# From Informal to Formal Proof in Geometry: a Preliminary Study of Scaffolding-based Interventions for Improving Preservice Teachers' Level of Proof

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**Abstract**: *This is a preliminary study of design research that investigates preservice mathematics teachers' proof level and the possible task of scaffolding-based interventions in proving the triangle theorem. The research subjects consisted of 58 second-semester mathematics education students at Universitas Negeri Surabaya, Indonesia. This research is descriptive using quantitative and qualitative approaches. Data collection uses a test to determine the level of proof of prospective mathematics teachers based on Miyazaki's classification. This method classifies four levels in constructing a proof, mainly Proof A, Proof B (deductive), Proof C, and Proof D (inductive). The results showed that there were 38% of students' answers in constructing proof with level Proof A, 5% of students' answers in constructing proof with level Proof B, 15% of students' answers in constructing proof with level Proof C, and the remaining 42% of students' answers in constructing proof with level Proof D. Furthermore, the scaffolding-based intervention task refers to the preservice teacher's difficulties in proving the triangle theorem, including a lack of understanding of concepts, not understanding language and mathematical notation and difficulties in starting proofs.*

**Keywords**: Level of Proof, Scaffolding, Geometry

## INTRODUCTION

Proof is at the heart of mathematical thinking and deductive reasoning (Cheng & Lin, 2009). Hernadi (2008) explains that proof is a series of logical arguments that explain the truth of a statement. Mingus and Grassl (1999) define proof as a collection of statements that are true and linked together in a logical way that serve as arguments to convince other of the truth of mathematical statements. Meanwhile, Griffiths (2000) states that mathematical proof is a formal and logical way of thinking that starts with axioms and moves forward through logical steps to a conclusion. In addition, proof is also a major component of understanding mathematics (Kogce et al, 2010). Proof is recognized as the core of mathematical thinking (Hanna et al, 2009). One cannot study mathematics without studying mathematical proofs and how to make them (Balancheff, 2010).

The role of proof for a mathematics learner is a determinant of the level of maturity in the process of thinking mathematics (Otten et al., 2014). This is because proof requires a person to use mathematical knowledge and write it down in a logical argument, so it requires a comprehensive mathematical thinking process (Cervantes-Barraza et al., 2020). Recently, several universities have begun to introduce lectures on the introduction of proof or mathematical reasoning programs (Epp, 2003, Selden & Selden, 2007), which aim to make it easier for students to understand the formal language of mathematics and its axiomatic structure. This can be seen in the first year students at Universitas Negeri Surabaya where this research took place, because the majority of students have been provided with the initial lecture program, namely in the fundamentals of Mathematics and number theory lectures. Clark and Lovric (2008) say that in the process of transitioning into constructing mathematical proofs for students there are many challenges to be faced. They suggest that this transition requires students to change the type of reasoning used, namely shifting from informal to formal language; for reasons of using mathematical definitions; to understand and apply theorems; and make connections between objects in mathematics.

Various research results have concluded that the learning process regarding proof of university students has not reached the optimal stage as expected (Azrou & Khelladi, 2019, Daguplo & Development, 2014, Jones, 2010, Michael et al., 2013). The research results of Reiss and Renkl (2002) revealed that there were still many student limitations in the proving process. Furthermore, Maarif et al. (2018) concluded from the results of their research that the limitations of student concepts in constructing geometric proofs included difficulties in sketching diagrams with proper geometric labels and difficulties in constructing conjectures in writing formal proofs. In addition, Moore (1994) also said that students were unable to understand and use language and mathematical notation in compiling proof. From this, it is necessary for us to optimize the process of exploring the ability to construct proof in order to improve preservice teachers' level of proof in geometry.

Proof in mathematics consists of several universally accepted methods. The methods used in the proof are divided into 2, namely the deduction method and the induction method (Siswono et al., 2020). Proof is recognized as the core of mathematical thinking and deductive reasoning (Cheng & Lin, 2009). In deductive proof, a conclusion must be true if the premises are true (Anderson, 1985). The deduction method involves several methods such as direct proof, proof with contraposition and proof with contradiction (Morali et al., 2006). Whereas in inductive proof, arguments whose conclusions are not necessarily true but are very likely to be valid (Anderson, 1985). Miyazaki (2000) classifies proof into four levels, namely Proof A, Proof B, Proof C, and Proof D. According to Miyazaki (2000), Proof A is a level of proof that involves deductive reasoning and functional language used in working on the proof, Proof B is a level of proof that involves deductive reasoning and does not use functional language, images, or manipulation of objects that can be used in the process of proving. Whereas Proof C is a level of proof that involves inductive reasoning and does not use functional language, images, or manipulation of objects that can be used in the process of making proofs, Proof D is a level of proof that involves inductive reasoning and functional language used in proving.

