Visualizing Math Word Problems: Impact on First-Grade Students' Problem-Solving Performance

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Abstract: It is important to work on verbal mathematical problem-solving skills, which is one of the high-level skills with which students have difficulty. Supporting problems with visuals has been suggested by mathematics educators to develop this skill. This study examined the effect of the visualization of mathematical problems on the problem-solving performance of primary school first grade students. The study group consisted of 41 students attending first grade in a public primary school located in a low socio-economic status region of Istanbul. Semi-structured individual interviews were conducted with 8 students. Problem-solving questions developed by the researchers were used for pretest and post-test, the results of which were analysed using the Wilcoxon Signed-Ranks test. For the experimental process, the Visually Supported Problem-Solving Worksheet was used as a data collection tool. Students' views on the implementation were analysed using a content analysis method. The results indicate that the problem-solving worksheet supported with visuals was effective in improving the mathematical problem-solving performance of the students. The majority of the students stated that the implementation contributed to improving their performance and their satisfaction.

Keywords: first grade, mathematics, mathematical word problem, problem solving, visualization

INTRODUCTION

Mathematical problem-solving, a skill that empowers students to apply mathematical concepts in diverse contexts, stands as an integral component of both mathematics and mathematics curricula (National Council of Teachers of Mathematics [NCTM], 2000). Educators hold the conviction that problem-solving not only enhances students' mathematical proficiency but also facilitates the transfer of knowledge to novel and unfamiliar problem-solving scenarios (Mršnik et al.,2023; Putri et.al, 2023). Mathematical word problem solving, with which students have difficulties at all levels of education (Fuentes, 1998; Mayer, 1998; Verschaffel et al., 2000: Verschaffel et al., 2009), requires reading and reading comprehension skills as well as mathematics. Although the roots of mathematics are observable prior to school (Sjoe et al., 2019), this skill, which is described as the heart of mathematics (Cockcroft, 1982), is first presented to children in the first grade of primary school, when students are expected to add and subtract numbers and to solve problems related to these two operations (Ministry of National Education [MoNE], 2018).

When the first-grade objectives of the curriculum related to verbal problem solving of the mathematics are examined, there are found to include objectives such as “Students are able to solve problems that require addition with natural numbers” and “Students are able to solve problems that require subtraction with natural numbers.” It has been stated in the explanations regarding these objectives that problems with these operations should only have one step (MoNE, 2018). When first grade mathematics textbooks are examined, it is apparent that addition and subtraction problems are generally presented to students with visuals, and students are asked to solve problems by writing mathematical sentences corresponding to the given images (e.g., Bahçivancı et al., 2021).

The related literature has noted that students’ have obstacles related to mathematical problem solving (Fuentes, 1998; Hudson & Miller, 2006; Mayer, 1998; Khoshaim, 2020).). The results of national and international exams of Turkey, which generally include the application of verbal mathematics problems to daily life, also support these findings. Looking at the 2019 results of the Trends in International Mathematics and Science Study (TIMSS), in which the science and mathematics achievements of fourth and eighth grade students are evaluated by the International Association for the Evaluation of Educational Achievement (IEA) over four-year periods, Turkish students performed significantly higher than the mean of the scale (500 points) for the first time. It was also observed that, although they showed higher performance in questions at the application level, they showed lower performance in verbal mathematics problems that require reasoning, which is one of the higher learning skills (MoNE, 2019). In 2018, the results of the PISA (Programme for International Student Assessment), another international assessment exam conducted by the Organization for Economic Co-operation and Development (OECD) in three-year periods were published. Although Turkish students increased their average score to 454 in mathematical literacy, it remained below the average score (459) of the participating countries (MoNE, 2019). In the national high school entrance exam, it has been observed that students have difficulty in solving mathematical problems and can only answer some of the questions (Karip, 2017; MoNE, 2019).

