

The Structure of Students' Mathematical Errors in Solving Calculus Problems Based on Cognitive Style

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Abstract

The understanding mathematical concept is an error that often occurs in classroom learning among students when solving mathematical problems. The most difficult part for students is solving problems, because it requires numeracy skills, high concept mastery, as well as the ability to use good language, and so on so that students don't make any more mistakes when working on math problems. Student errors in solving mathematics problems are (1) errors in connecting concepts, (2) errors in operations and (3) errors in constructing concepts. The problem is what is the structure of students' mathematical misconceptions in solving mathematical problems based on cognitive style. The method in this research, namely an exploratory descriptive approach, aims to determine the structure of students' errors based on cognitive style in solving mathematical problems. Analysis of research data, namely: (1) Data reduction, (2) Data exposure, (3) data triangulation and (4) drawing conclusions. The cognitive styles referred to are field dependent and independent. The conclusions are (1) the structure of conceptual errors with an applied field dependent cognitive style begins with disequilibrating, then solving by linking applicable concepts, and (2) the structure of conceptual errors with a field independent cognitive style begins with disequilibrating, then solving using analyse.

Keywords: conceptual errors, dependent, independent, mathematics education

1. Introduction

Mathematics is a lesson that requires concentration to remember and deepen the material studied so that students must be able to master mathematical concepts. Remembering that mathematics has several parts that are interconnected with each other, so the most important thing in learning mathematics is how a person's ability to understand mathematical concepts (Ratnayanti et al., 2021; Suharna et al., 2012). This type of error in understanding the question is made by all categories of student ability, where students do not write down the information that has been presented in the question. This type of transformation error is only made by students in the medium and low categories. In this type of error, students do not write down the strategy used as a solution, students make a mistake in determining the steps that must be taken first. Even though they can determine the formula correctly, students cannot carry out the solution steps. This type of skill process error was made by students in the medium and low categories. In this type of error, students are unable to solve the problem. Students get wrong results because they determine the wrong information. Meanwhile, in the type of error in writing the final answer, all students made errors. Students in the high category did not write conclusions from their answers, while students in the medium and low categories were because students did not complete the questions at the previous stage (Nurikawai et al., 2021; Suharna et al., 2013, 2014). Based on the results of research and discussion based on Newman's analysis of class IV students at SDS Pangkalan on flat shapes material, it was found that (1) the percentage of errors in reading the questions was 7.05% in the very low category, (2) the percentage of errors in understanding the questions was 41.17% with medium category, (3) the percentage of transformation errors is 29.41% in the low category, (4) the percentage of process skill errors is 16.15% in the low category. So it can be concluded that the most common mistakes made by class IV students at SDS Pangkalan in solving mathematics problems on plane figures based on Newman's analysis lie in errors in understanding the questions as much as 41.17%. This is due to the students' low ability to understand plane figures so that many students forget what you have been taught (Melisari et al., 2021; Suharna, 2014).

The low understanding of mathematical concepts proves that one of the factors causing student errors in solving mathematical problems. The most difficult part for students is solving problems, because it requires numeracy

skills, a high level of concept mastery, as well as the ability to use good language, and so on so that students don't make any more mistakes when working on math problems. Mistakes in solving questions are mistakes made by students because they only work on questions according to their abilities. Student errors in solving mathematics problems are conceptual errors, operational errors and careless errors, with the dominant error being conceptual errors.

2. Literature Review

Common mistakes made by students in doing mathematics assignments are lack of knowledge about symbols, lack of understanding of place value, use of incorrect processes, calculation errors, and writing that cannot be read so that students make mistakes because they are no longer able to read their writing alone (Hidayat, 2021; Suharna, 2013).

The meaning of student errors in working on mathematics problems is: (a) Translation errors are errors in changing information into mathematical expressions or errors in giving meaning; (b) Conceptual errors are errors in understanding abstract ideas; (c) strategy errors are errors that occur if students choose an inappropriate path that leads to a dead end; (d) systematic error is an error related to the wrong choice of extrapolation technique; (e) Sign error is an error in providing or writing mathematical signs or notation; and (f) Calculation errors are calculation errors in mathematical operations (Arti, 1994; Suharna, 2012, 2014, 2016, 2018).

