

# Professional Collaboration among Elementary School Teachers in Lesson Study

Adriana Richit<sup>1</sup>, João Pedro da Ponte<sup>2</sup>, & Mauri Luís Tomkelski<sup>2</sup>

1) *Universidade Federal da Fronteira Sul*, Brasil

2) *Universidade de Lisboa*, Portugal

## Abstract

Professional teacher collaboration, a fundamental characteristic of lesson study, has been pointed out as a principle of teacher personal and professional growth. This article aims to describe and analyze the development of professional collaboration in a lesson study, highlighting the main stages of this process. The investigation involved six elementary school teachers of Brazilian public schools, in planning, observing, and reflecting on a lesson on the curriculum topic of length measure at grade 4. The participants worked collaboratively planning a lesson about this topic. The methodology is qualitative, with data collected by records and field notes of sessions, conversations, and interviews. The analyses pointed out three remarkable stages of the development of professional collaboration, which constituted the categories of analysis: recognition of collaboration, development of collaboration, and valuing collaboration. The results show that the professional collaboration that developed during the lesson study was facilitated by the dynamics of the different steps of this process and, especially, by the relationship between participants and teacher educators. Collaboration emerged from the activities of planning, teaching, observing, and reflecting on the lesson, and was valued given the encouragement and strengthening of the group insofar as teachers felt confident, encouraged, and welcomed in the group.

## Keywords

Professional collaboration, elementary school, lesson study, professional practice, teacher education

---

**To cite this article:** Richit, A., Ponte, J. P., & Tomkelski, M. L. (2024). Professional collaboration among elementary school teachers in Lesson Study. *Journal of Research in Mathematics Education*, 13 (2) pp. 111-131. <http://dx.doi.org/10.17583/redimat.14337>

**Corresponding author(s):** Adriana Richit

**Contact address:** [adrianarichit@gmail.com](mailto:adrianarichit@gmail.com)

## Colaboración Profesional entre Maestras de Escuela Primaria en Estudio de Clases

Adriana Richit<sup>1</sup>, João Pedro da Ponte<sup>2</sup> y Mauri Luís Tomkelski<sup>2</sup>

1) *Universidad Federal de la Frontera Sur, Brasil*

2) *Universidad de Lisboa, Portugal*

### Resumen

La colaboración profesional, una característica fundamental de lo estudio de clases, ha sido señalada como un principio del crecimiento personal y profesional de los docentes. Este artículo tiene como objetivo describir y analizar el desarrollo de la colaboración profesional en un estudio de clases. La investigación involucró a seis profesores de enseñanza primaria de escuelas públicas brasileñas, en la planificación, observación y reflexión sobre una lección sobre el tema curricular de la medida de la longitud en el 4º grado. Los participantes trabajaron en colaboración planificando una lección sobre este tema. La metodología es cualitativa, con datos recolectados por registros y notas de campo de sesiones, conversaciones y entrevistas. Los análisis señalaron tres etapas notables en el desarrollo de la colaboración profesional: reconocimiento de la colaboración, desarrollo de la colaboración y valoración de la colaboración. La colaboración profesional que se desarrolló durante el estudio de clases se vio facilitada por la dinámica de las diferentes etapas de este proceso y, especialmente, por la relación entre los participantes y los formadores de docentes, y se valoró dado el estímulo y fortalecimiento del grupo en la medida en que los profesores se sintieron seguros, animados y acogidos en el grupo.

### Palabras clave

Colaboración profesional, escuela primaria, estudio de clases, práctica profesional, formación de profesores

---

**Cómo citar este artículo:** Richit, A., Ponte, J. P., & Tomkelski, M. L. (2024). Professional collaboration among elementary school teachers in Lesson Study. *Journal of Research in Mathematics*, 13(2), pp. 111-131. <http://dx.doi.org/10.17583/redimat.14337>

**Correspondencia Autores(s):** Adriana Richit

**Dirección de contacto:** [adrianarichit@gmail.com](mailto:adrianarichit@gmail.com)

Collaboration, conceived as a cooperative principle of association among teachers in administratively regulated and predictable forms (Hargreaves, 1998), is currently an emerging theme in scientific communities around the world, notably in mathematical education (Borko & Potari, 2020; Robutti et al., 2016). Collaboration characterizes a relational system among people or an emerging process characterised by unpredictability that implies negotiations and decisions and focuses on resources, goals, outputs, and outcomes (Morris & Miller-Stevens, 2016).

Professional collaboration characterizes the culturally established forms of interaction between people and the joint realization of professional activities, marked by principles such as effective communication, careful negotiation, joint decision (Boavida & Ponte, 2002; Robutti et al., 2016; Richit et al., 2024), dialogue, support, mutual encouragement, sharing, and trust (Hargreaves, 1998), overcoming professional hierarchies and valuing the voices and roles of all (Richit, Ponte, & Tomasi, 2021). Thus conceived, professional collaboration can promote the development of teachers to create opportunities for them to learn about teaching and to reflect on their role in school and society (Robutti et al., 2016; Richit, 2023), as well on the complexity of mathematical objects (Calle et al., 2022). In addition, professional collaboration mobilizes collective efforts to improve teaching, promote students' learning and promote educational changes.

Collaboration promotes professional growth and ensures educational change, because “it is a means to achieve a nobler end: a richest and most significant learning of students” (Lima, 2002, p. 8). In professional collaboration, collaboration among teachers is seen as part of their professional culture. Given its characteristics and possibilities of favoring the professional development of teachers, professional collaboration has been evidenced in different approaches to teacher education (Hargreaves & O'Connor, 2018; Richit & Ponte, 2017).

Lesson study is professional development process based on collaboration, and focused on school practice (Burroughs & Luebeck, 2010; Murata, 2011; Xu & Pedder, 2015; Ponte et al., 2016; Soto, et al., 2019; Richit, Ponte, & Tomkelski, 2020). In lesson study, teachers work collaboratively analyzing a learning difficulty of students and preparing a lesson to overcome this difficulty. Reflecting on the students' activity in the lesson, the teachers consider what was most appropriate to improve students' learning and what needs to be modified (Murata, 2011; Richit & Tomkelski, 2022). On the whole, this activity may be considered as a small research on teachers' own professional practice (Ponte et al., 2016). Lesson study gained much attention in Western countries as far as it begun to be developed in the United States and its results were disseminated in English language (e.g., Lewis et al., 2009). Professional collaboration, in the context of lesson study, constitutes a theme increasingly investigated by researchers from different countries (Hargreaves & O'Connor, 2018; Richit et al., 2021).