It is important to conduct research to try to solve the problems related to verbal mathematical problem-solving skills in the first years of primary school. Supporting problems with visuals is one of the strategies suggested by mathematics educators in the related literature to improve these skills (Cankoy & Özder, 2011; Krawec, 2014; van Garderen & Montague, 2003). Using images helps to eliminate the dependency on reading skills by making the problem more concrete or realistic. Visualization facilitates verbal problem solving by helping mental model formation (Múñez et al., 2013). For this reason, problems supported by visuals are frequently used in mathematics textbooks (Bahçivancı et al., 2021). Supporting problems with visuals is also an important aspect of Polya's (1957) problem-solving stages, which is emphasized in both the understanding and planning stages. According to Smith (1994), the visualization process matures between the ages of 8 and 11. Visualization occurs both internally (e.g., as a mental image) and externally (e.g., via the individual's use of a pencil or a different material) (Zimmerman & Cunningham, 1991; as cited in van Garderen & Montague, 2003, p. 246).

When the studies in the literature are examined, it appears that the effects of visualization on mathematical problem solving can differ. The reason for this difference may be related to the type of visualization used. Researchers have tried to explain the definition and types of visual images exactly. According to Arcavi (2003, p. 217), "visualization is the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings" (as cited in Presmeg, 2020). One of the earlier classification of visualization was defined by Presmeg (1986a, 1986b), who mentioned five different types of visualization: concrete imagery (a "picture in the mind"); kinesthetic imagery (of physical movement, e.g., "walking" several vectors head to tail with the fingers); dynamic imagery (the image itself is moved or transformed); memory images of formulae; and pattern imagery (pure relationships stripped of concrete details). A more recent classification has been made by Elia and Phillippou (2004), who identified four types of visualization: "(a) decorative, (b) representational, (c) organizational, and (d) informational. Decorative pictures do not give any actual information concerning the solution of the problem. Representational pictures represent the whole or a part of the content of the problem, while organizational pictures provide directions for drveawing or written work that support the solution procedure. Finally, informational pictures provide information that is essential for the solution of the problem; in other words, the problem is based on the picture (p. 328)".

Hoogland et al. (2016) presented problems to 31,842 students between the ages of 10 and 20 with depictive representations. They mostly used photographs—defined as decorative according to Elia and Philippou (2004)—and it was found that this had a positive effect on students' problem-solving performance. Cankoy and Özder (2011) found that fifth grade primary school students' performance in solving problems that included decorative visual expression was higher than their performance in solving problems that did not include such visual expression. In addition, adding a visual representation to problems with a context unfamiliar to the students increased their performance in solving these problems. Therefore, visual representations appear to be an important resource in making problems easier for students to solve (Cankoy & Özder, 2011). A study in which the quantities in the problem were visualized found that representational pictures had a strong and significant effect on the mathematical problem-solving performance of kindergarten students for addition and subtraction problems (Elia, 2020).

Matalliotaki (2012) compared oral and visual representation to support mathematical problems presented to five- to six-and-a-half-year-old children and found that visualization is more efficient than oral presentation in helping the children to solve problems. On the other hand, in studies conducted with primary school students, it has been observed that decorative visuals are not supportive in solving mathematical problems and the communication process. In the same study, other types of visualization were effective on mathematical problem-solving performance and communication skills (Elia & Philippou, 2004; Gagatsis & Elia, 2004).

There are also studies that indicate visualization has no effect on mathematical problem-solving performance. For example, Dewolf et al. (2017) found that visualization of problems did not have a positive effect on the performance of students aged 9–12. Indeed, Berends and van Lieshout (2009) showed that adding visuals to a verbal mathematical problem had a negative effect on the performance of students in the 9–10 age group. In their study, Hegarty and Kozhevnikov (1999) emphasized two different types of visuals: schematic and pictorial. They said that these two different types of visualization have different relations with mathematical problem solving. Like decorative visuals, pictorial visualization is defined as only coding the appearance of the object in the problem, while schematic visualization shows spatial relationships in parts of the problem, including spatial transformations. As a result of their study with sixth grade male students, they concluded that schematic visualization was positively related to mathematical problem solving, and pictorial visualization was negatively related. According to the results of another study conducted to examine the visual images used by students at different levels (gifted, medium, and with learning difficulties) while solving mathematical problems, it was concluded that gifted students used statistically significantly more visual spatial visualization than the students in the other two groups. Individuals with learning disabilities used more pictorial representations than other students. In other words, it was concluded that successful problem solving was positively related to visual spatial representations and negatively related to pictorial representations (van Garderen & Montague, 2003).

