Teachers' Efforts to Promote Students' Mathematical Thinking Using Ethnomathematics Approach

S. W. Danoebroto[1], Suyata[2], Jailani[3]

[1]Postgraduate Program, Yogyakarta State University, Sleman, Indonesia, [2]Postgraduate Program, Ahmad Dahlan University, Yogyakarta, Indonesia, [3]Faculty of Mathematics and Science, Yogyakarta State University, Sleman, Indonesia

sriwulandari@almaata.ac.id, suyata@mp.uad.ac.id, jailani@uny.ac.id

Abstract: *Mathematical thinking is closely related to students' success in solving mathematical problems. Local culture can be a resource to teach mathematical thinking skills. Nonetheless, a lot of teachers still struggle to help students develop their mathematical thinking abilities. This study aims to investigate the ethnomathematics approach applied by the teacher and the types of mathematical thinking that can be promoted through this approach. Case studies are used in this study with three junior high school mathematics teachers as key informants. They were selected based on their experience using the ethnomathematics approach in the classroom. Non-participant observation and in-depth interviews were used as data collection techniques. The findings show that different ethnomathematics approaches, with the use of different local cultures, will lead to different types of mathematical thinking skills. Based on these findings, conceptual and empirical implications are discussed for further recommendations for teachers and school stakeholders.*

Keywords: etnomathematics, mathematical thinking, problem-solving, high school

INTRODUCTION

Teaching mathematical thinking is a challenge for teachers in Indonesia (Marsigit, 2007, Sari et al, 2020, Rohati, et al, 2022). The 2019 and 2023 Program for International Students Assessment (PISA) study results indicate that Indonesian students perform worse than the global average. In mathematics, only 18% of Indonesian students reach Level 2, compared to the OECD average of 69%. Students can understand and identify the mathematical representation of a basic situation at this level. The inability to comprehend context-based tasks and convert them into mathematical problems (Wijaya et al., 2014), the inability to reason and communicate mathematically (Mahdiansyah & Rahmawati, 2014, Rahmawati, et al., 2021), and the challenge of developing mathematical models using symbols (Al Jupri & Drijvers, 2016) are a few factors that are believed to be the cause. Among these are mathematical thinking skills.

LITERATURE REVIEW

There is a strong correlation between students' success in solving mathematical problems and their mathematical thinking (Schoenfeld, 1992, Mason et al., 2010, Ellenberg, 2014). Diverse interpretations exist regarding mathematical thinking. According to Adam (2004), mathematical thinking is a process of reasoning that entails abstraction, generalization, symbolization, and logical reasoning. Mathematization, abstraction and its application, and mathematical sense-making are the processes involved in mathematical thinking, according to Schoenfeld (1992). According to Mason, Burton, and Stacey (2010), the process of mathematical thinking involves specializing, generalizing, conjecturing, justifying, and persuading. Several of these definitions identify mathematical thinking as the mental process that comes into play when applying mathematics to solve problems.

By dividing mathematical thinking into three categories—mathematical thinking attitudes (mindset), mathematical thinking methods, and mathematical thinking content—Isoda and Katagiri (2012) provide a more thorough explanation of the concept. When students think mathematically, they approach problems with an attitude of attempting to develop perspective, ask questions, think with data that can be applied, and strive to convey ideas succinctly and clearly. Abstract thinking, simplification, generalization, specific thinking, symbolic thinking, and thinking expressed in terms of numbers, quantities, and figures are a few examples of mathematical thinking techniques. Examples of mathematical thinking content are the idea of operation, the idea of approximation, the focus on rules and nature, finding the rules of the relationship between variables, and the idea of formulas.

Mathematical thinking activities can be promoted in several ways. It can be characterized as a reflective process that reorganizes the real world using mathematical concepts in an idealized setting (Marsigit, 2007). Mason et al. (2010) suggest that teachers can foster mathematical thinking by creating an environment that involves questioning, posing challenges, and encouraging students to think critically. According to Tall (2008), three main factors lead to the way we think mathematically: the ability to recognize patterns in data, such as similarities and differences; the ability to repeat actions in a sequence until they become automatic; and the ability to use language to explain to enhance our thought processes. Mathematics relies heavily on pattern recognition, which includes number and shape pattern recognition. Learning mathematical procedures requires repetition as well, but it is best if teachers help students understand the reasoning behind the procedures. Students have to understand the rationale behind this initial step as well as the obvious course of action. Students must apply mathematical terms and symbols correctly, concentrate on the important details, and utilize them when explaining.

Using an ethnomathematics approach can help develop mathematical thinking abilities. Students have the opportunity to study mathematics in diverse cultural contexts through the use of ethnomathematics, which broadens their comprehension of the subject's applicability (D'ambrosio, 2006). Empirical data from Utami et al. (2019) study demonstrates that the ethnomathematics approach improves students' critical thinking abilities. Through contextual learning with ethnomathematics, even students with different cognitive levels can be encouraged to develop mathematical problem-solving abilities (Nur, et al., 2020). According to Maasarwe et al. (2012), using artifacts in geometry instruction also improves students' creative problem-solving abilities.

One advantage of utilizing non-routine problem-solving concepts or procedures, similar to those taught in school, to solve mathematical problems in the context of students' socio-cultural environment is that the problems have meaning (Masingila, 2002). This demonstrates how teaching students to apply mathematical reasoning to real-world problems enhances their ability to think mathematically. Teachers must be familiar with the culture to help students understand the connection between mathematics and culture when integrating ethnomathematics into the classroom (Risdiyanti & Prahmana, 2020). The teacher incorporates issues from the students' everyday lives into the development of learning strategies. By applying pertinent cultural activities, these issues are transformed into a deeper understanding of real-life situations (Rosa & Orey, 2010).