From this perspective, it is relevant to analyze and discuss professional collaboration in lesson study, signaling key practices and situations of this process, and striving to understand how teachers experience collaborative professional practices. We examine the promotion of professional collaboration in a lesson study carried out with six elementary school teachers, involved in a professional development activity and a classroom practice quite different from what they are used to. This research aimed to answer the following question: How does professional collaboration develop in lesson study? The analysis contributes to deepen the

discussions concerning teacher professional collaboration, especially in lesson study contexts, highlighting fundamental situations that promote collaboration among peers from the perspective of professional culture.

## **Theoretical Framework: Professional Cultures and Professional Collaboration**

### **Professional Cultures and Professional Collaboration**

Professional cultures reflect the socialization process of teachers in the profession (Dubar, 1997) and also the sense of belonging to this profession and also to a specific school. The entry in the professional career, as well as the teacher professional development process, is strongly influenced by the prevalent professional culture in the school or in the subject group to which the teacher belongs (Marcelo, 2009). As Fialho and Sarroeira (2012) indicate, the prevalent ways of professional culture in the school, the norms, the beliefs, the ways to act are (or not) assimilated by teachers, and become reflected in their professional identity.

Hargreaves (1998) differentiates four ways of professional cultures by which the teacher work is undertaken and legitimized: individualism, collaboration, artificial collegiality and balkanization. Our study focuses on collaboration for its potential to favor the professional development of teachers to the extent that it allows them to work together, share experiences, ideas, information, decisions, as well as receive and provide support in several situations (Lima, 2002). In addition, collaboration favors the overcoming of individualism, which is a beacon of teacher development.

Professional collaboration, conceived as the new chorus line for innovation and improvement (Hargreaves & O'Connor, 2018), refers to an “ideal way to ensure the professional development of teachers throughout their careers, the learning par excellence of students and the transformation of schools into authentic learning communities” (Lima, 2002, p. 7). Collaboration has been presented as a viable solution to the problems of education to the extent that it is linked to the processes of improvement of the quality of teaching and development of the school (Hargreaves, 1998; Richit, 2023).

Etymologically, collaboration derives from “co-laborare”, working together with others, in projects or in solving problems, and cooperating derives from “co-operare”, carry out a task, guided by a well-defined plan (Boavida & Ponte, 2002; Morris & Miller-Stevens, 2016). Professional collaboration, according to Robutti et al. (2016), also implies co-learning (learning together), as it involves teachers in joint activities, common purposes, dialogue and critical questioning and mutual support in addressing issues that challenge them professionally. The conception of professional collaboration of Robutti et al. (2016) recovers and values teaching as an activity planned and developed by a professional collective, which is favored and enhanced in collaborative cultures, which underscore the purposes of teaching and education, the social role of the teacher and the school, teacher planning and the promotion of students’ learning. Collaboration requires equality and mutual assistance among participants in undertaking a work that promotes collective development, which is different from cooperation which just seeks to carry out a collective work to reach a specific aim.

Hargreaves (1998) underlines that the confidence that emerges from sharing and from collegial support among teachers “leads to a greater disposition to carry out experiences and take risks, and, with these, to a commitment of teachers in continual growth, as an integral part of professional obligations” (p. 209).

Borges (2007) considers that “a professional collaboration culture is that one in which everything is shared, is discussed, in which support and aid required to learn is sought” (p. 370-371). In his view, professional collaboration characterizes the interaction that teachers establish when they are involved in a joint activity, sharing professional goals. Hargreaves (1998) emphasizes that collaboration is based on dialogue and action among colleagues, shared decisions and mutual support in the face of which teachers are encouraged to diversify teaching strategies and thus take risks, face changes and reflect on their practice.

Professional collaboration, as a fundamental dimension of professional culture, constitutes a theme increasingly investigated, as it supports teacher professional development promoting professional learning and changes in professional practice (Xu & Pedder, 2015; Richit, 2021). We regard professional culture as the ways by which teachers relate to one another and act. That is, professional culture is represented especially in the “practice that they undertake, in the beliefs that underpin their way to understand the work that they did and they do, as well as the routines that motivate them in their actions in the school, with other teachers and, naturally, with their students” (Fialho & Sarroeira, 2012, p. 4).

Goulet and Aubichon (1997) indicate that the first step towards collaboration is trust, which develops in a context of respect and care, going through personal and professional levels. Trust is fundamental for members of a group to feel free to expose their ideas and questions, as well as to question the ideas and actions of others, respecting and valuing them, while their work and values are respected and valued. Therefore, without trust there is no possibility for collaboration (Boavida & Ponte, 2002).

Another aspect that may favor the development of participants engaged in an activity in which they share objectives is collaboration assuming different roles and positions in this process, such as collaboration between researchers and teachers (Miyakawa & Wisløw, 2009; Hargreaves & O’Connor, 2018). This form of collaboration favors overcoming professional hierarchies (Richit, 2020), the flexibility of roles (Lewis et al., 2009) and shared leadership (Hargreaves & O’Connor, 2018).

### **Lesson Study and Collaboration**

Lesson study was established in Japan in the transition from the nineteenth to the twentieth century as an approach to professional development within the process of implementing educational changes, expanding and involving the education system (Richit & Tomkelski, 2022). It usually develops based on four steps: identification of a learning problem, planning a lesson, conducting the research lesson and doing an after-lesson reflection. It is a process centered on school practice that provides the gradual implementation of changes in practice due its strong connection to practice. In the first step, the team defines a study question on a specific curriculum topic, considering the frequent learning difficulties of the students in that topic. In the planning, the most extended step, the teachers study curricular guidelines and

discuss research results on the learning of this topic, study about student learning, analyze teaching materials, study and elaborate tasks and materials for the research lesson, seek to anticipate the possible difficulties of students in solving the task, and plan in detail the dynamics of the lesson. The research lesson focuses on the activity of the students in solving the task, from which the observers seek to identify and record aspects that address the issue identified in the first step. In the reflection, the last step of the lesson study, the team gathers to analyze the actions, strategies, conclusions and difficulties of the students in solving the task, seeking to understand the learning process that took place. In this step is also carried out the review of the lesson, which can be taught again later to another class of students.

Collaboration is usually regarded as a salient feature of lesson study (Stigler & Hiebert, 1999; Takahashi & McDougal, 2016; Richit, 2023). Some studies also report the development of collaborative relationships as an outcome of lesson study, which is sometimes a complex non-linear process (Quaresma & Ponte, 2021). For example, in a study that examines professional collaboration in a lesson study, Robinson and Leikin (2012) analyze two classes conducted by a mathematics teacher in an Israeli school. The research shows that the lesson study provided the teacher the opportunity to develop collaborative observation, collaborative consciousness and collaborative discussion/reflection, which the authors regard as fundamental mechanisms to promote teacher's professional development. In a similar perspective, Wake, Swan and Foster (2016) examine lesson studies developed by English teams, aiming to show teachers' professional learning through the development of problem solving processes, using the lenses of historical-cultural activity theory. The research shows that the development of tasks for research lessons, undertaken in a collaborative way, constitutes an essential mechanism to promote teacher professional development.