3. Methodology

The type of research is qualitative exploratory. The subjects in this research are the subjects in this research are mathematics study program students. Khairun University Mathematics Education semester 4. The instruments in this research which are the instruments in qualitative exploratory research are (a) the researcher as the main instrument and (b) the auxiliary instrument, so in this research, the researcher is the main instrument and the auxiliary instrument is the test questions, (c) interviews, (d) think out aloud or think out aloud and in-depth observations (Creswell, 2010; Suharna, 2018, 2020). Instrument to see students' field dependent and independent cognitive styles through test questions. Selecting subjects repeatedly or continuously until data saturation is obtained. Data saturation means that subjects for each category have the same or constant pattern as several research subjects based on constant comparisons (Suharna, 2013, 2018).

Data analysis in this research uses research data analysis steps, namely (a) processing and preparing; (b) read all the data; (c) analyze in more detail by coding the data, (d) apply the coding process, describe and present these themes again in a qualitative narrative/report; and (e) interpret or interpret the data (Creswell, 2010; Suharna, 2018, 2020).

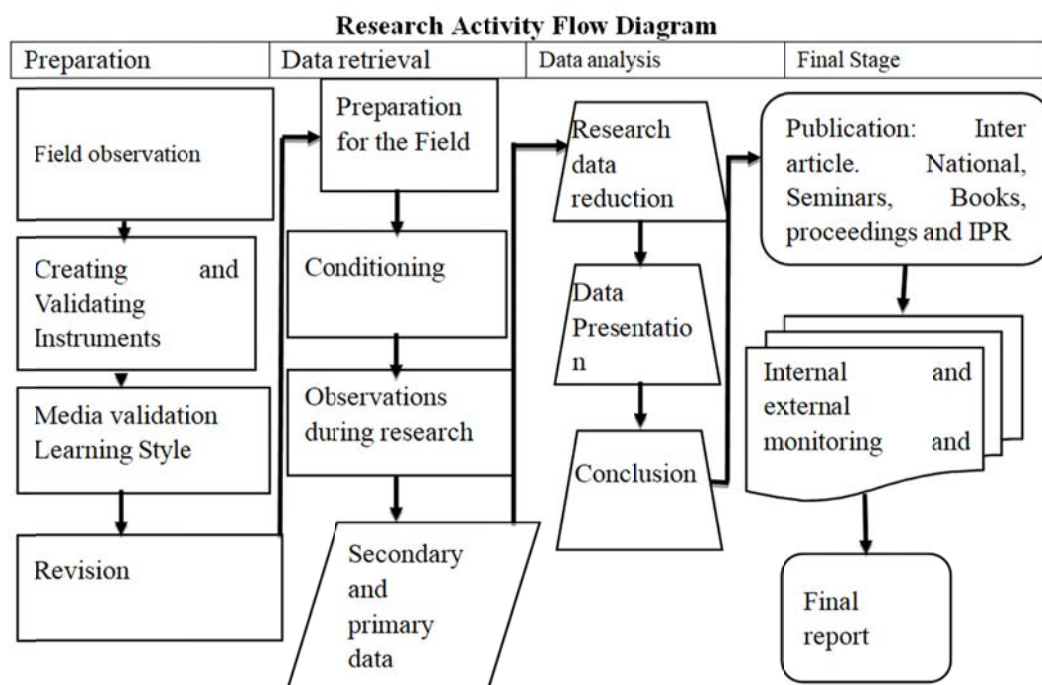


Figure 1. Research flow diagram

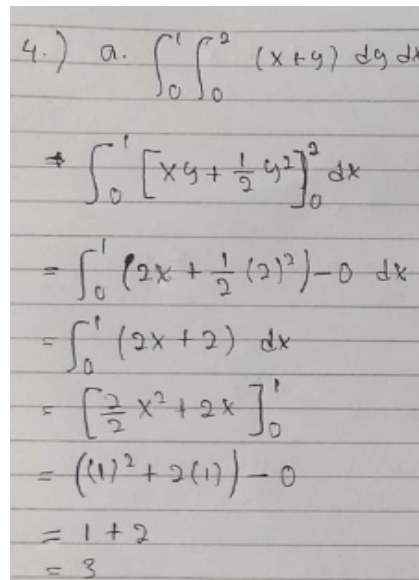
4. Findings and Results

The students who were research subjects in group 1 were FD1 and FD2. Group 2 is ID1 and ID2. Conceptual errors for group 1 subjects in solving calculus problems are dominated by the characteristics of making conceptual errors with an independent field cognitive style.