Mathematical thinking connects the application of mathematics in culture with traditional mathematical systems (Adam, 2004). To help students better understand the meaning of mathematical symbols used in the classroom, local culture can provide a context for interpretation (Abas, 2001, Gerdes 2011). To further encourage students to actively develop their thinking skills, teachers can also use local culture as a subject of study through hands-on and investigative activities (Maasarwe et al., 2012). Studies on the integration of realistic mathematics instruction with an emphasis on regional culture also revealed that students were able to investigate and expand their understanding of the region, which affected the variety of approaches they employed for solving geometric problems (Nirawati et.al., 2021).

Teachers have to encourage mathematical thinking exercises that connect students' early intuitive understanding of everyday situations with their formal education in mathematics. Despite the favorable opinion that Indonesian teachers have of the ethnomathematics approach (Mania & Alam, 2021), a considerable number of teachers lack the necessary resources and knowledge to plan mathematical thinking activities for their students' math classes (Marsigit, 2007). Teachers control the learning process, which hinders students' development of reasoning, critical, and creative thinking skills (Putra et al., 2020).

Several previous studies (Utami et al., 2019, Nur et al., 2020, Maasarwe et al., 2012) employed the researcher's design to develop learning employing an ethnomathematics approach. We have to understand the attempts made by teachers to teach mathematical thinking skills through the application of the ethnomathematics approach. To enhance learning and help teachers better teach mathematical thinking, knowledge about the efforts of the teachers can be a good place to start. Based on teachers' personal experiences, we can also explore different ways of thinking about mathematics that can be further developed through the use of an ethnomathematics approach.

Therefore, we concern on the following research questions: 1) How do teachers apply the ethnomathematics approach in learning mathematics? and 2) What types of mathematical thinking can be promoted from this approach?

METHOD

This study utilizes a qualitative design or a naturalistic research paradigm, allowing the researcher to apply an interpretive theoretical attitude to concentrate on findings, insights, and understandings (Merriam, 1998).

Design and Procedure

In Indonesia, ethnomathematics is a relatively recent phenomenon and is rarely utilized in the classroom by teachers. Case studies are suitable for analyzing modern phenomena, claims Yin (2014). Teachers who have implemented the ethnomathematics methodology can serve as case studies. Research questions can be addressed by examining what renders teachers who utilize an ethnomathematics approach special or different (Stake, 2009). Every informant experiences the same process. The researcher observed classroom teachers who employ an ethnomathematics approach. The researcher then conducted an in-depth interview with the teacher to gain further insight.

Research Context and Key Informant

Two junior high schools in Indonesia's Yogyakarta Special Province and one junior high school in the Central Java Province served as the research locations. Javanese culture is the prevalent cultural context at the research site. Yogyakarta is classified as a special region because it is a royal province governed by the Sultan. A governor's decree on culture-based education was released by the Sultan of Keraton, Jogjakarta.

Three teachers of mathematics served as the key informants. By selecting junior high school math teachers who were considered to have sufficient training and experience in applying the ethnomathematics approach, the informants for the research were deliberately selected as the sources of the data. The criteria for interview subjects involve junior high school math teachers who have been teaching for at least five years and who have employed the ethnomathematics approach for a minimum of one academic year.

The term "ethnomathematics" was introduced to the first informant during the 2015 National Seminar. She was motivated to utilize the internet to research ethnomathematics further and attempt to further refine her theories for use in the classroom. Since 2014, the second informant has been drawn to the ethnomathematics approach and has felt compelled to contribute to the preservation of Javanese culture. Subsequently, she performed an ethnomathematics investigation by mapping out the locations of cultural buildings or community centers, researching Javanese culture online or consulting with more knowledgeable colleagues, and creating her educational resources. The supervisor socialized the third informant about cultural values in education following Yogyakarta Province Governor Regulation Number 66 of 2013, and the informant then engaged in culture-based learning. The school also designated the development of local culture as one of its featured programs. Every Friday, mandatory Javanese language classes are held. Other activities include traditional dances, music, and crafts, as well as batik and crafts. Inspired by these developments, she attempted to understand how to implement culture-based mathematics education, including conducting online research for resources. She learned about the ethnomathematics approach through this search.

Data Collection

The objective of observation is to gather data on the application of the ethnomathematics approach for encouraging mathematical thinking in the classroom. Through non-participant observations, the researcher investigates three areas: 1) how local culture is used to teach mathematics, 2) how the learning flow works, and 3) what challenges teachers have when teaching mathematical thinking. Video is employed for recording the learning process so that it can be viewed again. Field notes are used to document the observations' findings.

In-depth interviews are conducted by the researcher with the following questions in consideration: 1) Why is this method of integrating ethnomathematics into learning utilized? 2) How does this method encourage the development of the mathematical thinking process? 3) What challenges do teachers have when it comes to imparting mathematical thinking? To obtain more detailed information, questions are created. Tape recorders are employed to record interviews, which occur in two or three sessions lasting at least sixty minutes each. See Table 1.

Table 1. Instrument Details.

Instrument	When	Why
Field Notes	1. While observing the teacher teach 2. While watching a video recorded	1. To formulate intriguing interview questions 2. To describe how teachers implement the ethnomathematics approach
Unstructured interview	After observing the teaching session and after watching video-recorded	To understand teacher perceptions and ideas about teaching mathematical thinking

The researcher verifies the validity of the research data by comparing the information discovered from the observations and the interviews, interpreting the results following pertinent theories, and then presenting the findings to the informants for responses and clarifications.

Data Analysis

Qualitative data were analyzed using the model developed by Miles and Huberman (1994) encompassing data reduction, data presentation, and conclusion. To draw conclusions and answer research questions, the data collected from field notes and interviews were evaluated in three phases.

