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Together in a Productive Struggle: Unpacking Student and Teacher Productive Struggle in Mathematics

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ABSTRACT

This article features a fifth grade mathematics exploration planned to facilitate students' productive struggle. The exploration was a catalyst for a team of educators to unpack the teacher's experience when facilitating students' productive struggle. The team called this *teacher productive struggle* and shares about the construct contextualized in the exploration.

Keywords: elementary, mathematics exploration, productive struggle

Introduction

In 2020, we collaborated as teacher educator, prospective teacher, and practicing teacher (respectively) in Moloney's fifth grade mathematics classroom. During one lesson, geoboards were used to facilitate unpacking the relationship between area and perimeter. This was Clark's first time observing students working with geoboards, and she noted how the tool was used to support students' productive struggle. It was also the first time our team of three educators considered what it might mean for a teacher to productively struggle during mathematics instruction that facilitates students' productive struggle. While the concept of students' productive struggle in mathematics had been read about, discussed, and supported in instruction, what unfolded in our team's post-lesson reflection was how Moloney felt as her students did, those who productively struggled with the mathematics. We called this personal experience her *teacher productive struggle*.

This article provides an overview of Moloney's lesson, which included geoboards, and her teaching that allowed for students' productive struggle. We share about this lesson exploration to provide insight into a teacher's feelings and thoughts during the facilitation of students' productive struggle. We unpack this teacher's insights throughout one exploration and within one context, so that other educators can begin to compare and identify their own *teacher productive struggle* and consider what that might encompass. Then, like our team, educators can collaborate with open and honest conversations about the apprehensions and benefits of facilitating students' productive struggle and the experience of their own teacher productive struggle.

What is Productive Struggle?

Student productive struggle occurs when students have opportunities to engage with appropriately challenging mathematics ideas and problems which require struggle and perseverance (Hiebert & Grouws, 2007). Student productive struggle is evoked through effective teaching strategies that encourage students to work through challenging mathematical ideas and relationships, with the goal of developing a greater sense of mathematical literacy and understanding, rather than just focusing on finding correct solutions (National Council of Teachers of Mathematics [NCTM], 2014). These teaching strategies may include observing and utilizing student strategies, activating students' prior knowledge, providing ample opportunities for students to pause and reflect on their own thinking through the use of strategic questioning, and highlighting that student struggle and perseverance in mathematics is recognized, valued, and ultimately productive (Baker et al., 2020). Teachers should be transparent about and reinforce with students that struggle is a natural and essential part of mathematical problem-solving (Wilson et al., 2019). To encourage more equitable spaces, teachers must work to ensure all students have access to cognitively demanding tasks that spur productive struggle and deep thinking, especially students from historically marginalized groups whose access has been limited (Lynch et al., 2018).

Since the construct of student productive struggle has been defined and discussed by mathematics researchers and educators, including its value and what it may look like in the context of a classroom for the student learner, our team began to reflect on a teacher's experience while facilitating students' productive struggle. Engaging students in appropriately challenging mathematical opportunities that encouraged their risk-taking opened the door for teacher risk-taking as well. We began to conceptualize the educator's thoughts and feelings during the facilitation of student productive struggle as *teacher productive struggle*, a term which serves to encompass both the apprehensions and benefits a teacher experiences alongside student productive struggle. We did not and do not use the term *teacher productive struggle* to mean we are struggling with the choice to offer tasks that induce student productive struggle; we believe students deserve to tinker with the mathematics and we work to "trust students with open-ended, multidimensional, challenging tasks" (Skinner et al., 2019). We do use the term to capture the momentary discomfort teachers may feel to as students experience momentary discomfort during challenges and problem solving.

What we share in the article highlights moments of student productive struggle in a fifth-grade mathematics Geoboard exploration in order to reveal Moloney's own teacher productive struggle. In the student exploration, we bring to attention two characteristics of student productive struggle as defined by Hiebert and Grouws (2007): (a) utilizing existing knowledge to engage in solving challenging problems which do not have immediate solutions, and (b) perseverance through problem solving in an effort to enhance mathematical understanding. We unpack Moloney's experience in response to facilitating these two characteristics in this specific mathematics exploration. Finally, we expand on teacher productive struggle beyond the specific exploration and classroom interactions.

