancing their professionalism when they become teachers. It would also contribute to the theoretical knowledge about the teaching and learning of mathematics in context.

METHOD

Research Design

This is an error analysis study with a summative qualitative content analysis approach that focuses on investigating the PSPSTs' solution accuracy in solving context-based mathematics problems. The PSPSTs' error indicates the difficulty they encounter in solving problems. In this summative content analysis, researchers quantify the PSPSTs' documents and interview data and then delve into their underlying significance to uncover latent meanings using qualitative content analysis (Hsieh & Shannon, 2005, Setiawan, 2023).

Participants

This study involved 87 pre-service primary teachers (N = 87; 73 females and 14 males) as participants. They were studying in the 3rd year of the Primary School Teaching Department of an Islamic University in South Sulawesi, Indonesia. Participants were purposively selected from students enrolled in a 'mathematics for primary school' course. Ten of the 87 participants, representing the high, middle, and low cognitive groups, were interviewed.

Instrument and Procedures

This study investigated the PSPSTs' difficulties encountered while solving context-based mathematics tasks. A test with five context-based mathematics problems about arithmetic, including numbers and their properties and operations, as fundamental mathematics concepts at the primary level, was employed to explore all 87 PSPTSs' problem-solving strategies and solution accuracy while solving such problems. A semi-structured interview approach was also adopted to collect useful qualitative information that would provide in-depth information about PSTSs' ways of thinking, rationale behind their answers, and type of error they made based on the error classification proposed in the Newman Error Procedure.

The five context-based arithmetic problems used in this study are as follows:

1. My mother has $3\frac{3}{4}$ kg of sago flour. $\frac{1}{3}$ part of the flour is used to make sago cheese cake. My mother then buys an additional 2 1/2 kg of flour. How many kilograms of flour does my mother have now?
2. During winter, the temperature in Alaska City experienced extreme changes. Today's weather report shows that the temperature in the afternoon reaches -2°C and drops dramatically to -19°C in the evening. What is the temperature difference (in °C) between the afternoon and evening in Alaska City?
3. Mrs. Rissa paid the price of three fans using 10 pieces of one hundred thousand rupiah, two pieces of fifty thousand rupiah, and one piece of two thousand rupiah without any change. Mrs. Rissa asked the cashier for a purchase receipt. Write the number representing the price of the fans that the cashier should write on the receipt!

Stimulus for questions 4 and 5

Mrs. Dika and Mrs. Jeni went to the traditional market to buy daily necessities. In the market, they checked the prices of several items they wanted to buy, and the results are as follows (see Table 1).

Table 1. Price of Item(s).

No	Item(s)	Price (in Rupiah)
1	Rice "Super Enak" (5 kg)	69.700
2	Rice "Putih Premium" (2 kg)	28.800
3	Egg (1 kg)	32.000
4	Sugar (1 kg)	13.000
5	Apple (1 kg)	42.000
6	Wheat Flour (1 kg)	13.500
7	Chicken (1 kg)	57.000

4. Mrs. Dika brought shopping money amounting to Rp 220,000.00 (two pieces of one hundred thousand rupiah and two piece of ten thousand rupiah). The first store Mrs. Dika visited was a store that sold sugar and wheat flour. If Mrs. Dika buys 1 kg of sugar and 2 kg of wheat flour, how much change will she receive from the cashier?
5. Mrs. Jeni only brought money amounting to Rp 100,000.00, and she intends to spend that money shopping. Mrs. Jeni decides to buy 1 kg of Rice 'Super Enak', 1 kg of sugar, and 1 kg of wheat flour. In your opinion, is Mrs. Jeni's decision correct to buy these items? Explain.

Data Analysis

The data were analyzed quantitatively and qualitatively using summative qualitative content analysis. Quantitative analysis was performed to determine the percentage of each type of error encountered by the PSPSTs in solving arithmetic problems. Newman (1977) identified five common types of errors made by students in solving written mathematics test: reading errors, comprehension errors, transformation errors, process skills errors, and encoding errors. Analyzing these errors comprehensively is known as Newman Error Analysis (NEA).