Phase one involves reading field notes to identify key learning moments and connecting them to pertinent theories to extract keywords. Every keyword has a code. To identify patterns or relationships, similar codes are grouped. Information not relevant to the study objectives is eliminated. The interview data undergoes the same process. For instance, the teacher could provide students instructions to search for traditional food examples that feature specific geometric shapes in their written field notes. Subsequently, the teacher creates a table on the whiteboard to record the outcomes of the class discussion. Finding examples of traditional foods is a crucial step at this point. The analysis produces the keyword "find", which is then assigned a learning flow code. Since they have no bearing on the primary subject of the research, details about the activities teachers engage in when explaining tables on the whiteboard are deleted.

Phase two involves verifying the field note analysis results with the interview analysis results. To enable the drawing and verification of conclusions, the information will be refined, arranged in a sequential manner, and concentrated on the research questions. To obtain a more detailed description of the learning flow and to understand its reasoning, for instance, the results of the interview analysis about the reason the teacher placed the activity at the end of the stage corroborate the field notes analysis results with the keyword "find" in the learning path.

We developed a code matrix (Table 2) to categorize the various forms of mathematical thinking. The Tall (2008) foundational set of mathematical thinking and the Ishoda and Katagiri (2012) type of mathematical thinking are the foundations upon which the matrix is constructed.

Table 2. Code Matrix for Type of Mathematical Thinking.

Mathematical Thinking Related to Attitude	Mathematical Thinking Related to Mathematical Method	Mathematical Thinking Related to Mathematical Content
1. Attempting to grasp one's problems or objectives or substance clearly, by oneself (A1) 2. Attempting to take logical actions (A2) 3. Attempting to express matters clearly and succinctly (A3) 4. Attempting to seek better things (A4) 5. Recognizing patterns, similarities, and differences (A5) 6. Repetition the action sequence (A6) 7. Language to describe (A7)	1. Inductive thinking (M1) 2. Analogical thinking (M2) 3. Deductive thinking (M3) 4. Integrative thinking (M4) 5. Developmental thinking (M5) 6. Abstract thinking (M6) 7. Thinking that simplifies (M7) 8. Thinking that generalizes (M8) 9. Thinking that specializes (M9) 10. Thinking that symbolizes (M10) 11. Thinking that express with numbers, quantifies, and figures (M11)	1. Idea of sets (C1) 2. Idea of units (C2) 3. Idea of expression (C3) 4. Idea of operation (C4) 5. Idea of algorithm (C5) 6. Idea of approximation (C6) 7. Idea of fundamental properties (C7) 8. Functional Thinking (C8) 9. Idea of formulas (C9)

In one learning exercise, for instance, students are asked to investigate an image of an artifact and then identify the artifact's nets to calculate its surface area. After reviewing Tall (2008) or Isoda and Katagiri (2012) mathematical thinking attributes, we ascertain which of these attributes are fulfilled by the thinking exercises. A code for the task was provided by each researcher. For instance, regarding the concept of units, Isoda and Katagiri (2012:71, author's highlight)

... figures are comprised of points (vertices), lines (straight lines, sides, circles, and so on), and surfaces (bases, sides, and so on). For this reason, thinking that concerns on these constituents, unit sizes, numbers, and interrelationships, is important.

The researcher provided the code C2 for the learning activities. We discussed to agree on the appropriate type of mathematical thinking.

During the third phase, we present the verified data for each case in an understandable table based on the research focus. The study's conclusions are derived through a cross-case synthesis since each case is independent of the others or does not impact others. Yin (2014) argues that we utilize argumentative interpretation to conclude after comparing and observing each profile to determine if there is a difference. Finally, thorough activity descriptions and quotes from teacher-student discussions are provided to further corroborate the findings.

RESULTS AND DISCUSSION

The results are presented initially for each informant teacher individually, denoted by the initials Case 1, Case 2, and Case 3. To address the research questions, a summary that compares and contrasts the informant teachers is then provided.

Case 1. Ethnomathematics Approach for Teaching Three-dimensional Shape

First, the teacher employs real objects, such as cans, plastic balls, and cones, to assist the ninth-graders in learning about three-dimensional geometric shapes. Students are first reminded of the prior subject by the teacher.

Teacher: How many three-dimensional shapes did we learn this semester?

Students answer: Three, ma'am. Cone, cylinder, and sphere.

Teacher: That's what we will learn today. Do you think there's any benefit in learning three-dimensional shapes?

Student: Yes, ma'am

Teacher: If there is, what are the benefits? (Students are silent).

This exchange demonstrates that the teacher's initial objective for the class was for them to comprehend the advantages of learning about three-dimensional shapes. Students struggle to provide examples of the advantages that three-dimensional shapes offer, despite their awareness of these advantages. Then, using questions to elicit responses, the teacher engaged the class by outlining the significance of learning about this subject.

Teacher: What is an example of a cylindrical object?

Student: Water container, ma'am...

Teacher: Well, if we want to know how much water is in a container, how do we do it?

Students: Calculate the volume of water in the container, ma'am.

Teacher: Well, we can calculate the volume of water using the volume of the container. That's one of the benefits of learning to the cylinder, what else?

Students: Calculate the surface area, ma'am.

Teacher: Have you ever seen the label on canned milk? If you remove the label, you can calculate the area.

Utilizing examples from daily life, Case 1 inspired students to learn about the advantages of studying three-dimensional shapes. Questions and answers based on students' experiences explore what aspects of mathematics are present in the objects around them.

Afterward, the teacher divides the class into groups of three or four people. To avoid confusing the class, the teacher told them not to open their books. Students were instructed to concentrate on group-based activities. By accomplishing this, students will be able to articulate their thoughts based on the tasks completed rather than the theory covered in the text.

Teacher: Do you know any examples of three-dimensional shapes around you?

Students: Examples of cylinders are milk cans, pipes... Examples of cones are caps, steamers, tumpeng, boiled peanut wrappers... Examples of spheres are basketballs.

Teacher: I have prepared three objects of different shapes. Your task is to observe the objects, then draw and fill in a table.