(1) Exposure and analysis of group 1 data, namely FD1 and FD2 in solving calculus problems.

Based on the selection of research subjects, the students who were subjects in group 1 were subject FD1 and subject FD2. In groups 1 FD1 and FD2 in solving mathematical problems, the structure of high ability mathematical conceptual errors is dominated by connective mathematical metacognition. Data exposure for group 1 subjects (FD1 and FD2) is as follows:

Group 1 subjects have received answers to the problems given, but are not sure about the answers they have obtained. The subject answers that have been obtained can be seen in Figure 2 below.

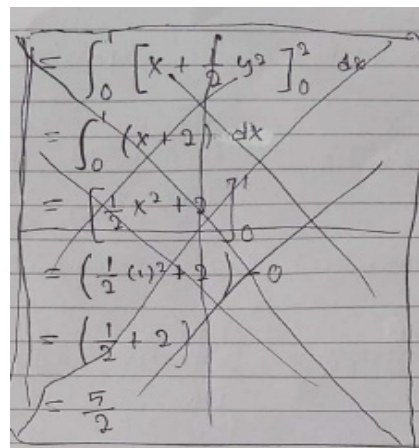


Handwritten mathematical solution for a double integral problem:

$$\begin{aligned}
 4.) \quad a. \quad & \int_0^1 \int_0^2 (x+y) \, dy \, dx \\
 & \rightarrow \int_0^1 \left[xy + \frac{1}{2} y^2 \right]_0^2 \, dx \\
 & = \int_0^1 \left(2x + \frac{1}{2} (2)^2 \right) - 0 \, dx \\
 & = \int_0^1 (2x + 2) \, dx \\
 & = \left[\frac{2}{2} x^2 + 2x \right]_0^1 \\
 & = (1)^2 + 2(1) - 0 \\
 & = 1 + 2 \\
 & = 3
 \end{aligned}$$

Figure 2. Results of group 1 subject exploration using field dependent style

Next, group 1 subjects experimented with the solutions that had been created by creating other ways of working on the questions. This trial and error process is carried out in order to be sure of the answers that have been worked out previously. The following are the results of the trials carried out as in Figure 3.



Handwritten mathematical solution for a double integral problem, crossed out with a large X:

$$\begin{aligned}
 & = \int_0^1 \left[x + \frac{1}{2} y^2 \right]_0^2 \, dx \\
 & = \int_0^1 (x + 2) \, dx \\
 & = \left[\frac{1}{2} x^2 + 2x \right]_0^1 \\
 & = \left(\frac{1}{2} (1)^2 + 2 \right) - 0 \\
 & = \left(\frac{1}{2} + 2 \right) \\
 & = \frac{5}{2}
 \end{aligned}$$

Figure 3. Results of group 1 subject exploration using field dependent style

From Figure 3, it can be seen that the subject made a mistake in the mathematical concept of integrating, so the subject crossed out the answer that he had just made the second integration. This shows that disequilibrating occurs before making improvements in completing answers and being confident in the answers made previously. This shows that an error occurred after doing the question correctly, but was not sure about the answer that had been done. The answer is yes understand the problem correctly but not sure about his understanding. This uncertainty was described by the subject when he asked “yes, sir” but the subject reassured himself by answering “if that’s not wrong, but it looks like the answer above is correct.” This shows that there is disequilibrating in group 1 subjects. Subjects are not sure about their understanding and make connections in carrying out applicable analysis by connecting several concepts to be convincing.

(2) Exposure and analysis of group 2 data, namely ID1 and ID2, in solving calculus problems.