Miyazaki's (2000) research explains more about levels in algebra, but in this study the focus will be on geometry. Even though proof is very important, there are still many students who experience difficulties in proof (Stylianides & Philippou, 2007, Weber, 2001). Because students often show difficulty in proving, researchers submit assignments to students, and in addition, provide scaffolding through Hypothetical Learning Trajectory (HLT) as a strategy to help students' difficulties in proving so that students can increase their level from informal to formal (Rahayu & Cintamulya, 2021). Anghileri (2006) divides the scaffolding hierarchy into three levels in learning mathematics. In scaffolding Level 1 is the most basic level. At this level, a suitable learning environment is needed that can support the learning process. Level 2 in scaffolding is known for several types, namely explaining, reviewing, and restructuring. Assistance provided at that level is used by students to achieve understanding. Level 3 in scaffolding is conceptual development, namely the level of scaffolding that develops concepts students already understand to build connections between concepts.

Scaffolding is given to students who experience difficulty in proving through Hypothetical Learning Trajectory (HLT). HLT is a description of students' thinking during the learning process in the form of conjectures and hypotheses from a series of learning designs to encourage the development of students' thinking so that mathematics learning objectives can be achieved as expected (Afriansyah & Arwadi, 2021, Sarama & Clements, 2004). The term hypothetical learning trajectory (HLT) itself was first proposed and used by Simon (1995) who stated that hypothetical learning trajectory consists of three components in the form of learning objectives, learning activities, and alleged learning processes - predictions about how students' thinking and understanding will develop in the future context of learning activities. The aim intended in this research is to achieve an understanding of the concept of proof. The intended learning activity is a series of tasks to find out how students can prove. The intended hypothesis of students' way of thinking is students' flow of thinking in understanding the concept of proof with the help of scaffolding according to Anghileri (2006).

HLT is very necessary in designing learning that will suit students' thinking patterns and characteristics (Rezky, 2019). In this research, HLT is a learning tool that contains a series of instructional tasks in the form of scaffolding and anticipation of possible difficulties that may occur for students in proving in order to help students understand the concept of proof so that students can increase their level from informal to formal. HLT with scaffolding is very rarely used by teachers in designing lessons, especially about geometric proofs in class. With this HLT, researchers hope to help teachers when teaching the concept of proving geometric theorems in class. Based on the description above, this study aims to investigate pre-service mathematics teachers' proof level and the possible task of scaffolding-based interventions through HLT in proving the triangle theorem.

## METHOD

### Research Approach and Design

The method used in this research is a preliminary study of design research. Researchers followed three research phases: the initial design stage (preliminary design); design testing through preliminary teaching and teaching experiments; and the retrospective analysis stage (Gravemeijer & Cobb, 2006). In this article, the discussion focuses only on the initial design stage (preliminary design). To explain a preliminary study, the researcher uses descriptive research using quantitative and qualitative approaches. At the preliminary stage, the researcher wanted to look at preserving teachers' levels of understanding of proof and preserving teachers' learning trajectories. The participants involved in this study were 58 second-semester mathematics education students at Universitas Negeri Surabaya, Indonesia. There were 2 classes, in which each class consisted of 29 prospective teachers. The choice of research location was based on the curriculum structure of the research location. There is a Basic Geometry course that accommodates proving geometry as an outcome of the learning process. In addition, the selection of the research location was carried out at the author's institution on the grounds that from previous experience teaching geometry, there were still many students who had difficulty in constructing of proof.

### Data Collection

The data collection technique to see teachers' levels of understanding of proof was carried out by giving a mathematical proof test to 58 students. The data was taken from the results of student work after the lecture process ended, then they were given a 15-minute mathematical proof test to construct geometric proofs. Afterwards, each prospective teacher's response was assessed to pre-service mathematics teachers' proof level of their deductive and inductive knowledge in constructing a proof. The present study tends to examine more on deductive and inductive proof without employing interviews like what Miyazaki (2000) did. The data were collected using a simple task of constructing one mathematical proof, namely to prove that the sum of the angles in a triangle is $180^0$. Actually, the task type could be more than one, such as the sum of the three external angles of a triangle is $360^0$, or prove the sum of the measures of the angles of a pentagon is $540^0$. However, the main point of this study was a proof method whether using deductive proof or inductive proof at each level of proof.