To determine how many sides, vertices, and edges an object has, the teacher assigns the students to discuss in groups. They are required to sketch the item as well. The teacher then asks the class to name the traditional dish that is in the same shape.

Each group received one item from the teacher. After about five minutes of observation, the second object changes places with the other group. After five minutes of observation, the second object was traded for the third object with the other group. This was accomplished to provide every group with an opportunity to observe three different kinds of objects.

The teacher designates a group representative for the presentation once the group discussion is considered to be finished. The teacher pays attention to mistakes made in the abstraction of geometric images throughout the presentation. The use of curved lines on an abstract image of a sphere is corrected by the teacher. Dotted curved lines should be used to depict the inside of the sphere to highlight invisible areas and denote a three-dimensional abstraction. Furthermore, the cone abstraction drawing's use of curved lines is corrected by the teacher. The definition of sides, vertices, and edges is subsequently employed by the teacher to explain the components of a cylinder, sphere, and cone. The teacher seemed to place a high priority on learning mathematics as a symbolic language, particularly when it came to writing.

Case 1 does not utilize local culture as a starting point for teaching, or not as a content base to explore its mathematical aspects. The teaching flow is presented in Figure 1.

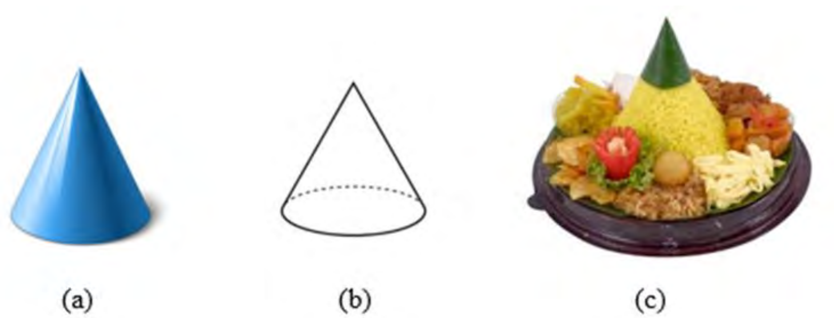


Figure 1. Teaching flow of Three-dimensional Shape

Props serve as a mathematical model and are the first teaching tool (a). Utilizing props instead of bringing in traditional food is more cost-effective for the teacher. The teacher's job is to explain mathematics in the context of symbols while assisting students in actively developing their understanding through discussion (b). Finally, the students are instructed to locate traditional foods that share the same shape (c).

Promoted Type of Mathematical Thinking

Since she believes that students learn mathematics by comprehending symbolic language, Case 1 does not examine any cultural aspects of mathematics. As a result, the teacher begins by outlining the components of three-dimensional shapes before connecting them to customary foods that share that shape. Using props, the teacher encourages students to think mathematically. The teacher assists the students in abstracting from actual objects to image forms or from three-dimensional to two-dimensional forms. Moreover, the abstraction is restored to its original form as an artifact.

Case 1 illustrates the application of abstract thinking to mathematical problem-solving. Students use concrete objects to think at first, then they abstract the objects' physical forms to create figures. Students employ recognition strategies to search for shape similarities between props and traditional foods when they recognize those foods in the same form.

A wrong answer could start a traditional discussion; thus, the teacher selected a group whose answers were incorrect to present. Other students disputed the presenters' understanding, which believed that cones are not symmetrical.

Questioning student 1: Why the cone does not have an edge? I think there is one.

Presenter: Wrong! ... the cone has no edges.

The student's explanation of why the cone lacks edges was not logical, and the presenter's explanation failed to make use of mathematical arguments. The same situation happens as in the following dialogue example.

Questioning students 2: In my opinion, there are two faces, how come only one?

Teacher: How are the other groups? Agree if the cone side is two? The presenter group accepts?

Presenter: Oh right ... I forgot to count the base.

The teacher did not direct attention toward correcting the presenter's reasoning during the presentation or the student-led question and answer sessions. The presenters' justifications have not been supported by mathematical evidence. The teacher did not refute the students' nonsensical claims when they clarified and addressed queries. Students may be inspired to think mathematically by logical arguments.

Based on the interview, the teacher realizes that for students to be able to argue logically they also require an understanding of language logic "*......sometimes there are but only one or two students who are quite special and can solve problems in a way other than mine, but it's very rare. When* *it comes to reasoning models, it is connected to language skills, for example, the result of subtracting this from this, but they are used to front minus back*"

An interesting case appears in the cone presentation dialog. There is a question from a student triggering a polemic in classical discussions.

Student: I am still confused about the vertex point. Is this the vertex point, ma'am?

Teacher: In the book, there are differences about cone vertices. Some books mention that the cone has one vertex, but some books say that the cone has no vertices, but edges.

According to Abrahamson (2009a), there needs to be a bridge between formal mathematical knowledge and intuitive knowledge. Liaisons are necessary, from an epistemological perspective, to resolve cognitive conflicts resulting from disparate perspectives on the same object. Regarding the cone's vertices, one intuitive perception is that it encompasses a single point, which is the pointed part at the top. However, another view maintains that the cone has no vertices at all since no angle is formed when two lines intersect. This other view is less intuitive but more formal. Students who experience cognitive conflict are more likely to develop mathematical thinking attitudes.

In this instance, the student demonstrates the mathematical thinking attitude of attempting to completely understand the subject matter. To elevate his thinking from a concrete to an abstract level, the student assesses and enhances his thinking. The portion of the cone that appears pointed is the vertex, as the student understands it intuitively. Formally speaking, though, a vertex point is where two edges converge. While the teacher's explanation is grounded in formal mathematical knowledge, the student's perception is based on his observations. To comprehend the vertices of the cone, the teacher and student engage in a negotiation-based educational dialogue.

Case 2. Ethnomathematics Approach for Teaching Surface Areas of Prisms

In the teaching of surface areas of prisms for grade 8, the teacher uses a picture of a building in the Yogyakarta Palace called *Kedhaton*, as shown in Figure 2.