Based on teacher learning theory, Lewis et al. (2009), in a study with teachers who taught in North American elementary and high schools, provided a framework for interpreting why lesson study contributes to improving teaching and student learning from changes in teacher knowledge and collaborative learning. The authors highlight that a lesson study is a system of collaborative learning from live instruction, which involves investigation, planning, observing research lessons, and reflecting, to create changes in teachers' knowledge and beliefs, professional community, and teaching resources. They indicate that "shared tools (such as curriculum and assessment) and individual and community characteristics (such as beliefs about students and structures to support collaborative work) mediate teacher learning" (Lewis et al., 2009, p. 399).

Xu and Pedder (2015), based on a systematic literature review of lesson study with both pre-service and in-service teachers, identify several benefits of this process: teacher collaboration and development of a professional learning community, development of professional knowledge, practice and professionalism, more explicit focus on pupil learning, and improved quality of classroom teaching and learning. Huang and Shimizu (2016) highlight that teacher learning contributes to improve teaching and student learning, implement curriculum changes, share instructional products and resources, and improve the relation between theory and practice. They add that a supportive and collaborative context in a lesson study can develop teachers' confidence in experimenting with innovative practices and ideas, with an increase in their self-efficacy making a positive impact on students' learning. In addition, Burroughs and Luebeck (2010) stress that lesson study allows teachers

to carry out collaborative work in their professional practice, characterized by sharing goals, discussion of ideas and collaborative development of teaching resources.

In a research on professional development for higher education, Soto et al. (2019) indicate how participation in a technology-facilitated lesson study provided the means for teachers from five higher education institutions across the United States to engage in professional development and evolve into a virtual community of practice. They conclude that lesson study contributed for building professional relationships:

As we continued our collaboration throughout the lesson study, our discussions broadened to include conversations about common struggles, interests and teaching or research-related resources we encountered. In a sense, our virtual community of practice evolved to include issues of importance to us in our tenure-track positions, rather than just focusing on the specific lesson we were researching. Through the use of technology, we discussed, shared, and analyzed teaching and learning processes, and supported each other in our professional growth (Soto et al., 2019, p. 15).

In this perspective, Richit, Ponte and Tomasi (2021), point that lesson study enhances the relationships among the participants, promoting a sense of belonging to a group that is confronted with similar challenges. In a lesson study, teachers have the opportunity to experience teacher education focused on teamwork, which fostered the proactive involvement of everyone in the process. The dynamics of the lesson study promoted the mutual support and encouragement, cooperation in carrying out tasks and reflection on teaching practice, so that these elements, taken as a whole, promoted collaboration.

Taking as context a lesson study that involved teachers and teacher educators of the university, Fiorentini, Honorato and Paula (2023) developed a study dedicated to describing and understanding how teachers learn collaboratively to make the teaching-learning management of mathematics based on exploratory tasks in a lesson study. Based on Dialogical Narrative Analysis, the authors sought to produce a holistic and diachronic understanding of the participation and learning movement of teachers in this experience, focusing on teachers' practice. The results indicate that the processes of collaborative management of exploratory tasks mobilized knowledge OF classroom practice. And the specialized knowledge, in turn, constituted mediations or fundamental tools for the groups to elaborate relevant exploratory tasks to develop students' mathematical thinking, as well as provided lenses that enabled the analysis of learning IN the practice of the participating teachers.

## **Methodology**

### **Nature and Goal of Research**

This qualitative research follows an interpretative perspective, with data collection by interviews, conversations, field notes, and gathering records (Denzin & Lincoln, 1989), aiming to describe and analyze the development of professional collaboration in a lesson study, by highlighting the main stages of this process. Qualitative research is appropriate to

study how professional collaboration develops in lesson study, attributing meaning to the dialogues and interactions among participants in this process and representing these meanings in the light of the assumptions of professional collaboration.

### Context and Participants

The participants (with pseudonyms Carla, Erika, Ivy, Janaína, Ranya and Yasmin) are teachers of public elementary schools of the state education network of Rio Grande do Sul, Brazil. The participants were invited for the lesson study through an e-mail sent to the schools belonging to a school district who taught in primary school. Eight female<sup>1</sup> teachers responded positively, however, two of them were unable to attend the activity due to timetable conflicts. Thus, the group consisted of six teachers, who voluntarily attended all sessions of the lesson study. They have between 5 and 20 years of professional experience, mostly in the early years of elementary school.

The lesson study was conducted by a team of three teacher educators from the Universidade Federal da Fronteira Sul, Brazil. The activities were undertaken along twelve meetings with three hours each, carried out two times a month in the facilities of the school district. Each lesson study session was conducted by the team of teacher educators, who strived to welcome participants through dialogue, listening and encouragement. Although lesson study, as a collaborative approach, presupposes voluntary engagement of the participants, we consider that this engagement can be strengthened through the effective and affective reception of all participants. To this end, we constituted a context of sensitive listening at the beginning of each session so that everyone could express their questions, expectations and concerns. This strategy was characterized in a space of listening, support and encouragement, strengthening teachers' individual voices and realizing the welcome at a high level, which transcends the professional to the relational. Table 1 indicates the organization of the lesson study and the activities set up for each session.

**Table 1**

*Organization and Activities of the Lesson Study*

Session	Activities developed
1	Presentation of the group and their expectations. Brief introduction to lesson study. Presentation, negotiation and approval of the program and schedule of the lesson study. Frequent learning difficulties of students in mathematics.
2	Detailed presentation of lesson study. Reading and discussion of an article about lesson study as a teacher professional development process (Ponte et al., 2016). Definition of the mathematics topic to deepen in the lesson study (decided it would be the meter unit of length measurement). Definition of an issue to deepen in the lesson study.
3	Review of students' difficulties and of the question defined in the second session. Definition of objectives for the research lesson. Analysis of the approaches adopted in teaching materials on the curricular topic meter. Analysis of curriculum guidelines on this topic (Brazilian Common National Curriculum Base).
4	Discussion of the article "Exploratory approach in the teaching of mathematics" (Canavarro, 2011). Selection and discussion of mathematical tasks on the topic meter.
5	Definition of the context for the development of mathematical tasks related to the topic meter: Gymkhana "Soletrando". Preparation of the first task.