Guiding Philosophies and Context of Our Mathematics Teaching

Our understanding of *teacher productive struggle* is grounded in our philosophy of mathematics education. Collectively we believe in conceptually based teaching that offers all students the opportunity to engage with and make sense of mathematical concepts (NCTM, 2014). These experiences are grounded in cognitively demanding tasks that allow for problem-solving with multiple strategies and connections across representations (Smith & Stein, 2018; Stein & Smith, 1998). We believe these learning experiences should be structured in a way that provides access to all students, disrupting patterns of marginalization that can exist in mathematics classrooms (Chao et al., 2014; Wilson et al., 2019). In addition, we believe discussion and student-to-student discourse supports students' understanding of mathematics (Chapin et al., 2013). In order to effectively facilitate these task-based, discussion-based, conceptually driven mathematics experiences for students, teachers need mathematical knowledge for teaching (MKT), a specialized knowledge of both the mathematical content and the pedagogical practices that support student learning (Ball et al., 2008). Our construct of teacher productive struggle is grounded in our acknowledgement that MKT is necessary; we cannot facilitate student productive struggle and give students the mathematical opportunities they deserve without investing in thoughtful problem selection, unpacking the mathematics content ourselves before exploring it with students, and considering the potential strategies students might use in their problem solving (Carpenter et al., 2014).

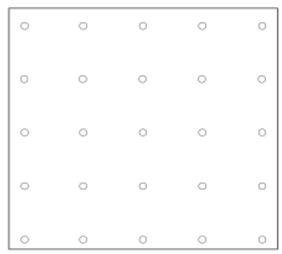
The particular exploration shared next is situated in Moloney's fifth grade classroom mid school year. At the time of this exploration, Moloney was in her fifth year of teaching, and was specializing in fifth grade mathematics teaching. The school was in the process of adopting the *Bridges* (2016) curriculum, and this was the first year with the curriculum for both the teacher and the students. The students in this class were familiar with experiences of productive struggle in mathematics. Throughout their fifth-grade year, students were exposed to cognitively demanding mathematical tasks and were given consistent opportunities to discuss mathematical concepts. Baker and Clark collaborated in university coursework and an undergraduate research project that explored humanizing mathematics (Gutiérrez, 2009, 2010; Yeh & Otis, 2019), and they collaborated with Moloney and her students to engage in mathematics both inside the classroom and in outside spaces. While this article features Moloney's classroom context and personal experience with teacher productive struggle in that context, our team acknowledged that we have each experienced teacher productive struggle within our varied mathematics teaching and learning contexts. As you read, keep in mind that teacher productive struggle is not unique to fifth grade mathematics teaching and learning, but rather can occur across grade levels and content areas, and consider how it might connect to other contexts.

The Geoboard Exploration

An overview of the Geoboard exploration and its intended goals is featured in Figure 1. Before Moloney introduced the geoboards to students, she first accessed students' prior knowledge by asking them to consider how perimeter, area, and multiplication are related. The students discussed these relationships in a turn-and-talk format and then shared some of their discussion points with the entire class. One student expressed that perimeter is "adding all sides of a shape," and another shared that "sometimes you can multiply to find perimeter if all sides of the shape are the same." Another shared that "area is the number of square units a shape is made up of." Moloney wrote these student definitions on the classroom white board for students to refer to during the exploration. She then launched the geoboard exploration, distributing the geoboards, and allowing students three minutes to explore, make observations, and consider "what it can and can't do to help think about math."