Figure 2. Kedhaton Building of Sultan Palace

The teacher identified the lesson's objectives, which are to compute the surface area of the prism and determine the formula for a vertical prism's surface area. She then reviewed the names of the different three-dimensional shape forms through questions and answers.

Teacher: Do you know what shape the building in the palace garden is?

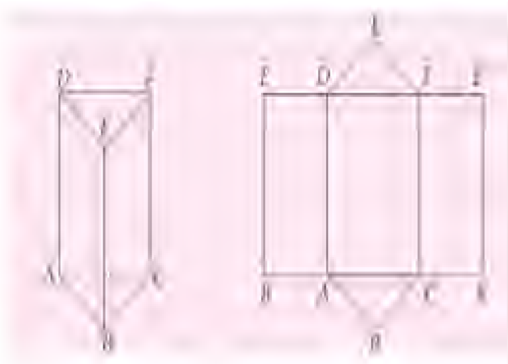
Student: Octagonal prism, ma'am.

Teacher: How did our ancestors know about octagonal prisms?. Students do not respond, they just listen.

The teacher illustrates the building's mathematical features, which include an octagonal prism without a base or roof. She provides that The King meets with royal employees in this building, which serves that purpose. The teacher then poses a problem,

Teacher: If the length of the base edge is 1.2 meters and the height of the building is 2.8 meters, how do you calculate the surface area of the building? To answer the problem posed, you need to understand the concept of the surface area of a prism.

The teacher utilizes a picture of a triangular prism and explains the formula analytically, as demonstrated in Figure 3.



The surface area of Prism = area of base plane + area of top plane + area of the lateral surface
= 2 × area of two planes + area of lateral surface
Area of lateral surface = AB × t + BC × t + AC × t = (AB + BC + AC) × t = (circumstance of base) × t
Surface area of Prism = (2 × area of base) + (circumstance of base × height)

Figure 3. Analytical Explanation of Surface area of Triangular Prism

The teacher elaborates the explanation into manageable steps, beginning with the composition of the prism's surface area, which is comprised of the total area of the sides. Afterward, the formula for the area of each side, expressed as a rectangle or triangle, is discussed to arrive at the prism's surface area total. To make sure the students comprehend the explanation, the teacher asks questions and provides answers. The teacher then provides an example of the problem of calculating the surface area of a prism and demonstrates how to solve it after the students have been assessed to ensure they understand the formula. The problem is that "*a prism whose base is in the form of an isosceles triangle has two equal sides of 10 cm and another side that is 12 cm. If the height of the prism is 15 cm, without drawing first, determine the surface area of the prism*".

The questions are displayed first, and then the teacher encompasses the students to solve the problem by asking questions to make students think, for instance *"Which one has a length of* 15?"; "*Where did the number* 8 *come from*?". The students answer these questions with tremendous enthusiasm; in addition to answering, they can also justify their answers with arguments. The teacher assigns a worksheet for group discussion. Students are provided with word problems regarding the prism's surface area to solve in the context of the local culture. The teaching flow is presented in Figure 4.

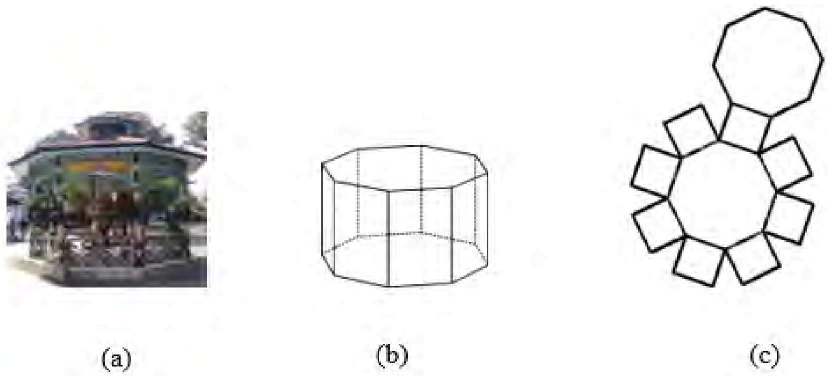


Figure 4. Teaching flow of Surface Area of Prisms

The teacher presents an octagonal prism-displaying cultural artifact (a). Based on the interview, we discovered that she generates students' attention to or interest in learning with a picture of the Kedhaton building. The geometry abstraction model is presented by the teacher as an illustration (b). The assignment for the students is to recognize the nets of an octagonal prism (c).

Promoted Type of Mathematical Thinking

Using images of artifacts, the teacher encourages students to employ mathematical skills. The teacher then assists students in identifying the elements and nets of the artifact. Students can comprehend the idea of surface area because artifact photos are utilized early in the lesson. Subsequently, the teacher uses an abstract model to explain the surface area formula. The teacher-student conversation is necessary to help the students comprehend the meaning of symbols.

There is a "leap" from the octagonal prism that is the Kedhaton building's physical form to the abstraction of a triangle prism and the formula for its surface area. The teacher no longer discusses the Kedhaton building's physical form; instead, she demonstrates how to obtain the triangular prism formula's surface area analytically. According to Abrahamson (2009b), it's a semiotic leap. The inquiry process is started when the perspective of the Kedhaton building transforms to an analytical justification of the triangular prism's surface area formula. Students investigate triangular prisms after attempting to understand octagonal prisms intuitively. These are inquiry activities, encompassing intimations, and implementations, which are critical (Sfard, 2002).

The teacher can use another artifact in the form of a triangular prism but she doesn't. Based on the interview, the teacher does not realize the need for interrelated learning flows so that there is no leap. The inquiry thinking process that occurs because of the leap is not designed by the teacher. In addition to not using the Kedhaton building to obtain the surface area formula, the teacher also does not discuss the Kedhaton's building surface area. This is a result of the teacher providing the formula explanation priority because she believes that her time is limited.