In a recent study conducted with students attending first grade in primary school (van Lieshout & Xenidou-Dervou, 2018) mathematics problems were supported pictorially, auditory, and both pictorially and auditory, and the effect of this support on student performance in solving addition and subtraction problems was examined. They found that combining pictorial information with auditory information reduced the cognitive load and consequently increased performance. In addition, these effects were observed to be most prevalent in children who scored below the average in the general mathematics test.

While verbal mathematical problem solving is first encountered by students in kindergarten, word problems involving reading comprehension first appear in the first grade of primary school. Solving addition and subtraction problems that require a one-step operation is among the first-year objectives for mathematics. The use of visualization in the first grade, when students are just learning to read, can be effective in helping students to acquire this skill by reducing the cognitive load in mathematical problem-solving. As Smith (1998) has pointed out, the visualization skills of children begin to mature in and after the second grade. In this context, the present research examined the effect of visualizing mathematical word problems on the problem-solving performance of primary school first-grade students. The visuals used in this research were created to explain the mathematical sentences that the students had to write. For this purpose, answers to the following questions were sought. (1) Do mathematical word problems supported by visuals have an effect on students' problem-solving performance? (2) What are the students' views on the implementation of mathematical word problems supported by visuals?

METHOD

Research Design

This research used the one-group pre-test–post-test pre-experimental model, which is an experimental research design (Karasar, 2013). In addition, an exploratory sequential design, a mixed research design, was used to answer the research questions. In exploratory sequential design, quantitative and qualitative data take place in two stages and sequentially. First, quantitative data, which are prioritized for answering the study's questions, were collected and analyzed. In the second phase, qualitative data were collected and analyzed to complement the initial data (Creswell & Clark, 2008).

Study Group

The homogeneous sampling method, a purposive sampling type, was used to determine the study group for this research. The study group consisted of 58 students attending the first grade in a public primary school located in a low socio-economic status region of Istanbul. According to the pre-test results, 12 students were successful in the test (100%) and 5 students did not want to participate in the study and were therefore not included. A total of 41 volunteer students, 25 boys and 16 girls, constituted the study group. The parents of these students were informed by the classroom teacher and their consent was obtained. Semi-structured individual interviews were conducted with 8 (5 boys and 3 girls) students who are volunteer to participate. One student's mathematical achievement was above the average, five were at an average level, and two of them were below the average.

Data Collection Tools

Problem-solving questions (pre-test–post-test)

The problem-solving questions developed by the researchers consisted of six problems related to the addition of natural numbers. The objective of these problems was that "Students will be able to solve problems that require addition with natural numbers" (MoNE, 2018). Based on this objective, three different types of problems requiring addition operations were presented to the students. According to the action in the content of the problem, the problems involving addition operations were grouped under three main headings. The unknown items in these headings are named as result unknown, start unknown, or change unknown (Olkun & Toluk Uçar, 2012), and two of each type were developed for the measurement tool. According to these headings, examples of problems related to the addition of natural numbers are given in Table 1 below.

	Example of Symbolic Model	Sample Problem	Number of Questions
Addition Result Unknown (ARU)	7+4=?	There were 7 fish in Nehir's aquarium. His mother bought 4 more fish for Nehir. How many fish were in the Nehir's aquarium altogether?	2
Addition Change Unknown (ACU)	8+?=15	Alp had 8 marbles. With the marbles his father gave as a gift, Alp had 15 marbles. In this case, how many marbles did his father give Alp as a gift?	2
Addition Start Unknown (ASU)	?+7=16	There were some cookies in the jar. By the time my mom added 7 more cookies, the number of cookies in the jar was 16. How many cookies were in the jar in the beginning?	2

(ARU: Addition Result Unknown, ACU: Addition Change Unknown, ASU: Addition Start Unknown)

Table 1. Examples of the Types of Problems

The rubric used in scoring the problems is as follows: 0 points, If the question is not solved at all; 1 point, if an attempt is made to solve the question; 2 points, if the problem is half solved or a computation error is made; 3 points, if there is only an answer without any calculation; and 4 points, if the question is completely correct. In this case, the minimum score that students can get is 0, and the maximum score is 24.