Reading errors occur when students misread or fail to accurately understand important information from a question. For example, students are wrong reading specific terms, symbols, numbers, words, or important information in the question, resulting in inappropriate answers. Comprehension error is the second type of error in which students have read the problem well, but do not understand the meaning of the question. Comprehension errors occur when students do not grasp the concept or intent of the question or material (e.g., they cannot identify known and asked about the problem), even after reading it correctly. In other words, reading errors are associated with mistakes in reading or understanding information accurately, while comprehension errors involve difficulties in understanding the concepts or meanings of the information, even after reading it correctly. Reading errors are more related to the accuracy of understanding words or phrases, while comprehension errors are more related to the overall understanding of concepts (Clemen, 1980).

Transformation error is the third type of error that occurs when students make mistakes in changing a problem into a mathematical model such as equations, drawings, graphics, or tables. Process skill error is a student's mistake in choosing rules/procedures or students already using correct procedures/rules, but errors occur in the calculation or computation. Encoding error is the fifth kind of mistake that students in this case make in writing the answer correctly, cannot show the truth of the answer, or do not write the conclusion of the answer (Clements & Ellerton, 1996).

The data analysis procedures in this study included several stages: analyzing all PSPSTs' answers, classification of error types based on NEP, calculating the percentage of each error type, qualitatively analyzing the error made by PSPSTs based on written responses, and analyzing the interview analysis to acquire in-depth information about PSPSTs' difficulty in solving problems. Data from in-depth interviews were qualitatively analyzed to obtain detailed information about the rationale behind PSPSTs' answers and the type of error they made based on the error classification proposed in NEP.

In the initial phase, the analysis focused on evaluating the errors made by PSPSTs by examining their responses in the test answer documents. These errors were independently identified, suggesting that in this situation, it is likely that PSPSTs may made multiple mistakes on different occasions. In every item of the problem, they could perform more than one error. For example, in Problem 1, a participant made errors in comprehension and transformation. Then, for every kind of error in NEA, the researchers calculated the percentage to identify the number of PSPTs who committed each type of error, dividing it by the total number of research participants ($N = 87$).

Furthermore, the data were analyzed qualitatively. Data on PSPSTs' written responses in solving context-based arithmetic problems were analyzed using the document content analysis method. Qualitative analysis was performed to analyze the error of PSPSTs based on NEP. Qualitative data analysis consists of three stages: data reduction, data display, and drawing conclusions (Huberman & Miles, 2019). The initial step of qualitative data analysis in this research involved analyzing the PSPSTs' written responses and in-depth interview results in solving context-based arithmetic problems. Coding was then conducted to ease the process of content analysis. During the data reduction phase, the researchers simplified the gathered information from the initial stage (comprising interview results and documents of students' written responses) by organizing and selecting essential data, recognizing themes and patterns, and excluding irrelevant information. Relevant information regarding PSPSTs' difficulties in solving context-based arithmetic problems was then grouped into themes or categories (i.e., categories of error types). Subsequently, researchers interpreted the reduced and displayed data. Finally, the researchers performed verification to guarantee the precision of the analysis outcomes, draw conclusions, formulate findings, and attribute significance derived from the data analysis. Atlas.ti (version 22.2.5) software was used to employ these qualitative analysis procedures, particularly in the coding, data reduction, and data display stages.

RESULTS AND DISCUSSION

Results

Written responses to the tasks were analyzed using the NEA framework. The analysis focuses on the error made by PSPSTs in answering context-based arithmetic problems. The representative answers indicate various types of errors from the 10 PSPSTs, which were further explored in-depth through the interview process. Using NEA, researchers identified the error type that PTPSs made when solving context-based arithmetic problems. The numbers of PSPSTs who gave incorrect answers, correct answers, and those who did not answer are provided in Table 2.

Table 2. Summary of the students' error analysis result ($N = 87$).

	Problem 1		**Problem 2**		**Problem 3**		**Problem 4**		**Problem 5**		**Average**
	n	%	n	%	n	%	n	%	n	%	%
Correct Answer	65	74.7	54	62.1	22	25.3	27	31.0	69	79.3	54.5
No Answer	0	0	0	0	3	3.4	2	2.3	0	0	1.1
Incorrect Answer	22	25.3	33	37.9	62	71.3	58	66.7	18	20.7	44.4
Reading Error	7	8.0	0	0	11	12.6	58	66.7	0	0	17.5
Comprehension Error	15	17.2	0	0	23	26.4	58	66.7	0	0	22.1
Transformation Error	0	0	6	6.9	58	66.7	0	0	0	0	14.7
Process Skill Error	0	0	27	31.0	4	0	0	0	11	12.6	8.7
Encode Error	0	0	0	0	0	0	58	66.7	18	20.7	17.5

A detailed explanation regarding examples and underlying reasons beyond the occurrence of the five types of errors is presented as follows, both in written and verbal forms.