Because the teacher uses a question-and-answer format in her explanation, the students do not seem to have any difficulty understanding it and can complete the worksheet's questions. Questions and answers encourage students' attitudes to think mathematically. When responding to the teacher's questions, they attempt to base their answers on evidence or presumptions. They also use abstract thinking, which is a type of mathematical thinking. Students first use the artifact's image to guide their thinking before abstracting the wall's shape to create an octagonal prism model. Students then consider describing an octagonal prism net using an octagonal prism abstraction. Students are encouraged to think in terms of numbers and figures when the teacher presents a contextual problem.

The following is a conversation excerpt that illustrates the question-and-answer technique used by the teacher to encourage students to think mathematically.

Teacher: What is the formula for the surface area of a prism?

Many students raised their hands, and the ones the teacher pointed at were able to provide the right response.

Teacher: Do you remember, yesterday I showed you a picture of the building in the Yogyakarta Palace Garden? What shape is the building?

Student: Octagonal prism.

Teacher: If the length of the base edge is 1.2 meters and the height of the Kedhaton building is 2.8 meters. How to calculate the surface area of a Kedhaton building?

One of the students said the surface area of an octagonal prism without a base and a roof is the length of the base times the height,

Student: Because the length of the base edge is 1.2 meters and the octagonal prism has 8 base edges, the length of the base is equal to the sum of each base edge length. So the surface area of the Kedhaton building is obtained from the length of the building base times the height of the building, namely $(1.2m+1.2m+1.2m+1.2m+1.2m+1.2m+1.2m+1.2m) \times 2.8m$.

Teacher: There is another way, isn't there?

The teacher proceeded to guide the students by demonstrating that the Kedhaton building's upright side has a rectangular shape (Figure 5). The form of the Kedhaton building's upright side was described by her as follows on the whiteboard.

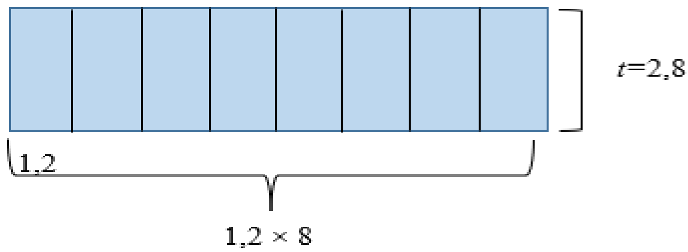


Figure 5. Calculation the surface area of a Kedhaton building

Teacher: Instead of counting one by one. Take a look, the length of the base is 1.2 meters. The Kedhaton building is shaped like an octagonal prism without a base and a roof, so the length of the edges around the building is $1.2\ m \times 8 = 9.6\ m$. The surface area of the Kedhaton building is $9.6m \times 2.8m = 26.88\ m^2$. So, the surface area of a prism without a base and a roof is equal to the perimeter of the base times the height.

Prism abstraction is the method the teacher employs to ensure that the students comprehend the symbols for the length of the base, the length of the edge, the relationship between symbols when they are presented with multiplication signs, the relationship between the surface area of the prism and the concept of the base's circumference, and so forth. Ishoda and Katagiri (2012) state that thinking symbolically entails a student's attempt to use symbols to express problems and make references to objects that have symbolic meaning. This occurs when learning how to comprehend the teacher's explanation of the formula for a prism's surface area when height is represented by the letter t (*tinggi* in Indonesian).

The teacher utilized dialogues and questions to motivate the class to argue with one another throughout the lesson. This indicates that analytical, problem-solving, and mathematical communication exercises—both oral and written—develop students' thinking skills. Based on prior mathematical knowledge—for instance, the surface area of a prism—the students addressed the teacher's problems by applying what they believed about Pythagorean triples and the areas of squares, rectangles, and triangles. In addition to teaching mathematics, the teacher includes the class in problem-solving exercises. To solve problems in the local cultural context, students are guided to be able to apply previously understood formulas through analytical explanations using geometric abstraction models.

When experienced with a shortage of teaching time, Case 2 prioritizes teaching mathematical structures and concentrates on content-based instruction. Early on in the learning process, local culture is used to inspire students and provide the mathematics material they will study later context and significance. Then, mathematical concepts are examined via their symbols. The teacher wants each student to become an expert in mathematical symbols utilizing arithmetic skills, calculation procedures, formula memorization, and symbolic reasoning. She explains while guiding students through interactive question-and-answer sessions that actively develop understanding. In this instance, the teacher introduces the symbols to the class first, and they jointly develop the meaning through conversation. Teachers' teaching practices remain focused on content to achieve material targets by the planned schedule and demands on the curriculum.

Case 3. Ethnomathematics Approach for Teaching Circle Intersections

The kawung batik motif (refer to Figure 6) is a widely recognized teaching tool for eighth-grade students when learning about circle intersections.

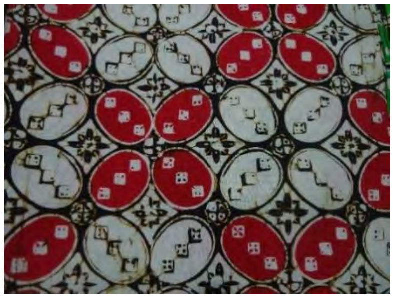


Figure 6. Batik motif Kawung

The teacher employed online resources to explain the origin of the kawung motif, stating that it was inspired by the shapes of four blossoming lotus petals or the kolang-kaling fruit, additionally referred to as coconut fruit. Then, a geometric arrangement of the motifs is established. The teacher displays a drawing of a batik kawung, similar to Figure 7.

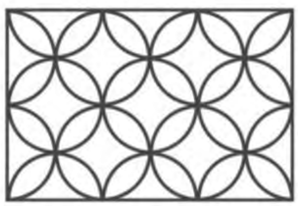


Figure 7. Batik motif kawung sketch

Teacher: How many circle patterns are there in the motif?