Session	Activities developed
6	Analysis of the first task and anticipation of possible doubts of the students. Preparation of the second task.
7	Discussion of the article “Mathematical reasoning in Elementary School and Higher Education (Ponte et al., 2012)”. Analysis, discussion and reformulation of the second task.
8	Test of the tasks of the research class. Discussion about the interesting aspects and weaknesses of the task pointed out by the teacher who solved them. Final adjustments.
9	Research lesson (part 1)
10	Research lesson (part 2)
11	Reflection about the research lesson. Modifications in the plan for the research lesson.
12	Reflection and evaluation about the dynamics and the activities of the lesson study. Closing.

Source. Elaborated by authors.

The dynamics of the development of the lesson study, throughout the sessions, prioritized activities stressing different aspects of collaboration: dialogue, sharing, trust, help and mutual incentive, joint work, decision making. To this end, each session had three main moments: welcome, peer-to-peer work and synthesis of activities. In the first part of the session, we promoted the group’s welcome through the sharing of the experiences of each participant in that week and the review of discussions and decisions of the previous meeting. From this, we carried out the study, discussion and deepening of the theme of the meeting through peer work. Finally, we systematized the understandings produced from the discussions and the main decisions made in relation to the planning of the research lesson.

The group of teachers decided to use of the Moodle platform to share documents, texts for reading, support material for the preparing the lesson, as well as for each participant to make available their ongoing logbooks. This platform favoured collaborative work on the task between sessions, since teachers were able to access it and make modifications.

During the lesson study meetings, the teachers planned a research lesson, aiming to promote the understanding of the mathematical concept of ‘meter’. The task elaborated to deepen this curricular topic adopted as a context a recreational and cultural activity that involves all students and is held annually in the school where the research lesson was developed.

The research lesson was carried out in a grade 4 class, organized in two sessions of two hours each on consecutive days. This decision was made jointly by all participants considering their interest to be as much time as possible observing the students’ autonomous work in order to better understand their strategies, questions, and conclusions.

### Empirical Material and Data Analysis

The lesson study sessions were recorded, as well as the research lesson. After each meeting we made a detailed description of activities and of work done by the group. After the end of the lesson study, we conducted a semi-structured interview with the participants, through questions related to the dynamics of the lesson study and the contributions of this process to their personal and professional growth.

The empirical material of the research consists of the researcher's field notes, the participants' logbooks, the transcripts of the sessions and the interviews conducted at the end of the lesson study. That material was analysed, interpreted and discussed (Erickson, 1986) seeking to depict the process which enabled the promotion of professional collaboration. The research was approved by the Research Ethics Committee of Brazil on 10 June 2021 (Process number 4.764.981) on, prioritizing the integrity of the participants.

The categories emerged in the process of analysis, following thematic analysis (Goetz & LeCompte, 1984) which went through the following steps: 1. careful reading of the empirical material seeking to identify reference units that could uncover the way in which the development of the collaboration in lesson study took place; 2. systematisation and pre-organisation of the reference units into thematic blocks that could contribute to answer the research question; 3. analysis and grouping by approximations of the reference units, constituting the final categories of analysis: recognition of collaboration, development of collaboration, and valuing collaboration.

### **Results: Promoting Collaboration in a Lesson Study**

Initially, the participant teachers found very strange all aspects of the dynamics of the lesson study, especially in relation to the work to be developed together with peers and the public teaching of the lesson. The teachers expressed their initial impressions in the logbook.

I was a little concerned about working together and someone giving the lesson in front of everyone, but at the same time curious, I accepted the challenge, so I have already made myself available, I would like to apply the lesson in my class (Erika, teacher logbook, session 1)

At first I was intrigued by the name lesson study, because we think about teacher education, the idea of improving the way of planning lessons and teaching practice immediately comes to mind. The first meeting has already awakened the desire to understand and get to know this process (Ivy, teacher logbook, session 1)

On the first day, when the facilitators explained what the lesson study is like, I found it strange that the whole group worked together to prepare only one lesson. I even thought it was a waste of time, because if each one prepared a lesson it would be more productive because we would have several lesson proposals. But then I realized the richness of planning together and thinking about every detail of the lesson (Ranya, session 12)

The professional routine of participant teachers, which was formerly framed by isolation and individualism (as the teachers indicated), was shaken when the group was challenged to become involved in a lesson study. They were very surprised by working collectively and through dialogue in the planning of the research lesson, observing classroom teaching and, especially, in being encouraged to support each other in overcoming professional challenges and conflicts. The construction of professional collaboration in this experience begun with the planning of the lesson and was intensified with the observation of the research lesson and the post-lesson reflection.

## **Recognition of Collaboration**

The initial milestone of the development of professional collaboration in the context of the lesson study was the recognition by the participants of collaboration as a dimension of the teacher's professional practice and also as a principle of teacher personal and professional growth. Erika and Ranya highlighted the possibilities of collaborative work for professional planning, indicating that this practice was not part of her routine:

As we finish our meetings I became very happy with that I learned through the planning of classes. At first, I was a bit confused about the real aim of the lesson study, but during sessions I could understand the importance of lesson planning and, especially, the observations and the evaluation of class enactment. (Erika, teacher logbook)

It will be very rich for us to be able to make exchanges, to work together, which is a practice that we do not have in school. I'm open to listening to colleagues, learning together and being criticized if I have to. (Ranya, session 2)

The isolation and individualism that prevail in teachers' professional culture, especially regarding classroom practice, was voiced in several interventions. Ivy contrasted the collaboration to the prevalent individualism in her former experience, stressing the importance of collective work for the quality of professional planning:

[Everyday] we are alone, we are lonely, we think alone, we analyse alone, and we find some way out. But what we made here in the group is thinking together and reflecting together. There are many people analyzing the same question, thinking in the same lesson. And this made a difference. Things can happen as such, but it is different to plan, think, apply and analyze alone. It is different to have a group. And, because of that, our group will be different at our school. (Ivy, session 10)

Ivy confirmed the prevalent individualism in schools, mainly concerning lesson planning, and concluded that the experience in the lesson study provided the teachers the opportunity to constitute a different group at their school. We must note that, when the teacher educators perceived that the group was not used to work collaboratively, they aimed to promote interaction, dialogue and collective work:

The group was unfocussed and confused about the task to do. There is some difficulty in working as a collective. Then, we suggested the teachers to search materials and activities related to the topic, to discuss with the other teachers. The outcome of this search was shared and discussed in the group so that each teacher could make comments about what had been presented. [...]. In addition, we stimulated the group to plan mathematical activities, designing and modifying them at the face-to-face meetings. (Researcher' field notes)