Based on the selection of research subjects, students who were subjects in group 2 were subject ID1 and subject ID2. In group 2 (ID1 and ID2) in solving mathematical problems, the error structure of high ability mathematical concepts is dominated by linear mathematical metacognition. Exposure data for group 2 subjects (ID1 and ID2) is as follows:

$$\begin{aligned}
 3. \quad F(x, y, z) &= x^2 + xy + y^2 \\
 \Rightarrow F(x, y, z) &\Rightarrow \frac{\partial^2}{\partial x} \cdot \frac{\partial^2}{\partial x} + \frac{\partial^2}{\partial y} \cdot \frac{\partial^2}{\partial x} + \frac{\partial^2}{\partial z} \cdot \frac{\partial^2}{\partial x} + \frac{\partial^2}{\partial x} \cdot \frac{\partial^2}{\partial x} \\
 &+ \frac{\partial^2}{\partial y} \cdot \frac{\partial^2}{\partial x} + \\
 &\Rightarrow \frac{\partial^2}{\partial x} \cdot \frac{\partial^2}{\partial x} + \\
 &= \frac{\partial^2}{\partial x} \cdot \frac{\partial^2}{\partial x} + \frac{\partial^2}{\partial y} \cdot \frac{\partial^2}{\partial x} + \frac{\partial^2}{\partial x} \cdot \frac{\partial^2}{\partial x} \cdot \frac{\partial^2}{\partial y} \\
 &\quad \frac{\partial^2}{\partial y} \quad + \frac{\partial^2}{\partial y} \cdot \frac{\partial^2}{\partial x} + \frac{\partial^2}{\partial x} \cdot \frac{\partial^2}{\partial x}
 \end{aligned}$$

Figure 4. Results of group 1 subject exploration using independent field style

From Figure 4, it can be seen that the subject made a mistake in the mathematical concept of operating more than one variable function, so that the subject crossed out the answer that had been made. The subject is not sure about the answers made while working. This shows that an error occurred after doing the question correctly, but was not sure about the answer that had been done. The subject answered by reflecting back on what had been done but was not sure of his understanding. This uncertainty was described by the subject when he asked “yes, sir” but the subject reassured himself by answering “if that’s not wrong, but it looks like the answer above is correct.” There was disequilibrium in group 2 subjects. Therefore, in understanding the problem, the subject was not sure of his understanding. In solving this problem, the subject carried out an applied analysis.

5. Discussion

(1) Structure of mathematical concept errors structure of mathematical concept errors with field dependent cognitive style

Based on the data and analysis, the structure of conceptual errors can be described with an applied field dependent cognitive style starting with disequilibrating, then solving by linking applicable concepts, as in Figure 5 below.