Reading error. This type of error occurs when students make errors when reading important words or crucial information in a question. These errors occur because of missing or unused important information from the question in problem-solving. An example of a reading error committed by the PSPST is presented in Figure 1.

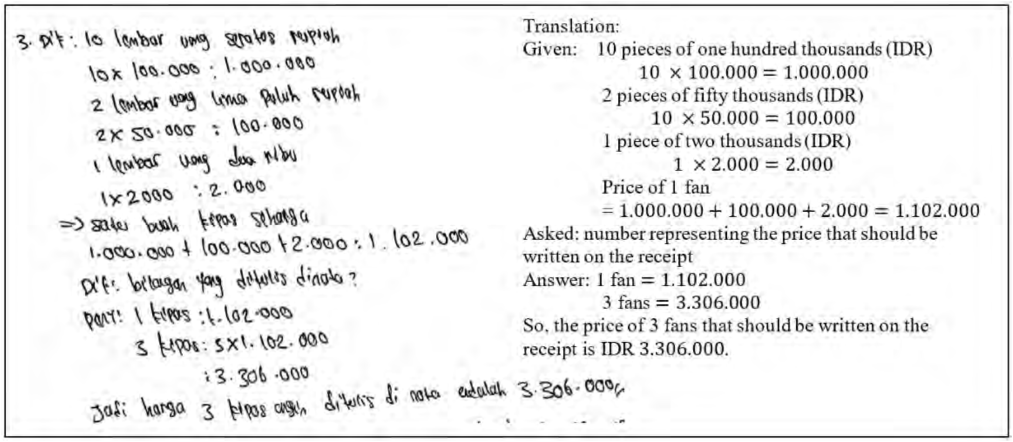


Figure 1. Written response of PSPST S-01 on problem 3

Figure 1 shows that there is an error of solution performed by S-01 on problem 3. By using the place value concept, S-01 was able to correctly calculate and write the number that represents the amount of money paid by Mrs. Rissa (IDR 1.102.000). From the information in figure 1, it is also apparent that S-01 is able to write what is asked in the question, which is the price of the three fans that should be written by the cashier on the receipt. However, an error occurred in the written problem solving strategy. Interviews were conducted to identify the cause of this error. The results of the interview between the researcher (R) and one of the PSPSTs (coded as S-01) are as follows:

R: Can you explain why it's 3×Rp 1.102.000,00

S-01: Because the question asked for the price of 3 fans. The price for one fan is Rp 1.102.000,00.

R: Oh, really? Can you show me where it states that the price of one fan is Rp 1.102.000,00?

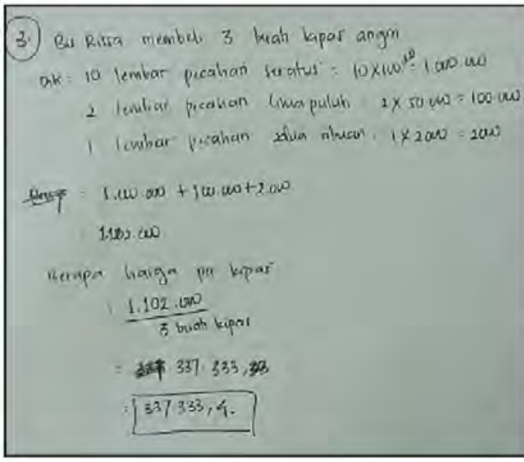
S-01: Carefully reading the question].

S-01: Oh, there's no information about the price of one fan. That means I didn't read it carefully. I thought the given information of Rp 1.102.000,00 was the price for one fan.

R: But do you understand the intention of the question? Where is the mistake?

S-01: Yes, I understand, Ma'am. Rp 1.102.000,00 is already the price for 3 fans. The question was asking for the price of 3 fans as well.

Comprehension error. This error type is related to the PSPSTs' understanding and interpretation of texts or questions. The inability of the test taker to understand accurately the given information and what is being asked in the question strongly indicates their difficulty in comprehending the problem. The following is an example of a comprehension error committed by PSPST S-02 (Figure 2).