The collective study was well evaluated by the teachers. The collective planning, based in the lesson study sessions, is a need that Ranya had already pointed to school administrators, because she considered that change in teaching is a process enacted at school, by teachers:

I have already suggested it several times to the Education Department. I said them: "you don't need to bring here lecturers to tell us what have to do". Organize study groups at schools. The

process takes place [in the group]. It doesn't need to come ready [...]. We need time to meet, study and plan: collective planning [...] If we now go back to school, we cannot meet [...] When to meet to plan? For this to succeed we need time. (Ranya, session 10)

Another important aspect was stressed by the teachers in the planning meetings for the research lesson. The systematization of activities for the research lesson which made up the lesson plan took place at face-to-face meetings. As the time was short to recall everything that had been discussed and defined, aiming to aid the teachers, we oriented the group to use a digital resource for sharing documents, teaching the teachers how use it:

Initially, the teachers had difficulty in using communication resources, as well as in storing and editing documents used in the lesson study (especially in Moodle). However, in planning the research lesson, the group began to use the Google Drive resource, by which [the teachers] developed a different collaboration perspective insofar as they began to work in a shared manner in the tasks for the research lesson, which were available in this resource. According to the teachers, this experience was excellent because they had the possibility to undertake the lesson study activities in a shared way, even if they were in different places. A teacher said: "This resource helped us a lot because we could work in the lesson plan and monitor what each one had done". (Researcher' field notes)

This way of work provided the teachers the opportunity to expand their collaboration experiences during the process of constructing the lesson plan, in collaborative writing, which is a difficult and complex activity, especially for teachers immersed in an individualist professional culture. This aspect enhanced the negotiation and joint decision-making related to the task they were preparing and the dynamics of the research lesson.

Therefore, professional planning was marked by dialogue, sharing, collective study and shared elaboration of the lesson plan, in a context framed by the respect for participants' ideas regarding activities which were designed by them:

[In several sections we dedicated to] planning for the research lesson, we sat down and dialogued. Initially, the regular teacher of the class gave us some descriptions of the student group and the school, such as the number of students and space available to the research lesson. We discussed some ideas for the activities. (Yasmin, teacher logbook)

In summary, the collective professional planning, enabled by the lesson study dynamic, was a context in which collaboration emerged, providing the teachers the opportunity to learn from each other's experiences, as well as to think and plan a lesson which was analyzed and discussed by the group and improved at each session. Also, it allowed the teachers to undertake professional activities such as to study about the students and about their social and family contexts, and, especially, the collaborative writing in a way very different of the individualism that they were used to. This process was mainly influenced by the way in which the teacher educators conducted the lesson study and how they worked to overcome hierarchies among teacher educators and teachers.

## Development of Collaboration

The lesson study dynamic provided the teachers the possibility to expand their collaborative experiences to classroom teaching. For the teachers, the fact they were involved in enacting classroom teaching, through the research lesson, was a real difference in the lesson study. They highlighted that the group collaborated in thinking and planning the research lesson and, most especially, in enacting it:

Different from lesson study cycles that took place recently, this was carried out in two days. The activities begun with the participation of people from the school community, with interviews aiming to find out about the units of measure which were used in the past. (Janaína, teacher logbook)

In planning the research lesson all teachers were involved in an intense and lasting way. First, the group analyzed the student and family contexts, aiming to plan the initial task of the research lesson, which was a survey of measuring strategies and instruments used by their families. Then, it was made a detailed study about the external and internal space of the school, aiming to plan a “treasure hunt” activity, for which the students would use measure units of length. All participants assumed important roles in different steps of this process:

A thing that marked me was that all of us went to the classroom and all of us were involved in something. When the students made the treasure hunt, all of us participated, even the school assistants and school administrators. After, when the students made the classroom activities, we were very involved. It was as if all of us were teaching the class. (Ivy, session 10)

Carrying out the treasure hunt, in the research lesson, also involved all the teachers and the school pedagogical staff in helping the students. The school pedagogical staff also participated in the research lesson, observing students’ actions. In addition, another aspect which contributed to the expansion of collaboration was the opportunity of co-learning from the observation of colleagues:

As we learn listening, as we learn with the colleagues, watching our colleague teaching! And it has to come somebody from [outside] to show us the importance of this that we know, but never do. Just the fact that we listen to each other, report [what we did] already is relevant learning. (Ranya, session 10)

The realization of the research lesson in two two-hour parts, on consecutive days, favored the deepening of the topic “meter” by students, as Janaína points out. In addition, it allowed the participants to be together for a longer time following the mathematics lesson. This strengthened trust among participants, as well as broadened shared impressions of this process.

Finally, the collaboration also took place in the development of the research lesson. In a moment when she felt challenged in role assumed, Janaína, the teacher in that led the lesson, sought support from the teacher educators:

I felt safe, not because I am a so confident person, but because you provided this tranquility. And I knew that when I could not say the [lesson] words I could say, “teacher, help me” as it happened when I began speaking of the history of the units of measurement and I said “teacher, will you lead the lesson now?” And you went there and led the class. Because sometimes we miss the words and it is good to have someone there, a support, a safety net, someone we may count on. And I knew that you were not there to evaluate, because you said very clearly that the aim of the research lesson was not to evaluate the student, not to evaluate the teacher. It would be a study about what the students would do. And we learn with that. (Janaína, session 10)

Therefore, with the research lesson, the collaboration, went beyond the physical and time boundaries of the school, because besides the six teachers, also the school pedagogical staff helped in different moments, as well as the students’ families of the class involved in the research lesson participated in a direct way, answering interviews about measuring procedures and instruments used in the past. It also involved the teacher educators that were called by the research lesson teacher to lead the class. This confidence among teacher educators and participants, favored by the way we conducted the process, was key aspect for promoting collaboration. In this perspective, the collaboration expanded to classroom teaching, transposing hierarchies and roles held by the different participants involved in the educational process at the school and in the lesson study context. Therefore, the research lesson constituted a context in which collaboration was expanded to classroom teaching, showing the possibilities of professional development from overcoming isolation regarding classroom work.

### **Valuing of Collaboration**

The teachers valued the collaboration insofar as it pervaded personal and professional relations. The teachers stressed the fact that teacher educators worked in equal grounds with the other participants, listening to their fears and professional needs, and especially, encouraging them to collaborate as a way to face professional challenges. This attitude from the teacher educators established a high level of confidence in the group. The teachers began to mutually support and encourage each other.