Expert opinions regarding the problem-solving questions and scoring were obtained from one primary school teacher, one mathematics educator, and one specialist with a doctorate in primary education. An expert opinion related to the language and expression of the problems was sought from an expert with a doctorate in Turkish education. This tool was used as both a pretest and a post-test. It was developed according to the expert opinions sought, and a pilot study was carried out with three students who are not in the study group. At the end of the pilot study, the tool was finalized. For the reliability of the problem-solving questions, the answers given by the students were scored according to rubric and the reliability coefficient between the raters was calculated. Reliability among researchers was calculated according to Miles and Huberman's (1994) formula (reliability = agreement / (agreement + disagreement) × 100), which takes into account the number of agreements and disagreements between the researchers. The inter-rater reliability coefficient was calculated as .97. A consensus was reached by the researchers for the questions scored differently. The Cronbach's alpha reliability coefficient for the reliability of the test was calculated as .83.

The visually supported problem worksheet

The visually supported problem worksheet developed by the researchers consisted of 10 mathematical word problems supported by visuals. The visuals illustrated in the worksheet are used to explain the mathematical sentence that the student has to write. Because the students' ARU (Addition Result Unknown) average was high (6.06 out of 8), it was decided that the worksheets supported by visuals should consist of 5 ASU (Addition Start Unknown) and 5 ACU (Addition Change Unknown) problems. Students were asked to solve the questions and fill in the boxes under each problem on the worksheet. The visuals were illustrated by a visual design expert. For the validity of the problem worksheet supported by visuals, expert opinions were obtained from one primary school teacher, one mathematics educator, one specialist with a doctorate in primary education, and one education technologist. An expert opinion related to the language and expression of the problems was sought from an expert with a doctorate in Turkish education. The answers given by the students to the worksheet were scored with the rubric prepared for the pre-test (Problem-Solving Questions). The Cronbach's alpha reliability coefficient, which checked the reliability of the questions, was calculated as .97. Examples of change unknown and start unknown problems with worksheet visuals are given in Figure 1.

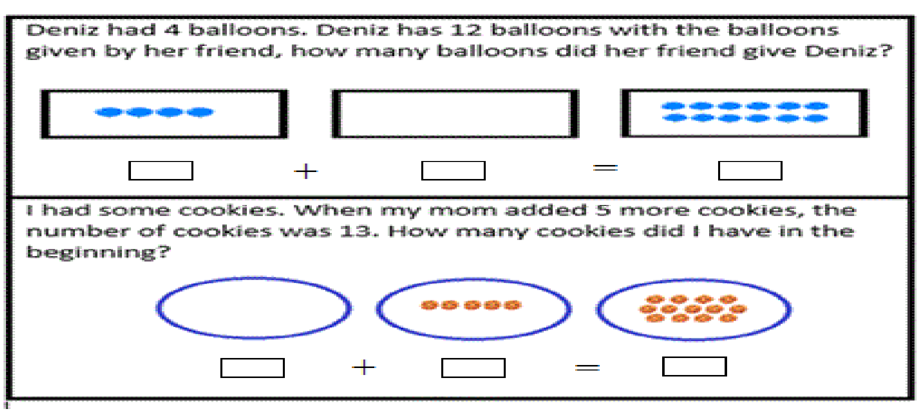

Figure 1. Examples of Change Unknown and Start Unknown Problems.

Interview questions

A semi-structured interview form with three questions was prepared by the researchers to get the students' views on the implementation. The opinions of a classroom teacher, a mathematics educator, and an expert with a doctorate in primary school education were obtained to assess the validity of the interview questions, and the interview form was finalized with three questions. With the necessary permissions, the interviews were audio-recorded. While the audio recordings were being analyzed by the two researchers, all of the words, pauses, and exclamations of the participants were included and converted into a written transcript. One of the audio recordings was randomly selected and transcribed separately by the researchers and compared. The similarity in the transcripts was determined as 100%. Voice recording analyses were performed separately by two researchers, and the inter-coder reliability coefficient was calculated as 98%. A consensus was reached by discussing a small number of encodings that were thought to be different.