Translation:
Mrs. Rissa bought 3 fans.
Given: 10 pieces of one hundred thousands (IDR)
$10 \times 100.000 = 1.000.000$
2 pieces of fifty thousands (IDR)
$10 \times 50.000 = 100.000$
1 piece of two thousands (IDR)
$1 \times 2.000 = 2.000$
$= 1.000.000 + 100.000 + 2.000 = 1.102.000$
Asked: how much is the price of one fan?
Answer: $= \frac{1.102.000}{3\ fans}$
$= 337.333{,}33$

Figure 2. Written response of PSPST S-02 on problem 3

Unlike S-01, who made a reading error in solving question 3 about a purchase receipt, another student, S-02, made a comprehension error in the same question. Figure 2 indicates that student S-02 failed to answer question 3 correctly. Looking at the given answers, S-02 was able to write down the information provided by the question. However, they made a mistake in writing down what was asked in the question. To investigate whether this error was due to a reading error or a comprehension error, an interview was conducted with the PSPST S-02, with the following results.

R: What is asked in problem 3?]

S-02: Price of one fan

R: What do you mean by the price of 1 fan? Can you explain?

S-02: Yes, Ma'am. 1 fan. Because Mrs. Rissa paid for 3 fans. The question asked for the price of 1 fan.

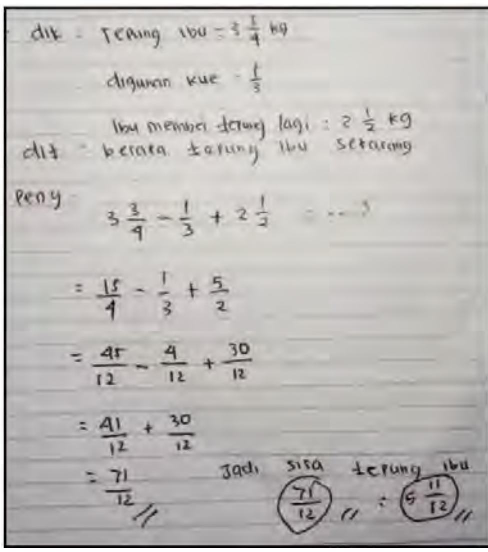
R: Is it written in the question that they are asking for the price of 1 fan?

S-02: No, it's not mentioned explicitly here about the price of 1 fan. Since it was stated that Mrs. Rissa paid for 3 fans, I assumed that the price for 3 fans was already known (more than 1 million). So, I thought they were asking for the price per fan.

The interview fragment indicated that S-02 was capable of reading the question but failed to comprehend it accurately. S-02 read all the information from the question but did not understand the meaning of the sentence "the price of the fan that should be written by the cashier on the receipt." S-02 interpreted this sentence as indicating that information about the prices of the three fans has already been given in the question, so it is unlikely that the same information is being asked again. This interpretation leads to the prediction that the question concerns the price per fan. Therefore, it can be concluded that S-02's comprehension error stems from an interpretation mistake in the given question sentence. S-02 had difficulty comprehending the meaning of the questions. Consequently, he was unable to identify accurately what was being asked. Therefore, the identification of problem-solving strategies does not proceed well, because the meaning of the question itself is not well understood.

Transformation error. This type of error occurs when test takers make mistakes in transforming a word problem into an appropriate mathematical model. Transformation errors will result in errors in selecting the appropriate formula or employing the calculation process, ultimately leading to students solving the problem incorrectly.

Figure 3 shows that S-03 committed a transformation error by being unable to convert or transform the information from Question 1 into a mathematical model. In the context of this question, a transformation error occurs because S-03 is unable to convert the narrative information in the contextual-based problem into a mathematical model. Upon further investigation through an interview, S-03 explained that: "I transformed the information from the question "My mother has $3\frac{3}{4}$ kg of sago flour. $\frac{1}{3}$ part of the flour is used to make sago cheese cake" into math sentence $3\frac{1}{3} - \frac{1}{3}$" From this statement, it can be perceived that the mathematical model created by S-03 is incorrect. This error occurred since S-03 did not thoroughly understand the meaning of "$\frac{1}{3}$ part of the flour". In other words, the transformation error that occurs in this context is the inability of the test taker to accurately create a mathematical model because of a lack of understanding of the question (comprehension error). However, not all transformation errors caused by comprehension errors. There is no guarantee that students who do not have difficulty understanding the meaning of the question can perform the transformation process accurately.