Figure 1

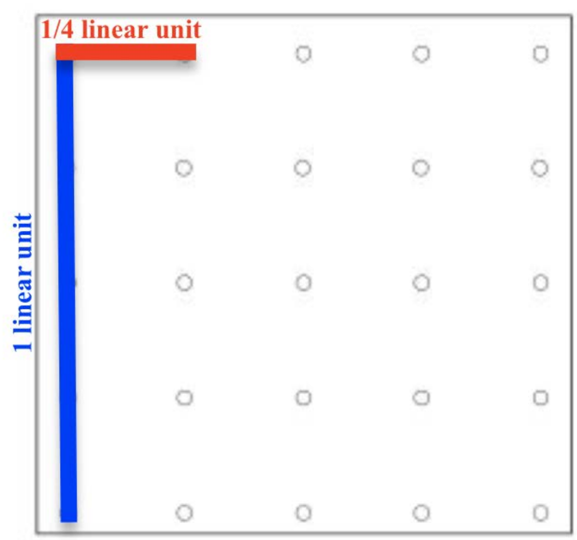
Fifth Grade Geoboard Exploration Overview

The Geoboard Exploration	**Content Goal:** Explore the relationship between area and perimeter using a geoboard as a visual representation and model. **Productive Struggle Goal:** Understand that not all problems are solved instantly or in one math session; be comfortable letting problems linger into future days; attempt different strategies if the current one is not working. **Student Experience**: Students begin to explore how to represent perimeter and area of fractional measurement lengths on a geoboard and model what that perimeter and area represents. **Model of Student 5 x 5 Geoboard:**  Lesson adapted from *Bridges* (2016) 5th Grade Curriculum

Next, Moloney displayed an enlarged geoboard and explained that the outline of the peg area of the board represented one square unit. She posed the question, "If the area is one square unit, what is the perimeter and how do you know?" Moloney's purpose for initiating the exploration with this question was to support students in seeing that each length of the side of the geoboard was one unit, and that each linear space from peg to peg was one fourth of a unit, as seen in Figure 2.

Figure 2

Geoboard Labeled with One Linear Unit and One-fourth of a Linear Unit.



Students were first invited to explore this question using rubber bands on their physical geoboard and with a printed copy of the geoboard image. After some independent work time with the materials, Moloney paused the class for a group discussion around initial thoughts about the value of the perimeter. A student offered, "the perimeter of the geoboard is four square units." Moloney caught that the perimeter would be four *linear* units, not four *square* units. However, rather than correct this, she wanted to see if the class could help refine the thinking and language. This would help the class review the connection between the concepts of perimeter and area. She prompted the entire class by saying, "If the area of the board is one square unit, [Student A] says the perimeter would be four square units. Do you agree or disagree?" She encouraged students to share their ideas with their table groups. As students discussed this prompt, they re-examined their own work and materials, and Moloney circulated the room, conversing with the small groups. A whole group discussion and class vote followed, revealing that students were indecisive about the validity of Student A's statement. Five students agreed with Student A's statement, nine disagreed, and multiple students refrained from voting on the validity of the statement just yet.

Faced with the group indecision, Moloney prompted students to share their reasoning as to why the perimeter of the geoboard would, or would not, be four square units. Moloney wanted to elicit various ideas so that students could use others' thoughts as evidence to support their own thinking. The following are examples of the students' contributions.

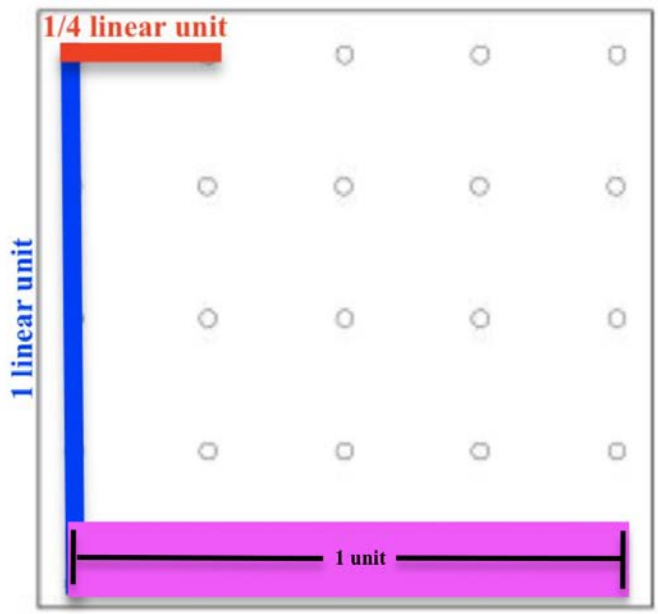
> **Student 1**: I reasoned that one unit divided by four (one side divided into four pieces) equals 1/4 unit. [Student goes up to the board to demonstrate that one side is 4/4 units which is equal to one whole.]
> **Student 2**: I know the perimeter represents the outside of the rectangle and I counted 16 [1/4th] segments. So, the perimeter equals 16 units.
> **Student 3**: I am confused by these answers because the total area is supposed to be one square unit, so I'm still not sure what the perimeter would be and how we would think about that.
> **Student 4**: I think that if the area is one square unit, the perimeter is four units, not square units. And that works with what Moloney said at the beginning about the area being one square unit total and us having to find perimeter, which is in units.