Student: Seven, ma'am.... four, ma'am.... (answer).

Teacher: Yes, ...let's see how the kawung motif can be produced from circle intersections.

The teacher elaborates on the blackboard on how the intersections of the circles produce the kawung motif, as presented in Figure 8.

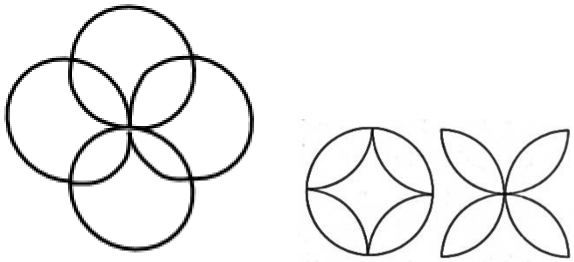


Figure 8. Batik Kawung as the intersection of the circle pattern

Teacher: So, how many circle patterns are there for the correct answer?

Student: Seven, Ma'am (in unison).

Drawing a kawung batik motif by joining circle patterns was the unstructured independent task assigned by the teacher. Students can design their own motifs. In addition to using the standard kawung pattern and color, students are free to add additional ornaments and colors to the pattern. Figure 9 displays a selection of student work.

Figure 9. Sample of Student work

The way that the teacher teaches students to sketch the kawung batik motif differs from how batik artisans do it. An alternative method for drawing a kawung batik motif involves drawing a square and a diagonal line, followed by the petals on the diagonal line (refer to Figure 10).

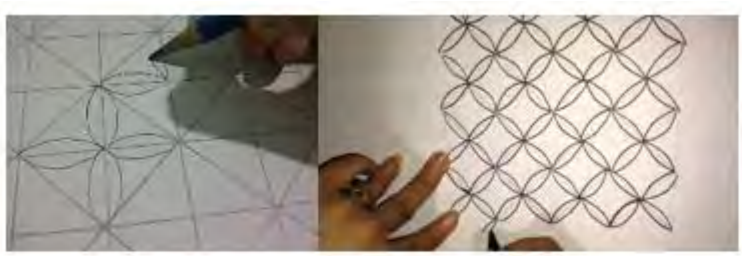


Figure 10. How Batik Artisan makes sketches of Kawung Motif

Through batik projects in after-school activities, students have acquired this technique. The teacher wants to convey to the students that a kawung motif shape can be generated when two circles intersect.

The teaching flow is presented in Figure 11.

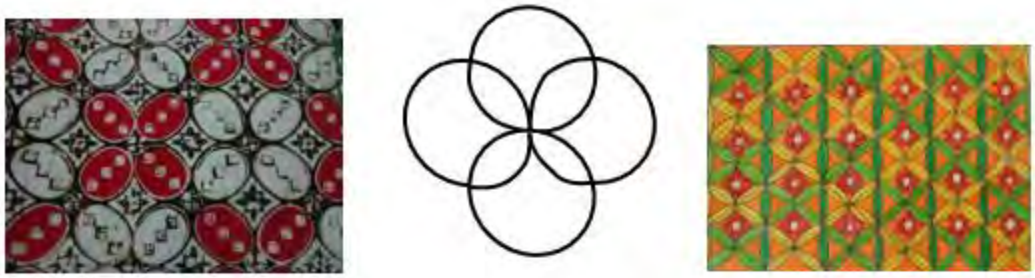


Figure 11. Teaching flow of Circle Intersection

Students are asked to examine how these motifs are applied mathematically, starting with cultural artifacts like batik Kawung (a). The teacher points out to the class that the kawung batik motif features a pattern of circles that are cut into or intersected by one another. After learning this pattern, students should understand that the ellipse, which serves as the kawung motif's fundamental shape, is the result of the intersection of several circle patterns (b). Students are then allowed to practice creating their kawung batik motifs, which further solidifies their understanding (c).

Promoted Type of Mathematical Thinking

Based on the interview, students understand the concept when using the local cultural context, but if the problem context is changed, students experience difficulties. The teacher recognized and comprehended this circumstance since the students were learning mathematical formulas through a direct approach, and their thinking abilities were at an intermediate level. Instead of teaching students how to discover formulas, teachers provide them to them along with examples of how to use them to solve mathematical problems. The teacher gives priority to the material that is simple for her students for them to master mathematics. The teacher also discussed how learning mathematics requires students to use not only their minds but also their emotions and will, which are expressed through action as a manifestation of their ideas and will.

The process of creating crafts involves mathematical thinking and activity (Gerdes, 2014). Teachers use cultural artifacts as a stepping stone for learning to help students develop their mathematical thinking skills. By repeating the steps involved in creating their batik, students can identify patterns on objects and practice creating them themselves. To create a kawung batik motif, they repeatedly create many circles intersection patterns. These activities are completed in the order listed. Tall (2008) identifies pattern recognition and sequence repetition as mathematical thinking.

Students do not independently research the integrated or intersecting circle pattern on the batik kawung; instead, the teacher demonstrates it to them. Students are not encouraged to refine their mathematical thought processes with this approach. The teacher said that, "*The students are* *enthusiastic when learning mathematics through practice. For example, on cube learning I invited them lèsèhan, while decorating the cubes they made, the opposite sides are given the same motif. But when the prism is pyramidal, they are confused. I've also used batik to teach the elements of a circle, I let them make other kawung batik motifs, but what the teacher teaches is what they imitate, the only difference is the color and decoration*."

Note: *Lèsèhan* (in the Javanese language) means sitting on the carpet or not employing a chair and for Javanese people, this way of sitting feels relaxing.

When given the challenge of creating kawung motifs out of circles, students will be motivated to expand their mathematical thought processes. By independent research, students can attempt to identify a solution. After that, students are free to share their ideas and solutions. Students can develop their mathematical understandings and be able to explain their work by engaging in investigation activities (Adam, 2004), which can foster mathematical thinking that is facilitated by the cultural context (D'Ambrosio, 2001).