Translation:
Given: Mother's wheat flour = $3\frac{3}{4}$ kg
Used for making cookies = $\frac{1}{3}$ kg
Buying additional flour = $2\frac{1}{2}$ kg
Asked: how many kg of flour does my mother have now?
Answer $3\frac{3}{4} - \frac{1}{3} + 2\frac{1}{2} = \cdots ?$

$= \frac{15}{4} - \frac{1}{3} + \frac{5}{2}$

$= \frac{45}{12} - \frac{4}{12} + \frac{30}{12}$

$= \frac{41}{12} + \frac{30}{12} = \frac{71}{12}$

So, the remaining flour of my mother now is $\frac{71}{12} = 5\frac{11}{12}$.

Figure 3. Written response of PSPST S-03 on problem 1

Process skill error. This error type is characterized by test-taker-making errors when applying a formula, making mistakes when performing algorithms or mathematical calculations, encountering errors in algebraic manipulation, and struggling to apply mathematical processes when solving a problem. The following is an example of a process skill error committed by the PSPST S-04.

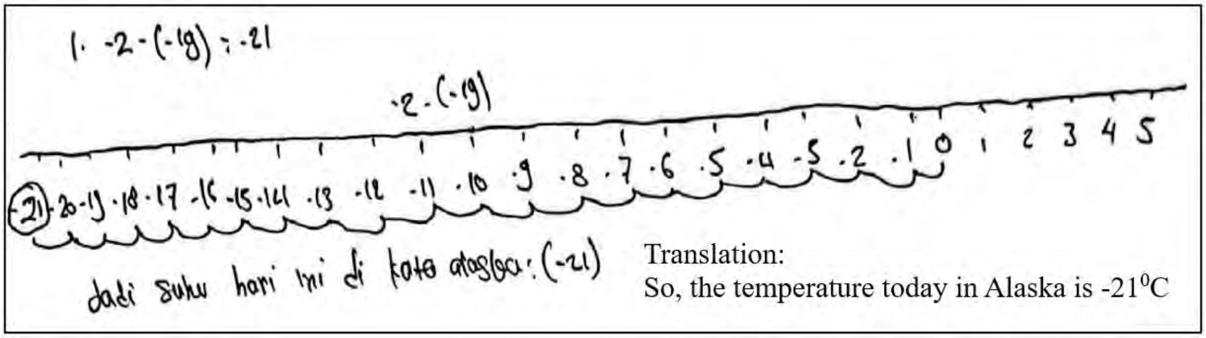
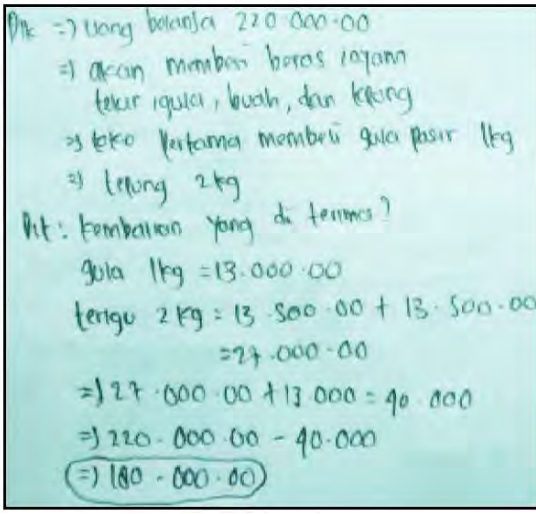


Figure 4. Written response of PSPST S-04 on problem 2

Figure 4 shows the process skill error. This written response was then confirmed through an interview. The interview results indicate that S-04 has good comprehension, being able to understand the meaning of the question well and accurately identify what is given and what is asked. Student S-04 stated, "The temperature in the afternoon is -2°C, and the temperature at night is -19°C. The question asks for the temperature difference, which means subtracting -2 from -19." This statement indicates that S-04 could interpret the words 'difference' and 'temperature decrease' accurately and can formulate them into the mathematical model $2-(-19)=\cdots$. S-04's verbal response indicates that he can explain the correct mathematical model and formulation to apply but fails to execute the procedure accurately. This error occurs because he does not understand the concept of integer operations that involve negative integers. The use of the number line does not help either because he does not understand the concept of subtracting negative integers.