After allowing time for students to listen and ask questions of one another, Moloney conducted a re-vote, which highlighted continued indecision and confusion among students. Three students shared that they believed "area = 1 sq. unit" [notation as verbally shared and as written out on board to present to the whole class] and "perimeter = 4 units", noting that no one thought the "square" should stay with the way the perimeter's value was being expressed. Five students now voted that they thought "area = 16 sq. units" and "perimeter = 16 units" after hearing that answer offered. Again, multiple students refrained from voting. At this point both the students and Moloney were visibly struggling to determine what the next move should be. This was expressed through confused and scrunched facial gestures, slumped body language, and shoulder shrugs. Moloney used a "Teacher Time Out" (Gibbons et al., 2017) with Baker to quickly run through options, considering the costs and benefits of stopping the lesson there for the day or attempting an alternative strategy for facilitating the lesson that would allow students to keep working. After considering the mathematical goals for the lesson, the team decided to introduce a tool that would help students focus on the distinction between linear and square units. Before jumping back into the mathematics, however, Moloney used a powerful and important teaching move for her classroom community by asking the class, "Who's confused? Be honest." As the majority of students in the class raised their hands, Moloney responded, "Great!" and thanked them for their honesty. She took this moment to share that she was also confused about how to proceed, and that both her confusion and their confusion was okay. She reassured students that they would work with one another to figure out a path and an answer. This reassuring moment served several purposes - to let students know it was okay to feel confused, to let students know that the confusion would not persist, and to let them know that she would support them to the other side of the initial confusion.

Moloney's next step was to guide students back to some of the critical mathematical moments shared in the discussion that could help move the class forward in their understanding (see Baker et al., 2020 for more on this reorientation strategy). She reemphasized to students that they were working in linear units, and she asked a student to explain what that meant in context. Once students had clarification of the units and expressed understanding of the difference between units and square units, Moloney introduced a new tool to further help students determine the perimeter of the geoboard. This new tool was a pink strip of paper, where "one strip of paper equals one unit". See Figure 3 for this tool.

Figure 3

Labeled Geoboard with Pink Ruler



While it was not necessarily pre-planned to introduce this specific tool during the lesson, the teacher had applicable tools on hand as a result of intentional pre-planning with MKT and mathematical learning goals in mind. Moloney shared, "the pink strip is like a ruler that measures one unit. What do we think the perimeter is now that we have our ruler?"

As students discussed and used this new tool with their geoboards, the class quickly concluded that the perimeter of the geoboard is four units and that this answer fit with the area being one square unit because, as one table group enthusiastically shared, "each side of the geoboard is one linear unit and the geoboard is actually one square!" Moloney ended the lesson by expressing to students that they would continue to use and learn with the geoboard in future lessons, exploring mathematics content with both fractional and whole number units of length. The long-term goal for this initial lesson with the geoboard was to lead into future learning experiences representing other more advanced questions of fraction multiplication on the geoboard. Moloney left her students with this final thought: "That was hard math with small numbers." She encouraged them to consider that "hard" does not always involve big numbers or intense problems, that challenging mathematics can happen anywhere, with any problem, and that collaboration is an effective way to productively struggle.