### Data Analysis

The process of assessing student answers is carried out by providing scoring coding following the level of proof of Miyazaki's classification (2000) in constructing a geometric proof. Since the subject of this study is the early mathematics education students who had received both methods in high school (Brady & Bowd, 2005) and these two methods have often been used by previous researchers in constructing a proof at the university level (Almeida, 2001). Furthermore, researchers try to make a student's learning trajectory (LT) for constructing a proof. This LT has yet to be tested on small-scale subjects; it was only made based on learning possibilities that can be used in constructing proof. See Table 1.

Table 1. Levels of proof in mathematics (Miyazaki, 2000).

| **Representation** | **Method** | |
|---|---|---|
| | **Deductive** | **Inductive** |
| Using functional language according to the theorem | Proof A | Proof D |
| Do not use functional languages, use images, or manipulate objects | Proof B | Proof C |

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In the preliminary design, the researcher designs the Hypothetical Learning Trajectory (HLT) to help students' difficulties in proving so that students can increase their level from informal to formal. HLT contains learning objectives (mathematical goals), teaching and learning activities, and the conjecture of student thinking (Simon, 1995). Before HLT is used in design testing through preliminary teaching and teaching experiments (further research), an expert review activity is needed, the instrument was reviewed by 2 experts who were lecturers from two universities with relevant knowledge. The selection of experts considers the length of service as a lecturer, the level of education, and the quantity and quality of research that has been carried out.

## RESULTS AND DISCUSSION

Based on data collecting technique, the findings of the research can be categorized according to the focus established at the beginning of the research, namely:

### Preservice teachers' levels of understanding of proof

In this study, data were collected through a mathematical proof test to assess pre-service mathematics teachers' proof level in geometry based on Miyazaki's (2000) classification. The results of this mathematical proof test (see Table 2) will be explained as follows:

Table 2. Teachers' proof level in geometry (Miyazaki, 2000).

| Level | Total of students | Percentage (%) |
|---|---|---|
| Proof A | 22 | 38 |
| Proof B | 3 | 5 |
| Proof C | 9 | 15 |
| Proof D | 24 | 42 |

Based on Table 2, Proof D has the highest score for preserving teachers' answers, totaling 24 preserved teachers' responses. This indicates that many preserved teachers' answers still utilize non-formal proof. Furthermore, the table reveals that 38% of preserved teachers' demonstrated Proof A, which requires deductive reasoning and the use of functional language to construct proofs. Meanwhile, 5% of the preserved teachers presented Proof B, utilizing deductive reasoning and manipulating objects or using sentences without functional language in their proofs. Additionally, 15% of preserved teachers exhibited Proof C, employing inductive reasoning and various languages, images, and manipulated objects to construct proofs. Moreover, 42% of preserved teachers displayed Proof D, using inductive reasoning and functional language for constructing proofs. The following section will provide some examples of preserved teachers' answers. See Figure 1, Figure 2, Figure 3 and Figure 4.


Buat sebuah segitiga sebarang dan beri nama tiap titik sudutnya A, B, dan C. Buat garis yang sejajar sisi AB dan melalui C, dan beri nama garis tersebut. Dalam kasus ini diberi nama DE.

Sudut CAB bersebrangan dengan sudut ACD, sudut CAB = sudut ACD = $x^0$

Sudut ABC bersebrangan dengan sudut BCE, sudut ABC = sudut BCE = $y^0$.

Dan besar sudut ACB yaitu $z^0$. Sehingga jumlah sudut ACD + ACB + BCE = $x^0 + y^0 + z^0 = 180^0$.

Translation

Create an arbitrary triangle and name each vertex A, B, and C. Draw a line parallel to side AB and through C, and name the line. In this case it is named DE.

Angle CAB is opposite angle ACD, angle CAB = angle ACD =$x^0$

Angle ABC is opposite angle BCE, angle ABC = angle BCE =$y^0$

And the size of the angle ACB is $z^0$. So the sum of the angles ACD +ABC+BCE = $x^0 + y^0 + z^0 = 180^0$.