Encoding error. This type of error is indicated by the test-taker’s error in drawing the conclusion. Encoding error refers to the mistake in accurately expressing the mathematical solution in a written format in a real-life context. Encoding errors can also occur because of the inability to interpret or validate whether the generated mathematical solution is appropriate in accordance with the context of the problem. The PSPST S-04 exemplifies an encoding error, as shown in Figure 5.



Translation:
Given: Mshopping money IDR 220.000
willing to buy rice, chicken, egg, sugar, fruit, flour
buying sugar 1 kg and flour in the first store
Asked: how much changes she receive?
Answer: Sugar 1 kg = 13.000 (IDR)
Flour 2 kg = 13.500 + 13.500 = 27.000 (IDR)
= 27.000 + 13.000 = 40.000 (IDR)
= 220.000 – 40.000 = 180.000 (IDR)

Figure 5. Written response of PSPST S-07 on problem 4

Upon initial observation, there seems to be no error in the response of S-07, as shown in Figure 5. S-07 demonstrated the ability to perform process skills using the correct algorithm and calculation procedures. However, upon closer examination of the given context of the problem, it was stated that Mrs. Dika has two pieces of one hundred thousand rupiah (IDR) and two pieces of ten thousand rupiah (IDR), which are used to purchase sugar and flour with a total price of IDR 40.000. The question asks for the change Mrs. Dika receives from the cashier. If we connect this with the real-world context, it can be assumed that the most likely scenario is that Mrs. Dika will pay for sugar and flour with one piece of one hundred thousand rupiah money. To pay for an item priced at IDR 40.000, Mrs. Dika does not need to use all the money she has (two pieces of one hundred thousand rupiah and two pieces of ten thousand rupiah). Based on this assumption, it can be concluded that the change Mrs. Dika receives is IDR 60.000.

The error that occurred was S-07's inability to interpret a mathematical solution in terms of a real solution. He was unable to verify whether the mathematical solution was reasonable or aligned with the context of the problem. Based on the interview results, S-07 made an encoding error because of misinformation while reading the problem. They overlooked specific details about Mrs. Dika's money (two pieces of one hundred thousand rupiah and two pieces of ten thousand rupiah). S-07 only focused on the total amount of IDR 220.000, which led them to simply interpret how much money Mrs. Dika had left after paying for sugar and flour. The main mistake made by S-07 was the inability to connect the given problem to a real-life context. In a real-life scenario, when we want to pay for an item worth Rp 40.000 while having two pieces of one hundred thousand and two pieces of ten thousand, we would typically use one piece of one hundred thousand (without using the entire IDR 220.000). S-07 acknowledged this mistake during the interviews.

DISCUSSION

The analysis of PSPSTs' written discourse and verbal responses through interviews indicated that many of them made errors in answering the given questions. In questions 3 and 4, the percentage of students who answered incorrectly exceeded 65% (see Table 2). Similarly, for the other three questions, the error percentage ranged from 20 to 40%. This clearly indicates that many students still face difficulties in solving context-based mathematical problems.

The data in Table 2 also indicate that many prospective teachers have difficulties with the five NEA procedures. Process skill errors are the least common type of error. Conversely, the most frequent errors were comprehension (22.1%), followed by reading and encoding errors, each at 17.5%. These findings are quite surprising, especially regarding the high occurrence of reading and encoding errors. These findings contradict with several previous studies (Alhassora et al., 2017; Fitriani et al., 2018; Wijaya et al., 2014) which mostly showed that reading errors are rarely, even do not not at all, committed by test takers. Similarly, in terms of encoding, multiple studies have shown that encoding is the least common type of error committed by students (Fitriani et al., 2018; Hadi et al., 2018; Wijaya et al., 2014). Similar findings were also revealed in several other studies (Hadi et al., 2018; Shinariko et al., 2020), which revealed that no reading and encoding errors were identified for all recorded questions. Alhassora et al. (2017) also reported that students generally had good reading and encoding abilities in their study.