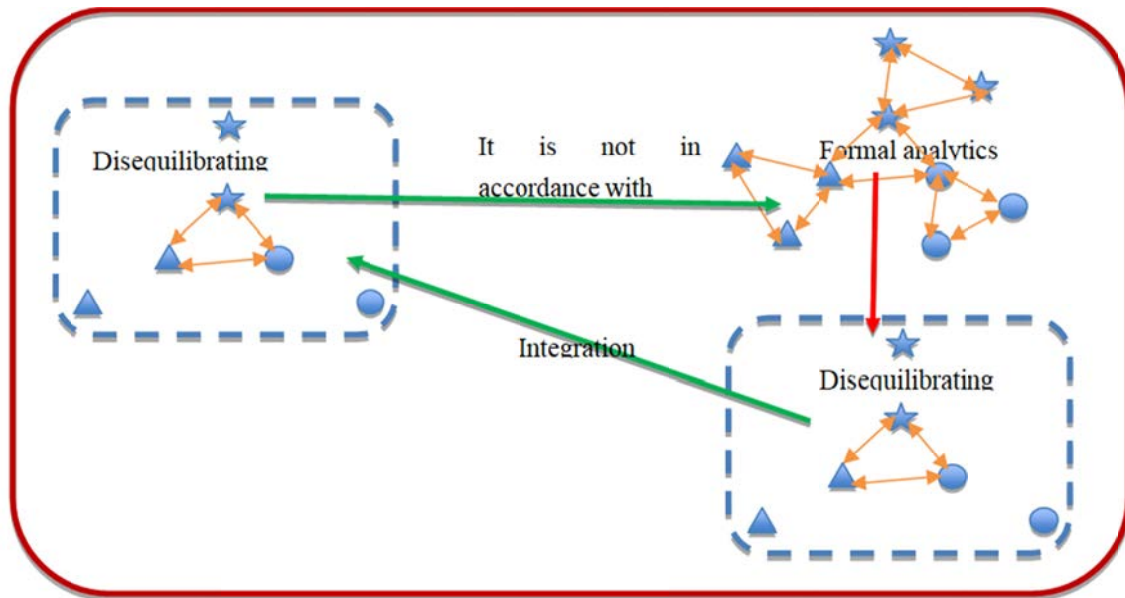


Figure 5. Structure of mathematical conceptual errors with field dependent cognitive style

(2) Structure of mathematical concept errors structure of mathematical concept errors with field independent cognitive style.

Based on the data and analysis, the structure of conceptual errors can be described using a field independent cognitive style, starting with disequilibrating, then solving it by analysing, as in Figure 6 below.

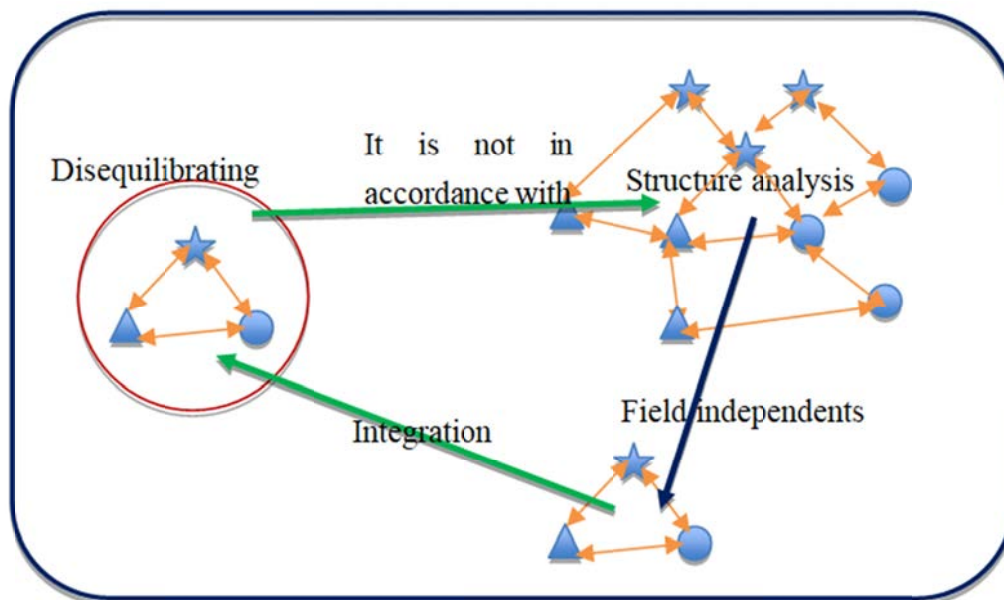


Figure 6. Structure of mathematical conceptual errors with field independent cognitive style

6. Conclusion

The conclusions in research on the structure of students' mathematical conceptual errors in solving calculus problems based on field dependent and independent cognitive styles are: (1) the structure of conceptual errors with an applied field dependent cognitive style begins with disequilibrating, then solves by linking applicable concepts, and (2) the structure of conceptual errors with field independent cognitive style begins with disequilibrating, then solving it by analysing.

7. Recommendations

Based on the results of research on the structure of students' mathematical conceptual errors in solving calculus problems based on field dependent and independent cognitive styles, it is recommended that (1) look at the structure of students' mathematical conceptual errors in solving calculus problems based on field dependent and independent cognitive styles which are assimilation and accommodation, and (2) researched further to see the structure of mathematical conceptual errors based on auditory, visual and kinaesthetic learning styles.

8. Limitations

Limitations in research on the structure of students' mathematical conceptual errors in solving calculus problems based on field dependent and independent cognitive styles are (1) research subjects that are difficult to find with various learning styles, such as auditory, visual and kinesthetic learning styles, and (2) problems that are encountered, raised regarding the structure of the problem, there are still not many references that can be used as references in this research.

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