**Figure 1**. Proof A


Jadi, karena ketiga sudut itu terletak pada garis lurus maka jumlahnya yaitu $180^0$

Translation

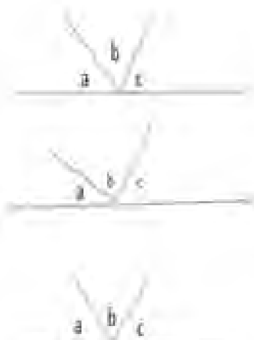
So, because the three angles lie on a straight line, their sum is $180^0$

**Figure 2.** Proof B

$30^0 + 60^0 + 90^0 = 180^0$
$45^0 + 65^0 + 70^0 = 180^0$
$60^0 + 60^0 + 60^0 = 180^0$
Jadi, jumlah ketiga sudut dalam segitiga sama dengan $180^0$

Translation
So, the sum of the three angles in a triangle is equal to $180^0$

**Figure 3.** Proof C




(∠ a,b,c adalah ∠ kurang dari $90^0$)

(∠ a,c adalah ∠ kurang dari $90^0$ dan ∠ b = $90^0$)

(∠ a,b,c adalah sama dan ∠ kurang dari $90^0$)

Translation
(∠ a, b, c is ∠ less than $90^0$)

(∠ a, c is ∠ less than $90^0$ and ∠ b = $90^0$)

(∠ a, b, c is equal and ∠ less than $90^0$)

**Figure 4.** Proof D

The preservice teachers perform Proof A, Proof B, Proof D, and Proof C types with the percentage of 38%, 5%, 42%, and 15%, respectively. Therefore, it shows that Proof D is the most commonly found in the prospective teachers' answers than those of other types. It aligns with the results of Kögce et al. (2010), in which the study results report that the inductive method is performed by most students than the other types of proof (51.2%). Researchers found that many students' answers indicated informal proof (62%) with several difficulties in proving, namely starting the proof (12%), understanding the concept (40%), and using symbols or language in compiling the proof (10%). In line with Baker (1996), many students experience difficulties in using symbols in constructing a proof. Harel and Sowder (1998) also concluded that many students had difficulty coming up with invalid deductive arguments and inductive arguments. Based on these difficulties, a learning trajectory is needed in the form of scaffolding to help students' difficulties in proving so that students can increase their level from informal to formal.

There are three difficulties experienced by students in compiling the proof of this theorem, namely in the starting of the proof, understanding concepts, and using symbols or language in compiling the proof. Based on Agustiani and Nursalim (2020), there are four activities in HLT for proof, namely reading proof, completing proof, examining proof, and constructing proof. Difficulties in starting the proof can be helped by reading proof activities in HLT, difficulties in understanding concepts can be helped by examining proof, and constructing proof activities, and difficulties in using symbols or language in compiling the proof can be helped by completing proof activities. This is in line with Anwar et al (2022), that students who have difficulty in the concept of proof can be helped by constructing a geometric figure. Then Miyazaki et al (2017) explained that reading of proof activities will help students in compiling or starting the structure of deductive proof.

**Preservice teachers' learning trajectories using scaffolding in Geometry**

After the researcher obtains quantitative data, the researcher can continue by analyzing student answers which include informal proof, to arrange scaffolding in the HLT so that this HLT can help students' overcome difficulties in proving, enabling them to increase their level from informal to formal. Then it is given to experts to provide input. Based on expert comments, researchers arrange things necessary to be discussed with experts. In outline, two things are subject to discussion between researchers and experts: students' understanding of using four levels of proof (Proof A, B, C, and D) that will be used in help organize HLT activities and the need to make separate steps. HLT, arranged as an initial design, is called the initial prototype. The initial prototype HLT consisted of four teaching-learning activities: reading proof, completing proof, evaluating proof, and constructing proof. In the expert review activity, the researcher intends to obtain an expert judgment on the relevance of the activities to achieve the expected goals along with the researcher's hypothesis about the conjecture of students' thinking. After the discussion with the experts, the following revision materials for the initial prototype HLT are in Table 3.

Table 3. HLT using scaffolding in Geometry.

| No | Activity | Goals | Students conjectured thinking | Type of scaffolding |
|---|---|---|---|---|
| 1 | Reading Proof | The purpose of the first activity "Reading Proof" is to introduce the parts that must be present in the sentence of proof and the levels of proof in constructing proof (deductive reasoning). | 1. Read carefully the proof of the following basic geometry theorems (Students are given complete proof, inductive proof for answer a question with Proof C)<br>2. After reading the proof of the theorem, then write down the premises (statement / closed sentence) of each statement of proof!<br>3. After reading the proof of the theorem, then write down the things you have understood | With student has difficulty in starting the proof<br><br>Level 2 (explaining, reviewing, and restructuring) |