Data Collection Process

The data collection process started with the application of the Problem-Solving Questions (pre-test) to the students by one of the researchers in the middle of the spring semester. The students were given 25 minutes for the six questions in the pre-test. According to the results of this test, 12 students were successful (100%), and 5 students who did not come to school on the application day were not included in the study. A Visually Supported Problem Worksheet was prepared, consisting of questions with change unknown and start unknown, since the remaining 41 students had a success rate greater than 60% for the result unknown questions. One of the researchers divided the 41 students into two groups and applied the Visually Supported Problem Worksheet over 40 minutes in two separate lessons. The researcher explained to the students that the questions in the prepared worksheet were supported by visuals, and the students answered the questions individually. A post-test was given to the students 3 weeks after the implementation. During these three weeks, students worked on data analysis and measurement issues in the curriculum. During this period, no problem solving studies involving addition and subtraction of natural numbers were carried out. To get students' opinions on the application, semi-structured interviews were conducted with 8 students who were not successful in the pre-test but were successful in the post-test and agreed to be interviewed voluntarily. In order not to distract the students, the interviews were carried out individually by one of the researchers in a room outside of the classroom. Each interview lasted approximately 15 minutes.

Data Analysis

Before testing whether there was a significant difference between the students' pre-test and post-test problem-solving scores, it was tested whether the distribution of the data was normal. Because the data were not normally distributed, the Wilcoxon Signed Rank test was used to compare the students' pre-test and post-test scores. The opinions of the students about the intervention were analysed using content analysis. Instead of the names of the students, numbers were given to the students, and they were coded as Student (S; e.g., the code S5 means student number five). The mathematics achievement of one of these students was above the average, five of them are at an average level, and two of them were below the average according to the opinion of the teacher who is also one of the researchers of this study. During the interviews, three questions were asked about the intervention. The codes and frequencies of the answers given by the students to the questions and examples from the answers of the students are explained in order.

RESULTS

The findings related to answering each of the research questions are presented separately. Before looking for an answer to the first research question—Do mathematical word problems supported by visuals have an effect on students' problem-solving performance? —the distribution of the data was checked, and it was concluded that the distribution of the data was not normal for both pre-test and post-test. Before the Wilcoxon Signed-Ranks test, the descriptive statistics for both pre-test and post-test scores are included and explained in Table 2.

	n	Min.	Max.	X	s
ACU - pre-test	41	2.00	8.00	2.78	1.75
ACU - post-test	41	0.00	8.00	4.39	2.84
ASU - pre-test	41	0.00	8.00	2.59	1.86
ASU - post-test	41	0.00	8.00	4.07	2.71
ARU - pre-test	41	1.00	8.00	6.07	2.45
ARU - post-test	41	5.00	8,00	7.46	1.00

Table 2. Descriptive Statistics for the Students' Problem-Solving Performance

Examination of the results shown in Table 2 reveals that scores improved from pre-test to post-test for the problem-solving questions. The average score was 6.07 for ARU in the pre-test, which rose to 7.46 in the post-test. The pre-test ACU average was 2.78, which rose to 4.39 in the post-test, and the pre-test ASU average was 2.59, which rose to 4.07 in the post-test. The Wilcoxon Signed-Ranks test results on whether students' problem-solving skills differed significantly before and after the intervention are shown in Table 3.

		n	Mean rank	Sum of ranks	z	p
ARU(pre-test post-test)	Negative ranks	4a	7.63	30.50	−2.98	0.003
	Positive Ranks	17b	11.79	200.50		
	Ties	20c				
ACU (pre-test − post-test)	Negative Ranks	7d	7.86	55.00	−2.54	0.01
	Positive Ranks	16e	13.81	221.00		
	Ties	18f				
ASU (pre-test − post-test)	Negative ranks	4g	7.63	30.50	−3.15	0.002
	Positive Ranks	18h	12.36	222.50		
	Ties	19i				

Table 3. Wilcoxon Signed-Ranks Test Results for Problem-Solving Test

The results of the analysis shown in Table 3 indicate that there is a statistically significant difference between the pre-test and post-test among the students participating in the study for all three problem types (result unknown, $z=-2.98$, $p < 0.01$; change unknown, $z=-2.54$, $p < 0.05$; and start unknown, z=−3.15, $p < 0.01$). When the mean rank and totals of the difference scores are considered, this difference appears to be in favour of the positive ranks—that is, the post-test score.