Teacher Productive Struggle During the Exploration

After the exploration, the educator team debriefed Moloney's intended teaching moves to facilitate students' productive struggle, and to also unpack what she was experiencing during the facilitation. Regarding the teaching practice of *facilitating students' use of existing knowledge to engage in solving challenging problems which do not have immediate solutions*, Moloney expressed her attempts to strike the balance between giving enough information, and guidance for students to draw conclusions about area and perimeter without giving the mathematics away or interfering with the students' thinking. In posing the initial question to students about finding the perimeter when knowing the area, she aimed to build upon existing knowledge about perimeter and area and also let students productively struggle to generate hypotheses about area, perimeter, and their relationships.

Moloney also expressed that she introduced a new mathematics tool, the geoboard, to facilitate student understanding of area and perimeter through a visual and physical exploration, because this tool would be beneficial to future lessons. She experienced a teacher productive struggle when considering how to facilitate effective use of the geoboard without letting it impede the mathematical sensemaking. Moloney shared that she felt confident in her choice to let students explore the tool initially, and make general observations, but was uncertain about how to guide students toward the mathematical goal without giving too much away. Balancing these turns in emotions around the same tool, in the same timeframe, was part of Moloney's teacher productive struggle.

In regard to *facilitating students' perseverance through problem solving in an effort to enhance mathematical understanding*, Moloney sought to adequately set up students for the geoboard exploration by exposing them to a productive struggle during the area and perimeter discussion, where they would hopefully gain skills and mindsets about persisting in mathematics. She was optimistic about her students' abilities to uncover important mathematical ideas, but began to question the timing of this exploration, and the pacing of its problem solving once students expressed confusion and frustration. She wondered if the task was not presented at the right point, or if it was too open-ended to support a mathematical goal (i.e., leading to many disconnected paths and ideas). She reflected on how her instructional decisions might influence a productive or unproductive student struggle. During the lesson, Moloney also experienced her own productive struggle as she balanced allowing students to grapple with the mathematics versus knowing when to step in with the whole class or individual learners. Constantly noticing students' affects and trying to determine the line between productive versus unproductive struggle added to Moloney's own productive struggle. Additionally, Moloney acknowledged that openly taking the Teacher Time Out during the geoboard exploration and debating the best teaching decision in the moment was an opportunity to highlight her own productive struggle alongside the students. She hoped this was a teaching moment that reassured students that it is acceptable to struggle, that Moloney supported them, and that they would have opportunities to reset and look at the problem from a fresh perspective. Ultimately, Moloney experienced encouragement and excitement by what students were sharing, knowing that they were growing in knowledge of the mathematics content and engaging in the productive struggle to persist through the geoboard exploration.

Recapping and Expanding on Teacher Productive Struggle

In review, Moloney experienced teacher productive struggle while she was trying to facilitate productive struggle for her students during a lesson. Table 1 overviews student productive struggle characteristics as well as the productive struggle that a teacher may experience in response during a lesson. Of note, in-the-moment teacher productive struggle that occurs while attempting to facilitate student productive struggle is only one layer of teacher productive struggle. In future debrief sessions, the team also discussed that productively struggling as a teacher encompasses decisions about the curriculum and interactions with colleagues.

Productively Struggling with Curriculum

As Moloney grew more comfortable in her understanding of what it meant to facilitate and support student productive struggle during individual lessons, she also challenged herself to consider and expand upon the opportunities that produce it. This meant considering how she utilized her school's mandatory curriculum in a way that served to facilitate her students' productive struggle (Drake et al., 2015). Moloney expressed that she experienced a teacher productive struggle in this effort, as she was fearful that attempting to manipulate the curriculum to allow for greater student productive struggle may harm student learning in the future, depending on how students' future teachers chose to access the curriculum. This was part of the teacher productive struggle that Moloney faced regarding her teaching choices and decision making, and how these choices impacted students. Even with this fear, Moloney recognized that if students followed a curriculum lockstep in future years, her emphasis on productive struggle now would still equip them with skills to evaluate mathematical solutions, and the chance to know what it feels like to persevere through a mathematical problem-solving scenario.