| | | | | |
|---|---|---|---|---|
| | | | (in a few points, if any) in the box below!<br>4. From the results of the class, discussion write in full the conclusions / new understanding that you get (If any)! | |
| 2 | Completing Proof | The purpose of the second activity "Completing Proof" is to train students to identify sentences/statements of proof that must be present in the proof sentence (incomplete), the use levels of proof in constructing the proof. | 1. Read carefully the proof of the following basic geometry theorems! (Students are given incomplete proof, inductive proof for answer a question with Proof D).<br>2. After reading the proof of the theorem in point 1, then write down the things that you think are incomplete (if any) of the proof of the theorem!<br>3. Write the complete proof of the theorem on point 1! | With limitations of students do not understand and use language and mathematical notation<br><br>Level 2 (explaining, reviewing, and restructuring.) |
| 3 | Examining Proof | The purpose of the third activity "Evaluating Proof" is to train students to evaluate the sentences/statements of proof presented by identifying errors | 1. Read carefully the proof of the following basic geometry theorems! (Students are given proof by logic/ wrong correct concept, deductive proof for answer a question with Proof B)<br>2. After reading the proof of the theorem, then write the things that are FALSE in your opinion (if any) in the box below!<br>3. Write the right proof of the theorem on point 1! | With difficulty understanding the concept students<br><br>Level 2 (explaining, reviewing, and restructuring) |
| 4 | Constructing proof | The purpose of the fourth activity "Constructing Proof" is to train students to construct their sentences/statements of proof from | 1. Read carefully the proof of the following basic geometry theorems! (Students are given proof by logic/wrong concept, deductive proof for answer a question with Proof A) | With difficulty understanding the concept students<br><br>Level 2 (explaining, reviewing, and |

| | | | |
|---|---|---|---|
| | several theorems provided with the correct sentence and proof of logic | 2. In your opinion, the theorem in point 1 is more effectively proven using deductive proof or inductive proof? Explain your reasons!<br>3. Write the right proof of the theorem on point 1! | restructuring) or Level 3 (conceptual development) |

Table 3 shows HLT activities starting from level C proof, namely, reading proof because preservice teachers need to introduce the parts that must be present in the sentence of proof. Then, it continues with the second activity, namely completing proof with level D, because preservice teachers to identify sentences/statements of proof that must be presented in the proof sentence (incomplete). The third activity is examining proof with level B, because preservice teachers to evaluate the sentences/statements of proof presented by identifying errors. Lastly, constructing proof is the last activity at level A because preservice teachers to train students to construct their sentences/statements of proof from several theorems provided with the correct sentence and proof of logic. Of these 4 activities, the learning trajectory used is from informal to formal proof. In line with Agustin and Nursalim (2020), constructing proof activities are activities with formal proof.

The scaffolding that will be used in HLT refers to the difficulties in starting the proof, understanding concepts, and using symbols or language in compiling the proof, namely level 2 (explaining, reviewing, and restructuring) and level 3 (conceptual development). Level 2 is used for all activities in HLT, and for level 3 only construction proof. According to Anghileri (2006), at level 3, there is making connections, namely making connections by encouraging students to use their mathematical knowledge in developing their own strategies in the problem-solving process so that they are suitable for use in constructing proof activities. This level really helps teachers in implementing HLT when applied in the classroom for learning proof.

## CONCLUSIONS

Most preservice teachers' answers in constructing proofs use inductive methods (Proof D and Proof C) rather than deductive methods (Proof A and Proof B). Researchers found that 62% of preservice teachers' answers were informal proof, and Proof D was the most commonly found in the prospective teachers' answers than those of other types. Some of the difficulties that preservice teachers in proving are starting the proof, understanding the concept, and using symbols or language in compiling the proof. From these three difficulties, an HLT was prepared containing a scaffolding that could help students' overcome difficulties in proving, enabling them to increase their level from informal to formal. HLT activities consist of reading proof, completing proof, examining proof, and constructing the proof. Each activity contains level 2 scaffolding and only the constructing the proof activity also contains level 3 scaffolding.

This research was limited to one problem, aiming to identify the level of proof by adopting a single problem to explore and investigated preservice mathematics teachers' proof level and the possible task of scaffolding-based interventions in proving the triangle theorem. Apart from that, this research is still being carried out in the initial design stage (preliminary design) in design research. Expected future research will focus on the second phase, namely design testing through preliminary teaching and teaching experiments. Researchers can implement HLT with students who have difficulty proving the triangle theorem in class so that it can help increase their level from informal to formal.

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