The fact that the collaborative experiences expanded to the dimension of personal relations, contributed to Janaína, who faced a stressful professional situation, not give up participating in the lesson study and accept to teach a research lesson with great commitment. The teacher indicated that from the beginning she felt very comfortable with teaching the research lesson, which is considered a major challenge in lesson study:

You as a group provided me this confidence [...]. Because at beginning I was very unsure. Because you know that I already was evaluated, I was criticized by my colleagues [referring a critique made by a member of the school administration]. And you showed up and proposed a lesson study. I felt afraid. But you explained, what we are going to do is a lesson study, it will be great. This safety that you provide to everyone, make us feel cool. I felt at home. I felt as nobody else was in the classroom. I felt very safe. At the beginning, it was scaring, but I felt safe. (Janaína, session 10)

The lesson study dynamics, especially the shared teaching in a perspective of support to the teacher who teaches the research lesson in the presence of peers, led the participants to

expand the collaboration in addition to working together and sharing. They were able to experience collaboration in a perspective of encouragement and mutual support that encompassed the personal and professional dimensions, support that encompassed the personal and professional dimension.

Erika confirmed this aspect stressing that, besides overcoming the challenge of teaching in front of a group, Janaína led well the lesson:

You saw as the universe conspires with us to favor good things. You were in a fragile moment. And this challenge arrived, and you went there and made that excellent work with the students. (Erika, session 10)

Furthermore, Ivy and Janaína evaluated this experience as very positive, mostly because of this aspect, and she was very thankful to the group for providing her this opportunity for personal improvement:

This [lesson study] came to [shaken us], because we go on automatic pilot and follow our routine without thinking too much about the way we teach and if this makes the student learn. And we end up forgetting that [we have] our colleagues, that they have a lot to teach us, that they can help us. And I felt that doing our job with our colleagues, doing it together for real, helps us improve (Yvy, interview).

And even if I learned nothing in this process, if it had added nothing to my professional career, just the fact that we are here talking now, we are listening to our colleagues, and you exposing your opinion, which is other reality, which is from outside, from another experience, that already adds much, already is worthwhile. I only have to thank you and the group for this. (Janaína, session 10)

The analysis suggests that at the beginning of the process, the teachers felt afraid about the work to be carried out in the lesson study, especially in relation to the purposes and the way of working collaboratively. However, after understanding that professional collaboration is change-oriented and presupposes teamwork around a common goal, they felt more engaged and committed to the activities of the lesson study, and from then on, they realized other principles of collaboration – trust, mutual help and support, leadership and shared responsibility, overcoming hierarchies, encouragement to take on new challenges – raising the level of collaboration in the group. At the end of the process, by feeling truly integrated as a team that works and learns collaboratively, they broadened their understanding of collaboration, valuing and expanding it beyond lesson study. This aspect indicates that the collaboration in the group surpassed the level of the basic principle of lesson study, characterizing it as a professional culture.

Therefore, the lesson study dynamic provided the teachers the opportunity to overcome the individualism and professional isolation, insofar as the process was oriented to promote professional collaboration. In the lesson study, especially in planning and undertaking the research lesson, professional relations were established and activities stimulating professional collaboration were promoted. This process begun in professional planning, went through classroom teaching, and consolidated in the personal dimension, facilitated by the way how the teacher educators conducted the process, as well as by the lesson study structure and dynamic.

Table 2 summarizes the categories of analysis that characterized the process of development of professional collaboration in the lesson study, highlighting the principles that underlie each category.

**Table 2**  
*Categories of Analysis*

Theme	Categories	Aspects evidenced
Development of Professional Collaboration	<i>Recognition of Collaboration</i>	Dialog, shared, negotiation. Collaborative work, study and writing Joint decision-making Shared planning
	<i>Development of Collaboration</i>	High level of confidence Confidence among teacher educators and participants Shared teaching Transposing hierarchies and roles
	<i>Valuing collaboration</i>	Encouraging, mutual support High level of confidence Overcome the individualism and professional isolation. Expanded to the dimension of personal relations

*Source.* Elaborated by authors.

We identified other aspects of teachers' professional development in lesson study, which were not addressed in this study, for example, the redefinition of the concept of collaboration. This and other aspects may be objects of investigation in future research.

## Discussion

Professional collaboration emerged in the context of the planning sessions for the research lesson in a lesson study, with teachers experiencing a teacher education approach geared to professional development. This was a different process from what they were used to, because it involved teachers in discussing ideas and collectively developing teaching resources (as in [Burroughs & Luebeck, 2010](#)) and collaborative management of the teaching and learning process ([Fiorentini et al., 2023](#)). It also enabled collective work about the research lesson and discussion about the students' characteristics and the school context. The teachers had a chance to experience collaborative planning, which contributed to the growth of the group, bringing them closer to each other (as in [Borges, 2007](#); [Teixeira, 1995](#)), with the administrators of the school, and, especially, with the teacher educators, creating opportunities to face professional situations external to the lesson study. An aspect repeatedly stressed by teachers was the discussion about the activities for the research lesson, the collaborative study and writing, which enabled the elaboration of the lesson plan. This



experience was enhanced by the way how the educators conducted the lesson study steps and activities.

Therefore, the lesson study fostered collaboration (as in Huang & Shimizu, 2016; Lewis et al., 2009; Quaresma & Ponte, 2021), as it enhanced the relationships between participants inside and outside the classroom in which they shared ideas and experiences in a dynamic that created trust among peers (as in Borges, 2007; Hargreaves, 1998; Richit et al., 2021), so that teachers felt comfortable to expose their thoughts, favoring peer work and contributing to professional growth.

The collaboration developed from planning to classroom teaching as all teachers involved in the research lesson took part in it, in shared teaching (as in Richit & Ponte, 2017). In the research lesson, participated the students' families, all teachers involved in the lesson study, and the school pedagogical staff, undertaking collaboration in different times, spaces and segments of the school community. In addition, the way in which the process was conducted established a higher level of confidence through dialog, sharing, and collegial support, which resulted in the disposition of participants to take risks and experience a new practice (as indicated in Hargreaves, 1998), that marked everyone very positively and, especially, Janaína, who got herself highly involved personally and professionally in this challenge.

Finally, the experiences of personal encouragement and empowerment constituted a context for the valorization of collaboration, providing the teachers with opportunities to share uncertainties, conjectures, fears, and goals, as well as to discuss different topics related to teaching and to feel welcomed in the group (as suggested by Borges, 2007). The stimulus and confidence manifested by colleagues and by teacher educators empowered the teachers (Hargreaves, 1998; Richit et al., 2020), just as encouraged Janaína to develop a research lesson in a critical professional moment (Borges, 2007). Thus, the confidence that was developed through the respect and care with the needs, interests and fears of all participants in the lesson study crossed the personal and professional levels (Hargreaves, 1998), facilitating the participants' personal growth. The promotion of collaboration was fostered by the teacher educators, who conducted this process prioritizing respect, dialog and confidence. In fact, the promotion of collaboration in a lesson study requires care, attention and appropriate intervention of those who lead it, so that the group feels welcomed, respected and valued in the collective.