Productively Struggling with Colleagues

Moloney's situation also highlights a third layer of teacher productive struggle, as it is more difficult for a teacher to feel confident in their abilities to facilitate student productive struggle if they do not have critical colleagues that can support, challenge, and problem solve with them when innovative lessons do not unfold as planned. Moloney recognized that she is more hesitant to try new approaches and pedagogies if she does not feel that she will have colleagues who will problem solve, or encourage her to continue to explore how productive struggle might be part of an equitable mathematics classroom space. She sought to adapt her school's adopted curriculum to her students' mathematical contributions and needs. However, she sometimes struggled to engage and persist in alternative pedagogical practices if she felt that she would not have others' support to do this.

Table 1

Characteristics of Student and Teacher Productive Struggle and How They Relate

Student Productive Struggle Characteristic	Teacher Productive Struggle Experiences	Possible Teacher Responses
When a student is engaging in solving challenging problems which do not have immediate solutions through the activation of prior knowledge...	• A teacher might feel tension trying to balance providing students with adequate mental and physical tools to solve the problems and giving too much away. • A teacher might feel encouraged to engage in strategic questioning of students as a means to allow students access to deeper mathematical knowledge and understanding, and to get to know their learners' affects and problem-solving personalities.	• A teacher may use a physical tool (e.g., a paper strip) and/or a visual to scaffold the given problems and allow students to access the key mathematical ideas. • A teacher may refer to pre-planned additional questions to probe student thinking and access key ideas. • A teacher may utilize talk moves (Chapin et al., 2013), such as student turn and talks, revoicing, or offering extended think time. • A teacher may consider the lived experiences, identities, and funds of knowledge (Moll et al., 1992) of students to contextualize the mathematical problem. • A teacher may draw upon past mathematical content experiences or mathematical backgrounds in framing or reframing the problem.
When a student is persevering through problem solving in the mathematics experience...	• A teacher might feel a pressure to move the mathematics along more quickly and guide students to the correct answer. • A teacher might feel excited to continue to push students to think deeper about the mathematics, as the students are effectively engaged and having critical mathematical insights.	• A teacher may take a "Teacher Time Out" (Gibbons et al., 2017) to determine how to respond to student thinking in the lesson. • A teacher may humanize the experience by making the shared productive struggle of both student and teacher known. • A teacher may discover a new line of questioning to drive student thinking in the midst of their own teacher productive struggle.

Why Persist with Productive Struggle?

Moloney recognized that despite the apprehensions that arise when facilitating students' productive struggle, her belief in the benefits of facilitating student productive struggle, as well as her motivation to challenge the dominant narratives in schools about which students have access to rich mathematics that facilitates productive struggle, outweighed these fears. She continued offering her students mathematics that facilitated productive struggle because she saw that they were able to engage in content in new and unique ways, and both Moloney and her students felt accomplished when they were able to persevere through a struggle. Student productive struggle in Moloney's classroom also served to foster a stronger community bond among students as they worked together to engage in challenging mathematics. Moloney was also aware that while she was fearful of using students during *the experimental phase*, when attempting new teaching practices, her students deserved her trust that they could accomplish big things in the mathematics space. Moloney has seen that her students think more critically and engage with the mathematics more deeply when they are given opportunities to productively struggle, and while this may not be the way that the curriculum was designed, she continues to engage in this work.

What's Next?

One suggestion for unpacking productive struggling in your own context is to read this article with a colleague or colleagues and compare which feelings you may have experienced while facilitating productive struggle for your students. In starting to name possible apprehensions, it might help to take a step together to facilitate an experience for students that promotes productive struggle. Another suggestion is to anticipate feelings while planning a lesson that includes productive struggle. Prior to introducing a lesson, generate lists about what your students might do, feel, or say when they are experiencing productive struggle during the lesson, and consider what you might do, feel, or say while facilitating it. Thinking about what productive struggle might look and feel like in your context may better support your ability to persist in the facilitation of it. As educators, we must strive to embrace the power of appropriate struggle as an opportunity for learning and growth for both students and educators. Our students can persist in their productive struggle of the mathematics content understanding if we trust them in the productive struggle and trust ourselves to persist in our facilitation of it.

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