Specifically, students experienced reading errors when solving questions 1, 3, and 4. The students' answers and interviews for these three questions indicated that reading errors occurred because students made mistakes in reading important words in the questions or misunderstood the main information. As a result, they did not use this information to solve the problem. These errors were caused by students' lack of attentiveness or disregard for crucial words or sentences, leading to misunderstandings of the entire question. Reading errors have implications for transformation errors, which involve mistakes when modeling a situation into mathematical sentences. As revealed by Isik & Kar (2012) in their study, pre-service elementary mathematics teachers face linguistic challenges when transitioning from symbolic to verbal representation. Based on the students' answers and interview results, it was found that reading errors directly impact comprehension errors, and consequently, the transformation stage, ultimately leading to incorrect final answers. These findings support Newman's (1977) argument regarding the hierarchical structure of error types. They suggested that when students encounter difficulties in a specific step of a task, it becomes a barrier that hinders their progress to the subsequent step.

The next type of error is comprehension error. Comprehending this question is an essential element of the problem-solving process. However, the findings of this study reveal that many PSPSTs still struggle to thoroughly understand the given information from context-based mathematics problems (more than 20% error). This finding reinforces the results of previous research (Burgos & Godino, 2022) that the difficulties most frequently identified by prospective teachers were those concerned with understanding the statement requirements from the problems.

The comprehension errors experienced by the PSPSTs in this study are caused by misinterpretation of the given question sentences. There is a misunderstanding about given keywords and instructions of the problem. The errors resulting from misunderstanding the instructions are closely related to reading errors. Due to the lengthy sentences and abundant information presented in context-based problem, PSPSTs experienced confusion to determine what should they do with the problems. Consequently, they struggled to extract the intended meaning from the problem, leading to potential misunderstandings that created uncertainty about the appropriate approach to solve the problems (Tambychik & Meerah, 2010).

However, it is crucial to highlight the fact that comprehension errors differ from reading errors. Reading errors are connected to inaccuracies in reading or understanding information, whereas comprehension errors involve difficulties in grasping the concepts or meanings of information even after reading it correctly (Clemen, 1980). As found in this study, the PSPSTs were able to correctly read information in questions, including words, sentences, and numbers. However, they struggled to comprehend the meaning of the sentences, failed to discern what the question was asking, and lacked a solid understanding of the concepts of the problem.

Furthermore, it should be emphasized that not all comprehension errors occur because of reading errors. Another case in this study is when students are able to read the problem comprehensively but fail to understand the meaning of the problem, resulting in an incorrect interpretation of what they are asked to do. In addition to misunderstanding the keywords of the problem, comprehension errors can also be caused by participants being unable to select relevant and irrelevant information given in the problem (Wijaya et al., 2014). Comprehension errors indicate mistakes in PSPSTs' understanding of the question when they have difficulty comprehending its meaning. Consequently, they are unable to identify what is being asked and what information is known from the given question. Errors in understanding the meaning of the question lead to students' inability to write down the important and relevant information required from the given question. The written answer provided by the PSPST did not represent what was asked or what was given in the test item, and it did not lead to the correct answer (Hadi et al., 2018). As observed in the error made by PSPST in answering questions 3 and 5 (see Figure 2 and 5), PSPST tends to utilize all the numbers provided in a task while neglecting the relevant information. These findings align with previous studies that indicate students face difficulties in understanding the wording of context-based tasks and identifying relevant information (Prakitipong & Nakamura, 2006; Wijaya et al., 2015).

Apart from understanding the problems, other difficulties encountered by PSPSTs is performing mathematics modeling. Often, PSPSTs understood the problems well, yet they still could not solve them. Multiple studies have demonstrated that transformation errors have become a major type of error experienced by test participants, particularly students, in solving mathematical problems (Shinariko et al., 2020). The findings of this study indicate that transformation errors occur because of students' inability to transform narrative information in contextual-based items into mathematical models. Students' failure to convert information from text to more abstract mental representations is attributed to their poor comprehension of the meaning of the problem. In other words, comprehending errors contribute to the occurrence of transformation errors. Transformation errors can indirectly result from reading errors, leading to comprehension errors. There is significant potential for misunderstanding the meaning of the problem, which can greatly contribute to errors in mathematical modeling or transformation errors.

However, it should be emphasized that not all transformation errors are caused by comprehension errors. The findings of this study also indicate that there are prospective teachers who make transformation errors even though they correctly understand the intent of the problem. This is because of the lack of ability of PSPSTs to model real-life situations into formal mathematical models. Transformation errors occur because participants are unable to construct a mathematical model by mathematizing a real-world problem that represents the given situation.