Collaboration, in this context, involved a sense of generosity, teachers being generous with their ideas and their time, supporting each other and encouraging themselves to take risks, such as promoting a new practice in the research lesson, looking at meaning in students' ideas and sharing successes with the group (as in Lewis et al., 2013; Warwick et al., 2016).

## Conclusion

Undertaking professional collaboration in this lesson study led the teachers to engage in an activity related to the daily life of the teaching profession (teaching), of common interest, and from which all participants benefited (Hord, 1997; Robuti et al. 2016; Richit et al., 2024), because they were able to reflect on the teaching of mathematics in the classroom and to

carry out professional planning in group (Burroughs & Luebeck 2010; Richit & Tomkelski, 2022), in a dialogued, negotiated and shared manner. And this joint work created the opportunity to all those involved for the mutual deepening of professional knowledge (Borko & Potari, 2020; Richit, 2023) and contributed for building professional relationships, promoting their professional development.

The results underline the possibilities of lesson study to promote the professional development of teachers who teach mathematics based in the development of professional collaboration. Collaboration that involves professional aspects (planning, designing tasks, classroom teaching, sharing of teaching materials and resources, reflection) and personal aspects (welcoming, encouraging, trust, valuing voices and experience, and strengthening dialogue), complementary to each other, has the potential to promote effective changes in education as it provides the teachers with a context to promote change and the necessary conditions to achieve them. However, the discussion on this theme is not limited to this investigation, because collaboration is a complex and dynamic form of work carried out by specific professional groups so that local and institutional cultures can influence it. Thus, the way collaboration develops in professional groups of different institutions may involve other elements, as well as configure different movements of development of this form of professional culture, thus requesting further research.

The first limitation of this research is related to the fact that the participants of the lesson study were not familiar with the teaching planning among peers. For this reason, the collaboration appeared more timidly at the beginning of the process and became more developed in the fourth session of the cycle (Richit et al., 2024). A second limitation concerns the categories of analysis (stages of collaboration) generated by a specific theoretical framework (Hargreaves, 1998). According to Breda et al. (2021), the categories generated from other theoretical frameworks could reach different results in relation to the stages of professional collaboration among teachers in lesson study.

### **Funding**

Conselho Nacional de Desenvolvimento Científico e Tecnológico (Process number: 402748/2021-2) and Universidade Federal da Fronteira Sul - UFFS (PES 2023 – 0253)

### **Acknowledgments**

We thank the teachers that participated in the lesson study.

### **Notes**

<sup>1</sup> In Brazil, the majority of elementary school teachers are women.

## References

- Burroughs, E. & Luebeck, J. (2010). Pre-service teachers in mathematics lesson study. *The Montana Mathematics Enthusiast*, 7(2/3), 391-400. <https://doi.org/10.54870/1551-3440.1196>
- Boavida, A.M. & Ponte, J.P. (2002). Investigação colaborativa: Potencialidades e problemas [Collaborative research: Potential and issues]. In GTI (Ed.), *Refletir e investigar sobre a prática profissional* (pp.43-55). APM.
- Borges, M.P.A. (2007). *Professores: Imagens e auto-imagens* [Teachers: Images and selfimages]. Doctoral Thesis, Instituto de Educação, Universidade de Lisboa, Lisboa.
- Borko, H. & Potari, D. (2020). Teachers of mathematics working and learning in collaborative groups (Proceedings of The Twenty-Fifth ICMI Study), Lisboa. Available at <http://icmistudy25.ie.ulisboa.pt>
- Breda, A., Hummes, V. B., Silva, R. S., & Sánchez, A. (2021). The role of the phase of teaching and observation in the lesson study methodology. *BOLEMA Boletim de Educação Matemática*, 35(69), 263-288. <https://doi.org/10.1590/1980-4415v35n69a13>
- Calle, E., Breda, A., & Font, V. (2022). The complexity of the notion to teach in the assessment of the pre-professional practice of future Ecuadorian mathematics teachers. *REDIMAT Journal of Research in Mathematics Education*, 11(3), 218-249. <https://doi.org/10.17583/redimat.10986>
- Canavarro, A.P. (2011). Ensino exploratório da Matemática: Práticas e desafios. *Educação e Matemática*, 115, 11-17. <https://em.apm.pt/index.php/em/article/view/1982>
- Denzin, N.K. & Lincoln, Y.S. (1989). *Handbook of qualitative research*. Newbury Park.
- Dubar, C. (1997). *A socialização: Construção das identidades sociais e profissionais* [Socialization: Construction of social and professional identities]. Porto Editora.
- Erickson, F. (1986). Qualitative methods in research on teaching. In M.C. Wittrock (Eds), *Handbook of research on teaching* (pp.119-16). Macmillan.
- Fiorentini, D., Honorato, A. H. A., & Paula, A. P. M. (2023). Experiências de Aprendizagem Docente na Gestão Colaborativa do Ensino-aprendizagem de Matemática baseado em Tarefas Exploratórias. *Perspectivas da Educação Matemática*, 16(42), 1-30. <https://doi.org/10.46312/pem.v16i42.18404>
- Fialho, I. & Sarroeira, L. (2012). Cultura profissional dos professores numa escola em mudança [Professional culture of teachers in a changing school]. *Educação: Temas e problemas*, 9. <http://hdl.handle.net/10174/8187>
- Goetz, J. P. & LeCompte, M. D. (1984). *Ethnography and qualitative design in educational research*. Academic Press.
- Goulet, L. & Aubichon, B. (1997). Learning collaboration: Research in a First Nations teacher education program. In: H. Christiansen, L. Goulet, C. Krentz, & M. Maeers (Eds), *Recreating relationships: Collaboration and educational reform* (pp.115-127). State University of New York Press.
- Hargreaves, A. (1998). *Professores em tempo de mudança: O trabalho e a cultura dos professores na idade pós-moderna* [Teachers in changing times: The work of teachers in the postmodern age]. McGraw-Hill.