A comparison of these overall findings is demonstrated in Table 3 as a summary of the three cases, based on research questions.

Table 3. Summary of The Three Cases.

Case	Teaching Flow with Ethnomathematics Approach	Type of Mathematical Thinking
Case 1	The teacher asked students to: 1. Observe Props (three-dimensional shapes) 2. Identify the number of edges, the number of vertex points, and the number of sides 3. Draw abstraction of the props 4. Discover traditional food with the same shape	Student attitude: ▪ Attempt to understand about substance clearly ▪ Strive to escalate thinking from a concrete level to an abstract level and evaluate their thinking to refine ▪ Recognize to identify the similarities in shape between props and traditional food Mathematical thinking related to the mathematical method: ▪ abstract thinking Mathematical thinking related to mathematical content: ▪ Focusing on constituent elements (units) and their sizes and relationships (Idea of units) ▪ Focusing on basic properties
Case 2	Teacher: 1. Display a picture of the Kedhaton Building as an	Student attitude: ▪ attempt to think based on the data or assumption

	Octagonal Prism Model 2. Ask students to identify the Kedhaton building elements 3. Ask students to identify nets of an octagonal prism 4. Explain the surface area of the prism by using an analytical approach 5. Explain an example of a problem and how to solve it 6. Apply the question-and-answer technique 7. Ask students to discuss word problems with cultural context	▪ describe the way they think about Kedhaton Mathematical thinking associated with the mathematical method: ▪ abstract thinking ▪ thinking that express with numbers and figures ▪ thinking that symbolizes Mathematical thinking related to mathematical content: ▪ The idea of a unit when identifying artifact elements ▪ Functional thinking when determining the relationship between octagonal prism and triangular prism ▪ Idea of operation when solving word problems with cultural context
Case 3	The teacher asked students to: 1. Recognize patterns in batik motif 2. Observe how to make the motif from the intersection of the circle 3. Draw batik by applying a circle intersection	Student attitude: ▪ recognizing pattern ▪ repetition of sequences of action Mathematical thinking associated with mathematical content: ▪ The idea of a unit when identifying the component of batik (circle, line, etc.) ▪ Functional thinking when finding the relationship between the batik motif and the intersection of the circle

Through the use of artifacts, the three teachers employ an ethnomathematics approach. Case 1 encourages students to identify cultural practices by using their understanding of the components of three-dimensional shapes. To help students better understand how mathematics is employed in their culture, teachers should be able to investigate more topics related to traditional foods and mathematics.

Gaining knowledge of ethnomathematics serves as a foundation for improving comprehension of school mathematics concepts (Gerdes, 1996). For various reasons, Cases 2 and 3 employ this strategy. Case 2 draws students' attention to the local culture at the outset of the lesson and uses it as a stepping stone to formal mathematics.

The teacher utilizes a question-and-answer format in addition to providing extensive explanations of mathematical concepts to encourage students to share their understanding. This finding is consistent with research by Kurniasih and Hidayanto (2022), which discovered that studentcentered activities like asking for an explanation of their thought processes and a justification for their reasoning demonstrate a teacher's efforts to encourage mathematical thinking skills.

Case 3 presents mathematics as a product of learning while accounting for the cognitive capacities of lower-middle-class students. Practices involving student creation replicate the process of mathematical discovery. However, because they are still developing their reflective thinking skills, students should only learn inductive mathematical concepts or principles.

The problems that Case 3 encountered demonstrate that while motivating students through familiar local culture can be effective, motivation is insufficient to help them grasp mathematical concepts. Students must possess mathematical qualities that call for the use of deductive reasoning. They must be able to accurately represent the knowledge acquired under somewhat different circumstances. In addition, basic mathematical abilities like counting have an impact on their capacity to resolve the presented issues.

CONCLUSIONS

According to the study, teachers implement the ethnomathematics approach to teach mathematics in a variety of ways. This includes: 1) utilizing the local culture to motivate students to learn the subject and create a sense of purpose in their learning; and 2) utilizing the local culture as a mathematical object, the context for mathematical problems, and real-world examples of mathematics.

The teachers' ethnomathematics approach, which incorporates local culture and learning flow, fosters the development of diverse mathematical thinking styles. The application of investigation and problem-solving in the ethnomathematics approach stimulates mathematical thinking more. Encouraging students to think mathematically through similarity recognition is an ethnomathematics approach with a learning path that initiates with the understanding of school mathematics knowledge and then looks for examples in cultural practice. The action sequence is repeated by the students with the assistance of an ethnomathematics approach and practice in batik making. It helps to promote meaningful learning and practical thinking to introduce local culture at the beginning of a lesson. It encourages students to participate in inquiry-based learning even though local culture serves as a stepping stone for formal mathematics. Using artifacts to teach mathematics will encourage mathematical thinking related to the concept of a unit.

The design of instruction using an ethnomathematics approach can benefit from further development of these findings. Teachers must examine mathematical concepts in the context of the local culture to help students connect with their culture on a deeper level. Mathematical expression in culture can take many forms, encompassing concepts and customs that permeate everyday life in addition to tangible objects. Considering this, the teacher will be able to facilitate learning mathematics on other topics in addition to teaching geometry using the ethnomathematics approach. Teachers can use projects and teaching aids in the ethnomathematics approach to allow students to engage in practical, investigative learning in a mathematics classroom. Then, the teacher encourages the class to think mathematically by using abstraction, generalization, symbolization, and logical reasoning.

The quantity of cases examined in this study places limitations on it. It is advised that more research be conducted and that the mathematics topics be changed or added to, incorporating algebra and other subjects in addition to geometry. Therefore, it may provide a more profound comprehension of how to encourage mathematical thinking using an ethnomathematics approach.

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