The second research question was: What are the students' views on the implementation of mathematical word problems supported by visuals? The analyses of the semi-structured interviews with eight students are explained below, including the codes and frequencies of the answers given, as well as example answers. The answers to the question of whether the visuals provided with the problems contributed to problem-solving are given in Table 4.

View		f	Student Number
Contribute	understand the problem more easily	3	S1, S4, S7
	solve the problem by counting the images	3	S3, S6, S8
Did not contribute	confused me	1	S5
	made it difficult for me to solve the problem	1	S2

S1, S2, S3, S7, and S8 average; S4 and S5 below average; S6 above average.

Table 4. Contribution of the Visuals to Problem Solving

In response to the first interview question, six students stated that presenting the problems with visuals contributed to the solution. Three of these students stated that the reason for the contribution was that the visuals made it easier to understand the problem, while the other three stated that they solved the problem by counting the images and benefited in this way. One of the two students who stated that they saw no contribution stated that it confused them, and the other stated that it made it difficult to solve the problem. One of the students who stated that they saw no contribution but gave more correct questions when given the visuals had low mathematics achievement and the other an average level. Some of the example answers include: "I did it by looking at the balloons and cookies given with the problems. These helped me to understand" (S1); "I counted the items given while solving the problem. Counting it that way was easy. I did it" (S6); and "[the] cookies and pencils confused me while solving the problem. I did not understand the problem" (S5). Based on these results, it appears that the students thought that presenting the problems with visuals contributed to more easily finding the solution to the problem. The opinions of the students regarding the presentation of the problems with the visuals are given in Table 5.

View	f	Student Code
I like that being given the images	6	S1, S3, S4, S6. S7, S8
I didn't like being given the images	2	S2, S5

Table 5. Student Views on Giving Problems with Visuals

In the student interviews, six students stated that they were satisfied with the presentation of the problems with the visuals, while two students were not satisfied. Sample answers included: one student said, "I like the fact that there are pens and cookies within the problems" (S1), while another student said, "I didn't like having visuals within the problems" (S5). It can thus be said that the students were mostly satisfied with the presentation of the problems with the visuals.

Finally, the opinions of the students regarding the use of visuals in future problems they may solve and their reasons are given in Table 6.

Preference	Reason	f	Student Code
I prefer it to be given with images.	It is easy to solve the problem when the problem is presented with visuals.	4	S1, S3, S7, S8
	When the problem is presented with visuals, it provides support for solving the problem.	2	S4, S6
I do not prefer it to be given with images.	When the problems are presented with visuals, the solution to the problems becomes even more difficult.	1	S2
	When the problems are presented with visuals, it becomes difficult to understand the problems.	1	S5

Table 6. Reasons for Preferences Related to Visually Presented Problems

According the results displayed in Table 6, six students stated that they preferred see the problems with visuals, because it provided ease of problem solving (n=4) or supported problem solving (n=2). Two students did not prefer problems with visuals and argued that the visuals make the solution and understanding difficult. For example, one student said, "It is very nice that problems are presented like this, and they are solved very easily. I would like it to be like this" (S7), while another student said, "I want it to be plain. When there are visuals, I get confused" (S5). Based on the overall results, it appears that most of the students prefer to see the problems presented with visuals.