- Hargreaves, A. & O'Connor, M. (2018). *Collaborative professionalism: When teaching together means learning for all*. Sage.
- Hord, S. M. (1997). *Professional learning communities: Communities of continuous inquiry and improvement*.
- Huang, R. & Shimizu, Y. (2016). Improving teaching, developing teachers and teacher educators, and linking theory and practice through lesson study in mathematics: An international perspective. *ZDM Mathematics Education*, 48, 393-409. <https://doi.org/10.1007/s11858-016-0795-7>
- Lewis, C., Perry, R., & Hurd, J. (2009). Improving mathematics instruction through lesson study: A theoretical model and North American case. *Journal Mathematics Teaching Education*, 12, 285-304. <https://doi.org/10.1007/s10857-009-9102-7>
- Lewis, J. M., Fischman, D., Riggs, I., & Wasserman, K. (2013). Teacher Learning in Lesson Study. *The Mathematics Enthusiast*, 10(3), 583-620
- Lima, J.A. (2002). *Culturas colaborativas nas escolas: Estruturas, processos e conteúdos* [Collaborative cultures in schools: Structures, processes and contents]. Porto Editora.
- Marcelo, C. (2009). Desenvolvimento profissional docente: Passado e futuro. *Sísifo-Revista de Ciências da Educação*, 8, 7-19. <http://sisifo.ie.ulisboa.pt/index.php/sisifo/article/view/130>
- Miyakawa, T. & Winsløw, C. (2009). Didactical designs for students' proportional reasoning: an "open approach" lesson and a "fundamental situation. *Educational Studies in Mathematics*, 72, 99-218. <https://doi.org/10.1007/s10649-009-9188-y>
- Morris, J.C. & Miller-Stevens, K. (2016). *Advancing collaboration theory: models, typologies, and evidence*. Routledge.
- Murata, A. (2011). Introduction: Conceptual overview of lesson study. In L.C. Hart, A. Alston, & A. Murata (Eds.), *Lesson study research and practice in mathematics education: Learning together* (pp. 1-12). Springer.
- Ponte, J. P., Mata-Pereira, J., & Henriques, A. (2012). O raciocínio matemático nos alunos do ensino básico e do ensino superior. *Praxis Educativa*, 7(2), 355-377. <https://doi.org/10.5212/PraxEduc.v7.i2.003>
- Ponte, J. P., Quaresma, M., Mata-Pereira, J., & Baptista, M. (2016). O estudo de aula como processo de desenvolvimento profissional de professores de matemática [Lesson study as a professional development process for mathematics teachers]. *BOLEMA Boletim de Educação Matemática*, 30(56), 868-891. <https://doi.org/10.1590/1980-4415v30n56 a01>
- Quaresma, M., & Ponte, J. P. (2021). Developing collaborative relationships in lesson studies. *PNA Revista de Investigación en Didáctica de la Matemática*, 15(2), 93-107. <https://doi.org/10.30827/pna.v15i2.16487>
- Richit, A. (2023). Professional development of professors in lesson study. *Educação Unisinos*, 27, 1-20. <https://doi.org/10.4013/edu.2023.271.20>
- Richit, A. (2021). Desenvolvimento profissional de professores: um quadro teórico [Teacher professional development: A theoretical framework]. *Research, Society and Development*, 10(14), 1-15. <https://doi.org/10.33448/rsd-v10i14.22247>
- Richit, A. (2020). Estudos de Aula na Perspectiva de Professores Formadores. *Revista Brasileira de Educação*, 25(2), 1-24. <https://doi.org/10.1590/S1413-24782020250044>

- Richit, A., Agranionih, N. T., Zimer, T. B., & Neves, R. B. (2024). Professional collaboration in a lesson study with university mathematics professors. *International Electronic Journal of Mathematics Education*, 19(2), em0772. <https://doi.org/10.29333/iejme/14290>
- Richit, A., & Ponte, J. P. (2017). La colaboración docente en estudios de clase en la perspectiva de profesores participantes [Teaching collaboration in lesson studies from the perspective of participating teachers]. *Paradigma* (Maracay), 38(1), 330-352. <https://dialnet.unirioja.es/servlet/articulo?codigo=7302923&orden=0&info=link>
- Richit, A., Ponte, J. P., & Tomasi, A. P. (2021). Aspects of professional collaboration in a lesson study. *International Electronic Journal of Mathematics Education*, 16(2), em0637. <https://doi.org/10.29333/iejme/10904>
- Richit, A., Ponte, J. P., & Tomkelski, M.L. (2020). Desenvolvimento da prática colaborativa com professoras dos anos iniciais em um estudo de aula [The Development of Elementary School Teachers' Collaborative Practice in a Lesson Study]. *Educar em Revista*, 36, 1-24. <https://doi.org/10.1590/0104-4060.69346>
- Richit, A., & Tomkelski, M. L. (2022). Meanings of mathematics teaching forged through reflection in a lesson study. *Eurasia Journal of Mathematics, Science and Technology Education*, 18(9), em2151. <https://doi.org/10.29333/ejmste/12325>
- Robinson, N. & Leikin, R. (2012). One teacher, two lessons: The lesson study process. *International Journal of Science and Mathematics Education*, 10, 139-161. <https://doi.org/10.1007/s10763-011-9282-3>
- Robutti, O., Cusi, A., Clark-Wilson, A., Jaworski, B., Chapman, O., Esteley, C., Goos, M., Isoda, M., & Joubert, M. (2016). ICME international survey on teachers working and learning through collaboration. *ZDM Mathematics Education*, 48, 651-690. <https://doi.org/10.1007/s11858-016-0797-5>
- Stigler, J. & Hiebert, J. (1999). *The Teaching Gap: Best Ideas from the World's Teachers for Improving Education in the Classroom*. The Free Press.
- Soto, M., Gupta, D., Dick, L. & Appelgate, M. (2019). Bridging distances: Professional development for higher education faculty through technology-facilitated lesson study. *Journal of University Teaching & Learning Practice*, 16(3). <https://doi.org/10.53761/1.16.3.7>
- Takahashi, A. & McDougal, T. (2016). Collaborative lesson research: Maximizing the impact of lesson study. *ZDM Mathematics Education*, 48, 513-526. <https://doi.org/10.1007/s11858-015-0752-x>
- Teixeira, M. (1995). *O professor e a escola: Perspectivas organizacionais*. McGraw Hill.
- Xu, H. & Pedder, D. (2015). Lesson study: An international review of the research. In P. Dudley (Ed.), *Lesson study: Professional learning for our time* (pp. 29-58). Routledge.
- Wake, G., Swan, M., & Foster, C. (2016). Professional learning through the collaborative design of problem-solving lessons. *Journal Mathematics Teaching and Education*, 19, 243-260. <https://doi.org/10.1007/s10857-015-9332-9>
- Warwick, P., Vrikki, M., Vermunt, J.D., Mercer, N., & Halem, N. (2016). Connecting observations of student and teacher learning: an examination of dialogic processes in lesson study discussions in mathematics. *ZDM Mathematics Education*, 48, 555-569. <https://doi.org/10.1007/s11858-015-0750-z>