The next error type is the process skill error. Process skill errors were not major errors caused by the PSPSTs in this study. It appears that students do not struggle much to perform mathematical calculation procedures. Technical calculation errors were also minimal. This is because the calculation algorithms required to solve context-based mathematics problems in this study can be considered very simple for prospective mathematics teachers, involving basic arithmetic operations, such as addition, subtraction, and multiplication. The high occurrence of process skill errors in this study (see Figure 4) depicts, the failure to perform the procedure accurately, which is caused by PSPSTs' lack of understanding of the concept of integer operations involving negative numbers. Operations involving negative numbers are often considered challenging topic for many students.

The next error type is the process skill. However, process skill errors were not the major errors committed by the PSPSTs in this study. It appears that students did not encounter significant difficulties in performing mathematical calculation procedures. Technical calculation errors were also minimal. This is because the calculation algorithms required to solve context-based mathematics problems in this study can be considered quite simple for prospective mathematics teachers, involving basic arithmetic operations such as addition, subtraction, and multiplication. The process skill error that occurred most frequently in this study (see Figure 4) reflects the inability to accurately perform the procedure, which is caused by PSPSTs' lack of understanding of the concept of integer operations involving negative numbers. Operations involving negative numbers are often considered to be challenging for many mathematics learners. Previous research has also indicated challenges in the conceptual understanding of negative integers (Almeida & Bruno, 2014, Fuadiah et al., 2019) and their depiction of the number line among pre-service teachers (Widjaja et al., 2011).

The last type of error is encoding. Encoding errors can occur because of students' inability to interpret and evaluate mathematical solutions obtained in real-world applications. The students struggled to accurately understand and verify the mathematical solution in relation to real-life problems. This mistake is indicated by an impossible or unrealistic answer (Wijaya et al., 2014).

In this study, encoding errors were frequently found in the PSPSTs' responses in solving problems 4 and 5. Based on the analysis of students' written responses, encoding errors were not the least common type of error committed by the PSPSTs. More than 17% of participants had encoding errors. These findings tend to contradict previous research indicating that test takers made fewer errors in the interpretation of the mathematical solution in real-world situations (Alhassora et al., 2017, Fitriani et al., 2018, Wijaya et al., 2014).

The findings of this study revealed that the occurrence of errors committed by participants has a hierarchical structure. This means that when participants make errors at the initial stages (e.g., reading or comprehension errors), there is a high possibility that they will become stuck or unable to proceed with the problem-solving process in the subsequent stages (e.g., transformation, process skill, and encoding). Alternatively, under different conditions, there is a high probability that they will cause errors in subsequent stages. When PSPSTs are already stuck in the early stages (e.g., reading, comprehending, or modeling process), they will not arrive at the stage of carrying out mathematical procedures and performing encoding accurately. Additionally, the research findings also revealed that reading comprehension significantly influences the ability of PSPSTs to solve context-based mathematics problems correctly. Reading and comprehension errors are two types of errors that act as initial barriers to PSPSTs, preventing them from accurately solving problems.

CONCLUSIONS

The analysis of PSPSTs' written and verbal responses through interviews revealed that many encountered difficulties in solving context-based mathematics problems. This is indicated by the number of errors made. Based on the error types in the NEA framework, the most common error committed is comprehension error (22.1%), followed by reading and encoding errors, each accounting for 17.5%, transformation error (14.7%), and process skill error as the least common type among the others (8.7%). Furthermore, the study findings indicate a hierarchical structure in the occurrence of errors among test-takers. Errors in the initial stages significantly impact the progression of the problem-solving process in subsequent stages. Test takers who encounter challenges in the early stages (e.g., reading and comprehension) are highly likely to find difficulties and make errors when performing transformation, process skill, and encoding in the subsequent stages. The hierarchical identification of specific cause-and-effect relationships between error types could be a topic for further research.

In light of these findings, it is recommended to focus on addressing identified error types to improve the problem-solving abilities of PSPSTs. Emphasizing strategies and interventions to enhance reading, comprehension, transformation (modeling), processing skills, and encoding skills can significantly help PSPSTs to be more ready to teach mathematics in real class situations. Furthermore, future research should explore the hierarchical relationships among error types in greater depth. Investigating specific cause-and-effect relationships between error types can provide valuable insights into instructional design and tailored interventions to enhance problem-solving proficiency. Overall, these recommendations aim to enhance the overall performance of PSPSTs by addressing specific error types, fostering a comprehensive understanding of mathematical problems, and improving their ability to solve context-based problems.

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