DISCUSSION AND CONCLUSIONS

This study examined the effect of visuals to support mathematical word problems on the problem-solving performance of first grade students. First, in this context, addition problem-solving questions containing three different problem types (ARU, ACU and ASU) were applied to the students as a pretest. When the student answers were examined, it was found that the mean score for ARU was 6.07 (out of 8), while the means for ASU and ACU were much lower (2.78 and 2.58, respectively, out of 8). Prior studies have found that students are generally more successful in ARU problems than other types of addition problems (Peterson et al., 1989; Tarım, 2017). In a study conducted with children attending kindergarten and primary school in the United States, all children were successful in problems with ARU. In the problems with ACU, 61% of the kindergarten children and 56% of the first-grade children were successful, and in the ASU problems, 9% of the kindergarten students and 26% of the first grade students were successful (Riley et al., 1983 as cited in Erdoğan and Özdemir Erdoğan, 2009, p.39). Because the students in the present study already appeared, as in other studies, to be confident with ARU problems, the focus of the worksheets supported with visuals was on ASU and ACU problems.

According to the post-test results, the problem-solving worksheet supported with visuals appeared to be effective in improving the performance for both ACU and ASU problems. In fact, although ARU problems were not used in the worksheet, an increase was also observed in the students' ARU performance in the post-test. The students' views on the implementation also support this finding. According to the results of the interviews with eight students (who were not successful in the pre-test but succeeded in the post-test), most of the students (n=6) said that presenting the problems with visuals contributed to solving the problem, and they were pleased with the visual presentation of the problems, which facilitated finding the solution. Although the mathematical problem-solving performance of other the two interviewed students improved, they expressed a negative opinion about the intervention. Despite the negative opinions of these two students, the fact that their performance improved may indicate that the study achieved its purpose.

In a study in which the quantities in the problem were visualized, it appeared that visualization had a strong and significant effect on the performance of kindergarten students in addition and subtraction problems (Elia, 2020). Matalliotaki (2012) has also shown that visualization is more efficient than verbal presentation in solving verbal or visualized problems presented to five- to six-and-a-half-year-old children. Similarly, in studies conducted with primary school students, it has been observed that all kinds of visuals, except decorative ones, help mathematical problem solving and communication processes (Elia & Philippou, 2004; Gagatsis & Elia, 2004). There are also studies stating that even adding decorative visuals to the problems may contribute to student performance. For example, 31,842 students aged 10–20 were presented with decorative representations—mostly photographs—and it was found to have a positive effect on their problem-solving performance (Hoogland et al., 2018). Similarly, Cankoy and Özder (2011) have argued that adding decorative visuals to problems with a context that students are not familiar with increases student performance in solving these problems.

According to the related literature, it has been seen that the ways to visualize problems differ from each other, and this affects the study results. In their study with sixth grade students, Hegarty and Kozhevnikov (1999) concluded that (decorative) pictorial visualization and mathematical problem solving were negatively related, while schematic visualization was positively related to mathematical problem-solving success. Dewolf et al. (2017) also found that adding visuals to mathematical word problems did not have a positive effect on the performance of students aged 9–12. In a similar study, Berends and van Lieshout (2009) showed that visualization had no effect on the problem-solving process for students aged 9–10, and may even slow it down. In addition, there are also studies emphasizing that this kind of (decorative) visualization is negatively related to mathematical problem-solving success (Hegarty & Kozhevnikov, 1999; van Garderen & Montague, 2003). Although the results of the studies on problem visualization differ from each other, it can be said that visualization, which contributes to the understanding and solution of the problem, has a positive effect on the performance in solving mathematical word problems.

To increase students' mathematical word problem-solving performance, it is important to use visualization, especially in the first grade, when students first encounter word problems. The results of this research indicate that adding visuals that model the mathematical sentence necessary to solve the word problems improves student performance. In fact, when we look at the books prepared by the MoNE, such examples are often included (e.g., Bahçivancı et al., 2021). However, problems supported by visuals related to ACU and ASU word problems as given during the implementation process in this study are not encountered in MoNE textbooks (Olkun & Toluk Uçar, 2012; Peterson et al., 1989; Tarım, 2017). It is important to have questions supported by visuals that include all types of additional problems. This may be one of the reasons why students have difficulties, especially in ACU and ASU type problems. Because there may be difficulties in reading comprehension in the first grade, giving word math problems with visuals can reduce students' cognitive load. Future studies should consider monitoring the change in students' cognitive load. In addition, studies in which the control group is included in testing the effectiveness of the study and which has a longer duration could also be recommended for future research.

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